# Funneling Electromagnetic Waves with PTD Symmetric Metastructures


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***Abstract*—This paper investigates a new class of metastructures for funneling electromagnetic energy within arbitrary narrow channels. This class is based on structures that are invariant under the Parity Time-reversal Duality (PTD) transformation. Such structures produce zero backscattering under the illumination of a plane wave at normal incidence for all incident polarizations. These reflectionless and arbitrary polarizations features are exploited to design metastructures for concentrating electromagnetic energy within subwavelength channels. Numerical results of the proposed metastructures are also discussed.**



***Index Terms*—PTD symmetric structures, reflectionless media, zero backscattering.**


## I. Introduction

Concentrating and funneling electromagnetic waves within narrow channels is of great importance in a variety of applications, such as sensing, energy harvesting, subwavelength imaging, and optical nonlinearity. In recent years, several proposals on squeezing electromagnetic waves through channels with subwavelength cross-section have been put forward. Transmission of light through subwavelength holes in metallic screens, known as extraordinary optical transmission (EOT), has been extensively investigated [1]–[3]. The concept of EOT is typically based on resonant mechanisms, such as Fabry-Perot resonances [1]–[3], and plasmonic resonances [4]. As a result, EOT is inherently narrow band, which limits the range of potential applications. To overcome this issue, the phenomenon of Brewster full transmission at air-dielectric interface was extended to plasmonic gratings [5], [6]. In this proposal, an incoming electromagnetic waves can propagate through electrically small slits over a wide range of frequency. However, it is limited to incoming waves that are TM (transverse-magnetic) polarized. Recently, it has been shown how self-dual metastructures can funnel an arbitrary-polarized incoming waves through very narrow channels with zero backscattering. The bandwidth of such metastructures is only limited by the constituents materials [7], [8]. Along this line of research, this paper investigates another type of symmetry to design a class of metastructures with extreme funneling effects. The class of proposed metastructures are invariant under the Parity, Time-reversal, and Duality (PTD) operators [9]–[12]. The structures are inherently matched to free space and allow funneling an arbitrary-polarized incoming waves into arbitrary-narrow channels regardless of their length. The bandwidth is related only to the constituent materials.

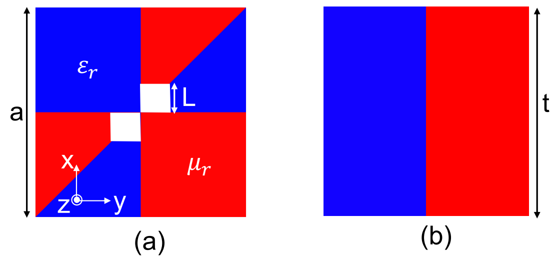


Fig. 1: Unit cell of the PTD symmetric metastructure. (a) top and (b) front view.

## II. PTD symmetric metastructures

PTD symmetric metastructures are defined as structures invariant under the sequential action of the parity, time-reversal, and duality operators. Considering reciprocal lossless media, which are symmetric with respect to time reversal, the PTD invariance condition translates in the following relation between the relative permittivity and permeability of the structure [9]

$$\varepsilon_r\left(x, y, z\right) = \mu_r\left(-x, -y, z\right) \tag{1}$$

In the previous equation, we assumed the parity transformation $(x, y, z) \rightarrow (-x, -y, z)$. In this paper, we concentrate in a PTD symmetric metastructure whose unit cell is displayed in Fig. 1. Blue and red colors indicate electric and magnetic materials, respectively. We denote with $a$ and $t$ the period and the thickness, respectively. Around the center, there are two channels of square cross-section with side $L$ (see white squares in Fig. 1(a)). For simplicity, we assume the two channels are filled with air. One can observe that such a unit cell remains unchanged if it is rotated by 180° with respect to the $z$ axis and the electric permittivity and the magnetic permeability are swapped. It can be shown that this condition, which is exactly what Eq. (1) tells us, ensures that the metastructure with the unit cell shown in Fig. 1 is perfectly matched to free space. A normally incident plane wave is forced to propagate throughout the metastructure regardless of the spatial discontinuity and of the thickness. It is worth emphasizing that this reflectionless condition holds also for $z$-variant structures as long as any cross-section satisfies Eq. (1).

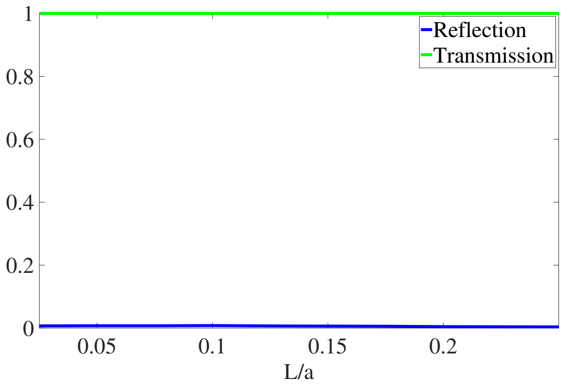


Fig. 2: Reflection and transmission coefficients from the metastructure with the unit cell shown in Fig. 1.

## III. Numerical results

To test the validity of the proposed PTD-symmetric metastructure (Fig. 1) in funneling an incoming electromagnetic wave through the air-filled channels, we consider an extreme case. The electric material shown in blue in Fig. (1) is replaced with a perfect electric conductor (PEC) and the magnetic material shown in red Fig. (1) is replaced with a perfect magnetic conductor (PMC). The period of the unit cell is set to be much smaller than the wavelength ($a = \lambda/8$ with $\lambda = 3$cm). As a result, only the fundamental Floquet mode contributes to the far fields. The thickness ($t$) is assumed to be $\lambda/4$. The unit cell was analyzed with the commercial software COMSOL Multiphysics® by imposing periodic boundary conditions on the vertical walls. The simulated reflection and transmission coefficients as a function of the side length, assuming a normally incident plane wave polarized along $y$, are shown in Fig. 2. It is worth noting that the transmission (reflection) coefficient is one (zero) highlighting the proposed metastructure's reflectionless feature and that all the incoming power propagates through the channels regardless of their side length. Since the structure is illuminated with an $y$ polarized wave, all the incoming energy flows only on the right channel. Fig. 3 shows the amplitude of the electric field for two different side lengths of the channel. As expected, the narrower the channel is, higher is the intensity of the electric field inside the channel. The energy of normally incident plane wave polarized along $x$ will flow only on the left channel. Whereas, both channels will be activated for incoming circularly-polarized plane wave.

## IV. Conclusion

A new class of metastructures based on the PTD symmetry for funneling electromagnetic waves has been presented. The proposed structures can perfectly funnel a normally incident arbitrarily polarized plane wave with zero backscattering. Numerical results have also been presented. Other salient features of this type of metastructures, such as bandwidth, the dependence on the angle of incidence, and practical implementation, will be presented at the conference.

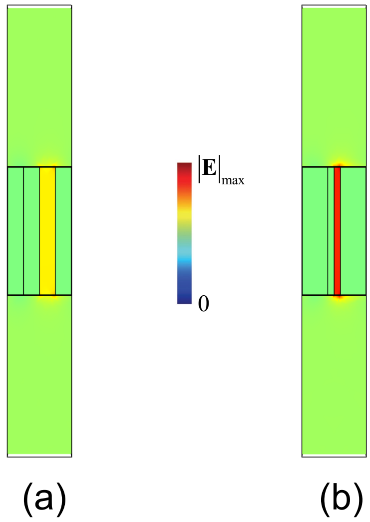


Fig. 3: Distribution of the amplitude of the electric field when the metastructure with the unit cell shown in Fig. 1 is illuminated with an $x$ polarized plane wave. (a) $L = a/4$ and (b) $L = a/10$.

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