# Dispersion Characteristics of Additive-Manufactured Metasurfaces

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***Abstract*—This paper presents an efficient and accurate approach for the analysis of additive-manufactured metasurfaces (MTSs) consisting of dielectric posts grown on a grounded slab. The formulation is based conventional effective medium theory that allows modeling such MTSs as uniaxially anisotropic grounded slabs. This procedure can be used to effectively characterize the surface waves supported by additive-manufactured MTSs, as needed in the design of planar lenses and leaky-wave antennas.**

***Index Terms*—surface waves, metasurfaces (MTSs), planar lenses, homogenization, Acrylonitrile Butadiene Styrene(ABS) posts, additive manufacturing.**

## I. Introduction

In recent years, metasurfaces (MTSs) —the 2D equivalents of metamaterials— have been at the center of attention for providing a compact platform to control, manipulate, and tailor electromagnetic (EM) waves almost over the entire EM spectrum [1]. In the microwave and millimeter-wave regime, MTSs can be realized by patterns of subwavelength metallic elements. These artificial structures have been proposed for a variety of applications, including tailoring surface waves [2]–[4], controlling the phase and polarization of spatial waves [5]–[7], and transforming surface waves into leaky waves [8]–[10].

The advent of 3D printing technology has led to the emergence of a new class of MTSs entirely additive manufactured [11]. Designing EM devices based on such MTSs have the significant advantage of reducing the overall cost and the manufacturing time. The transformation of surface waves into leaky waves [10] has been demonstrated using additive-manufactured MTSs consisting of elliptical cylinders with different heights, orientation, and axial ratio. This proposal has shed a new light on MTS antennas [8], [9]. Fano resonance has been investigated in the microwave range in MTSs consisting of 3D printed dielectric resonators [12]. Highly transmissive MTSs consisting of circular posts made of Acrylonitrile Butadiene Styrene (ABS) plastic, which is one of the most common material utilized by the commercial 3D printers, can provide an alternative platform to control the phase of spatial EM waves [13]. A minor research effort has been dedicated in designing additive manufactured MTSs for manipulation of surface waves. To this end, this paper presents an accurate and efficient approach to derive surface-wave dispersion curves of MTSs consisting of dielectric posts grown on a ground plane.

The paper is organized as follows. Section II presents the modeling of the MTSs as a uniaxially, anisotropic grounded slab and the dispersion equation for the fundamental TM (transverse magnetic) mode. Section III shows the accuracy of the proposed approach in studying the dispersion characteristics of surface waves. Section IV concludes our paper.

## II. Formulation

Let us consider the MTS unit cell shown in Fig. 1(left). It consists of a dielectric post with radius $r$ and height $h$ on a ground plane. A rectangular reference system is assumed with the $z$ axis orthogonal to the post cross-section and the origin at the ground-plane level. The post is assumed to be made of ABS plastic that presents a relative permittivity ($\varepsilon_r$) about 3 in the frequency range from few GHz to 35GHz [13]. The unit cell of the MTSs possesses a period $a$ along both $x$ and $y$ axes. In the long wavelength limit, the subwavelength MTS can be accurately modeled as a uniaxially, anisotropic, homogeneous grounded slab [see Fig. 1(right)] with the following relative permittivity tensor

$$\underline{\underline{\varepsilon}}_r = \begin{bmatrix} \varepsilon_{r,\perp} & 0 & 0 \\ 0 & \varepsilon_{r,\perp} & 0 \\ 0 & 0 & \varepsilon_{r,||} \end{bmatrix} \qquad (1)$$

$\varepsilon_{r,\perp}$ and $\varepsilon_{r,||}$, which represent the relative permittivity for the TM (transverse magnetic) and TE (transverse electric) waves, respectively, can be obtained through Maxwell-Garnet mixing equations [14]. The dispersion equation of the fundamental TM mode supported by such a slab can be derived by the transverse resonance technique with the help of the equivalent transmission line depicted in Fig. 2. It reads

$$\frac{k}{\eta\sqrt{k_\rho^2 - k^2}} - \frac{\varepsilon_{r,\perp} k}{\eta k_{z1}} cot\left(k_{z1} h\right) = 0 \qquad (2)$$

where $\eta$ and $k$ are the free-space impedance and wavenumber, respectively, $k_\rho$ is the surface-wave wavenumber, and $k_{z1} = \sqrt{\varepsilon_{r,\perp} k^2 - k_\rho^2}$. The dispersion relation in [Eq. 2] is a transcendental equation. The explicit $k_\rho$ dispersion curves as a function of the frequency ($f$) can be obtained numerically.

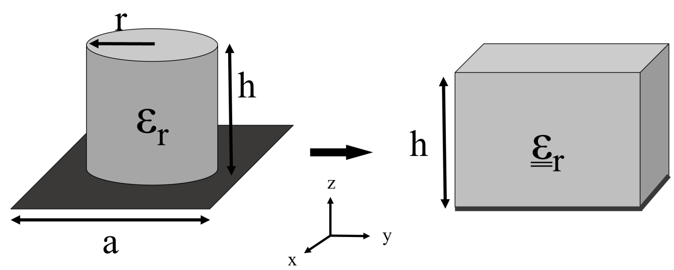


Fig. 1. Left: geometry for the unit cell of the ABS post MTS. Right: Equivalent uniaxially anisotropic grounded slab modeling the MTS.

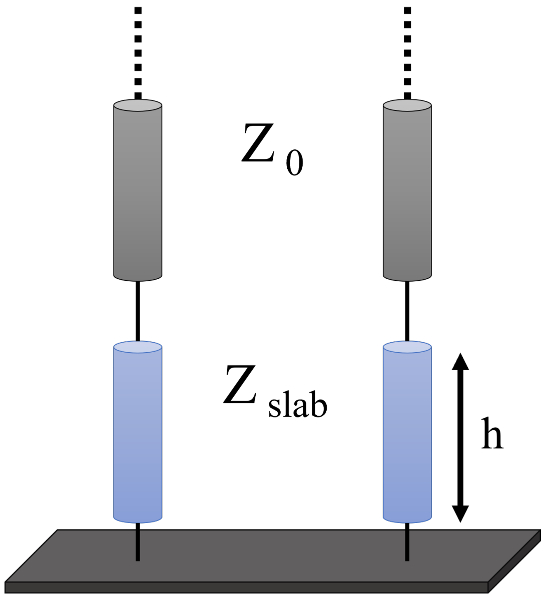


Fig. 2. Transmission line model for the fundamental TM mode of the slab in Fig. 1(right) with $Z_0 = -j\frac{\eta}{k}\sqrt{k_\rho^2 - k^2}$ and $Z_{slab} = \frac{\eta}{\varepsilon_{r,\perp} k}\sqrt{\varepsilon_{r,\perp} k^2 - k_\rho^2}$.

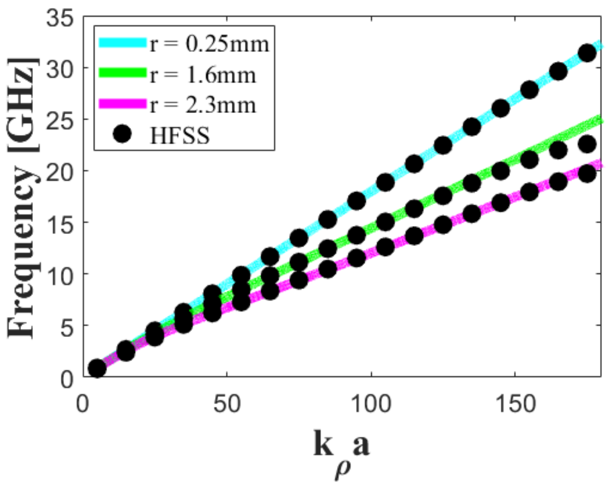


Fig. 3. Dispersion curves for an ABS-post MTS characterized by $a = 4.6$mm and $h = 10$mm. Comparison between the dispersion curves provided by Eq. 2 (solid lines) and the numerical results of HFSS (black dotted lines) for thee different values of the post radius.

## III. Numerical Results

The proposed approach has been applied to the analysis of an ABS post MTS with geometry characterized by $a = 4.6$mm, and $h = 10$mm. Fig. 3 shows the surface-wave transverse wavenumber ($k_\rho$) as a function of frequency for three different values of the post radius. The results provided by Eq. 2 are compared with the results of HFSS, showing an excellent agreement.

## IV. Conclusion

An effective approach for the characterization of the surface waves supported by ABS-post MTSs has been presented. This kind of analysis is important in the design of devices based on additive-manufactured MTSs, like planar lenses and leaky wave antennas. Examples of designs performed with the proposed approach will be presented at the conference.

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