Memristor based Online Learning Neuromorphic Processor for Adaptive Modulation Spectrum Sensing in Communication Jammed Environments

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***Abstract*— A memristor based neuromorphic processor for on-chip training is presented. Additionally, a novel approach utilizing in-situ learning to improve wireless signal modulation classification under adversarial jamming is described. The neuromorphic system is over 50× energy efficient than optimized digital systems at this wireless signal modulation task for similar accuracy levels.**

Keywords—memristor, Neuromorphic, analog computing, adversarial jamming, AMC, wireless modulation spectrum, online learning, in-situ learning.

I. INTRODUCTION

Wireless communication uses various types of modulation techniques, such as Phase Shift Keying (PSK), Frequency Shift Keying (FSK), Quadrature Amplitude Modulation (QAM). Modulation techniques help to compress the signal, so that the message can be sent with limited band widths. Automatic Modulation Classification (AMC) is a technique to detect the wireless signals and identify the modulation class. AMC has a wide range of applications in intelligent and Cognitive Radio (CR) [2]. CR networks detect the received signal without knowing the type of modulation technique used. AMC classifies the modulation types from complex-valued raw radio signals without knowing the signal or the channel parameters.

The state-of-the-art approach of AMC technique is utilizing artificial intelligence, in particular Deep Neural Networks (DNN) for better performance in dynamic spectrum access [3]. However, the DNN training is performed in Graphics Processing Units (GPU) based high power computing facilities, such as in cloud computing or data centers. These are expensive computing facilities and these GPU based computing facilities are not feasible for low powered edge devices.

The traditional pretrained DNN models for AMC are severely affected by various noise interference, and thus their performance deteriorates after deploying. This is particularly problematic in adversarial jamming situations, where a strong interference signal is used to prevent communications. To address this issue, Zhang et al [4] recently proposed an online learning approach to help improve AMC in a strong interference or jamming situation. In this approach, the transmitter periodically sends out a known "dictionary" of the different modulations it uses, and the receiver uses this dictionary to train a neural network that is customized for AMC in the current high interference environment. The receiver knows when the dictionary will be sent out and is thus able to assign class labels to the signals received in order to carry out the training.

Deep network learning is computationally expensive and thus is typically done on high performance computing systems. In a high interference environment, however, communications to the cloud would be disrupted and thus the receiver would have to do the training directly on its own hardware. This would require highly energy efficient on-chip training DNN chips. No commercial edge processor offers online learning for deep learning applications so far.

This work proposed an extremely low power AMC system with online learning capabilities in a wireless adversarial jamming environment. The system utilizes the emerging memristor devices for developing the Computing in-Memory (CIM) neuromorphic processor for on-chip training. The memristor crossbar circuit is capable of computing multiplication and addition simultaneously in highly parallel fashion to perform the dot-product of artificial neural networks. This work used a transposable two-column memristive kernel circuit for implementing Convolutional Neural Networks (CNNs). Analog to digital converters (ADCs), digital to analog converters (DACs), and on-chip memory units are also needed to perform on-chip training in the analog domain. We developed a custom python-numpy platform to determine the training accuracy in such memristor systems. We compared the performance of the memristor system with a highly energy

efficient digital computing system that would be computing in 16 bit floating point (FP16) precision. We assumed a 40 nm process technology was used for both memristor and digital systems.

Our experimental setup used two physical transmitters and a physical receiver for the online learning experiment. We generated our own dataset and used it for training our networks. One transmitter sends a sequence of 12 modulation classes while the second transmitter sends an adversarial jamming signal to block the original signal. Both transmitters were at the same distance from the receiver and the jamming signal was much stronger than the main transmission (0dB vs -30dB). Both transmitters used the same center frequency. The receiver used two LeNet-5 networks [5] to identify the modulation class. LeNet-5 was used to reduce the training simulation time in memristor based systems. One of the networks was preprogrammed, while the other was trained online during at real-time in the adversarial jamming environment.

We found that the AMC system using the pretrained network did poorly at AMC during jamming, with its accuracy dropping from 97% down to down to 15% and 12% for digital and memristor based systems at 0 dB interference. On the other hand, with online learning, the AMC accuracy dropped down to 71% and 69% for digital and memristor systems respectively at 0 dB interference. At -12 dB interference, the online learning AMCs have shown 85% and 81.5% accuracy.

Our memristor based online learning chip design was much more efficient than the digital FP16 chip design. We show that to achieve the same level of accuracy, the memristor system needed 40 times less energy than the FP16 system. Additionally, the memristor system had a 20 times higher throughput compared to the FP16 system, leading to faster training speeds. This makes the memristor system significantly more efficient than the digital system.

As far as we know, this is the first study of using memristor based analog neuromorphic computing paradigm for implementation of DNN based AMC systems for wireless modulation signal detection in an adversarial jamming environment. The rest of the article is organized as follows: section II presents the related works. Section III described the wireless modulation dataset, section IV presents memristor based analog computing systems. Section V presents the experimental setup of online learning. Section VI describes the results of this study, and section VII presents the processor performance and energy efficiency, and section VIII presents a brief conclusion.

II. Related Works

Traditional approaches for AMC include likelihood and feature based techniques [6]. The drawback of these is that they need precise understanding of received signals and are computationally expensive. Recently, deep learning based approaches for AMC have become popular due to their better performance.

F. Meng et al. is one of the pioneers of using DNN models for implementing AMC. They achieved about 63% accuracy with 24 modulation classes [7]. T. Huynh-The et al. and S. -H. Kim et al. used asymmetric kernels and customized convolutional blocks to achieve 93.59% and 94.94% accuracy respectively [8]. Almost all of the recent works on AMC using DNNs are based on pretrained models and their performance declines in real-world applications due to noise interference. To address this, S. Zhang et al. proposed an AMC model with online learning capabilities to maintain a good automatic spectrum detection accuracy [4]. Their experiments used a complex ResNet network model for the experiment. This model is difficult to train at the edge at ultra low power because of the huge amount of intermediate data produced during training, and the large amount of computations needed for this data.

The works described above are implemented in DNN frameworks and trained in GPU based high performance computing systems. This work proposed a memristor based non-von Neumann hardware implementation of a deep learning based AMC system with online learning capability. The memristor based analog CIM neuromorphic system is presented in our prior work [9]. This work is the first implementation of memristor based AMC systems. Our proposed work implemented online training of an intelligent AMC system for edge platforms.

III. Wireless Spectrum Data

The dataset for this study was obtained using a three PlutoSDR radio setup as shown in Fig. 1. Further details on this are available in [4]. Two radios were set up as transmitters (Tx-1, and Tx-2) and one as a receiver (Rx). In this experiment, both transmitters are situated at the same distance from the receiver. In this setup, transmitter Tx-1 is the primary source that receiver Rx is trying to communicate with. Transmitter Tx-2 is an interference source to jam the communications between Tx-1 and Rx. Radio Tx-2 transmitted a 128-QAM interference signal with a gain that was varied from -28 to 0 dB and the target transmitter (Tx-1) sent signals with a constant gain of -30dB. The 12 modulation classes shown in Table I were used in this study by transmitter Tx-1. The radios transmit signals with a 902 MHz center frequency.

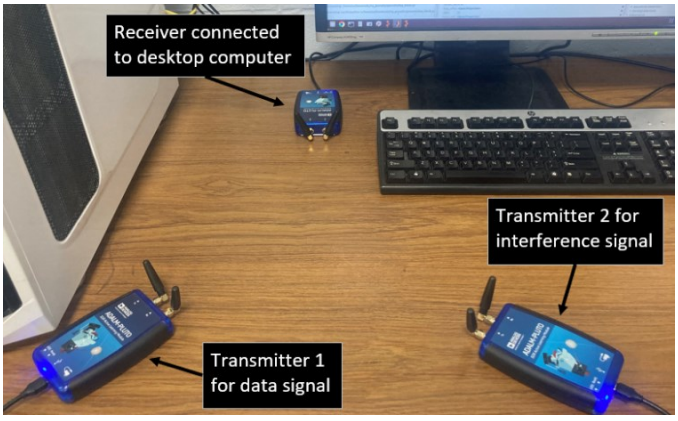

Fig. 1: Experimental setup consisting of three Analog Devices PlutoSDRs, with transmitter, and source, and receiver unit.

To allow adaptability to the radio environment, transmitter Tx-1 periodically sends out a dictionary of training data at predetermined times. This dictionary consists of the signals of different modulations being transmitted in a predetermined sequence for a predetermined amount of time, as shown in Fig.

2. As the receiver knows what modulation classes each of the dictionary samples received is, it can assign class labels and train on the received dictionary signals.

Once the dictionary is received by the receiver, the in-phase and quadrature components of the signal are split into 2×1024 arrays analogous to RADIOML 2018.01A dataset [10]. The dataset has a total of 30,000 samples with 2500 in each modulation spectrum class. Three sets of transmissions are collected, a clean signal transmission (with only Tx-1 transmitting), a dictionary transmission under interference (with both Tx-1 and Tx-2 transmitting), and a jammed signal transmission (with both transmitters transmitting).

Fig. 3 an organizational view of our setup. Receiver Rx uses two separate DNN networks: one pretrained (Net-1) and one online trained (Net-2). Net-1 is a pretrained model, which is trained offline, perhaps using a large dataset on powerful computing systems. It is capable of detecting a large variety of modulation distortions. Net-2 is programmed through online learning using the dictionary transmitted by Tx-1. As a result, Net-2 is able to adapt to a specific network environment and thus have better accuracy than Net-1 for unknown or unusual jamming situations.

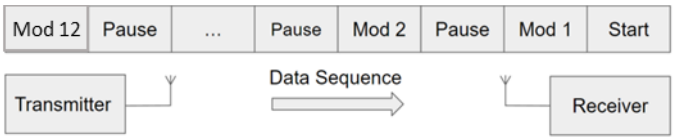

Fig.2 Data sequence for neural network training.

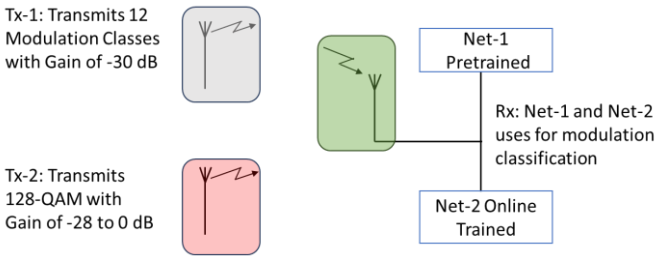

Fig. 3: Online training in wireless jamming environment.

TABLE I. WIRELESS MODULATION SPECTRUM CLASSES

Class Number	Modulation Category
1	BPSK
2	QPSK
3	8PSK
4	16QAM
5	64QAM
6	GFSK
7	CPFSK
8	PAM4
9	B-FM
10	DSB-AM-WC
11	DSB-AM-SC
12	SSB-AM

IV. MEMRITOR BASED ANALOG COMPUTING SYSTEMS

Memristors are resistive memory devices, whose resistive state can be programmed and they then retain this resistance level. State-of-the-art memristor devices can be programmed up to thousands of states for deep learning applications [11]. Memristors are generally used in crossbar circuits to perform neural network computational primitives of Matrix-Vector-Multiplication (MVM) in the analog domain. A simple crossbar kernel or neuron circuit is presented in Fig.4. This is a two column kernel representation to accommodate positive and negative synaptic weights. The algebraic difference of conductance of two adjacent devices represents a synaptic weight. Op-amp circuits are used as summing amplifiers for carrying out the resultant dot-product of the neural system and give a corresponding voltage output. The crossbar circuit outputs can be represented as Eq. (1). Fig. 5 is composed of M kernels connected to form a neural network layer.

A DNN model can be mapped onto multiple inter-connected crossbar circuits. A transposable crossbar circuit is implemented in Fig. 5 for performing CNN training operations. The crossbar maps the M kernels with the dimension of k. The input feature has C channels which gives a (k×k×C) by M crossbar size for computing a convolution layer in one shot. The crossbar circuit is orchestrated with digital to analog converters (DAC), analog to digital converters (ADC), weight update circuits, buffer storage for storing activations, and analog sensing circuits to read column outputs. The details of all these circuit elements are outside the scope of this study. Instead, we focus on the CNN training process on memristor systems for online learning and its application to adaptive wireless spectrum detection.

$$y_j = f\left(\sum_{i=1}^{N+1} x_i.(\sigma_i^+ - \sigma_i^-)\right)$$

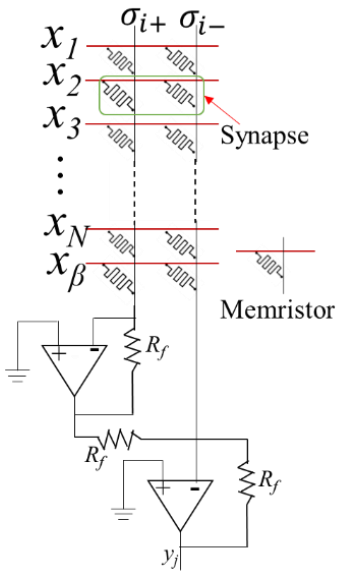

Fig. 4: Memristor based analog computing kernel circuit.

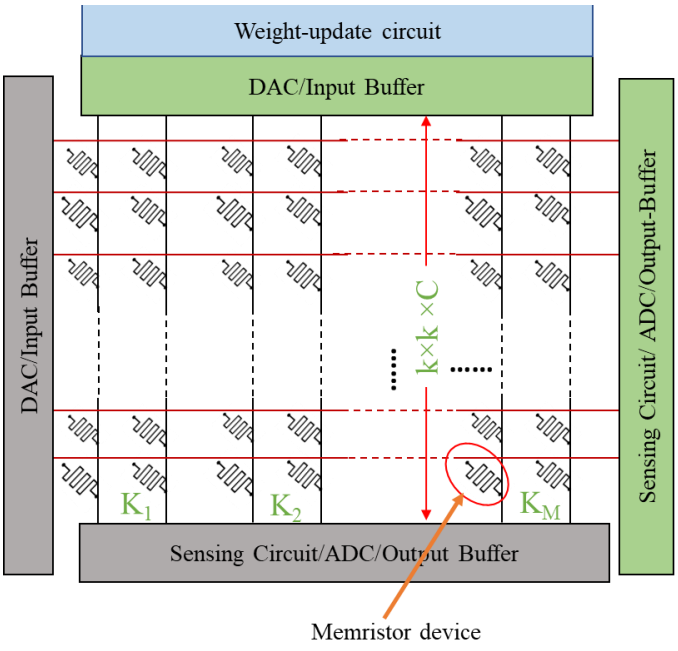

Fig. 5: The crossbar circuit presenting a CNN layer with M number of Kernels and input channel width C.

Fig.6 presents the layout of a memristor based analog in-memory computing processor. The design of the processor level architecture is inspired by H. Jiang et al. [12]. The high-level architecture consists of an array of interconnected tiles and each tile consists of multiple process engines. A process engine may contain one or multiple crossbar array(s). The processor has separate activation, pooling, and weight update units. The accumulator unit is an array of op-amp circuits that work as summing amplifiers. The I/O buffer and global buffer are used for storing intermediate parameters during the on-chip training processes. No off-chip memory is used in the current architecture to remove the off-chip access energy and data transmission latency. The weight update module is activated during the weight update phase of a DNN layer.

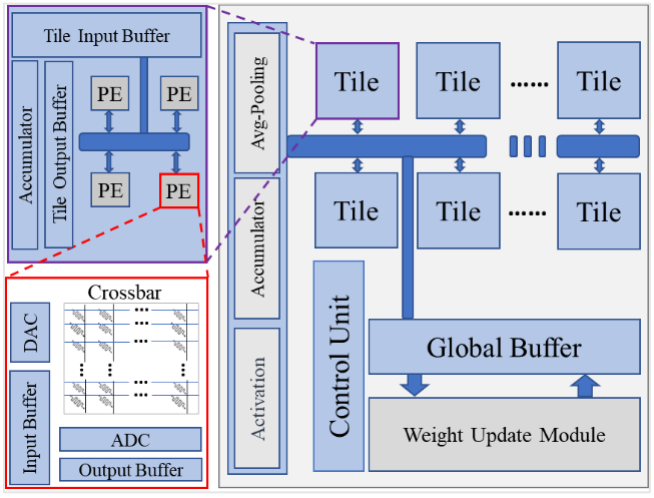

Fig.6: Layout of memristor based on-chip training system.

V. Experimental Setup For Online Learning

In this study, we assumed a memristor and digital system for evaluation. All the hardware parameters were estimated based on a 40 nm process technology. The memristor system was specified in detail (see below) while the digital system is described at the end of this section.

In this study, we used the LeNet-5 network with three convolutional layers, two average pooling layers, and two fully connected layers to process the wireless modulation data. The feature space of the wireless modulation spectrum is an asymmetric vector array, which is 2×1024. Thus, asymmetric kernels are implemented in the crossbar array. The layer-wise kernel dimensions are (1×3)×1×32, (1×3)×32×64, and (2×3)×64×64 and the average-pooling window is 1×2. There are two fully connected layers, one with 64 and the other with 12 output neurons. The full CNN architecture implementing LeNet-5 is presented in Fig. 7.

The crossbar sizes for three convolution layers are 3×(32×2), 92×(64×2), and 384×(64×2). The output feature size after the third convolution layer is 252, which makes the weight matrix size 16128×(64×2) for FC-1 and the array size for FC-2 is 64×(12×2) as the classification layer. The ADC and DAC bit-widths are set to 10-bits for both forward and backward propagation. The high resistive state and low resistive state of the memristor devices are considered 2 MΩ and 5KΩ. The experiment computes the activation and error gradient in the digital domain. The training system assumes there are *M* ADCs for the faster training process.

For both the pretrained (Net-1) and the online trained (Net-2) networks in Fig 3, we calculated the accuracy for modulation classification. For the digital system, we used TensorFlow to calculate the accuracies. For the memristor systems, we calculated the accuracies using a custom software we developed.

The use of ADCs in the memristor circuits leads to a quantization of the op-amp outputs and reduces the training accuracy of the memristor system compared to a digital training system. It is essential to capture the effect of this to ensure accurate modeling of training in memristor circuits. Thus we developed a python-numpy based deep learning training software that modeled the training of the LeNet-5 network in our memristor crossbar based training circuits. This software is flexible enough to model other types of networks and ADC/DAC bit widths.

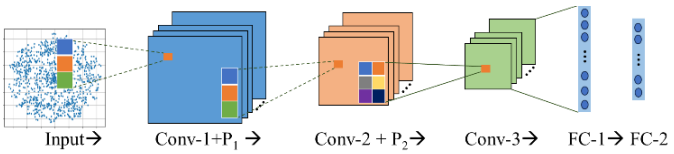

Fig. 7: LeNet-5 architecture with 3 convolutional layers, 2 fully connected layers and 2 average pooling layers.

For the digital system, we considered only the energy consumption of key memory and compute elements. We assumed that both the memristor and digital systems were able to compute the same number of operations per cycle, due to the digital system having an array of compute units. The array sizes for the digital system were the same as the memristor crossbar

sizes specified below. The digital compute arrays perform MVM operations in FP16 systems but all the parameters are stored using an off-chip memory. The memory and digital arrays are connected with the data bus system. A controller controls all data movement and MVM operations. We have considered SRAM memory utilized for storing training parameters in both memristor and FP16 systems. The energy consumption of the control system is not considered, but the area is.

For the digital system, but took only the memory or computation energies for that system. We ignored all other energies, including control. Thus our digital system energy would be the equivalent of a highly optimized digital system. All digital computations were assumed to be done in FP16 for energy and timing considerations. However the accuracies for the digital system were calculated using TensorFlow, which is using FP64.

The computing speed of the FP16 system was estimated using parameters from L. Li et al. [13]. The memory energy consumption was estimated with a 40 nm ultra-low leakage memory SRAM memory design by J. Wang et al. [14], and the memory area was estimated with Hewlett Packard's CACTI-P memory estimation software [15]. The ADCs are often one of the most energy hungry pieces of hardware in analog processors. The ADC energy consumption and area was scaled and estimated based on the experimental results from Y. -H. Tsai et al. [16].

VI. Results and Discussion

A. *Offline Pretraining and Performance*

The offline training is performed both in memristor and digital computing systems in Net-1 (see Fig. 3). The digital computing utilized traditional deep learning frameworks. The memristor systems performed analog MVM computing with 10-bit ADCs and DACs. The minibatch training methodology is utilized with a minibatch size of 64. The received dataset had 30,000 samples, and was split 90% and 10% for training and testing respectively. The training results are presented in Fig. 8.

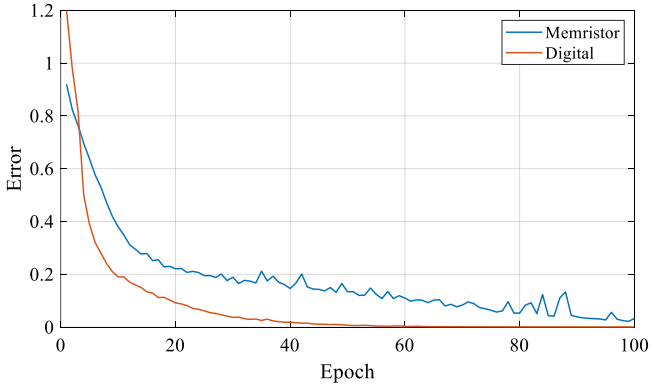

Fig. 8: Offline pretraining error convergence of memristor and digital AMC systems.

The memristor based training converged a little slower than traditional digital computing systems. After each epoch the testing dataset was used for model validation. Fig. 9 presents the accuracy vs. epoch curve of a pretrained model. The pretrained model achieved 98.4% and 97.2% accuracy respectively for digital and memristor based analog processors. This accuracy is recorded after 100 epochs of training for both systems.

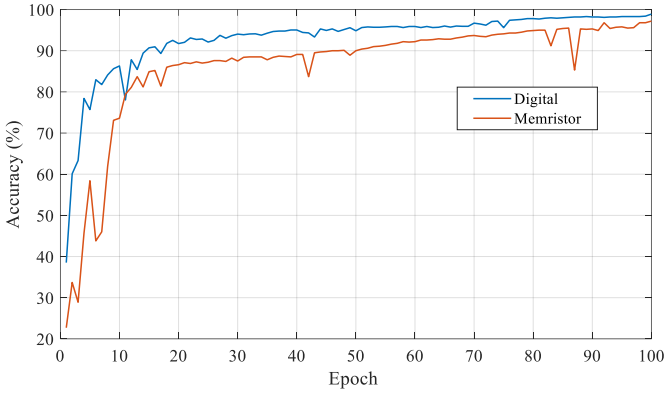

Fig. 9: Accuracy vs. epoch of the pretrained AMC models.

B. *Online Training*

The online learning model for AMC is presented in Fig. 1. Tx-1 transmits the dictionary sequence of predetermined wireless modulation signals with a constant -30dB gain. Tx-2 transmits a jamming signal that interferes with Tx-1's transmissions and makes it challenging for Rx to classify the modulations used by Tx-1. This is because the pretrained network on Rx (Net-1) may not have had the current configuration of interference in its training dataset.

The online learning technique equips the receiver radio with knowledge about the current environment and thus allows it to adapt to the interference signals. The receiver can fine tune the AMC network for better performance using the dictionary sent by the transmitter. Although the online learning helps to improve the performance in jamming environments compared to the pretrained AMC model, the wireless detection accuracy does decline with higher interference (ie Tx-2 gain). as the jamming signal becomes stronger and learning on this signal becomes more challenging. Implementing more complex DNN networks could help to maintain better accuracy, but it will be an expensive computation on the edge devices.

Fig.10 presents the accuracy for varying Tx-2 gain. Tx-2 transmits a 128 QAM modulation signal from -28 dB to 0 dB. At the beginning, Tx-2 was in OFF mode, meaning there was no jamming signal, and thus the AMC detection accuracy was similar for the pretrained and online learning models. Once the adversarial jamming signal was applied, the accuracy of the pretrained AMC model dropped down to 15% and 12% for digital and memristor based systems at 0 dB. On the other hand, with online learning, the AMC accuracy dropped down to 71% and 69% for digital and memristor systems respectively at 0 dB. At -12 dB gain on Tx-2, the online learning AMCs have shown 85% and 81.5% accuracy. The accuracy of the online learning network could be improved by using deeper networks, but these require much more powerful computing facilities. The higher energy costs related to the deeper networks would likely make them too expensive for edge applications.

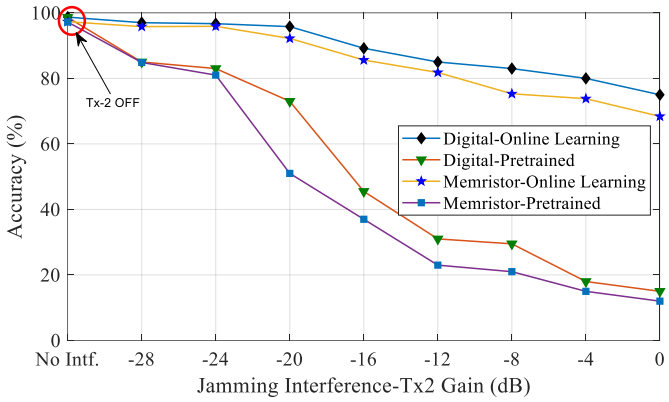

Fig. 10: Online learning and AMC wireless detection accuracy vs. strength of jamming signal.

VII. System Performance And Energy Analysis

Energy consumption and performance analysis is crucial for measuring the robustness of any hardware. We estimated the energy, power, and processor performance using detailed system evaluations. In the analog processor design, the data conversion modules, and memory modules were the most expensive pieces of hardware. The system needs 357 KB of on-chip memory for training a modulation signal. This is needed to store intermediate training parameters generated in the forward pass and to be consumed during the backward propagation. To compute the energy consumption of the memory module, both the static and dynamic energy consumptions are considered for more realistic energy estimation. We found that memory modules consume and data conversion modules take about 48% and 31% of total energy respectively in the memristor system.

Table II shows the high-level hardware parameters of the FP16 and memristor based analog in-memory and neuromorphic computing processors with 10-bit ADC/DAC. We see that memristor in-memory computing systems perform about 20 times more operations/sec for wireless modulation spectrum training operations while consuming 50.6 times less energy than the FP16 system in one training epoch.

TABLE II. Performance and Cost of Memristor and FP-16 Training Systems

Parameters	Memristor	FP-16
Energy/Epoch (J)	0.098	4.96
Time/sample (us)	26.1	2069
Power (W)	0.0023	0.075
Performance (GOPS)	258	13.2
Power Efficiency (TOPS/W)	5.67	0.82
Area (mm^2)	5.49	6.76
Performance/unit area (GOPS/mm^2)	25.4	1.66

The FP16 system needs more memory for storing intermediate training parameters and weight gradients. In this system, the memory occupies about 80% of the total chip area. The ALU arrays, control units, data bus, and other digital circuits occupy the remaining 20% of the FP16 chip area. As the memristor system stores the weight matrix within the memristor crossbars, this chip needs less memory than the FP16 chip. Note that the memristor systems still needs I/O buffer memory, which is about 47% of the overall chip area.

Fig. 11 presents the energy consumption vs. accuracy loss of a memristor based neuromorphic computing system and an FP16 processor when training on the same network and dataset. The data shows that to reach an accuracy loss of 2.8% in the pretrained model, the FP16 chip consumes 40 times more energy than a memristor based analog processor

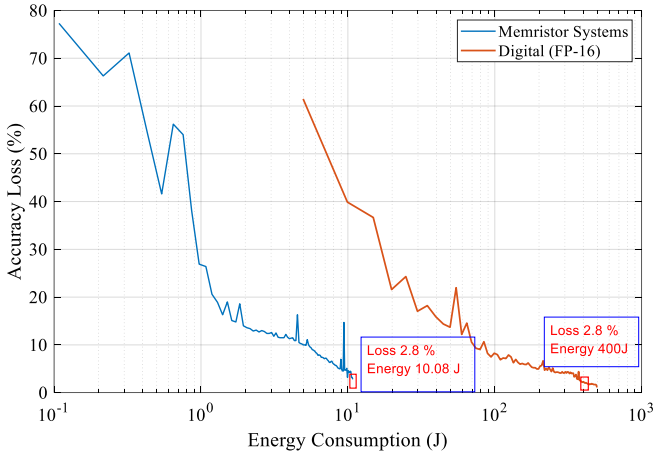

Fig. 11: Energy consumption vs. accuracy loss during training operation. Energy consumption for memristor and FP-16 systems 10.08 J and 400 J to reach 2.8 % accuracy loss.

VIII. Conclusion

In this article we have developed a low power memristor based analog computing processor with online learning capability. We have applied it to wireless modulation signal classification under adversarial jamming environments. Our results show with the main transmission at -30dB and the jamming at 0dB (both at the same distance from the receiver and using the same center frequency), our online learning algorithm was able to achieve a modulation classification accuracy of 69%, while a pretrained system had an accuracy of only 12%. We also compared our memristor based online learning chip with a digital FP16 chip, and show that to achieve the same level of accuracy, the memristor system needed 40 times less energy than the FP16 system. Finally, the memristor system had a 20 times higher throughput compared to the FP16 system, leading to faster training speeds. This makes the memristor system significantly more efficient than the digital system.

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