Relative Rate of Expansion Controls Speed in 1D Pedestrian Following

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Abstract

Patterns of crowd behavior are believed to result from local interactions between pedestrians. Many studies have investigated the local rules of interaction, such as steering, avoiding, and alignment, but how pedestrians control their walking speed when following another remains unsettled. Most pedestrian models assume the physical speed and distance of others as input. The present study compares such “omniscient” models with “visual” models based on optical variables. We experimentally tested eight speed control models from the pedestrian and car following literature. Walking participants were asked to follow a leader (a moving pole) in a virtual environment, while the leader’s speed was perturbed during the trial. In Experiment 1, the leader’s initial distance was varied. Each model was fit to the data and compared. The results showed that visual models based on optical expansion ($\dot{\theta}$) had the smallest RMS error in speed across conditions, whereas other models exhibited increased error at longer distances. In Experiment 2, the leader’s size (pole diameter) was varied. A model based on the relative rate of expansion ($\dot{\theta}/\theta$) performed better than the expansion rate model ($\dot{\theta}$), because it is less sensitive to leader size. Together, the results imply that pedestrians directly control their walking speed in 1D following using relative rate of expansion, rather than the distal speed and distance of the leader.

Keywords: pedestrian following, optical expansion, visual control of locomotion

Introduction

A number of pedestrian models have been developed to characterize the interactions between walking humans. For example, the dominant Social Force Model and its variants (Helbing & Molnar, 1995; Lakoba, Kaup, & Finkelstein, 2005; Yu, Chen, Dong, & Dai, 2005) take the distance and velocity of objects and other pedestrians as input to compute the acceleration of the simulated agent. Velocity-based models of collision avoidance (Berg, Lin, & Manocha, 2008; Van Den Berg, Guy, Lin, & Manocha, 2011; Guy, Lin, & Manocha, 2010) calculate the velocity space (set of velocities) that will lead to collisions, then find a velocity outside the space based on some optimization criteria. However, physical variables such as distance and velocity are not directly available to human observers, nor is there evidence that pedestrian interactions are governed by hypothesized forces or global optimization. Such “omniscient” phenomenological models are approximations of pedestrian movements, but do not offer plausible explanations of human behavior grounded in the perceptual coupling between individuals. Pedestrian interactions are likely based on optical variables and governed by control laws for locomotion (Warren, 2006).

In this paper we focus on one-dimensional (1D) pedestrian following, in which a follower walks behind a leader and regulates their speed to stay with the leader. 1D following is a phylogenetically ancient behavior dating to the Cambrian explosion, as revealed by fossils of trilobites processing in single file, coupled via mechanoreception (Vannier, et al., 2019). Models of pedestrian following are often inspired by car-following models of vehicular traffic (Brackstone & McDonald, 1999), which are based on physical variables (variable names appear in Table 1, models in the left two columns of Table 2; see Rio, Rhea & Warren, 2014, for detailed descriptions). For example, the follower could maintain a constant distance from the leader (Kometani & Sasaki, 1958), maintain a distance that increases with speed (Herman, Montroll, Potts & Rothery, 1959; Pipes, 1953), use a linear combination of distance and speed (Helly, 1959), or match the leader's speed (Lee & Jones, 1967). Lemercier et al. (2012; Fehrenbach, et al., 2015) proposed that pedestrian followers use the ratio of speed difference to leader distance to control their speed, similar to the car-following model of Gazis, Herman & Rothery (1961), with an explicit time delay between leader and follower. The model was able to reproduce stop-and-go waves that are commonly observed in dense one-way pedestrian traffic. Bruneau, Dutra, and Pettre´ (2014) subsequently proposed a model in which the speed of the follower is based on the predicted future distance of the leader, which can also replicate stop-and-go waves. However, these pedestrian models are again omniscient, relying on the leader's physical distance and/or speed as input.

Rio, Rhea, and Warren (2014) tested a variety of omniscient physical models against data on following in pedestrian dyads and found that the simple speed-matching model fit the human data as well or better than other models. In this model, the follower's acceleration ($\ddot{x}_f$) is proportional to the difference between the speed of the leader and the follower ($\Delta\dot{x}_{lf}$), and the gain parameter (c) is fit to the data:

$$\ddot{x}_f = c\Delta\dot{x}_{lf} \tag{1}$$

Rio, Rhea & Warren (2014) also proposed a visual control law in which the follower adjusts their speed to cancel the leader's optical expansion and contraction (cf. Andersen & Sauer, 2007; Lee & Jones, 1967, for car following). Specifically, the follower's acceleration is proportional to the rate of change in the leader's visual angle ($\dot{\theta}_l$), and the gain parameter (b) is fit to the data:

$$\ddot{x}_f = -b\dot{\theta}_l \tag{2}$$

This Rate of Expansion (RE) model did not fit their data quite as well as the physical speed-matching model, but there are two possible reasons it may have fallen short.

First, the theoretical advantage of the RE model is that it exploits the non-linearity of visual angle, which decreases with distance (d) as $tan^{-1}(1/d)$ (see Figure 1a). Whereas the speed-matching model is independent of leader distance, the rate of expansion decreases nonlinearly with distance (Figure 1b). Consequently, the RE model inherently depends on the leader's distance (Ducourant, et al., 2005) without explicitly recovering physical distance. It is possible that the speed-matching model fit the data better because the range of distances tested by Rio, Rhea & Warren (2014) was small (1-4m), and a larger distance range might produce a distance effect consistent with the RE model. Second, the RE model predicts an asymmetric response to leader acceleration and deceleration. For a given initial distance, decelerating decreases the leader's distance, producing a higher expansion rate, whereas accelerating increases the leader's distance, producing a lower contraction rate. Rio, Rhea & Warren (2014) did not report this asymmetry in their first experiment with a human leader, perhaps because the leader's acceleration and deceleration were uncontrolled. But in a second experiment with a virtual leader, they reported a significant asymmetry in the follower's response: speed changes were twice as large to leader deceleration as to leader acceleration. In the present study, we tested a larger range of distances and also matched the acceleration/deceleration of the leader using Virtual Reality (VR).

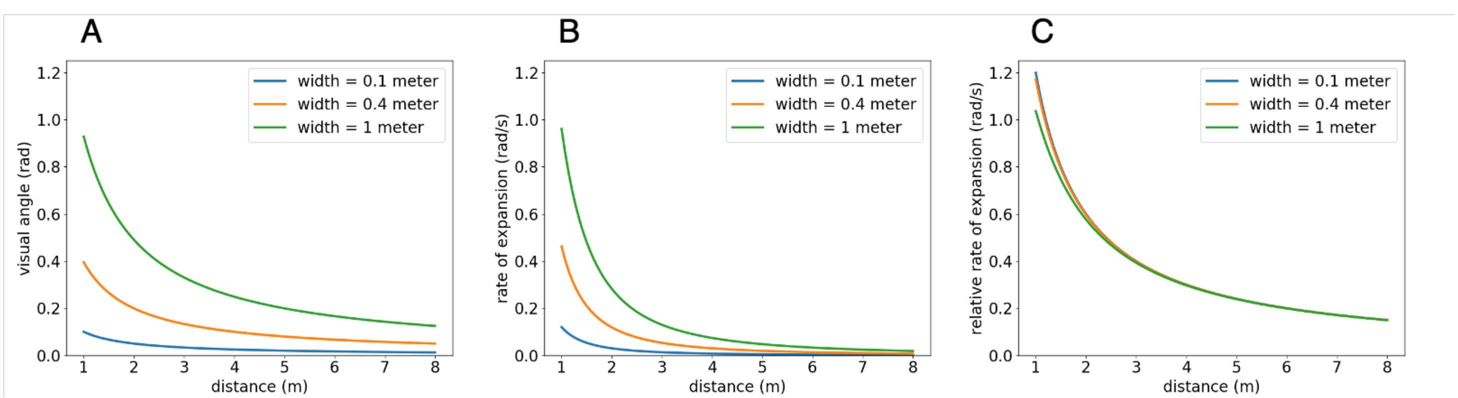

Figure 1. (a) The visual angle of the leader (θ) increases nonlinearly as distance decreases, at a rate that depends on leader size. (b) Similarly, the rate of change in visual angle (optical expansion, $\dot{\theta}$) increases nonlinearly as distance decreases, and also depends on size. (c) The relative rate of expansion (RRE, $\dot{\theta}/\theta$) also increases nonlinearly, but is less dependent on size. Graphs are

computed with a constant relative speed between leader and follower ($\Delta\dot{x} = 1m/s$).

A limitation of the RE model is the fact that the rate of expansion not only depends on the leader's distance and relative speed, but also its size: for a given relative speed and distance, an object with a larger diameter generates a higher rate of expansion or contraction at the follower's eye (Figure 1b). We thus test an alternative model based on the Relative Rate of Expansion (RRE) (Wagner, 1982). The RRE normalizes expansion rate ($\dot{\theta}$) by the visual angle (θ) of the leader, thus partially compensating for variation in leader size (Figure 1c):

$$\ddot{x}_f = -b\frac{\dot{\theta}_l}{\theta_l} \tag{3}$$

Note that RRE is the inverse of the time-to-contact variable τ (assuming the small angle approximation) (Lee, 1976). The τ variable is a poor control variable for following, however, for as the follower brings the rate of expansion to zero, τ goes to infinity. Its inverse is more useful, for RRE ~ 0 indicates successful following, RRE < 0 indicates falling behind the leader, RRE > 0 indicates gaining on the leader, and RRE >> 0 indicates an imminent collision. The relative rate of expansion thus provides information about temporal 'immediacy'.

In Experiment 1, we compared the speed-matching and RE models by varying the leader's distance over a larger range (1-6m) and controlling the leader's speed perturbation (±0.3 m/s). If distance influences the follower's response, the RE model should show a higher goodness-of-fit than the speed-matching model, and vice versa. In Experiment 2, we compared the RE and RRE models by varying the diameter of the leader (0.2, 0.6, 1.0 m), while controlling the speed perturbation (±0.3 m/s). If leader size influences the follower's response, the RRE model should fit the data better than the RE model, and vice versa. We also compared six physical models using the data from both experiments: speed matching, constant distance, speed-based distance (SBD), linear combination, ratio, and Lemercier models (see Table 2). Finally, we did not investigate the role of binocular disparity or vergence angle here, which can contribute to time-to-contact judgments (Gray & Regan, 1998; Heuer, 1993) and ball catching (Rushton & Wann, 1999; Savelsbergh, Whiting & Bootsma, 1991). Previously, Rio, Rhea & Warren (2014) dissociated disparity and vergence from optical expansion, and found little influence of the former variables on pedestrian following.

Notation	Meaning
x_f	Position of follower.
Δx	Distance between follower and leader.
$\dot{x}_f$	Speed of follower.
$\Delta\dot{x}$	Speed difference between leader and follower.
$\ddot{x}_f$	Acceleration of follower.
θ	Visual angle of leader in the perspective of follower.
$\dot{\theta}$	Rate of expansion (RE).
$\frac{\dot{\theta}}{\theta}$	Relative rate of expansion (RRE).
d_0	Initial distance between leader and follower
Δv	Speed perturbation on leader

Table 1: Variable names used in this article.

Model	Equation	Parameters	BIC	Mean RMSE (m/s)
Relative rate of expansion (RRE)	$\ddot{x}_f(t) = -b\frac{\dot{\theta}}{\theta}$	$b = 0.920$	-3182[a]	0.092 (SD=0.049)[a]

Ratio	$\ddot{x}_f(t) = c{\dot{x}_f}^M \frac{\Delta\dot{x}}{\Delta x^L}$	c = 1.810 M = −0.052 L = 1.509	-3162[b]	0.091 (SD=0.053)[a]
Rate of expansion (RE)	$\ddot{x}_f(t) = -b\dot{\theta}$	b = 8.463	-3136[c]	0.091 (SD=0.056)[a]
Linear	$\ddot{x}_f(t) = c_1\Delta\dot{x} + c_2[\Delta x - (a + b\dot{x}_f)]$	$c_1 = 0.255$ $c_2 = 0.010$ a = −6.946 b = 10.665	-3043[d]	0.102 (SD=0.054)[b]
Speed	$\ddot{x}_f(t) = c\Delta\dot{x}$	c = 0.219	-3021[e]	0.105 (SD=0.054)[b]
Lemercier et al (2012)	$\ddot{x}_f(t) = c\frac{\Delta\dot{x}(t+\tau)}{\Delta x(t)^{\gamma}}$	τ = 1.000 c = 2.466 γ = 1.439	-3008[f]	0.102 (SD=0.056)[b]
Speed-based distance (SBD)	$\ddot{x}_f(t) = c[\Delta x - (a + b\dot{x}_f)]$	c = 0.026 a = −17.461 b = 19.750	-2494[g]	0.144 (SD=0.087)[c]
Distance	$\ddot{x}_f(t) = c(\Delta x - \Delta x_0)$	c = 0.004	-2355[h]	0.152 (SD=0.096)[d]
Null	$\ddot{x}_f(t) = 0$	None	-2355[h]	0.154 (SD=0.101)[d]

Table 2: The fitting and testing results of 9 models on perturbation trials (n=696) of Experiment 1. Parameter values and Bayesian Information Criterion (BIC) values were acquired by fitting the models to all trials, while minimizing the Root-Mean-Square Error (RMSE) on speed. BIC values were computed based on equation 1. The models were also tested using Leave-one-subject-out cross-validation, in which each model was trained on 11 participants and tested on the one left out until all combinations of training set and test set were used. The test results (mean and standard deviation of RMSE) of 12 iterations of cross-validation are shown in the table. The letters in the superscript of BIC values indicate the rank of model based on BIC, whereas those in the superscript of cross-validation error indicate Duncan group in Duncan's multiple range test. T in Lemercier et al (2012) was capped at 1 second because a reaction time longer than 1 second is unlikely for human pedestrian.

Experiment 1

Method

Participants

Twelve students from Brown University (4 M, 8 F) participated in the experiment. All participants had normal or corrected to normal vision. The protocol was approved by Brown University's Institutional Review Board, in accordance with the Declaration of Helsinki. Informed consent was obtained from all participants, who were paid for their participation.

Apparatus

The experiment was conducted in the Virtual Environment Navigation Laboratory (VENLab) at Brown University. Participants walked freely in a 11m x 9m area while immersed in a virtual environment presented in a head-mounted display (HMD, Oculus Rift CV1). The HMD provided stereoscopic viewing with a 94 H x 93 V binocular field of view (okreylos, 2016), resolution of 1080 x 1200 pixels in each eye, and a refresh rate of 90 Hz. Displays were generated on an MSI VR One backpack PC (New Taipei City, Taiwan, weight 3.3 kg) at a frame rate of 90 fps, using the Vizard 5 3D animation package (WorldViz, 2014). Head position was recorded at a sampling rate of 90 Hz by a hybrid inertial-ultrasonic tracking system (IS-900, Intersense, Billerica, MA). Head orientation was tracked by the built-in inertial sensor of the HMD. Head position and orientation were used to update the display with a latency estimated to be less than 50ms.

Displays

The virtual environment consisted of a ground plane with a granite texture and a blue sky; a blue home pole (radius 0.12 m, height 1.35 m) with a granite texture on the ground plane, where participants started each trial; a stationary red orientation pole (radius 0.2 m, height 3 m) appeared in front of the participant, which participants faced before the trial began; and a moving green leader pole (radius 0.2 m, height 2 m) appeared during the trial, which served as the leader in the following task (see Figure 2).

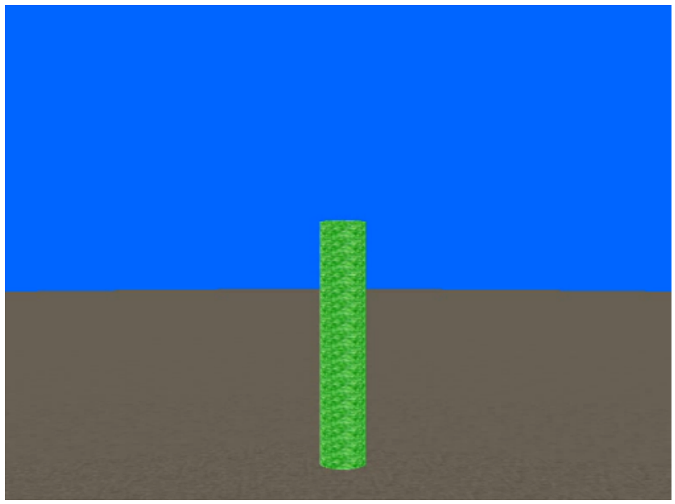

Figure 2. Virtual display of the leader pole from the participant's viewpoint.

Procedure

Prerecorded instructions were played through the HMD's built-in headphones at the beginning of the experiment. A session began with four self-paced walking trials, in which the participant simply walked to the red orientation pole in the virtual environment at their preferred speed, allowing us to measure their normal walking speed. The participant then received four practice trials to learn the following task. These pre-experimental trials helped participants adapt to walking in VR and rescale perceived distance in the virtual environment (Mohler, Creem-Regehr, & Thompson, 2006; Richardson & Waller, 2007). They were followed by 90 experimental trials.

In both practice and experimental trials, participants were asked to "walk behind the green pole as if you were following someone down the street, while trying to keep a constant distance". This instruction was intended to encourage participants to keep up with the leader pole, and was also used by Rio, et al. (2014). Their observation that distance was not actually held constant is consistent with reliance on some other control variable. Participants were not told how closely or quickly to follow the pole.

At the beginning of each trial, the participant stood at the blue home pole and faced the red orientation pole. After 3 s, the green leader pole appeared (1, 3, or 6 meters in front of them) and immediately started to move away from the participant on a straight path at a constant initial speed (1.2 m/s), while the prerecorded instruction "begin" was played over the headphones. After a random interval (3-4 s), a speed perturbation (-0.3, 0, or +0.3 m/s) was applied to the leader pole, with an average acceleration of 1 m/s^2. After the perturbation, the leader pole moved at a new constant speed (0.9, 1.2, or 1.5 m/s) until the end of the trial. The trial ended when the participant had walked for 12.2 m or 12 s, whichever came first. The blue home pole then reappeared nearby and the participant walked to it, which triggered the next trial.

Design

Experiment 1 had a within-subject factorial design: 3 initial distance (d_o = 1, 3, 6 m) x 3 speed perturbation (Δv = -0.3, 0, +0.3 m/s), yielding 9 conditions. There were 10 repetitions in each condition, for a total of 90 trials, which were presented in different random order for each participant.

Data processing

The time series of the participant's head position in the horizontal plane was recorded, which was reduced to uni-dimensional trajectory along the axis of leader motion. Given that there were minor fluctuations in sampling rate, the uni-dimensional data were then linearly interpolated to 90 Hz to ensure equal time intervals between frames. The time series for each trial was then filtered using a fourth-order low-pass Butterworth filter (1 Hz cutoff) to reduce anterior-posterior oscillations due to the step cycle and tracker noise. To eliminate endpoint error caused by the filter, each time series was extended by two seconds using linear extrapolation before filtering and was truncated by three seconds after filtering. The filtered time series of position were then differentiated to produce a time series of speed for each trial, which provided the data for model fitting. Dependent variables for statistical analysis included the participant's final speed (the mean speed in the last 2s) and final distance from the leader (the mean distance in the last 2s), and final speed difference with the leader (mean difference in speed during the last 2s) on each trial. Twenty out of 1080 trials (1.85%) were excluded due to tracking failures.

Statistical analysis

To analyze the relationships between independent and dependent variables, linear mixed effects regression analyses were performed in R (R Core Team, 2019) and lme4 (Bates, Mächler, Bolker, & Walker, 2015). The advantage of a linear mixed effects model is that it has higher statistical power, partials out individual differences, and is robust to missing data (Baayen, Davidson, & Bates, 2008). Visual inspection of residual plots did not reveal any obvious deviations from homoscedasticity or normality. P-values were obtained from Wald Chi-Squared Test.

Model fitting and comparison

Only trials in which the speed of the leader pole was perturbed (-0.3 and +0.3 m/s) were used for model fitting because trials with no speed perturbation drive model parameters to zero. To avoid initial and final transients, we fit the follower's speed time series from 0.5 seconds before the perturbation to 5.5 seconds after the perturbation in each trial. Based on these criteria, 696 trials were used in the fitting procedures, while 11 were excluded for being shorter than 5.5 seconds after the perturbation.

Two fitting procedures were used to serve different goals. Numerical optimizations were achieved by a derivative-free method (Lagarias, Reeds, Wright, & Wright, 1998) for both procedures. The first procedure searched for the optimal parameter values over all trials; although this procedure introduces the risk of over-fitting, it allows us to have a set of optimal parameters. Over-fitting will be addressed by model comparisons using the Bayesian Information Criterion (BIC) because it punishes models that have more free parameters (i.e., higher chance of over-fitting). The nine candidate models in Table 2 were fit to and tested on the time series of follower speed for all trials, including a null model that made no response to the leader. Each trial was simulated using the optimal parameter values for a model, the mean squared error between the model time series and human time series was computed for each trial, and the mean squared error (MSE) was computed over all n trials. The Bayesian Information Criterion (Equation 4) was used to penalize models based on the number of free parameters (k). The BIC value indicates the overall goodness of fit for each model and models can be compared by computing the difference in BIC scores, where a ΔBIC of 2-6 indicates positive evidence, 6-10 strong evidence, and >10 very strong evidence in favor of the lower BIC.

$$BIC = n ln(MSE) + k ln(n) \tag{4}$$

The second fitting procedure used leave-one-subject-out cross validation to avoid over-fitting, and allowed frequentist model comparisons. The nine candidate models were fit to the time series of speed for 11 participants, and were tested on the data from the 12th participant. This procedure was repeated twelve times, such that each participant was left out and tested once. In the test, each trial was simulated using the parameter values from fitting the other 11 participants, and the root mean squared error (RMSE) between the model time series and human time series was computed. This resulted in a mean RMSE for each subject in each condition, which was compared using frequentist statistical tests. Note that the RMSE does not penalize free parameters.

Results

Mean time series of speed for each condition appear in Figure 3A and B. It is apparent that the participant's response decreased with leader distance on deceleration trials (Panel A) and was greater on deceleration trials than acceleration trials (panel B).

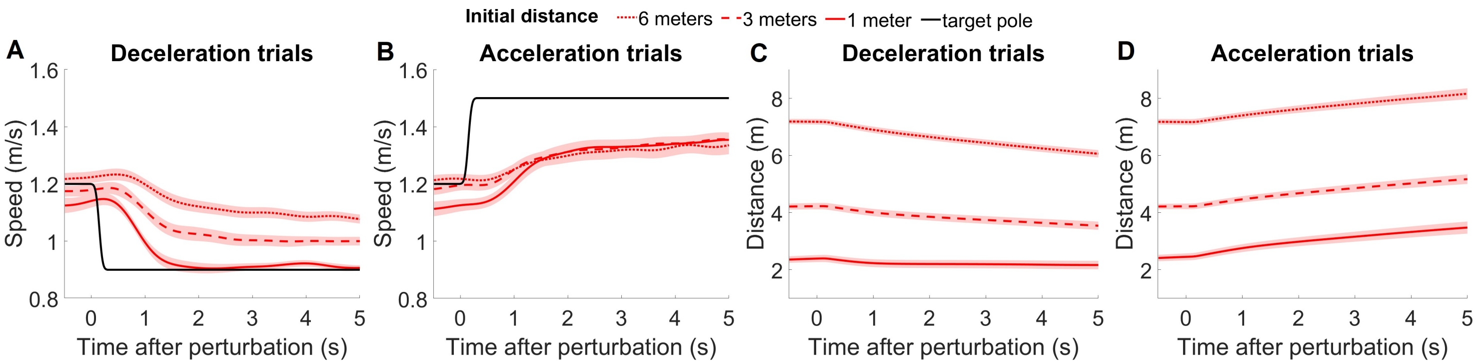

Figure 3: Mean time series of participant speed on (A) deceleration (n=355) and (B) acceleration (n=341) trials in Experiment 1. Mean time series of participant distance from leader on (C) deceleration and (D) acceleration trials. Black curves indicate the leader's speed, red curves indicate participant responses at each initial distance, shading indicates 95% confidence intervals computed on all trials. All time series are aligned at the time of perturbation.

Final states

The follower's mean final distance, final speed, and final speed difference with the leader are plotted for each experimental condition in Figure 4. A linear mixed effects regression, in which initial distance (d_0) and speed perturbation (Δv) were fixed effects predictors and subject was a random effect, was used to analyze each of the three dependent variables. The results reveal significant effects of initial distance, speed perturbation, and their interaction on all final states, all $p<0.001$ (see Table 3).

Trials with greater initial distance showed greater final distance (Figure 4A). There was also a significant effect of initial distance on final speed (Figure 4B): closer distances tended to produce slower final speeds. However, this effect did not occur when the leader sped up, as reflected in a significant distance by speed perturbation interaction. This asymmetry will be considered further below. Finally, the difference between the final speed of the follower and the leader increased with initial distance (Figure 4C), as revealed by the significant distance effect. This effect was largely due to leader deceleration, yielding a significant interaction. Participants thus did not match the speed of the leader, except in the constant-speed control condition.

On deceleration trials (red curves), final distance was smaller (Figure 4A) and final speed was slower, but depended on initial distance (Figure 4B). Acceleration trials (blue curves) showed greater final distance (Figure 4A) and faster final speed (Figure 4B). Thus, our manipulations on initial distance and speed perturbation effectively influenced the participants' behavior.

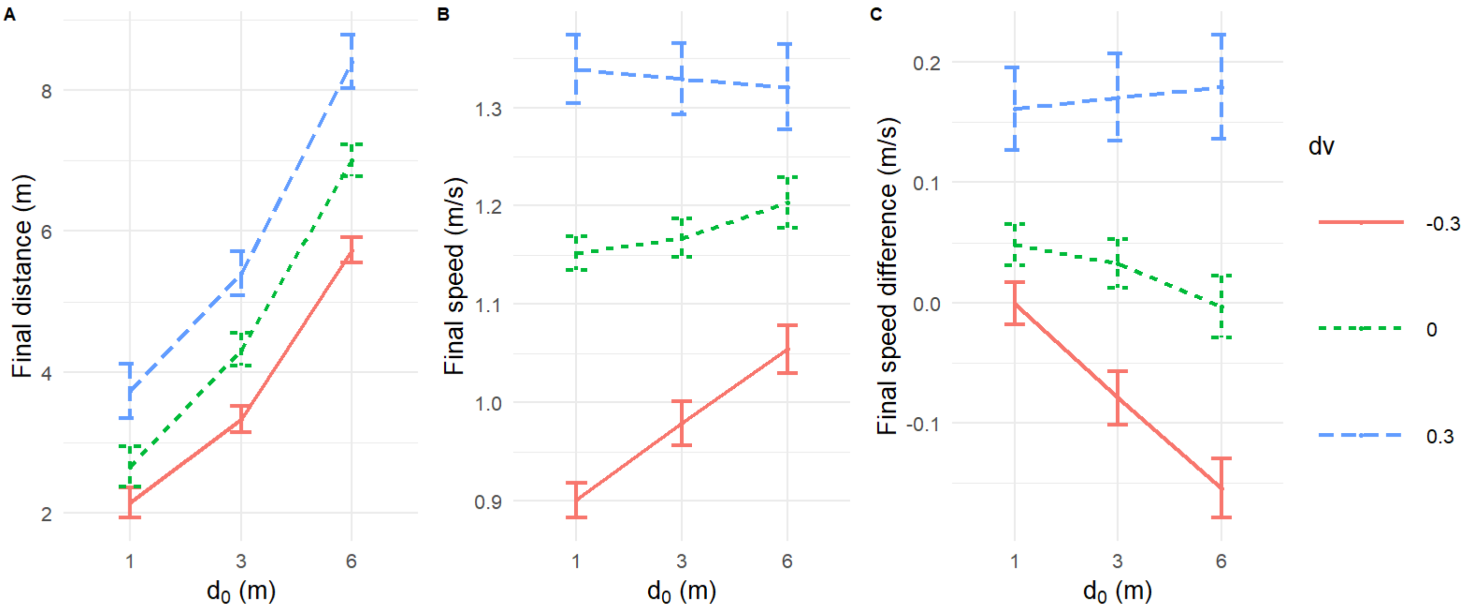

Figure 4: Mean final distance to leader (A), final speed of follower (B), and final speed difference between follower and the leader (C), in each condition of Experiment 1. In C, positive values mean that the leader was faster than the follower. Error bars represent the standard error of the mean (SEM).

Dependent Variable	d_0	Δv	$d_0 \times \Delta v$
Final distance	$\chi 2(2, N = 1060) = 669.74$ ***, $\eta^2=.98$	$\chi 2(2, N = 1060) = 115.13$ ***, $\eta^2=.85$	$\chi 2(4, N = 1060) = 148.16$ ***, $\eta^2=.13$
Final speed	$\chi 2(2, N = 1060) = 21.57$ ***, $\eta^2=.64$	$\chi 2(2, N = 1060) = 103.95$ ***, $\eta^2=.83$	$\chi 2(4, N = 1060) = 208.84$ ***, $\eta^2=.17$
Final speed difference	$\chi 2(2, N = 1060) = 21.79$ ***, $\eta^2=.64$	$\chi 2(2, N = 1060) = 73.48$ ***, $\eta^2=.77$	$\chi 2(4, N = 1060) = 208.80$ ***, $\eta^2=.17$

Table 3: The results of Wald Chi-Squared Test on fixed effects on final states. *** $p < .0001$

Response asymmetry

The results for walking speed revealed an asymmetry in the response to leader acceleration and deceleration (Figure 3A, 3B, 4B, C). Although the leader increased and decreased speed by the same amount, participants had a larger response to deceleration than acceleration, especially at smaller initial distances, consistent with the asymmetry in the rate of optical expansion and contraction. A linear mixed effects regression was performed on the absolute value of the follower's speed adjustment, with initial distance (d_0) and leader speed perturbation (Δv = -0.3 and +0.3 m/s only) as fixed effects, and subject as a random effect. There was a significant main effect of leader acceleration/deceleration (Figure 5), confirming the asymmetry in the magnitude of the follower's response ($\chi^2 =7.60$, $p < .01$, $\eta^2=.39$). There was also a main effect of initial distance ($\chi^2 =49.42$, $p < .01$, $\eta^2=.77$). The interaction was not significant ($\chi^2 =4.31$, $p = .12$).

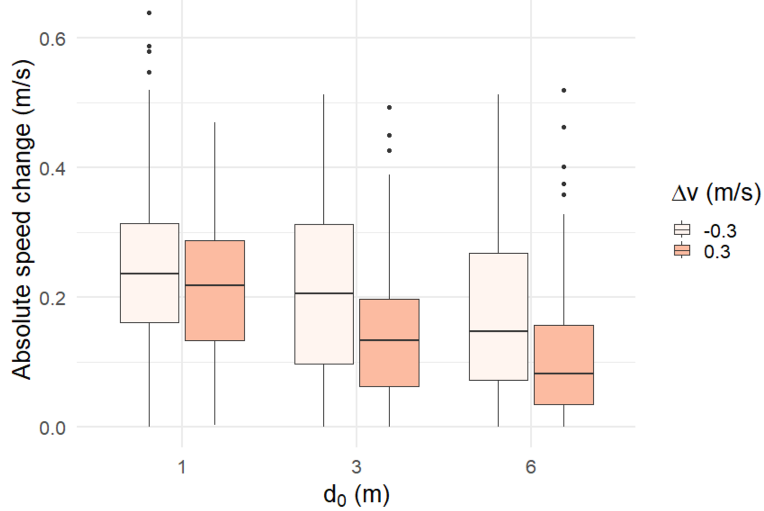

Figure 5. The asymmetry in absolute speed change of participants in response to the leader's deceleration or acceleration (Δv).

Finally, despite instructions to follow the leader at a constant distance, the actual following distance increased by the time of speed perturbation, and was generally not maintained after perturbation (Figure 3C, D), except when the initial distance was 1m and the leader decelerated.

Model comparisons

Results from the two fitting procedures appear in the right two columns of Table 2. Based on BIC (lower is better), the ranking of models is RRE < Ratio < RE < Linear < Speed < Lemercier < SBD < Distance = Null. For all inequalities, ΔBIC > 10, indicating very strong evidence in favor of this ordering.

Similar results were found for the mean RMSE from cross-validation (right column of Table 2; smaller is better), confirmed by frequentist tests (Figure 6). A one-way ANOVA on RMSE revealed a main effect of model, $F(8, 6255) = 102.26, p < .0001, \eta^2 = 0.116$. Duncan's multiple range test (de Mendiburu, 2019) showed the following RMSE ranking: (RE, RRE, Ratio) < (Speed, Linear, Lemercier) < SBD < (Null, Distance), where all inequalities are $p < .05$.

The purpose of Experiment 1 was to compare the RE and speed-matching models, because Rio, Rhea & Warren (2014) found that these two models best explained their data. We thus compared them in a linear mixed effect regression with Model, initial distance (d_0), and speed perturbation (Δv) as fixed effects, and subject as a random effect. The results revealed (1) significant main effects of Model $\chi^2(1, N = 696) = 31.32$, $p = .020$, $\eta^2 = .02$, (2) the Model × d_0 interaction, $\chi^2(2, N = 696) = 7.83$, $p = .020$, $\eta^2 < .01$, (3) the Model × Δv interaction, $\chi^2(1, N = 696) = 5.26$, $p = .022$, $\eta^2 < .01$, and (4) the three-way interaction of Model × d_0 × Δv, $\chi^2(2, N = 696) = 24.73$, $p < .0001$, $\eta^2 = .02$. The Model × d_0 interaction indicates that the difference between the RE Model and speed-matching model depends on d_0: the speed-matching model shows larger error than RE Model, especially when d_0 is 1 or 6 meters, which is due to its distance invariance (Figure 7).

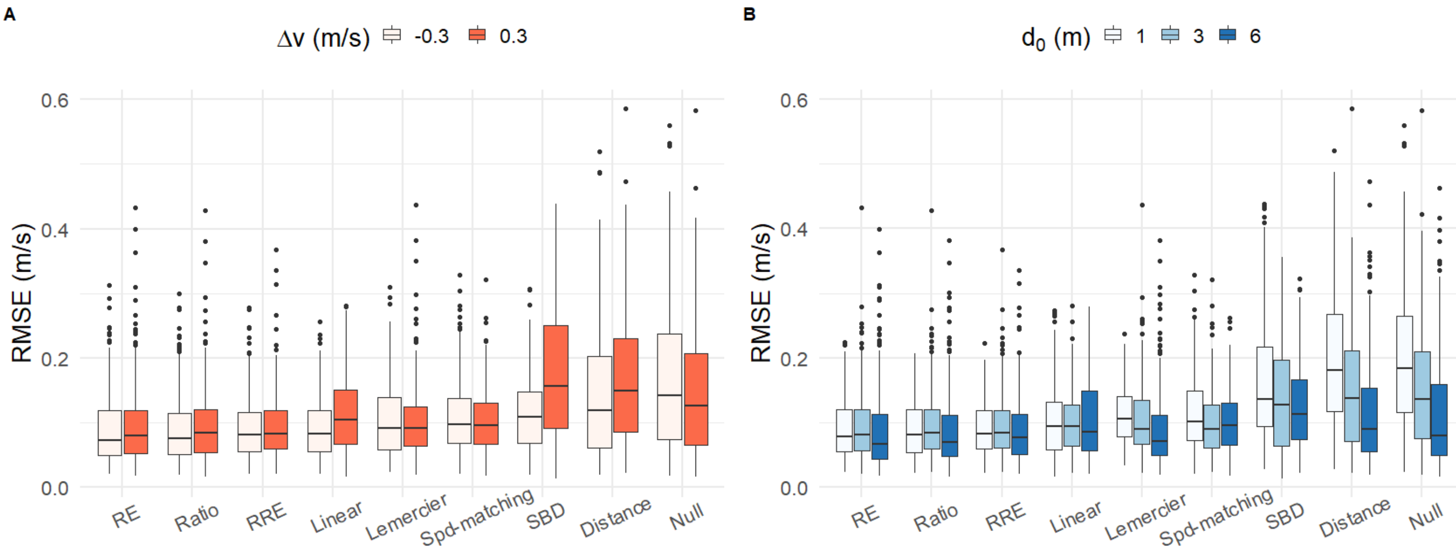

Figure 6: Box plots of RMSE for each model from leave-one-subject-out cross-validation in Experiment 1. (A) RMSE on acceleration and deceleration trials. (B) RMSE in each initial distance condition. The heavy black bar is the median, boxes represent the interquartile range, vertical whiskers represent the range of the data excluding outliers (black dots), which were defined as lying outside 1.5 times the interquartile range.

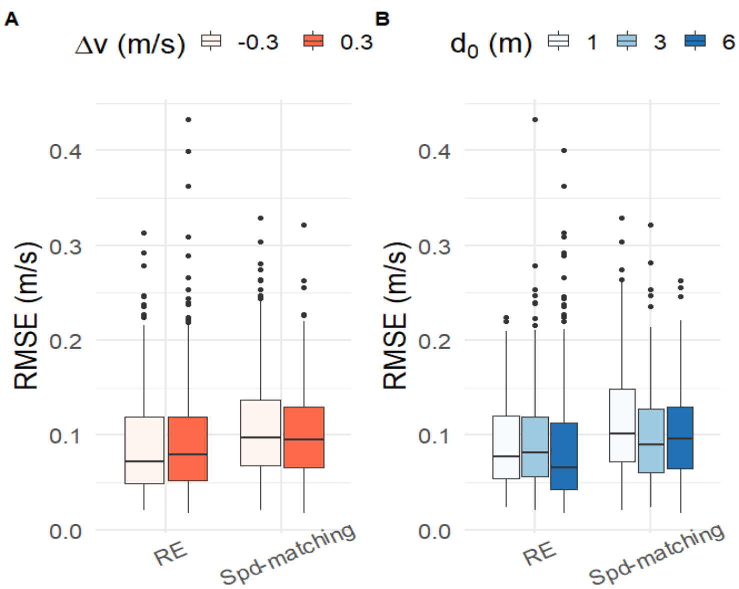

Figure 7. Box plots of RMSE highlighting the effects of (A) speed perturbation and (B) initial distance on the RE and Speed-Matching models. Data are the same as Figure 6.

The mean time series of speed for the speed-matching model and the RE model in each condition (Figure 8) reveal why the speed-matching model has a poorer fit. The predictions of the RE Model (green curves) vary with initial distance, whereas the speed-matching model (blue curves) makes similar predictions across conditions (compare columns in Figure 8). For the speed-matching model, minimizing error at the middle distance ($d_0 = 3$, panels B,E) means increasing error at short ($d_0 = 1$, panels A,D) and long ($d_0 = 6$, panels C,F) distances, because the behavior of participants changed with distance.

Discussion

The rate of expansion (RE), relative rate of expansion (RRE), and ratio models describe pedestrian following better than other models. They can predict the speed of pedestrians across different conditions without changing parameter values. In contrast, the speed-matching model makes very similar predictions at different following distance, which does not match the behavior of the participants.

This was not observed in Rio et al. (2014) because the range of following distance used in their experiments was not large enough to reveal the limitation of the speed-matching model.

Although distance can influence following behavior, it did not do so as described by the distance model or the speed-based distance (SBD) Model. Participants did not maintain the initial distance from the leader, nor did they maintain a distance that increases with their speed. Moreover, linearly adding a distance term to the speed-matching model (linear model) did not significantly improve the performance. The data and fitting results from Experiment 1 suggest a nonlinear relationship between distance and following behavior.

The models that use nonlinear forms of distance are the RE, RRE, ratio, and Lemercier models. The Lemercier model is a version of the ratio model with a constant delay or reaction time, but it had a lower goodness-of-fit. Although a delay can be observed in the data, it is not a constant value. In particular, the delay is shorter at smaller distances than at larger distances; we suspect this is due to lower rates of optical expansion/contraction at greater distances. In addition, whereas the Lemercier model does not consider the speed of the follower, the ratio model takes it into account. The RE and RRE models both rely on the change in visual angle, which is a nonlinear function of distance and depends on the relative speed of leader and follower. Not only do they predict behavior as well as the ratio model, but they have more concise form with only one free parameter. Given that humans rely on vision, they also offer biologically plausible control laws for pedestrian following. In contrast, the ratio model is omniscient, relying on distance, absolute speed, and relative speed, variables that are not immediately available in vision and are not accurately perceived.

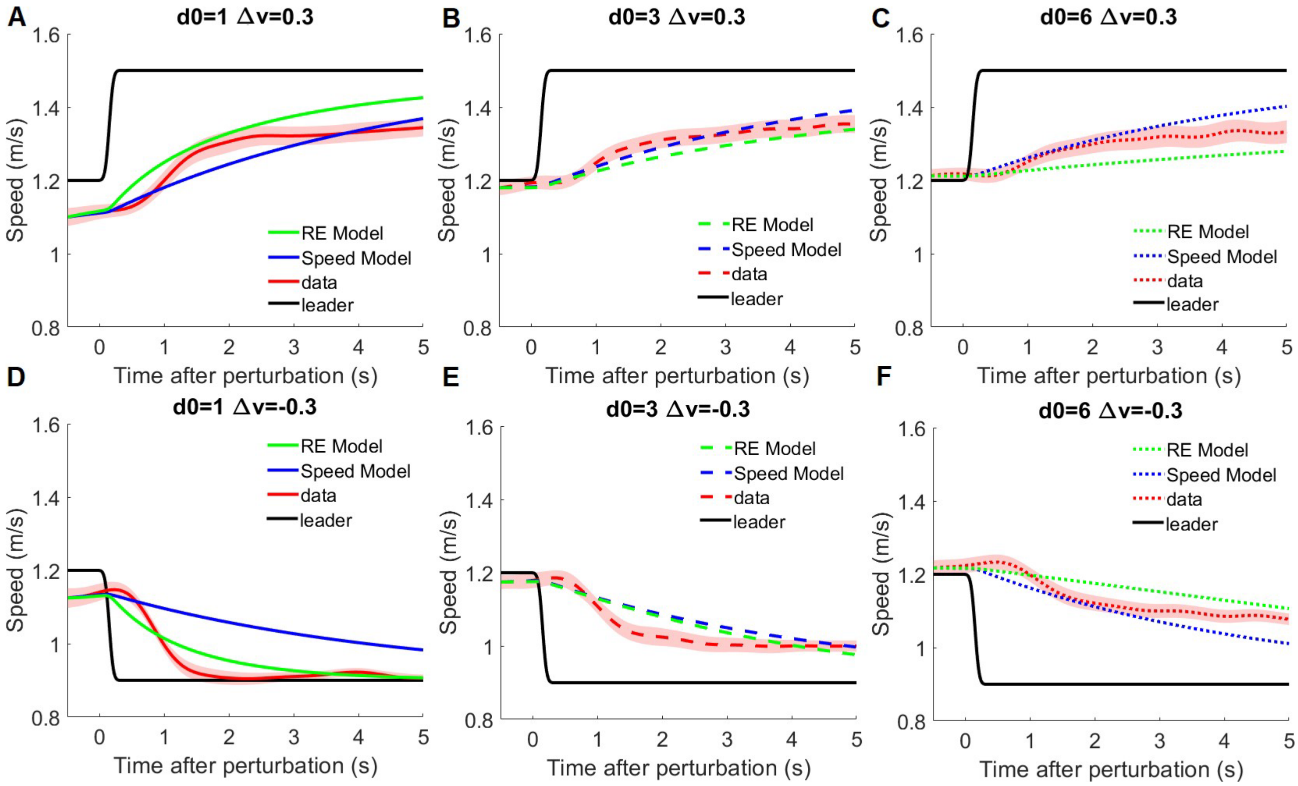

Figure 8: The real and predicted time series of speed in Experiment 1. D_0 is initial distance, Δv is speed perturbation. The shading indicates 95% confidence intervals for the human data. The predictions of RE Model are distance-dependent, whereas those of Speed-matching model are not.

Experiment 2

The RE model is sensitive to the size of the leader, because the optical expansion rate depends on the leader's visual angle, for a given distance and speed. The model thus predicts that a larger leader will produce a stronger response in the follower, all other things being equal. This size-dependance is attenuated by the RRE model because the expansion rate is normalized by the visual angle; in effect, RRE specifies the nearness of a collision in time at the current relative speed. Experiment 1 did not strongly distinguish these models because the leader pole was a constant size, so they made similar predictions after parameter fitting. In Experiment 2, we manipulated the size (diameter) of the leader pole.

Method

Participants

Twelve students from Brown University, five males and seven females who had not participated in Experiment 1, participated in Experiment 2. All participants had normal or corrected to normal vision. The protocol was approved by Brown University's Institutional Review Board, in accordance with the Declaration of Helsinki. Informed consent was obtained from all participants, who were paid for their participation.

Apparatus and displays

The equipment and displays were the same as in Experiment 1, with two exceptions. First, the diameter (i.e. width) of the leader pole was manipulated (0.2, 0.6, or 1 m), while its height was 3m in all trials. Second, the initial distance of the leader pole was held constant at 2m.

Procedure and design

The instructions and procedure were the same as before. Experiment 2 had a within-subject factorial design: 3 leader width (0.2, 0.6, 1 m) x 3 speed perturbation (-0.3, 0, +0.3 m/s), yielding 9 conditions. There were 10 repetitions in each condition, for a total of 90 trials, presented in a different random order for each participant. A test session lasted about 40 minutes.

Data processing and model fitting

The data were processed as in Experiment 1. Thirty-six (3.33%) out of 1080 trials were excluded due to tracking failures. The data used for model fitting is the time series of follower's speed from 0.5 seconds before the perturbation to 5.5 seconds after the perturbation in each trial. Only trials with a speed perturbation (n = 694) were used for model fitting, After removing twenty-six trials that were shorter than 5 seconds after perturbation. Participants' final speed (the average speed in the last two seconds) and final distance to the leader pole (the average distance in the last two seconds) were calculated for each time series. Models were fitted and tested using the same procedures of Experiment 1.

Results

Mean time series of speed and distance in each condition appear in Figure 10. Leader size (separate curves) clearly influenced the participant's distance from the leader (Panels C, D), although it had little influence on the participant's speed (panel B).

Final states

The mean final distance, final speed, and final speed difference between the participant and the leader are plotted as a function of leader width in Figure 9. A linear mixed effect regression with leader width (w) and speed perturbation (Δv) as fixed effects, and subject as a random effect, revealed a significant effect of leader width on final distance ($p < .001$) (Table 4). Specifically, distance increased with the width of the leader (Figure 9A); this effect cannot be explained by a constant visual angle, for the mean final angle was four times larger with a 1m width than a 0.2m width in each speed perturbation condition. In contrast, there was no effect of leader width on final speed or final speed difference (Figure 9B,C). This finding is inconsistent with the RE model, in which the speed response depends on leader size, but consistent with the RRE model, in which the response is normalized by leader visual angle.

On the other hand, the speed perturbation had a significant effect on all final states, confirming that it effectively influenced behavior. In particular, participants followed at a closer distance when the leader slowed down than when the leader sped up (Figure 9A). Participants also walked slower when the leader decelerated than when the leader accelerated (Figure 9B), resulting in a slower and faster speed than the leader, respectively (Figure 9C). The $w \times \Delta v$ interaction was not significant for any final state.

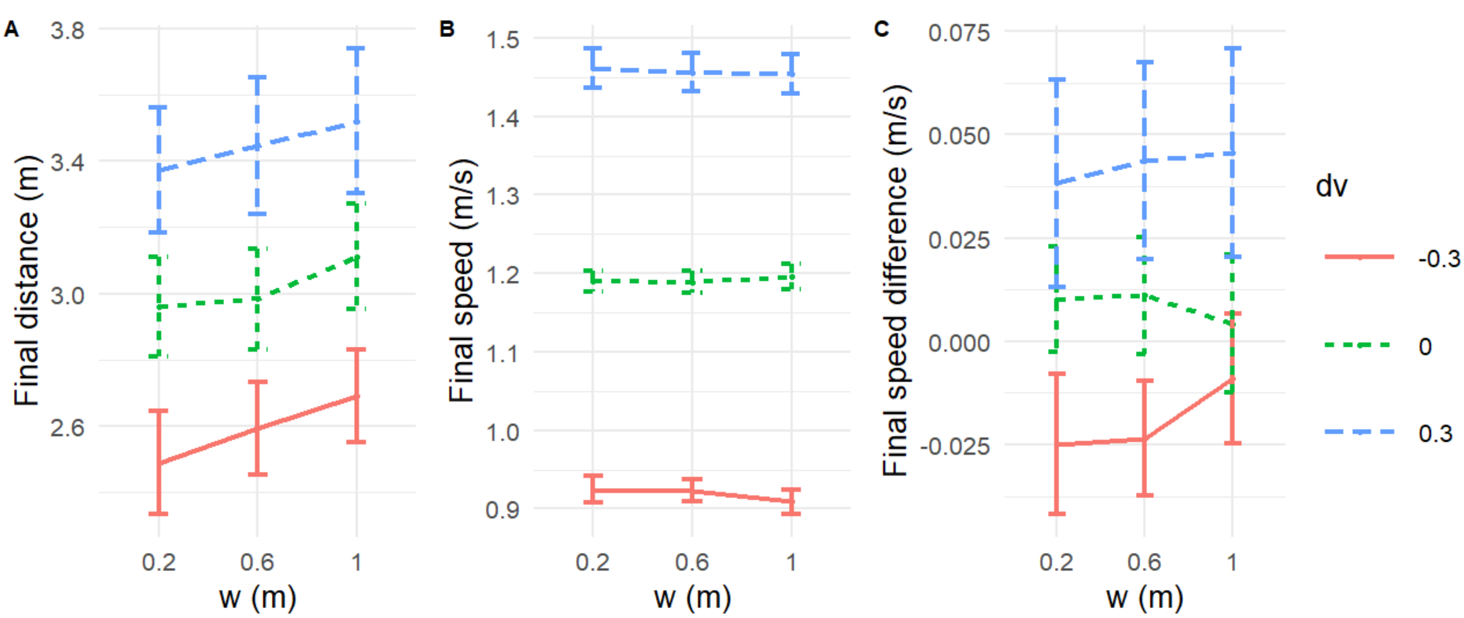

Figure 9: Mean final distance to (the surface of) leader (A), final speed of follower (B), and final speed difference between follower

and the leader (C) as a function of leader width (w) and speed perturbation (Δv) in Experiment 2. In C, positive values on the ordinate mean that the leader was faster than the follower. Error bars represent the standard error of the mean (SEM).

	w	Δv	$w \times \Delta v$
Final distance	$\chi^2(2, N=1044)=23.72^{***}$, $\eta^2=.52$	$\chi^2(2, N=1044)=63.68^{***}$, $\eta^2=.83$	$\chi^2(4, N=1044)=1.18$
Final speed	$\chi^2(2, N=1044)=1.15$	$\chi^2(2, N=1044)=2173.35^{***}$, $\eta^2=.99$	$\chi^2(4, N=1044)=6.19$
Final speed difference	$\chi^2(2, N=1044)=1.16$	$\chi^2(2, N=1044)=23.23^{***}$, $\eta^2=.55$	$\chi^2(4, N=1044)=6.19$

Table 4: The results of Wald Chi-Squared Test on fixed effects on final states. *** p < .0001.

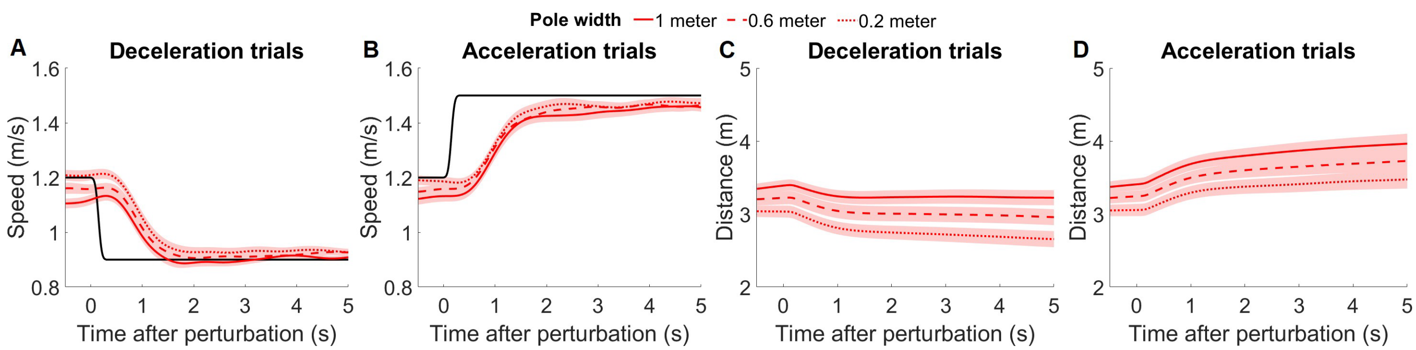

Figure 10: Mean time series of participant speed in Experiment 2 for (A) deceleration (n=346) and (B) acceleration (n=351) trials. Mean time series of participant distance from leader in (C) deceleration and (D) acceleration trials. Black curves indicate leader speed, red curves indicate participant responses in each width condition, shading indicates 95% confidence intervals. All time series are aligned at the time of perturbation.

Following speed and distance

As is apparent from the mean time series (Figure 10), the width of the leader did not influence participants' speed (panels A, B) but did affect the distance of participants from the leader (panels C, D). Once again participants did not maintain the initial distance of 2m to the leader, nor did they keep a constant distance after perturbation.

Model comparisons

Results from the two fitting procedures are shown in Table 5. Based on BIC (lower is better), the ranking of models is RRE < (Ratio = Linear) < Speed < RE < Lemercier < SBD < Null < Distance. For all inequalities, ΔBIC > 8, indicating strong evidence in favor of this ranking.

The mean RMSE for each model (smaller is better) from leave-one-subject-out cross-validation appears in Figure 11. One-way ANOVA on RMSE revealed a main effect of Model, $F\ (8,\ 6237) = 636.2$, $p < .0001$, $\eta^2 = 0.449$. Duncan's multiple range test (de Mendiburu, 2019) showed the following ranking of models: (RRE, Speed, Linear, Ratio) < RE < Lemercier < SBD < Distance < Null, where inequalities are $p < .05$ (means and standard deviations are shown in Table 5).

To compare the RE and RRE models, a linear mixed effect regression was performed on RMSE with Model, leader width (w), and speed perturbation (Δv) as fixed effects, and subject as a random effect. It revealed a significant main effect of Model, $\chi^2(1,\ N = 694) = 23.39$, $p < .001$, $\eta^2 = 0.02$, Model × w interaction, $\chi^2(2,\ N = 694) = 7.92$, $p = .019$, $\eta^2 < .01$, and a significant Model × Δv interaction,

$\chi^2(1, N = 694) = 9.52$, $p = .002$, $\eta^2 < .01$. It can be seen from Figure 11B that the difference between the RE and RRE model depends on w. The error of RE Model is larger than that of RRE model especially when w is 0.2m or 1m, which is due to RE Model's sensitivity to size of the leader pole.

Mean time series for simulations of the RE and RRE models in each leader acceleration/deceleration condition (Figure 12) reveal that the RE model over-reacted to the large leader (w = 1m) and under-reacted to the small leader (w = 0.2m). The RRE Model overcame this sensitivity by normalizing the expansion/contraction rate with the visual angle of the leader.

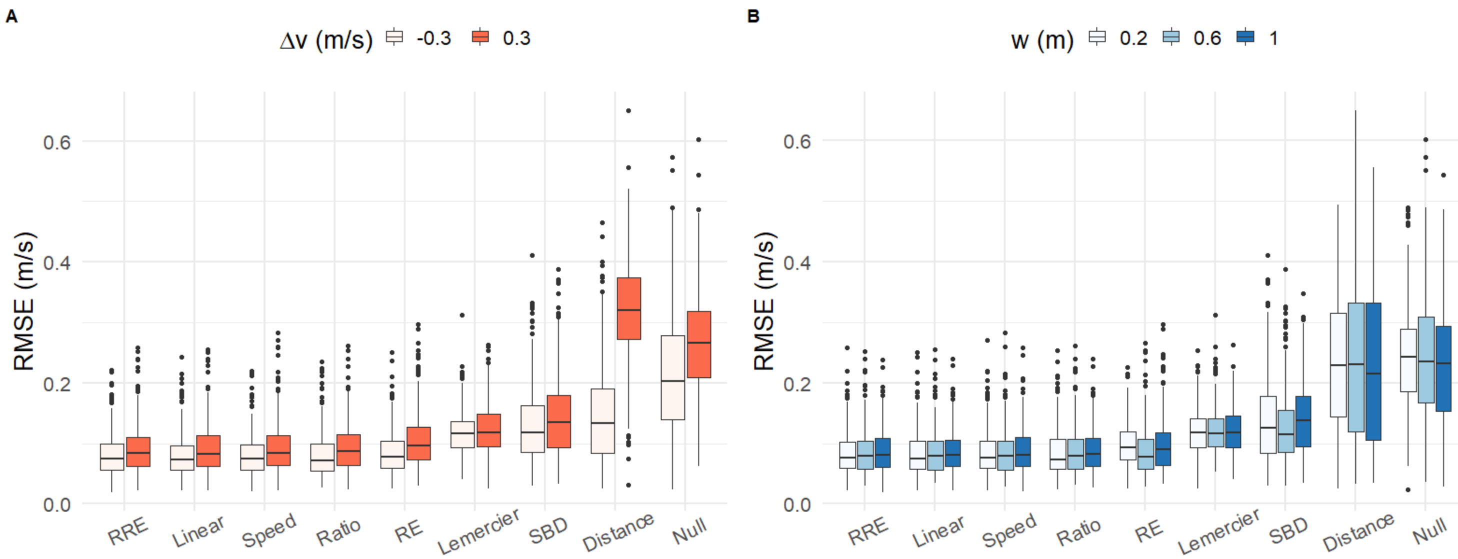

Figure 11: Box plots of RMSE for each model from leave-one-subject-out cross-validation in Experiment 2. (A) RMSE on acceleration and deceleration trials, (B) RMSE in each leader width condition.

Model	Equation	Parameters	BIC	Mean RMSE (m/s)
Relative rate of expansion (RRE)	$\ddot{x}_f(t) = -b\dfrac{\dot{\theta}}{\theta}$	b = 2.629	-3289[a]	0.086 (SD=0.038)[a]
Ratio	$\ddot{x}_f(t) = c\dot{x}_f{}^M \dfrac{\Delta\dot{x}}{\Delta x^L}$	c = 3.698 M = −1.760 L = 1.014	-3281[b]	0.088 (SD=0.040)[a]
Linear	$\ddot{x}_f(t) = c_1\Delta\dot{x} + c_2[\Delta x - (a + b\dot{x}_f)]$	c_1 = 0.894 c_2 = −0.035 a = 2.080 b = 0.652	-3281[b]	0.087 (SD=0.038)[a]
Speed	$\ddot{x}_f(t) = c\Delta\dot{x}$	c = 0.831	-3267[c]	0.087 (SD=0.039)[a]
Rate of expansion (RE)	$\ddot{x}_f(t) = -b\dot{\theta}$	b = 20.443	-3141[d]	0.095 (SD=0.043)[b]
Lemercier et al (2012)	$\ddot{x}_f(t) = c\dfrac{\Delta\dot{x}(t+\tau)}{\Delta x(t)^{\gamma}}$	τ = 1.000 c = 1.833 γ = 0.796	-2863[e]	0.121 (SD=0.039)[c]
Speed-based distance (SBD)	$\ddot{x}_f(t) = c[\Delta x - (a + b\dot{x}_f)]$	c = 2.644 a = 1.231 b = 1.746	-2647[f]	0.137 (SD=0.066)[d]

Null	$\ddot{x}_f(t) = 0$	None	-1887[g]	0.238 (SD=0.097)[f]
Distance	$\ddot{x}_f(t) = c(\Delta x - \Delta x_0)$	$c = 0.011$	-1868[h]	0.231 (SD=0.120)[e]

Table 5: The fitting and testing results of 9 models on perturbation trials (n=694) of Experiment 2. Parameter values and Bayesian Information Criterion (BIC) values were acquired by fitting the models to all trials, while minimizing the Root-Mean-Square Error (RMSE) on speed. BIC values were computed based on equation 1. The models were also tested using Leave-one-subject-out cross-validation, in which each model was trained on 11 participants and tested on the one left out until all combinations of training set and test set were used. The test results (mean and standard deviation of RMSE) of 12 iterations of cross-validation are shown in the table. The letters in the superscript of BIC values indicate the rank of model based on BIC, whereas those in the superscript of cross-validation error indicate Duncan group in Duncan's multiple range test. T in Lemercier et al (2012) was caped at 1 second because a reaction time longer than 1 second is unlikely for human pedestrian.

Discussion

Experiment 2 found that the width of the leader did not influence the participant's speed in 1D following. Speed responses to the leader did not change significantly based on its diameter, at least over the range of 0.2 to 1.0 m. Whereas the RE Model was sensitive to this range of leader size, the RRE Model normalizes the expansion rate by the leader's visual angle, and thus had a smaller error.

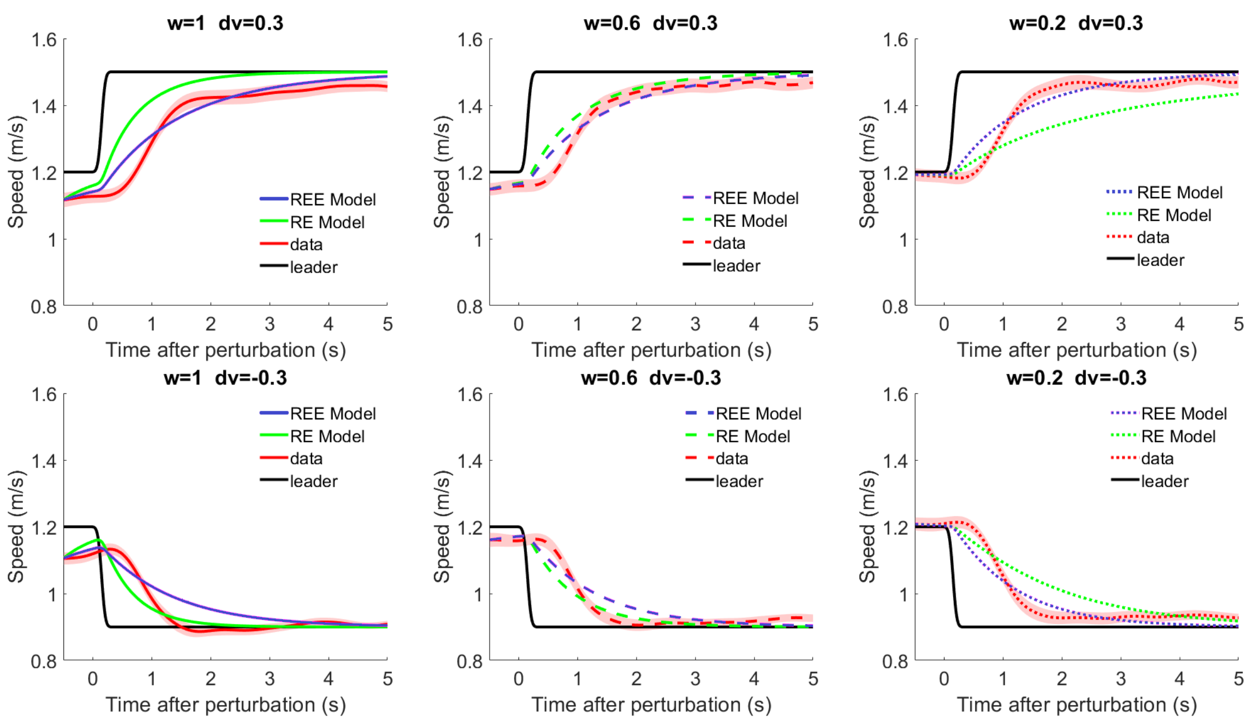

Figure 12: The real and predicted time series of speed in Experiment 2. D_0 is initial distance. Δv is speed perturbation. The shading indicates the 95% confidence intervals for the human data. The predictions of RE Model change with the width of the leader pole, whereas those of RRE Model do not to the same degree.

Note that the final distance did vary by the width of the leader. Figure 9A shows that the final distance to the surface of the leader increased as the width of the leader increased. This is due to the speed difference after initial acceleration. The time series of speed in Figure 10A and 10B show that participants' average speed before the perturbation was lower when the leader is larger, which caused longer distance at the time of perturbation. This reluctance to get close to a larger object could be a result of safety concern. However, the width of the leader did not influence how participants change speed in response to the perturbation. Therefore, the width of the leader influenced final distance but not final speed.

Experiment 2 also replicated the results of Rio et al. (2014), who found that the speed-matching, ratio, and linear models all performed at a comparable level, with a significantly lower RMSE than the distance and speed-based distance models. However, we find that the RRE model is also comparable to these omniscient models by the frequentist criterion, has the smallest RMSE overall, and performs the best by the Bayesian Information Criterion (BIC), because it only has one free parameter. The RRE Model does not rely on separate inputs of distance and speed but an optical variable that depends on both. Therefore, it is free from assumptions about the accuracy of distance and speed perception. The RRE Model thus offers a biologically plausible explanation of how human pedestrians use visual information to control walking speed while following a leader.

General discussion

Pedestrian following behavior

The present study showed that the speed control of pedestrian following depends on the distance and speed, but not the size, of the leader. Although participants were asked to maintain a constant following distance, the actual distance varied. When the leader accelerated to 1.5 m/s, participants did not fully match that speed, which caused the distance to increase; conversely, when the leader decelerated to 0.9 m/s, participants also did not fully match that speed until the distance decreased to less than 3 meters. In addition, the present study replicated the asymmetry in the response to leader deceleration and acceleration reported by Rio, et al (2014): participants exhibited a greater change in speed when the leader slowed down than when the leader sped up. This asymmetry was greater as the leader distance decreased because the rate of expansion increased, given the same deceleration. Finally, the follower's speed does not depend on leader size (over a range of 0.2 to 1.0m), contrary to the prediction of the RE model but consistent with the RRE model.

We note that the human data have a sigmoidal speed profile in response to the leader's change in speed, whereas all the models produce an instantaneous speed change (Figure 3, 8, 10, 12). This contributed to visible prediction errors. The discontinuous change in speed is due to the fact all the models are second-order models, which control instantaneous acceleration. In the future, use of third-order models could reduce this error by controlling the derivative of acceleration (jerk). Such an approach would allow the model to generate a gradual change in speed (acceleration) and match the sigmoidal speed profile of the human data.

Models of Pedestrian following

Previously, Rio, et al. (2014) reported that the speed-matching model approximated the human data more closely than the RE Model, at least over a small range of leader distance (1 to 4m). However, the present Experiment 1 found that both the RE and RRE models perform better than the speed-matching model when tested on a wider range of distances (1 to 6m). Moreover, the follower's speed is better predicted by the relative rate of expansion (RRE) than the rate of expansion (RE) per se in both of the present experiments. When the size of the leader was varied in Experiment 2, the error of RE model increased, whereas the error of the RRE model remained smaller than all other models, with ΔBIC indicating strong evidence in favor of RRE.

The closest competitor to the RRE model is the ratio model. However, it is predicated on physical rather than optical variables, and has three free parameters as opposed to one. The ratio model takes leader distance, follower speed, and speed difference as input, and is thus omniscient. In addition, the optimal value for each of its three parameters differed between Experiment 1 and Experiment 2, for reasons that are hard to interpret. In contrast, the RRE Model takes one optical variable and its rate of change as input, and its one free parameter has a straightforward interpretation as the gain or sensitivity of speed control. Most important, the RRE model provides a biologically plausible explanation of speed control based on vision, whereas the ratio model relies on distance and speed, which are not accurately perceived.

Conclusion

The relative rate of expansion (RRE) model explains speed control in pedestrian following better than the seven other models tested in the present study. The model generalizes over a range of leader distances, speeds, and sizes. Most importantly, it provides a concise, vision-based explanation of 1D pedestrian following, which can be applied to related problems such as 2D following, or controlling heading and speed in the horizontal plane (Dachner & Warren, 2017), and collective crowd motion (Dachner, Wirth, Richmond & Warren, 2022).

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