Benchmarking the User-Centric Clustering and Pilot Assignment Problems in Cell-Free Networks

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***Abstract*—This paper addresses the user-centric clustering and pilot assignment problems in cell-free networks, recognizing the need to solve both problems simultaneously. The motivation of this research stems from the absence of benchmarks, general formulations, and the reliance on subjectively designed objective functions and heuristic algorithms prevalent in existing literature. To tackle these challenges, we formulate stochastic non-linear binary integer programs for both the user-centric clustering and pilot assignment problems. We specifically design the pilot assignment formulation to incorporate user-centric clusters when evaluating the desirability of pilot assignments, resulting in improved efficiency. To solve the problems, the proposed methodology employs sample average approximation coupled with surrogate optimization for the user-centric clustering problem and the genetic algorithm for the pilot assignment problem. Numerical experiments demonstrate that the optimized solutions outperform baseline solutions, leading to significant gains in spectral efficiency.**

***Index Terms*—Cell-Free Networks, Massive MIMO, User-Centric Clustering, Pilot Assignment, Stochastic Optimization**

I. Introduction

Cell-free Massive Multiple-Input-Multiple-Output (MIMO) has emerged as a promising physical layer technology for supporting future deployments in beyond 5G and 6G networks. The main concept is to go beyond the cellular paradigm by employing an ultra dense deployment of small-sized multi-antenna access points (APs) which cooperate to serve users in the coverage area, eliminating the notion of boundaries between cells. The most practical form of this paradigm is user-centric cell-free massive MIMO [1]–[5]. Instead of allowing all the APs to serve all the users in the network as envisioned in the first cell-free massive MIMO paper [6], each user is served by a subset of the APs which ensures that network operation is scalable as the number of users grows. The main objective of this paper is to provide an optimal benchmark for designing the cluster of APs that serve each user which is known as the user-centric clustering problem. On the pursuit to solve the clustering problem, there is another problem which is tightly connected to it, the pilot assignment problem. Both problems must be solved together to ensure satisfactory network-wide performance.

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The majority of research on the user-centric clustering problem [7]–[12] and the pilot assignment problem [13]–[20] has been centered on designing heuristic algorithms. However, there is a notable lack of optimal benchmarks in these studies. As a result, our focus is on formulating each problem as an optimization problem and developing optimized benchmark solutions that can be used to evaluate heuristic algorithms. By establishing these benchmarks, we can better understand the effectiveness of existing algorithms and develop new approaches that lead to significant performance gains. Moreover, the optimized solutions can be used as a reference for machine learning algorithms.

Our contributions in this paper are summarized as follows:

- Proposing a novel formulation of the user-centric clustering problem as a stochastic non-linear binary integer program. The proposed formulation has the flexibility to accommodate different per user performance metrics and network-wide utility functions;
- Proposing a novel formulation of the pilot assignment problem as a stochastic non-linear binary integer program which takes into account the user-centric clusters in evaluating the desirability of pilot assignments;
- Providing nearly optimal solutions for both problems by employing sample average approximation coupled with surrogate optimization to solve the user-centric clustering problem, and the genetic algorithm to address the pilot assignment problem.

The remainder of the paper is structured as follows: Section II introduces the correlated Rayleigh fading channel model, the channel estimation procedure and performance metrics. Section III formulates the use-centric clustering and pilot assignment problems as stochastic non-linear binary integer programs. It also highlights the connection between the two problems. Section IV presents baseline heuristic methods for both problems from the literature and highlights the use of the genetic algorithm and surrogate optimization to realize the optimized solutions. Section V discusses numerical experiments to evaluate the proposed methodology. Finally, Section VI concludes the paper.

II. Network Model and Operation

In line with [1]–[5], we consider a cell-free network with L geographically distributed APs throughout the coverage area, each equipped with N antennas, such that the total number of antennas is $M = LN$. The APs cooperate to serve K users residing within the coverage area. A massive MIMO operating regime is considered where the number of APs is considerably larger than the number of users' equipment (UEs) $L \gg K$ implying that $M \gg K$. The APs are connected to a central processing unit (CPU) using high-speed fronthaul links which manages AP cooperation. The CPU is connected using backhaul links to the core network. There are two paradigms of cell-free network operation: centralized and distributed; herein, we only focus on centralized operation where almost all signal processing is delegated to the CPU.

Each UE is served by a subset of the APs which are referred to as user-centric clusters. Following the dynamic cooperation clustering (DCC) framework [21], we define a set of diagonal matrices $\mathbf{D}_{kl}$ for $k = 1, \cdots, K$ and $l = 1, \cdots, L$ such that

$$\mathbf{D}_{kl} = \begin{cases} \mathbf{I}_N & \text{if AP } l \text{ serves UE } k \\ \mathbf{0}_{N\times N} & \text{otherwise} \end{cases} \tag{1}$$

where $\mathbf{I}_N$ is the N-by-N identity matrix and $\mathbf{0}_{N\times N}$ is the N-by-N zero matrix.

We assume that each channel use is constrained to a channel coherence block which is a time-frequency block time whose duration is equal to the coherence time τ_c and its frequency width is equal to the channel coherence bandwidth B_c. It is also assumed that channel realizations of different blocks are uncorrelated. Each coherence block is utilized for both uplink and downlink transmissions in a time-division-duplexing (TDD) protocol which makes it sufficient to transmit pilots only in the uplink [1]–[5]. Each coherence block is split: τ_p for transmission of pilots, τ_{ul} for uplink data transmission and τ_{dl} for downlink data transmission such that $\tau_p + \tau_{\text{ul}} + \tau_{\text{dl}} = \tau_c$. The channel between AP l and UE k is represented by an N-dimensional vector $\boldsymbol{h}_{kl} \in \mathbb{C}^N$ and the collective channel $\boldsymbol{h}_k \in \mathbb{C}^M$ of UE k is written as

$$\boldsymbol{h}_k = \left[\boldsymbol{h}_{k1}^T \cdots \boldsymbol{h}_{kL}^T\right]^T. \tag{2}$$

A. Channel Model

The correlated Rayleigh fading channel model is used such that the channel between AP l and UE k is generated as $\boldsymbol{h}_{kl} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_N, \mathbf{R}_{kl})$ where $\mathcal{N}_{\mathbb{C}}$ is the complex circularly-symmetric gaussian distribution. $\mathbf{R}_{kl}$ is the positive semi-definite spatial channel correlation matrix which captures large-scale fading characteristics including path loss, shadowing and spatial channel correlation. The large-scale fading coefficient β_{kl} of AP l and UE k is defined to be

$$\beta_{kl} = \frac{1}{N}\,\mathrm{tr}(\mathbf{R}_{kl}) \tag{3}$$

where tr is the trace operator.

B. Channel Estimation

In the uplink, pilot sequences are transmitted by the UEs for the purpose of channel estimation. Because pilot sequences span τ_p samples, there are only τ_p mutually orthogonal pilot sequences $\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_{\tau_p} \in \mathbb{C}^{\tau_p}$ which are designed to have unit-power $\|\boldsymbol{\phi}_i\|^2 = \tau_p$. The number of pilots is less than the number of users $\tau_p < K$ in any practical network. Hence, pilot reuse is necessary. To estimate the channel between of AP l and UE k, the received pilot signal is projected onto $\phi_{t_k}/\sqrt{\tau_p}$ which is the pilot sequence assigned to UE k with $t_k \in \{1, \cdots, \tau_p\}$ being the index of the pilot assigned to UE k. This removes interference from UEs which do not share the same pilot sequence as UE k. The projection results in the decision statistic $\boldsymbol{y}_{t_k l}^{\text{pilot}} \in \mathbb{C}^N$ which is written as

$$\boldsymbol{y}_{t_k l}^{\text{pilot}} = \underbrace{\sqrt{\eta_k \tau_p}\boldsymbol{h}_{kl}}_{\text{desired channel}} + \underbrace{\sum_{i\in\mathcal{P}_k/\{k\}} \sqrt{\eta_i \tau_p}\boldsymbol{h}_{il}}_{\text{interference}} + \underbrace{\frac{1}{\sqrt{\tau_p}}\mathbf{N}_l \boldsymbol{\phi}_{t_i}}_{\text{receiver noise}} \tag{4}$$

where $\mathcal{P}_k$ denotes the set of UEs that share the same pilot sequence as UE k including UE k, η_i is the pilot transmit power of UE i and $\mathbf{N}_l \in \mathbb{C}^{N\times\tau_p}$ is the independent additive receiver noise. The MMSE estimate of $\boldsymbol{h}_{kl}$ given $\boldsymbol{y}_{t_k l}^{\text{pilot}}$ is

$$\hat{\boldsymbol{h}}_{kl} = \sqrt{\eta_k \tau_p}\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_k l}^{-1}\boldsymbol{y}_{t_k l}^{\text{pilot}} \tag{5}$$

where $\boldsymbol{\Psi}_{t_k l}$ is the received pilot correlation matrix for pilot t_k and AP l defined as

$$\boldsymbol{\Psi}_{t_k l} = \sum_{i\in\mathcal{P}_k} \eta_i \tau_p \mathbf{R}_{il} + \sigma_{\text{ul}}^2 \mathbf{I}_N \tag{6}$$

The relation between the channel estimate $\hat{\boldsymbol{h}}_{kl}$ and the actual channel $\boldsymbol{h}_{kl}$ is $\boldsymbol{h}_{kl} = \hat{\boldsymbol{h}}_{kl} + \tilde{\boldsymbol{h}}_{kl}$ where $\tilde{\boldsymbol{h}}_{kl}$ is the channel estimation error. The channel estimate and the channel estimation error are independent random variables and their distribution is [1],

$$\hat{\boldsymbol{h}}_{kl} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_N, \eta_k\tau_p\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_k l}^{-1}\mathbf{R}_{kl}) \tag{7}$$

$$\tilde{\boldsymbol{h}}_{kl} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_N, \mathbf{C}_{kl}) \tag{8}$$

where

$$\mathbf{C}_{kl} = \mathbf{R}_{kl} - \eta_k\tau_p\mathbf{R}_{kl}\boldsymbol{\Psi}_{t_k l}^{-1}\mathbf{R}_{kl} \tag{9}$$

The channel estimate $\hat{\boldsymbol{h}}_{kl}$ is computed only if AP l serves UE k.

C. Performance metrics

The main performance metrics that are utilized as an optimization objective to design receive combining and transmit precoding vectors, power control and power allocation coefficients are the uplink and downlink spectral efficiencies of every UE k denoted as SE_k^{ul} and SE_k^{dl}; respectively. We utilize the spectral efficiency expressions derived using the *use-and-then-forget* bounding technique which are widely used in cell-free massive MIMO literature [22], [23].

An achievable uplink and downlink spectral efficiency of UE k is

$$\mathrm{SE}_k^{\mathrm{ul}} = \frac{\tau_{\mathrm{ul}}}{\tau_c} \log_2\left(1 + \mathrm{SINR}_k^{\mathrm{ul}}\right) \tag{10}$$

$$\mathrm{SE}_k^{\mathrm{dl}} = \frac{\tau_{\mathrm{dl}}}{\tau_c} \log_2\left(1 + \mathrm{SINR}_k^{\mathrm{dl}}\right) \tag{11}$$

where $\mathrm{SINR}_k^{\mathrm{ul}}$ and $\mathrm{SINR}_k^{\mathrm{dl}}$ are the uplink and downlink signal-to-interference-and-noise ratios (SINR); respectively, which are defined as follows.

$$\mathrm{SINR}_k^{\mathrm{ul}} = \frac{p_k \left|\mathbb{E}\left\{\boldsymbol{v}_k^H \mathbf{D}_k \boldsymbol{h}_k\right\}\right|^2}{\sum_{i=1}^K p_i \mathbb{E}\left\{\left|\boldsymbol{v}_k^H \mathbf{D}_k \boldsymbol{h}_i\right|^2\right\} - p_k \left|\mathbb{E}\left\{\boldsymbol{v}_k^H \mathbf{D}_k \boldsymbol{h}_k\right\}\right|^2 + \sigma_{\mathrm{ul}}^2 \mathbb{E}\left\{\|\mathbf{D}_k \boldsymbol{v}_k\|^2\right\}} \tag{12}$$

$$\mathrm{SINR}_k^{\mathrm{dl}} = \frac{\left|\mathbb{E}\left\{\boldsymbol{h}_k^H \mathbf{D}_k \boldsymbol{w}_k\right\}\right|^2}{\sum_{i=1}^K \mathbb{E}\left\{\left|\boldsymbol{h}_k^H \mathbf{D}_i \boldsymbol{w}_i\right|^2\right\} - \left|\mathbb{E}\left\{\boldsymbol{h}_k^H \mathbf{D}_k \boldsymbol{w}_k\right\}\right|^2 + \sigma_{\mathrm{dl}}^2} \tag{13}$$

where $\mathbf{D}_k = \mathrm{diag}(\mathbf{D}_{k1}, \cdots, \mathbf{D}_{kL})$ is a block diagonal matrix containing the clustering matrices of UE k, p_k is the uplink transmit power of UE k, $\boldsymbol{v}_k$ and $\boldsymbol{w}_k$ are the centralized receive combining and transmit precoding vectors of UE k, respectively. Moreover, σ_{ul}^2 and σ_{dl}^2 are the average uplink and downlink noise powers, respectively.

III. Problem Formulation

A. User-Centric Clustering

We define K binary assignment vectors $\boldsymbol{x}_k \in \{0,1\}^L$, $k = 1, \cdots, K$ to represent the user-centric clusters.

$$x_{kl} = \begin{cases} 1 & \text{AP } l \text{ serves UE } k \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Note that the clustering matrix $\mathbf{D}_k$ of every UE k has a one-to-one relationship with the clustering vector $\boldsymbol{x}_k$. The user-centric clustering problem is concerned with designing the K clusters to maximize a utility function or a performance metric. There are two viewpoints for assessing the performance of a clustering algorithm: each individual UE performance, and network-wide performance which is a collection of simultaneously achievable UE performances. Each UE k is assumed to have a performance function $g_k : \mathcal{R} \to \mathcal{R}$ of the SINR which measures the degree of satisfaction of the UE by its quality of service. The mathematical structure of the SINR differs between the uplink and downlink, leading to distinct downlink and uplink performances. Accordingly, we define the individual performance of each UE k as the set

$$\left\{g_k(\mathrm{SINR}_k^{\mathrm{ul}}), g_k(\mathrm{SINR}_k^{\mathrm{dl}})\right\}. \tag{15}$$

which includes the uplink and downlink performances given by $g_k(\mathrm{SINR}_k^{\mathrm{ul}})$ and $g_k(\mathrm{SINR}_k^{\mathrm{dl}})$, respectively.

Design of user-centric clusters is adapted on a larger time scale compared to the coherence time, primarily to match variations in large-scale fading characteristics. Denote the time interval for which the clusters are kept fixed as $T = n_c \tau_c$ where n_c is the number of coherence frames within T which we refer to as the clustering interval. Without loss of generality and inspired by the approach in [24], the user-centric clustering problem can be formulated as the following multi-objective optimization problem (MOP)

$$\max_{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_K} \left\{g_k\left(\mathrm{SINR}_k^{\mathrm{ul}}(n)\right), g_k\left(\mathrm{SINR}_k^{\mathrm{dl}}(n)\right) : k = 1, \cdots, K, \quad n = 1, \cdots, n_c\right\} \tag{16}$$

where $\mathrm{SINR}_k^{\mathrm{ul}}(n)$ and $\mathrm{SINR}_k^{\mathrm{dl}}(n)$ are the uplink and downlink SINR at time n, respectively. The MOP can be interpreted as searching for the clusters $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_K$ that maximize the performance of all UEs during the clustering interval T. Since the performances of different UEs are coupled, there is generally no single transmit strategy that simultaneously maximizes the performance of all UEs. Furthermore, the clusters are designed before the start of the clustering interval T. Hence, the performance metrics of the UEs defined by the set in (16) are unknown. In such case, we need to deal with the uncertainty in the objective functions. We address the uncertainty by optimizing the expected value of the performance functions rather than the instantaneous value. Accordingly, we reformulate the problem defined in (16) as follows:

$$\max_{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_K} \left\{\mathbb{E}\left\{g_k\left(\mathrm{SINR}_k^{\mathrm{ul}}\right)\right\}, \mathbb{E}\left\{g_k\left(\mathrm{SINR}_k^{\mathrm{dl}}\right)\right\} : k = 1, \cdots, K\right\} \tag{17}$$

where $\mathbb{E}$ is the expectation operator. The expectation is computed with respect to the time index n which we dropped for convenience in the notation.

As with any MOP, there are many operating points that we can choose from. Therefore, we need a way to assess the desirability of each operating point. The common approach is to choose an aggregate system utility function $f(\boldsymbol{g})$ which takes an operating point $\boldsymbol{g} = (g_1, \cdots, g_K)$ as an input where g_i is the value of objective i and outputs a scalar value. Candidates of the aggregate system utility function are weighted arithmetic mean $f(\boldsymbol{g}) = \sum_k w_k g_k$ and weighted geometric mean $f(\boldsymbol{g}) = \prod_k g_k^{w_k}$. The weights w_k can be used to prioritize certain objectives over others [24]. We adopt the ergodic spectral efficiency $\mathbb{E}\{g_k(\mathrm{SINR}_k)\} = \log_2(1 + \mathbb{E}\{\mathrm{SINR}_k\})$ as a performance function and the sum spectral efficiency as the aggregate system utility function which is equivalent to the weighted arithmetic mean with all the weights set to one. This results in the following single objective optimization problem

$$\max_{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_K} \sum_{k=1}^K \log_2\left(\left(1 + \mathbb{E}\left\{\mathrm{SINR}_k^{\mathrm{ul}}\right\}\right)\left(1 + \mathbb{E}\left\{\mathrm{SINR}_k^{\mathrm{dl}}\right\}\right)\right) \tag{18}$$

The problem is regarded as a binary integer non-linear program which belongs to the class of *NP*-complete problems. It is usually hard to compute closed forms of the SINR expected values. However, we can generate random realizations of the SINR which can be used to estimate their expected values as follows:

$$\mathbb{E}\left\{\mathrm{SINR}_k^{\mathrm{ul}}\right\} \approx \frac{1}{N_r} \sum_{n=1}^{N_r} \mathrm{SINR}_k^{\mathrm{ul}} \tag{19}$$

$$\mathbb{E}\left\{\text{SINR}_k^{\text{dl}}\right\} \approx \frac{1}{N_r}\sum_{n=1}^{N_r}\text{SINR}_k^{\text{dl}} \tag{20}$$

where N_r is the number of generated realizations. The technique is known as sample average approximation.

B. Pilot Assignment

The main purpose of transmitting pilots in the uplink is to utilize the received pilots in channel estimation. From the decision statistic $\boldsymbol{y}_{t_k l}^{\text{pilot}}$ given in equation (4), there are four ways to improve channel estimation performance: 1) Increase the energy of the desired channel $\eta_k \tau_p \|\boldsymbol{h}_{kl}\|^2$ by boosting the transmit power η_k; 2) Decrease the interference energy by limiting the transmit powers of pilot sharing UEs η_i for $i \in \mathcal{P}_k/\{k\}$, 3) Choose AP l to serve UE k if $\|\boldsymbol{h}_{kl}\|^2$ is large enough; and 4) Reduce the number of pilot-sharing UEs.

The first two strategies appear to be conflicting as prioritizing one user by increasing its transmit power while limiting the power of other pilot-sharing users may lead to a considerable decrease in network-wide performance. The third strategy involves the selection of user-centric clusters, which is the problem that we aim to solve in this paper. The fourth strategy pertains to pilot assignment, which is a separate problem. Nevertheless, both pilot and cluster assignment are closely intertwined, as evident from the arguments presented here.

We define a set of binary assignment vectors $\boldsymbol{a}_k \in \{0,1\}^{\tau_p}$, $k = 1, \cdots, K$ to represent the pilot allocation such that

$$a_{kj} = \begin{cases} 1 & \text{pilot } j \text{ is assigned to UE } k \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

Using the assignment vectors, we can rewrite equation (6) and (9), respectively, as follows:

$$\boldsymbol{\Psi}_{jl} = \sum_{i=1}^{K} \eta_i \tau_p \mathbf{R}_{il} a_{ij} + \sigma_{\text{ul}}^2 \mathbf{I}_N \tag{22}$$

$$\mathbf{C}_{kl} = \mathbf{R}_{kl} - \eta_k \tau_p \sum_{j=1}^{\tau_p} \mathbf{R}_{kl} \boldsymbol{\Psi}_{jl}^{-1} \mathbf{R}_{kl} a_{kj} \tag{23}$$

$$j \in \{1, \cdots, \tau_p\}$$

We adopt the expected value of the L_2 norm of the channel estimation error as the optimization criterion which is written as follows:

$$\begin{aligned}\mathbb{E}\{\tilde{\boldsymbol{h}}_{kl}^H \tilde{\boldsymbol{h}}_{kl}\} &= \text{tr}(\mathbf{C}_{kl}) \\ &= \text{tr}(\mathbf{R}_{kl}) - \text{tr}\left(\eta_k \tau_p \sum_{j=1}^{\tau_p} \mathbf{R}_{kl} \boldsymbol{\Psi}_{jl}^{-1} \mathbf{R}_{kl} a_{kj}\right)\end{aligned} \tag{24}$$

As discussed, there is a strong connection between the clustering and pilot assignment problems. Consequently, our formulation of the pilot assignment problem considers the user-centric clusters. Hence, we refer to this as the clustering-aware pilot assignment problem formulation. Assuming that the clusters $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_K$ are known, we formulate the clustering-aware pilot assignment optimization problem as follows:

$$\min_{\boldsymbol{a_1}, \cdots, \boldsymbol{a_k}} \sum_{k=1}^{K} \sum_{l=1}^{L} \mathbb{E}\{\tilde{\boldsymbol{h}}_{kl}^H \tilde{\boldsymbol{h}}_{kl}\} x_{kl} \tag{25}$$

$$\begin{aligned}&\mathbb{E}\{\tilde{\boldsymbol{h}}_{kl}^H \tilde{\boldsymbol{h}}_{kl}\} = \text{tr}(\mathbf{R}_{kl}) - \eta_k \tau_p \\ &\sum_{j=1}^{\tau_p} \text{tr}\left(\mathbf{R}_{kl}\left(\sum_{i=1}^{K} \eta_i \tau_p \mathbf{R}_{il} a_{ij} + \sigma_{\text{ul}}^2 \mathbf{I}_N\right)^{-1} \mathbf{R}_{kl}\right) a_{kj}\end{aligned} \tag{26}$$

$$\sum_{i=1}^{\tau_p} a_{ki} = 1 \tag{27}$$

$$\forall k \in \{1, \cdots, K\}$$

The objective function in (25) is the sum of channel estimation errors between each AP l and UE k, only if AP l serves UE k. If we ignore the user-centric clusters (assuming $x_{kl} = 1$ for all k and l), the objective function becomes the sum of all channel estimation errors between every AP l and UE k, regardless of whether or not AP l serves UE k. However, by taking into account the user-centric clusters, we gain more degrees of freedom and potentially improve overall performance. The constraints in equation (27) ensure that each UE is assigned only one pilot. Like the user-centric clustering problem, the pilot assignment problem is a binary integer non-linear program, which is known to be *NP*-complete. However, a closed-form expression of the expected value $\mathbb{E}\{\tilde{\boldsymbol{h}}_{kl}^H \tilde{\boldsymbol{h}}_{kl}\}$, given in equation (26), is available, eliminating the need for sampling.

IV. Methodology

A. Baseline

We utilize the joint pilot assignment and clustering algorithm proposed in [1] as baseline. The algorithm employs a greedy strategy for pilot assignment and user-centric clusters design. Initially, it assigns orthogonal pilot sequences to the first τ_p UEs. For the remaining UEs, each UE k is assigned a pilot that causes the least interference at the best AP. The best AP for UE k is the one with the highest average channel gain β_{kl} with UE k. Subsequently, each AP l serves only τ_p UEs to avoid pilot-sharing on the same AP. Each UE k is served by AP l on pilot t, if it has the highest channel gain β_{kl} among all the UEs sharing the same pilot.

B. Optimized Solution

Both the user-centric clustering and pilot assignment problems are classified as non-linear binary integer programs. There is generally no optimization algorithm that can find the global optimum in a reasonable timeframe. We use the genetic algorithm to solve the pilot assignment problem and surrogate optimization to solve the user-centric clustering problem. Both algorithms can be readily applied to this type of optimization problems. The genetic algorithm, which is a structured random search optimization method inspired by natural selection, proved to be effective in solving the pilot assignment problem. However, it had limited efficacy with the user-centric clustering problem due to the high computational cost of the objective function. Hence, surrogate optimization, which is designed to deal with time-consuming objective functions, is used for the user-centric clustering problem. Detailed explanation of the algorithms is outside the scope of the manuscript. However, the reader is encouraged to refer to [25], [26], and references therein.

V. Results and Discussion

This section examines the effectiveness of optimized solutions for the user-centric clustering and pilot assignment problems, compared with the baseline algorithm outlined in Section IV-A. Section V-A introduces the simulation setup utilized in this paper. Section V-B presents the numerical experiment conducted to evaluate the performance of the algorithms.

A. Simulation Setup

In line with [1], we assume ultra-dense deployment in an urban area where the APs are deployed in a plane ten meters above the UEs. This matches the 3GPP Urban Microcell Model defined in [27]. The path loss coefficient is computed as

$$\beta_{kl}\ [\text{dB}] = -30.5 - 36.7\log_{10}\left(\frac{d_{kl}}{1m}\right) + F_{kl} \tag{28}$$

where d_{kl} is the three dimensional distance between AP l and UE k. $F_{kl} \sim \mathcal{N}(0, 4^2)$ represents the shadow fading. The shadowing terms are correlated as

$$\mathbb{E}\{F_{kl}F_{ij}\} = \begin{cases} 4^2 2^{-\delta_{ki}/9\ m} & l = j \\ 0 & k \neq i \end{cases} \tag{29}$$

where δ_{ki} is the distance between UE k and UE i.

To evaluate the performance of the optimized solutions, we define a simulation setup which will be maintained throughout the paper. The total coverage area is 0.5 km $\times$ 0.5 km, the number of APs is $L = 30$, each equipped with $N = 4$ antennas, and the number of users is $K = 12$. Each coherence block extends for $\tau_c = 200$ samples and the length of pilot sequences is $\tau_p = 5$. Both APs and UEs are uniformly distributed throughout the coverage area, and wrap-around topology is used to avoid cell-edge effects. The full parameters of the simulation setup are summarized in Table I.

For power control, we consider that the UEs transmit with full power in both pilot and data transmission phases which has been shown to be nearly optimal in many scenarios [1], i.e.,

$$\eta_k = p_k = p_{\max}, \quad k = 1, \cdots, K \tag{30}$$

where $p_{\max}$ is the maximum uplink transmit power. Furthermore, we use the following heuristic to determine the power allocated to each UE during the downlink which is commonly known as fractional centralized power allocation [1], i.e.,

$$\rho_k = \rho_{\max}\frac{\left(\sqrt{\sum_{l\in\mathcal{M}_k}\beta_{kl}}\right)^{-1}\left(\sqrt{\omega_k}\right)^{-1}}{\max_{l\in\mathcal{M}_k}\sum_{i\in\mathcal{D}_l}\left(\sqrt{\sum_{l\in\mathcal{M}_i}\beta_{il}}\right)^{-1}\left(\sqrt{\omega_i}\right)} \tag{31}$$

TABLE I
Parameters of the simulation setup.

Parameter	Value
Network Area	0.5 km $\times$ 0.5 km
AP distribution	Uniformly distributed
Users distribution	Uniformly distributed
Number of APs	30
Number of users	12
Number of antennas per AP	4
Samples per coherence block	200
Samples per pilot	5
Bandwidth	20 MHz
Receiver noise power	-94 dBm
Maximum uplink transmit power	100 mW
Maximum downllink transmit power	200 mW

with $\omega_k = \max_{l\in\mathcal{M}_k}\mathbb{E}\left\{\|\bar{\boldsymbol{w}}_{kl}\|^2\right\}$ where $\bar{\boldsymbol{w}}_{kl}$ is the direction of the portion of the centralized precoding vector $\boldsymbol{w}_k$ assigned to AP l, $\rho_{\max}$ is the maximum downlink transmit power, ρ_k is the downlink transmit power assigned to UE k, $\mathcal{D}_l$ is the set of UEs served by AP l, and $\mathcal{M}_k$ is the set of APs that serve UE k.

B. Numerical Experiments

We conduct an experiment to assess the performance of optimized solutions relative to the baseline, outlined in Section IV-A. The performance is evaluated for the Minimum Mean-Square-Error (MMSE), Partial MMSE (PMMSE), Partial-Regularized-Zero-Forcing (PRZF) and Maximum-Ratio (MR) combining/precoding methods. Figure 1 displays the cumulative distribution function (CDF) of the sum spectral efficiency achieved by UE k for different combining/precoding schemes. The optimized solutions exhibit superior performance compared to the baseline solutions for all combining and precoding schemes.

To further illustrate these observations, we present the 95% likely sum spectral efficiency which is computed by drawing a

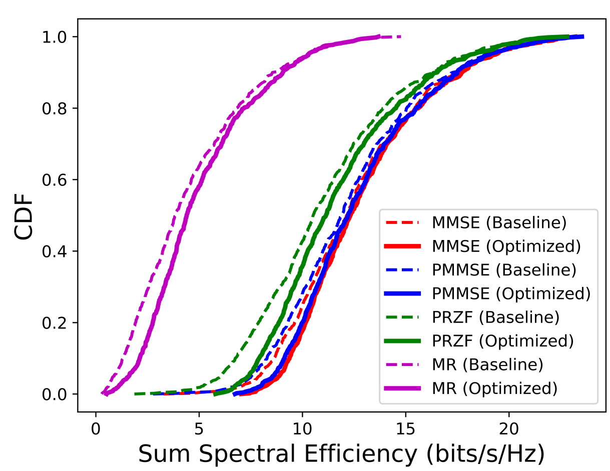

Fig. 1. CDF of the sum spectral efficiency of UE k.

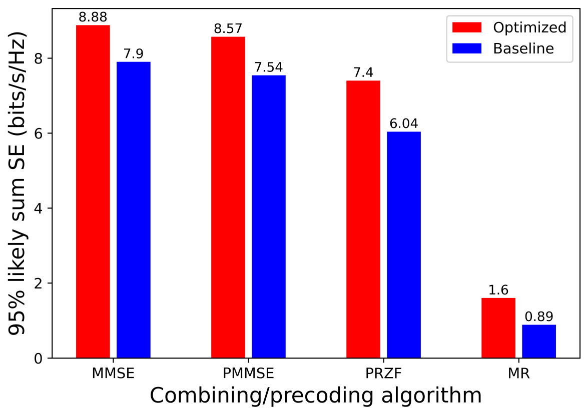

Fig. 2. Summary of the 95% likely Sum SEs achieved in the experiment by the optimized and baseline methods.

horizontal line at CDF $= 0.05$ on Figure 1 and computing the sum SE from the intersection points with each curve. The results are shown in Figure 2. The optimized solutions consistently show superior performance. It is worth noting that even small enhancements in spectral efficiency can translate to significant gains in the information rate. For instance, a 0.1 increase in spectral efficiency corresponds to a 0.1×20 Mbps increase in the information rate.

VI. Conclusion

In this work, we studied the user-centric clustering and pilot assignment problems in cell-free networks, recognizing the necessity of solving both problems together. The main motivation for approaching these problems is the lack of benchmarks and general formulations, as well as the subjectively designed objective functions and heuristic algorithms used in most literature. Stochastic non-linear binary integer programs were formulated for both the user-centric clustering and the pilot assignment problems. The pilot assignment formulation was developed to consider user-centric clusters when evaluating the desirability of the pilot assignment, making it more efficient. There is no known algorithm that guarantees optimal solutions. However, surrogate optimization and the genetic algorithm were used to solve the user-centric clustering and the pilot assignment problems; respectively. Numerical experiments showed that the optimized solutions outperformed baseline solutions, resulting in significant spectral efficiency gains.

References

[1] O. T. Demir, E. Bjornson, and L. Sanguinetti, *Foundations of user-centric cell-free massive MIMO*. Now Publishers, 2021.

[2] G. Interdonato, E. Björnson, H. Q. Ngo, P. Frenger, and E. G. Larsson, "Ubiquitous cell-free massive MIMO communications," *EURASIP Journal on Wireless Communications and Networking*, vol. 2019, no. 1, 2019.

[3] H. Q. Ngo *et al.*, "Cell-free massive MIMO: Uniformly great service for everyone," in *2015 IEEE 16th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2015.

[4] E. Björnson and L. Sanguinetti, "Making cell-free massive MIMO competitive with MMSE processing and centralized implementation," *IEEE Transactions on Wireless Communications*, vol. 19, no. 1, pp. 77–90, 2020.

[5] E. Bjornson and L. Sanguinetti, "Scalable cell-free massive MIMO systems," *IEEE Transactions on Communications*, vol. 68, no. 7, pp. 4247–4261, 2020.

[6] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO versus small cells," *IEEE Transactions on Wireless Communications*, vol. 16, no. 3, pp. 1834–1850, 2017.

[7] C. F. Mendoza, S. Schwarz, and M. Rupp, "Cluster formation in scalable cell-free massive MIMO networks," in *International Conference on Wireless and Mobile Computing, Networking and Communications*, 2020.

[8] S. Biswas and P. Vijayakumar, "Ap selection in cell-free massive MIMO system using machine learning algorithm," in *International Conference on Wireless Communications, Signal Processing and Networking*, 2021.

[9] R. Wang, M. Shen, Y. He, and X. Liu, "Performance of cell-free massive MIMO with joint user clustering and access point selection," *IEEE Access*, vol. 9, pp. 40 860–40 870, 2021.

[10] Y. Al-Eryani, M. Akrout, and E. Hossain, "Multiple access in cell-free networks: Outage performance, dynamic clustering, and deep reinforcement learning-based design," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 4, pp. 1028–1042, 2021.

[11] C. D'Andrea and E. G. Larsson, "User association in scalable cell-free massive MIMO systems," *arXiv*, vol. abs/2103.05321, 2021.

[12] C. Wei, K. Xu, X. Xia, Q. Su, M. Shen, W. Xie, and C. Li, "User-centric access point selection in cell-free massive MIMO systems: A game-theoretic approach," *IEEE Communications Letters*, vol. 26, no. 9, pp. 2225–2229, 2022.

[13] X.-T. Dang *et al.*, "A genetic algorithm based pilot assignment strategy for cell-free massive MIMO system," in *2020 IEEE International Conference on Communications and Electronics (ICCE)*, 2021, pp. 93–98.

[14] Y. Hao, J. Xin, W. Tao, S. Tao, L. Yu-xiang, and W. Hao, "Pilot allocation algorithm based on k-means clustering in cell-free massive MIMO systems," in *2020 IEEE 6th International Conference on Computer and Communications (ICCC)*, 2020, pp. 608–611.

[15] R. Sabbagh, C. Pan, and J. Wang, "Pilot allocation and sum-rate analysis in cell-free massive MIMO systems," in *2018 IEEE International Conference on Communications (ICC)*, 2018.

[16] M. Attarifar, A. Abbasfar, and A. Lozano, "Random vs structured pilot assignment in cell-free massive MIMO wireless networks," in *2018 IEEE International Conference on Communications Workshops*, 2018.

[17] Y. Zhang, H. Cao, P. Zhong, C. Qi, and L. Yang, "Location-based greedy pilot assignment for cell-free massive MIMO systems," in *IEEE International Conference on Computer and Communications*, 2018.

[18] S. Buzzi, C. D'Andrea, M. Fresia, Y.-P. Zhang, and S. Feng, "Pilot assignment in cell-free massive MIMO based on the hungarian algorithm," *IEEE Wireless Communications Letters*, vol. 10, no. 1, pp. 34–37, 2021.

[19] X. Zhu, Z. Wang, L. Dai, and C. Qian, "Smart pilot assignment for massive MIMO," *IEEE Communications Letters*, vol. 19, no. 9, pp. 1644–1647, 2015.

[20] H. Liu, J. Zhang, X. Zhang, A. Kurniawan, T. Juhana, and B. Ai, "Tabu-search-based pilot assignment for cell-free massive MIMO systems," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 2, pp. 2286–2290, 2020.

[21] E. Bjornson, N. Jalden, M. Bengtsson, and B. Ottersten, "Optimality properties, distributed strategies, and measurement-based evaluation of coordinated multicell OFDMA transmission," *IEEE Transactions on Signal Processing*, vol. 59, no. 12, pp. 6086–6101, 2011.

[22] M. Bashar, K. Cumanan, A. G. Burr, M. Debbah, and H. Q. Ngo, "On the uplink max–min sinr of cell-free massive mimo systems," *IEEE Transactions on Wireless Communications*, vol. 18, no. 4, pp. 2021–2036, 2019.

[23] E. Nayebi, A. Ashikhmin, T. L. Marzetta, and B. D. Rao, "Performance of cell-free massive mimo systems with mmse and lsfd receivers," in *2016 50th Asilomar Conference on Signals, Systems and Computers*, 2016.

[24] E. Björnson and E. Jorswieck, "Optimal resource allocation in coordinated multi-cell systems," *Foundations and Trends® in Communications and Information Theory*, vol. 9, no. 2–3, pp. 113–381, 2013.

[25] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*, 1st ed. Addison-Wesley Longman Publishing Co., Inc., 1989.

[26] H.-M. Gutmann, "A radial basis function method for global optimization," *Journal of Global Optimization*, vol. 19, no. 3, p. 201–227, 2001.

[27] 3GPP, "Further advancements for E-UTRA physical layer aspects (release 9)," 3GPP TS 36.814.