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Profit-based unit commitment models with price-responsive decision-dependent uncertainty

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ABSTRACT

Highlighting the increasing importance of demand elasticity in electricity markets and its impact on the revenues of power generating companies, this paper proposes new profit-based unit commitment models that effectively capture the uncertainty in the willingness-to-pay the price set for the elastic demand. To develop a new revenue scheme for power generating companies, we use a coupling function to model the willingness-to-pay response of the elastic demand as a decision-dependent source of uncertainty. The coupling function reflects how power generating companies' pricing decisions may influence the market appeal (i.e., the buyer's willingness-to-pay) and how it affects their revenues. The optimization models are stochastic mixed-integer nonlinear problems with nonconvex continuous relaxations and are not amenable to a numerical solution in their original forms. We devise a convexification reformulation method and derive valid inequalities to strengthen the formulation. We propose a learning framework to parameterize the willingness-to-pay functions and the concept of the value of the decision-dependent solution to quantify the value of the uncertainty modeling approach. Numerical tests on power systems of various sizes, demand portfolios, and price elasticity levels show (i) how the valid inequalities speed up the solution process, (ii) the benefits of properly modeling decision-dependent uncertainty and demand elasticity, and (iii) how the incorporation of decision-dependent uncertainty in demand elasticity can change the power generating companies' decisions and revenue estimation.

1. Introduction

1.1. Background and motivation

Unit commitment (UC) practices have been approached for decades in the short-term operation of electric power systems with the objective of optimizing the power generating unit schedules to meet the electricity demand. Depending on the purpose, the UC is solved under centralized or competitive environments, from self-scheduling to centralized auction-based market clearing, over a time horizon ranging from one day to one week. There exist three main classes of UC problems in practice, two of which are performed by power generating companies (GENCOs) – i.e., the cost-based UC (CBUC) and profit-based UC (PBUC) – and the other performed by the independent system operator (ISO) – i.e., the security-constrained UC (SCUC).

- Of interest to ISOs is the SCUC: a decision-making problem that is solved to minimize the cost of system operation by scheduling the status of generating units. Different from the CBUC/PBUC problems and in addition to the generating units' constraints, SCUC takes into account the power network topology, captures the transmission line security constraints, and involves day-ahead and real-time market clearing processes.
- Of primary interest to individual GENCOs are the CBUC and PBUC models. CBUC minimizes the cost of power generation over the decision horizon by scheduling the status of generating units while satisfying the generating units' ramp up/down limits, minimum/maximum generating capacity, minimum up/down times, and reserve constraints. We refer the reader to Ackooij, Lopez, Frangioni, Lacalandra, and Tahanan (2018) and Zheng, Wang, and Liu (2015) for reviews of the extended literature. In contrast, PBUC – the focus of this study – maximizes GENCO's profit over the scheduling horizon by taking into account the price-dependent revenue and cost of power generation. Different from the SCUC problem solved by the ISO capturing the network constraints and involving market clearing processes, the GENCO solves a day-ahead PBUC problem to decide how to bid in the market so as to be more likely favored in the market and result in the maximum profit for the GENCO.

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The demand for electricity encompasses the aggregate demand from all end-users, including households, businesses, and industrial customers, and is represented by the utility companies (or demand-serving entities) in the wholesale electricity market. This demand is forecasted by market participants, and it is used as a basis for determining the supply of electricity that will be made available for purchase in the day-ahead market. Market participants, including electric utilities, ISOs, and power traders or their agents, use this demand forecast information to make informed decisions about buying and selling electricity in the market. In a deregulated environment, GENCO's expected profit in the PBUC problem is heavily driven by the price signals, including the fuel purchase price, energy sale price, ancillary service sale price, etc. More active participation of the elastic demand (through demand-serving entities) in the wholesale electricity market can significantly challenge GENCO's own revenue management, i.e., scheduling optimization and profit estimation. With the traditional setting in which the energy sale price, set by a GENCO in the PBUC model, was assumed to be acceptable by buyers (e.g., electric utility companies), the challenge stems from the fact that the GENCO's price may or may not be favored in the market, thereby resulting in an uncertain willingness-to-pay (WTP) response to the seller's (GENCO's) offers. If the buyer's response to GENCO's sale price is not properly modeled, this could lead to misleading generating unit schedules and overestimation of GENCO's revenue and profit. We propose new stochastic models for the day-ahead PBUC problem employing the concept of WTP function to account for demand elasticity, the uncertainty about whether the buyer accepts to pay the set price, and to provide new pricing schemes for individual GENCO. Under a day-ahead market setting (Liu & Wu, 2007, 2008), we consider a GENCO that aims to fulfill a certain volume of the elastic demand at a price set by the proposed model. Depending on its set price and how it compares with the ones submitted by rival GENCOs, the GENCO of interest will be able to make revenue on the elastic demand conditional to its price being favored in the market. Accordingly, this price-conditional elastic demand setting results in a stochastic PBUC model described in Section 3.

We here provide an example that illustrates the day-ahead electricity market setting we consider in this study. There are three parties in the market (see Fig. 1): the GENCOs which sell the energy product; the utility companies which buy the energy product; and the ISO which matches the supply and demand through an auction and decides the settled price. Through the example, we assume that GENCO-B uses the proposed PBUC models to price its elastic supply. To illustrate how the auction procedure works, let us assume that the ISO receives offers including price and supply volume from four GENCOs: GENCO-A (40MW at $41), GENCO-B (40MW at $44), GENCO-C (20MW at $42) and GENCO-D (10MW at $45). On the buyers' side, the total elastic demand received by the ISO amounts to 100MW for the next day. To meet the demand, ISO starts to accept offers with the lowest price at first, moving to the second lowest one until the total electricity demand is met (i.e., accepted supply volume equals the total demand). In this example, offers from GENCOs A, C, and B are accepted sequentially, and the offer from GENCO-D is declined. The settled wholesale price is equal to the highest price among the accepted offers, which means GENCOs A, B, and C would all be paid $44 per MW.

In the appended Supplementary Material H, we include three alternative examples that illustrate how changes in the bidding prices by GENCO-B and rival GENCOs (i.e., GENCOs A, C, and D) could impact the acceptance of offers and the final settlement prices.

The remainder of the paper is structured as follows. Section 2 presents a review of the literature on the PBUC problem, pricing schemes, as well as demand elasticity models and algorithms. Section 3 presents the proposed SP PBUC models with DDU where several WTP functions capture the demand elasticity to price signals. Section 4 is devoted to the reformulation of the proposed nonconvex MINLP problems via a concavification approach. Section 5 introduces the learning framework used to parameterize and fine-tune the WTP coupling functions. Section 6 presents the valid inequalities and is followed by extensive numerical tests and evaluations in Section 7. Section 8 summarizes the findings.

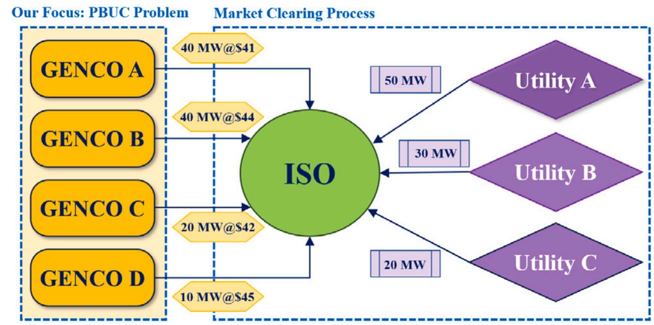

Fig. 1. Positioning of the proposed approach within a day-ahead electricity market setting.

2. Literature review

We present in this section a review of the literature devoted to the PBUC problem structured along two axes: pricing mechanisms and demand elasticity models.

2.1. PBUC problem and pricing schemes

Research on the application and solutions of the PBUC problem has been conducted over the past decades. GENCOs can be distinguished either as price-taker or price-maker entities in the decision-making process. In the former, the price-taker GENCO (i) participates in the day-ahead electricity market, (ii) is small enough to have a negligible influence in the market and clearing mechanisms, and (iii) the electricity price is exogenous and a forecasted parameter in the PBUC models. However, in the latter which is the focus of our research, a price-maker GENCO (i) is big enough to be influential on the energy market and the electricity price, and (ii) pursues strategic bidding strategies for its own revenue management and affect the market clearing prices. The literature is rich in both directions offering a variety of formulations and solution techniques (Morales-España, Gentile, & Ramos, 2015; Taktak & D'Ambrosio, 2017; Wang, Wang, & Guan, 2013a). An overview of deterministic mathematical programs for the PBUC problem of a GENCO is provided in Taktak and D'Ambrosio (2017). Since effective revenue management within the PBUC problem requires knowledge of the selling and buying price of energy at each time interval within the scheduling horizon, a variety of forecast models have been proposed to develop pricing strategies for GENCOs. The literature on GENCOs' optimal pricing for PBUC optimization strategies under different market structures is very rich. Amongst, bidding strategies using PBUC in a deregulated electricity market are presented in Yamin and Shahidehpour (2004). Given the price quota curve, Torre, Arroyo, Conejo, and Contreras (2002) present an MILP PBUC model for the self-scheduling of a price-maker GENCO to achieve maximum profit in a pool-based electricity market. Morales-España et al. (2015) propose tight MIP formulations and apply them to the PBUC problem for GENCOs' self-scheduling. Wang et al. (2013a) utilize a sample average approximation approach to solve a two-stage stochastic PBUC problem for price-taker GENCOs with chance constraints considering the price and wind power uncertainties. Note that there exists some two-stage stochastic optimization models (e.g., De, Tan, Li, Huang, & Song, 2018; Zheng, Chen, Xu, Liang, & Chen, 2020) and bi-level optimization models (e.g., Hobbs & Helman, 2011; Shafiekhani, Badri, Shafie-khah, & Catalao, 2019; Wei et al., 2018) for the UC problem (but mostly not the focused PBUC) or involve strategic bidding problems.

2.2. Demand elasticity models and algorithms

Inspired by the fast-growing demand response (DR) programs, electricity providers (i.e., GENCOs) strive to account for the demand behavior and offer variable prices for serving different classes of customers. DR models capture the demand elasticity, i.e., changes in the electricity usage by customers from their normal consumption patterns for financial benefits and in response to incentives or variations in the electricity price over time. If provided with sufficient incentives or acceptable prices, customers are willing to change (reschedule or reduce) their energy usage patterns and to trade off comfort and electricity bills. Accordingly, price-elastic and price-inelastic demand are distinguished as those that respectively do and do not participate in the DR programs. While demand for electricity has traditionally exhibited little price elasticity (Burke & Abayasekara, 2018; Farhar, 1996), the rapid deployment of DR programs and the existence of large and (aggregated) small price-responsive commercial, industrial, residential, and agricultural customers may bring about challenges and opportunities in short-term commitment, operation, and decision-making around generating units. A most recent comprehensive review of the literature on DR programs is provided in Motta, Anjos, and Gendreau (2024). Su and Kirschen (2009) propose a method to rigorously quantify the impact of the increased demand-side participation in the electricity market and on market participants. In particular, this study suggests a day-ahead market clearing mechanism that allows buyers to submit complex bids. The demand elasticity can affect the generation schedule, where a more elastic demand is found to generally reduce GENCO's profits. Jhala, Natarajan, Pahwa, and Wu (2019) set up discrete-time nonlinear autonomous system model to capture the interaction and dynamics of the electricity prices and the total demand including the elastic sector by deriving an equilibrium. Kirschen, Strbac, Cumperayot, and de Paiva Mendes (2000) investigate the potential effects of market structure on the elasticity of the electricity demand. Real-world examples also show that neglecting the buyers' response when setting electricity prices may lead to some patently unfair scenarios. Zhao, Wu, and Song (2014) study The impact of different DR price elasticity characteristics and DR participation levels on the convergence of volatile power markets. Duan (2016) considers a price-based DR scheduling problem in a day-ahead electricity market and analyzes the impact of DR and price-responsive demand on the gross surplus from load-serving entities. Bompard, Ma, Napoli, and Abrate (2007) analyze the impact of demand elasticity on the strategic bidding behavior of the electricity producers and on the oligopoly market performance. A hierarchical model predictive multi-period power dispatch and control strategy for modern power systems with price-elastic controllable demand is presented in Shi, Wen, Cao, and Yu (2019) with a price-elastic utility function incorporated in a bilevel optimization model. The stochastic UC problem with uncertain DR is addressed in Wang, Wang, and Guan (2013b). Baldick (2016) studies the role of the demand WTP functions in the context of mitigating the market power scenarios. Zoltowska (2016) proposes clearing and pricing models suitable for demand-shifting bids in non-convex pool-based auctions and develops guidelines on how responsive demand can best participate in such auctions. We refer the reader to Bernstein and G. (2005) and Burke and Abayasekara (2018) for discussions on the price elasticity of the US electricity demand.

The state-of-the-art literature has primarily considered deterministic PBUC models for inelastic electricity demand. In particular, strategic bidding behaviors in the literature mostly considered the demand with given curves (see, e.g., Kirschen et al., 2000; Li & Shahidehpour, 2005) while only a few accounted for the influence of the demand elasticity on the gaming behavior in the oligopoly electricity markets, in which the effects of strategically changing the consumers' load profile in different trading intervals – characterized by the cross-price elasticity of the demand – were the only focus of concern (Kirschen et al., 2000). Game theory models have also been utilized by individual market participants (see, e.g., Devine, Gabriel, & Moryades, 2016; Ferrero, Rivera, & Shahidehpour, 1998) to simulate the bidding behaviors of other GENCOs (and market participants) and develop Nash equilibrium bidding strategies. A few have proposed deterministic models that take into account the impact of demand elasticity on electricity markets (Su & Kirschen, 2009), on day-ahead scheduling of generating units (Wu, Shahidehpour, & Khodayar, 2013), and on the strategic behavior of electricity producers (Bompard et al., 2007). Wang et al. (2013b) study the stochastic UC problem with endogenously-defined demand response. To the best of the authors' knowledge, accounting for decision-dependent uncertainty (DDU) in the WTP response of the elastic consumers has not been researched in general and in the context of GENCO's PBUC pricing in particular. This study is the first one that considers an SP model with DDU represented with a WTP function. In contrast, the related literature usually studies closed-form, unconstrained optimization formulations and derives closed-form optimal policies. Such closed-form optimal policies cannot be used in the PBUC problem due to the complexity of the constraint set. It, therefore, calls for the development of tractable reformulations which can be solved numerically for large instances.

3. Stochastic programming PBUC models with DDU

3.1. Stochastic programming PBUC-DDU models

Traditional SP problems, which deal with exogenous uncertainty sources, are notoriously challenging to solve (see Lejeune & Prékopa, 2021; Prékopa, 2003). We present here risk-neutral SP models that incorporate an additional layer of complexity since they account for DDU which is modeled via a *coupling function* defining the impact of decisions on random variables. We propose DDU SP problems with *decision-dependent probabilities*, also called Type 1 DDU, in which decisions impact the probability distribution of random variables. They must be differentiated from problems with Type 2 DDU (i.e., problems with decision-dependent information structure), where decisions affect the time at which information is revealed and uncertainty gets resolved. We refer the reader to Hellemo, Barton, and Tomasgard (2018) who propose a taxonomy of DDU SP problems and to Section B in the Supplementary Material which provides a more in-depth characterization of the various forms of DDU.

We focus on the PBUC problem through which GENCOs seek to maximize their profit by selling energy products in the electricity market. A critical element, yet often overlooked, by the GENCOs is the DR, or more precisely, the propensity of the price and supply being accepted by the demand. This approach is related to price responsiveness (Kirschen et al., 2000; Wang et al., 2013b; Zhao et al., 2014). We tackle this question by using a *willingness-to-pay probability* or *price acceptance probability* function to model how buyers respond to price signals. This approach has been used in revenue management (Phillips, 2005, 2013) and price discrimination (Besbes & Zeevi, 2015) for example.

3.2. Willingness-to-pay function

From GENCO's perspective who aims at strategic revenue management to maximize profit by solving a day-ahead PBUC decision-making optimization, the buyers' response to the price set for the elastic demand is obviously uncertain and not known, i.e., the GENCO's offer may or may not be accepted by the buyers (i.e., price-conditional elastic demand). The GENCO may, however, affect the likelihood of the price being favored in the market. This can be achieved by varying the price offered to the elastic demand, which is one of the decisions in the PBUC problem. The acceptance or not of the GENCO's price is therefore a DDU that is affected by the pricing decisions.

Accordingly, the uncertainty as to whether the buyer will accept to pay the price set by the GENCO for the elastic demand at time t is modeled with a Bernoulli random variable ξ_t.

Definition 1. The Bernoulli random variable ξ_t

$$\xi_t = \begin{cases} 1, \text{ with probability } q_t \\ 0, \text{ with probability } 1 - q_t, \end{cases} \quad t \in \mathbf{T} \tag{1}$$

where outcome 1 (resp. 0) signifies that the buyer accepts to pay GENCO's price for the elastic demand at time t. The Bernoulli random variable ξ_t is defined by the parameter q_t representing the probability of acceptance that depends on the price π_t:

$$q_t = \Upsilon(\pi_t) \,,\ t \in \mathbf{T}.$$

The function Υ with argument π_t is called the (WTP) coupling function and defines how the price π_t affects the price acceptance probability q_t.

The GENCO proposes to satisfy a volume d_t^e of the elastic demand at a price π_t. The acceptance (WTP) by the market participants (demand entities) to pay the price π_t is a monotone decreasing function of the price and is not set in stone. The WTP the price set by the GENCO is therefore modeled as a Bernoulli random variable ξ_t with the expected value q_t, which also represents the probability of acceptance. The use of a Bernoulli random variable is consistent with the auction mechanism employed in the wholesale market. To be more specific, a GENCO's offer is either accepted or declined in its entirety (i.e., the whole supply volume is accepted or declined) thus justifying the use of a binary random variable. The volume of elastic demand that the GENCO will be able to sell on the market is, therefore, a random variable, called price-conditional elastic demand, taking value d_t^e with probability q_t and equal to 0 with its complement (1-q_t).

The coupling function represents the dependency of the endogenous uncertainty (price accepted by the buyer) on the pricing decision and takes the form in the proposed PBUC problem of a *willingness-to-pay* function. Two observations worth underlining follow from Definition 1. First, the price acceptance probability is not a fixed parameter but depends on the price π_t. The acceptance probability varies with the price and is accordingly defined as a decision variable in the DDU SP formulation. Second, the form of DDU is of *Type 1* with decision-dependent parameters.

3.3. Formulation

The proposed SP model for PBUC with decision-dependent price acceptance probability determines the optimal generation output and the schedules of the generating units as well as the optimal price for the elastic demand so as to maximize GENCO's profit. The problem takes the form of the stochastic mixed-integer nonlinear programming (MINLP) model **GF:**:

$$\textbf{GF:} \quad max \ \mathbb{E}_{\mathcal{P}(\boldsymbol{\pi_t})}\left[\sum_{t\in\mathbf{T}} \xi_t \pi_t d_t^e\right] + \sum_{t\in\mathbf{T}} \rho_t d_t^f + \sum_{g\in\mathbf{G}}\sum_{t\in\mathbf{T}} (l_t^s r_{g,t}^s + l_t^n r_{g,t}^n)$$
$$- \sum_{g\in\mathbf{G}}\sum_{t\in\mathbf{T}} (a_g \lambda_{g,t}^2 + b_g \lambda_{g,t} + c_g \alpha_{g,t} + C_g^u y_{g,t} + C_g^d z_{g,t}) \tag{2a}$$
$$\text{s.to} \quad q_t = \Upsilon_t(\pi_t) \qquad t\in\mathbf{T} \tag{2b}$$
$$0 \le q_t \le 1 \qquad t\in\mathbf{T} \tag{2c}$$
$$(1-\gamma)\rho_t \le \pi_t \le (1+\gamma)\rho_t \qquad t\in\mathbf{T} \tag{2d}$$
$$\mathbf{x} \in \mathcal{X} \tag{2e}$$

The objective function maximizes GENCO's total profit. The terms in the first line of the objective function (2a) represent the total revenue from selling energy products. The first term represents the expected revenue stemming from the elastic demand d_t^e whose price π_t is a decision variable. The Bernoulli random variable ξ_t indicates that it is unsure if the GENCO's set price appeals to the buyers in the market. The second term reflects the GENCO's income generated by the inelastic demand d_t^f for which the forecasted price ρ_t is fixed. The last two terms on the first line of Eq. (2a) represent the revenue generated from the spinning and non-spinning reserve products for which l_t^s and l_t^n are the corresponding forecasted market prices. The terms in the second line of (2a) represent the total costs of power generation by the GENCO's generating units. The first three terms represent the quadratic power generation costs, while the last two terms are the start-up and shut-down costs for operating the generating units, respectively.

The equality *linking* constraint (2b) enforces the dependence of ξ_t on π_t. More precisely, it can be seen from (2b) how q_t, which specifies the distribution of ξ_t, depends on the pricing decision π_t, which shows that the DDU is of *Type 1* with decision-dependent parameter. Constraint (2c) forces q_t to take a value between 0 and 1 as it denotes the probability of acceptance. Note that (2c) is redundant as its satisfaction is implicitly guaranteed from the definition of the WTP function $\Upsilon(\pi_t)$ (see Section 3.4). Constraint (2d) does not allow the price set for the elastic demand to differ by more than a certain percentage γ, set by the GENCO, from the price for the fixed demand. GENCOs may strive for an increase in prices with respect to the competitive values, providing higher producer surplus at the expense of market efficiency. As a mitigation approach to avoid any potential market power behavior of the GENCOs, we bound the prices not to exceed the competitive levels (Baldick, 2016). Accordingly, the price set for the elastic demand cannot differ by more than a certain percentage γ from that of the fixed demand (2d). The notation $\mathbf{x}$ denotes the aggregated decision vector which is the concatenation of all decision variables (with the exception of π_t and q_t) in the PBUC problem (see Supplementary Material A). Constraint (2e) states that the aggregated vector $\mathbf{x}$ of decision variables must belong to the mixed-integer linear feasible set $\mathcal{X}$

$$\mathcal{X} = \mathcal{X}^B \cap \mathcal{X}^R \cap \mathcal{X}^M \cap \mathcal{X}^E \tag{3}$$

defined as the intersection of $\mathcal{X}^B, \mathcal{X}^R, \mathcal{X}^M$, and $\mathcal{X}^E$ respectively referring to the feasible sets defined by the power flow balance, the generating units' ramp rate, the min-up/min-down time, and the reserve constraints presented in details in Supplementary Material C. The above model is a stochastic MINLP problem whose key properties will be analyzed in the next section.

3.4. DDU modeling and coupling functions

The proper modeling of DDU relies on a coupling function (Dupačová, 2006) that defines the dependency of the probability distribution of the random variables on (some of) the decisions. It is an encompassing concept that has been employed to model DDUs in multiple contexts, such as oil field exploitation (Jonsbråten, Wets, & Woodruff, 1998), network survivability (Peeta, Salman, Gunnec, & Viswanath, 2010), military medical evacuation (Lejeune & Margot, 2018), etc. In the PBUC context studied here, the coupling function takes the form of a WTP function. Definition 2 presents the key properties of the WTP functions. To ease the notation, we drop the subscript t and use $l = (1-\gamma)\rho$ and $u = (1+\gamma)\rho$ to refer to the lower and upper bounds (see (2d)) on π.

Definition 2. Any willingness-to-pay function $\Upsilon(\pi) : [l, u] \subset \mathbb{R}_{++} \mapsto [0, 1]$ used in the PBUC problem must have the following properties:

1. Bounded with range $[0, 1]$.
2. Continuous on $[l, u]$.
3. Antitone (monotone decreasing) π: $\pi^1 \le \pi^2 \Rightarrow \Upsilon(\pi^1) \ge \Upsilon(\pi^2)$.

We consider three WTP functions used previously in the literature and in practice (Besbes, Phillips, & Zeevi, 2010; Besbes & Zeevi, 2015; Lau & Lau, 2003).

Definition 3. Let $\tau > 0$ denote the price elasticity and $\pi \in [l, u]$. Three functional forms are considered for the WTP function:

1. Linear WTP function:

$$\Upsilon^1(\pi) = 1 - \tau\pi \tag{4}$$

for $\tau \le 1/u$ with u denoting the largest admissible price.

2. Exponential WTP function:

$$\Upsilon^2(\pi) = e^{-\tau\pi} \ . \tag{5}$$

3. Logit WTP function:

$$\Upsilon^3(\pi) = \frac{e^{\nu-\tau\pi}}{1+e^{\nu-\tau\pi}} \ . \tag{6}$$

See an example of each function in Figure 5, 6, and 7 in Section G of the Supplement Material. A larger value for τ indicates a higher price elasticity, which implies that the buyer is more sensitive to the price. The parameter ν is the intercept of the log odds ratios defined as $log[\frac{\Upsilon^3(\pi)}{1-\Upsilon^3(\pi)}]$.

3.5. Specific WTP formulations

Having defined the specific functional forms of the considered WTP functions, we derive the explicit formulation of the MINLP problem corresponding to each of them that can be obtained by successively substituting $\Upsilon^1(\pi)$, $\Upsilon^2(\pi)$, and $\Upsilon^3(\pi)$ for the generic WTP notation $\Upsilon(\pi)$ in **GF** (2a). The three resulting formulations differ in their objective functions but have the same feasible set.

We recall that $\mathbf{x}$ denotes the aggregated vector of decision variables. To further ease the notation, we introduce $s(\mathbf{x})$ to refer to the deterministic part of the objective function (2a):

$$s(\mathbf{x}) = \sum_{t\in\mathbf{T}} \rho_t d_t^f + \sum_{g\in\mathbf{G}} \sum_{t\in\mathbf{T}} (l_t^s r_{g,t}^s + l_t^n r_{g,t}^n - a_g\lambda_{g,t}^2 - b_g\lambda_{g,t} - c_g\alpha_{g,t} - C_g^u y_{g,t} - C_g^d z_{g,t}) \ . \tag{7}$$

Proposition 1. *The SP formulations for the PBUC problem with linear WTP function $\Upsilon^1(\pi_t)$, exponential WTP function $\Upsilon^2(\pi_t)$, and logit WTP function $\Upsilon^3(\pi_t)$ can be reformulated as the MINLP problems* **M1**, **M2**, *and* **M3**, *respectively:*

$$\mathbf{M1}: \left\{ \max_{\pi_t,\mathbf{x}} \sum_{t\in\mathbf{T}} \pi_t d_t^e - \sum_{t\in\mathbf{T}} \tau d_t^e \pi_t^2 + s(\mathbf{x}) : (1-\gamma)\rho_t \le \pi_t \le (1+\gamma)\rho_t, t\in\mathbf{T} \ , \ \mathbf{x}\in\mathcal{X} \right\} \tag{8}$$

$$\mathbf{M2}: \left\{ \max_{\pi_t,\mathbf{x}} \sum_{t\in\mathbf{T}} e^{-\tau\pi_t}\pi_t d_t^e + s(\mathbf{x}) : (1-\gamma)\rho_t \le \pi_t \le (1+\gamma)\rho_t, t\in\mathbf{T} \ , \ \mathbf{x}\in\mathcal{X} \right\} \tag{9}$$

$$\mathbf{M3}: \left\{ \max_{\pi_t,\mathbf{x}} \sum_{t\in\mathbf{T}} \frac{e^{\nu-\tau\pi_t}}{1+e^{\nu-\tau\pi_t}}\pi_t d_t^e + s(\mathbf{x}) : (1-\gamma)\rho_t \le \pi_t \le (1+\gamma)\rho_t, t\in\mathbf{T} \ , \ \mathbf{x}\in\mathcal{X} \right\} \tag{10}$$

Since ξ_t in **GF** is a Bernoulli random variable, we can replace its expected value by q_t which is the probability of acceptance, and, which, due to (2b), can then be replaced by the considered willingness-to-pay functional form. The detailed proof is given in the Supplementary Material D.1. The next step is to analyze the computational challenges posed by the three MINLP formulations derived. In particular, Proposition 2 analyzes whether the continuous relaxation of each of these MINLP formulations is a convex programming problem.

Proposition 2. *The continuous relaxation of the MINLP problem* **M1** *(8) is convex. The continuous relaxations of the MINLP problems* **M2** *(9) and* **M3** *(10) are nonconvex.*

The proof is given in Supplementary Material D.2. While one can be reasonably hopeful to solve the convex quadratic MINLP problem **M1** with state-of-the-art solvers, the solution of the nonconvex MINLP problems **M2** and **M3** for practice-sized instances poses severe computational challenges. In the next section, we derive computationally efficient reformulations for **M2** and **M3**.

4. Concavification

In this section, we design a *concavification* method to derive equivalent convex programming reformulations (i.e., convex continuous relaxation) of the nonconvex MINLP problems **M2** and **M3**.

We will next show that the nonlinear nonconcave functions $h^2(\pi_t) = \pi_t e^{-\tau\pi_t}$ and $h^3(\pi_t) = \frac{\pi_t e^{\nu-\tau\pi_t}}{1+e^{\nu-\tau\pi_t}}$ in the objective functions of **M2** and **M3** are *concavifiable*, i.e., can be *transformed* into concave functions. Section 4.1 gives an overview of the concavifiability concept while Section 4.2 presents the convex reformulations of **M2** and **M3** obtained using this concept.

4.1. Overview

Concavification is the transformation of a nonconcave function into a concave one. In this study, we use a *domain transformation* approach (Boyd & Vandenberghe, 2004) which involves a combination of both range and domain transformations (Li, Wu, Lee, Yang, & Zhang, 2005). First, it proceeds to a one-to-one transformation of the domain of a function so that its upper sets are transformed into convex ones, which implies that the "transformed" function is quasi-concave. Second, a monotone increasing range transformation of the quasi-concave function is carried out in order to obtain a concave function.

It is well known that a concave monotonic transformation of a concave function is itself concave. While such results for convexity – or concavity – preservation via a monotone increasing function are well known, much less is known about *concavity-inducing* (or convexity-inducing) functions, in which we are particularly interested here since the nonlinear components in the objective functions of problems **M2** and **M3** are not concave. The challenge here is not to find a concavity-preserving transformation, but to uncover a concavity-inducing one.

We shall now determine whether there exists a concavity-inducing transformation $F : \mathbb{R} \to \mathbb{R}$ for the nonlinear components $h^2(\pi_t)$ and $h^3(\pi_t)$ of the objective functions of **M2** and **M3** such that $F(h^2(\pi_t))$ and $F(h^3(\pi_t)), t \in \mathbb{T}$ are concave and provide equivalent *convex reformulations* for the continuous relaxations of **M2** and **M3**. To determine whether $h^2(\pi_t)$ and $h^3(\pi_t)$ are concavifiable, we shall examine whether they are endowed with the g-concavity and quasi-concavity properties.

Definition 4. Let $f : C \to \mathbb{R}$ be defined on $C \subseteq \mathbb{R}^n$ and with range $I_f(C)$. The function f is said to be *g-concave* if there is a continuous real-valued monotone increasing function $F : I_f(C) \to \mathbb{R}$ such that $F(f(x))$ is concave over C. This is the case if

$$F(f(\lambda x^1 + (1-\lambda)x^2)) \ge \lambda F(f(x^1)) + (1-\lambda)F(f(x^2)) \tag{11}$$

holds for any $x^1, x^2 \in C, 0 \le \lambda \le 1$.

The g-concavity property is a *sufficient condition* for the concavification of a function. As g-concavity is a demanding property and may be difficult to prove, we shall also look at the quasi-concavity of the functions. We note in that respect that every g-concave function f on a convex set C is quasi-concave, but the converse is not true (Avriel, Diewert, Schaible, & Ziang, 2010); not every quasi-concave function f is g-concave.

Definition 5. Let $0 < \lambda < 1$ and x^1, x^2 be two arbitrary points in C.

A function $f : C \to \mathbb{R}$ is quasi-concave if and only if

$$f(\lambda x^1 + (1-\lambda)x^2) \ \ge \ \min(f(x^1), f(x^2)) \ . \tag{12}$$

The function f is strictly quasi-concave if the inequality is strict whenever $x_1 \neq x_2$.

A strictly quasi-concave function is such that it has a unique global maximum on any compact convex domain. Strict quasi-concavity is a necessary condition for the concavification of a function; quasi-concave functions that are not strictly quasi-concave are never concavifiable. We now determine the key features (Proposition 3 with proof in Supplementary Material D.3) of the nonlinear functions $h^2(\pi_t) = e^{-\tau\pi_t}\pi_t$ and $h^3(\pi_t) = \frac{\pi_t e^{\nu-\tau\pi_t}}{1+e^{\nu-\tau\pi_t}}$, and especially whether they are strictly quasi-concave.

Proposition 3. *The univariate functions $h^2(\pi_t) = e^{-\tau\pi}\pi$ and $h^3(\pi_t) = \frac{\pi e^{\nu-\tau\pi}}{1+e^{\nu-\tau\pi}}$ are non-monotone and strictly quasi-concave.*

Taking the monotone logarithmic transformation of $h^2(\pi_t)$ and $h^3(\pi_t)$ gives a concave function, thereby proving that $h^2(\pi_t)$ and $h^3(\pi_t)$ that $h^2(\pi_t) = e^{-\tau\pi_t}\pi_t$ and $h^3(\pi_t) = \frac{\pi_t\ e^{\nu-\tau\pi_t}}{1+e^{\nu-\tau\pi_t}}$ are g-concave and concavifiable.

4.2. Convex reformulations

We now derive the convex MINLP reformulations of the nonconvex MINLP problems **M2** and **M3**. The results are presented in Theorems 1 and 2. The equivalence relationship is demonstrated using the logarithmic transformation of the epigraphic formulations of **M2** and **M3**.

Theorem 1. *Let $w_t, t \in \mathbf{T}$ be a set of continuous decision variables.*
(i) The MINLP problem **RM2**

$$\mathbf{RM2}: \quad \max \sum_{t\in\mathbf{T}} w_t + s(\mathbf{x}) \tag{13a}$$

$$s.to \quad -\tau\pi_t + ln(\pi_t d_t^e) \geq w_t \qquad t \in \mathbf{T} \tag{13b}$$

$$(1-\gamma)\rho_t \leq \pi_t \leq (1+\gamma)\rho_t \qquad t \in \mathbf{T} \tag{13c}$$

$$\mathbf{x} \in \mathcal{X} \tag{13d}$$

is equivalent to **M2** *and has a convex continuous relaxation.*
(ii) The optimal solution(s) $(\mathbf{x}^*, \pi^*, w^*)$ of **RM2** *is identical to the optimal solution(s) of* **M2**.
(iii) There is a one-to-one mapping between the optimal values of **RM2** *and* **M2**. *The optimal value $z^*_{\mathbf{M2}}$ of* **M2** *is: $z^*_{\mathbf{M2}} = \sum_{t\in\mathbf{T}} e^{w_t^*} + s(\mathbf{x}^*)$.*

The proof is provided in Supplementary Material D.4.

Theorem 2. *Let $v_t, t \in \mathbf{T}$ be a set of continuous decision variables.*
(i) The MINLP problem **RM3** *is equivalent to* **M3***:*

$$\mathbf{RM3}: \quad \max \sum_{t\in\mathbf{T}} v_t + s(\mathbf{x}) \tag{14a}$$

$$s.to \quad ln(\pi_t d_t^e) + \nu - \tau\pi_t - ln(1 + e^{\nu-\tau\pi_t}) \geq v_t \qquad t \in \mathbf{T} \tag{14b}$$

$$(1-\gamma)\rho_t \leq \pi_t \leq (1+\gamma)\rho_t \tag{14c}$$

$$\mathbf{x} \in \mathcal{X} \tag{14d}$$

(ii) The continuous relaxation of the MINLP problem **RM3** *is a convex programming problem.*
(iii) The optimal solution(s) $(\mathbf{x}^*, \pi^*, v^*)$ of **RM3** *is identical to the optimal solution(s) of* **M3**.
(iv) There is a one-to-one mapping between the optimal values of **RM3** *and* **M3**. *The optimal value $z^*_{\mathbf{M3}}$ of* **M3** *is: $z^*_{\mathbf{M3}} = \sum_{t\in\mathbf{T}} e^{v_t^*} + s(\mathbf{x}^*)$.*

The proof is similar to the one given for Theorem 1.

5. Learning the WTP function

In this section, we design a data-driven learning approach to specify the parameters of the WTP functions. The suggested learning approach is particularly beneficial to GENCOs' decision-making in two ways: (i) the GENCO could be guided on the choice of the WTP function – and accordingly, the PBUC-DDU model – as the power system and electricity market structures evolve and the varying (typically increasing) participation rate of the elastic demand may potentially require different choices of the WTP function over time; (ii) the GENCO could continuously fine-tune the parameters of a selected WTP function over time and as new data on the buyer behavior and market response become available. The proposed PBUC-DDU analytics are intended to be used in practice by GENCOs when they could employ their historical data to train the learning algorithm and dynamically fine-tune the WTP parameters.

We employ the least square estimation method to learn the parameters of the WTP functions. The WTP function $Y(\pi)$ depends, in some fashion (i.e., linear, exponential, logit form), on the price π which is the explanatory or predictor variable in the proposed least square estimation models (see Proposition 4). The least-square estimation models are nonlinear optimization problems that each minimize the sum of the squared residual values between the observed and the predicted WTP. The unknowns or decision variables in the nonlinear optimization problems are the parameters τ and ν of the WTP functions. Proposition 4 analyzes the convexity (or not) nature of the optimization problems that allow us to learn the value of the parameters of the WTP functions.

Proposition 4. *Let K be the training set of data points, $\pi^{(k)} \in [l, u]$ be the observed price, and $Y^{(k)}$ the observed WTP probability for data point k.*

1. *The parameter τ of the linear WTP function (4) can be found by solving the quadratic convex optimization problem* **L1***:*

$$\mathbf{L1}: \quad \min_{\tau} \quad \sum_{k\in K} \left(Y^{(k)} - (1 - \tau\pi^{(k)})\right)^2 \tag{15}$$

$$s.to \quad 0 \leq \tau \leq \frac{1}{u} \tag{16}$$

with u denoting the largest price in the training set: $u = \max_{k\in K} \pi^{(k)}$.

2. *The parameter τ of the exponential WTP function (5) can be found by solving the nonlinear optimization problem* **L2***:*

$$\mathbf{L2}: \quad \min_{\tau\geq 0} \quad \sum_{k\in K} \left(Y^{(k)} - e^{-\tau\pi^{(k)}}\right)^2 . \tag{17}$$

Problem **L2** *is convex for low elasticity levels*

$$\tau \leq \min_{k\in K} \frac{ln(2/Y^{(k)})}{\pi^{(k)}} \tag{18}$$

and is nonconvex otherwise.

3. *The parameters ν and τ of the logit WTP function (6) can be found by solving the unconstrained nonlinear nonconvex optimization problem* **L3***:*

$$\mathbf{L3}: \quad \min_{\tau\geq 0,\nu} \quad \sum_{k\in K} \left(Y^{(k)} - \frac{e^{\nu-\tau\pi^{(k)}}}{1 + e^{\nu-\tau\pi^{(k)}}}\right)^2 . \tag{19}$$

The proof is given in Supplementary Material D.5.

6. Valid inequalities

In this section, we derive several families of valid inequalities to strengthen the continuous relaxation of the MINLP problems **M1**, **RM2**, and **RM3** (see Sections Section 3.5, and 4) and speed up their solution. In preliminary computational experiments, we considered a number of possible valid inequalities. We refer the reader to Huang, Pan, and Guan (2021) and the references therein for a recent review of the related literature. The valid inequalities described next are those that have the strongest impact on the solution of the problem instances considered in this study.

Let $\alpha_{g,0}$ be a variable representing whether the generating unit g was online the day before the start of the last period, which we referred to by the subscript 0.

The first two types of valid inequalities (20) stipulate that any unit g can start up and shut down at most once over the minimal duration $\overline{S_g} + \underline{S_g}$ of a cycle.

Proposition 5. *Let $V_t = \min\left(|T|, t + \overline{S_g} + \underline{S_g} - 1\right)$. The linear constraints*

$$\sum_{k=t}^{V_t} y_{g,k} \le 1 \quad \text{and} \quad \sum_{k=t}^{V_t} z_{g,k} \le 1 \quad t \in \mathbf{T} \setminus \{1\}, g \in \mathbf{G} \tag{20}$$

are valid inequalities for problems **M1**, **RM2**, *and* **RM3**.

The valid inequalities (21) and (22) use logical relationships between the decisions to use, start-up, and shut down a generating unit g to derive a valid upper bound on: (i) the decision to turn on generating unit g over a number $(\overline{S_g} + 1)$ of consecutive periods (21) and (ii) the decision to shut down generating unit g at a specific period t (22).

Proposition 6. *The linear constraints*

$$\alpha_{g,t} \ge \sum_{k=t-\overline{S_g}+1}^{t} y_{g,k} \qquad t = \overline{S_g} + 1, \ldots, |\mathbf{T}|, g \in \mathbf{G} \tag{21}$$

$$\underline{S_g} z_{g,t} \le t - \sum_{k=1}^{t} \alpha_{g,k} \qquad t \in \mathbf{T}, g \in \mathbf{G} \tag{22}$$

are valid inequalities for problems **M1**, **RM2**, *and* **RM3**.

The valid inequalities (23) stipulate that non-spinning reserves from offline generating units cannot be activated if a generating unit was started more times than it was shut down.

Proposition 7. *The linear constraints*

$$\beta_{g,t} \le 1 - \sum_{k=1}^{t} (y_{g,k} - z_{g,k}) - \alpha_{g,0} \quad t \in \mathbf{T}, g \in \mathbf{G} \tag{23}$$

are valid inequalities for problems **M1**, **RM2**, *and* **RM3**.

The valid inequalities provide tighter and equivalent formulations for **M1**, **RM2**, and **RM3**. They are equivalent in the sense that they do not eliminate any integer feasible solution. They are tighter since they eliminate fractional solutions that would be otherwise feasible for the continuous relaxations of the problems. The above valid inequalities will be included up-front in the formulations of the mixed-integer problems **M1**, **RM2**, and **RM3**, thereby providing the strengthened formulations **SM1**, **SRM2**, and **SRM3**:

$$\mathbf{SM1}: \left\{ \max_{\pi_t, \mathbf{x}} \sum_{t \in \mathbf{T}} \pi_t d_t^e - \sum_{t \in \mathbf{T}} \tau d_t^e (\pi_t)^2 + s(\mathbf{x}) : \text{(2d)–(2e); (20)–(23)} \right\} \tag{24}$$

$$\mathbf{SRM2}: \left\{ \max_{\pi_t, \mathbf{x}, w} \sum_{t \in \mathbf{T}} w_t + s(\mathbf{x}) : \text{(13b)–(13d) ; (20)–(23)} \right\} \tag{25}$$

$$\mathbf{SRM3}: \left\{ \max_{\pi_t, \mathbf{x}, v} \sum_{t \in \mathbf{T}} v_t + s(\mathbf{x}) : \text{(14b)–(14d) ; (20)–(23)} \right\} \tag{26}$$

The strengthened formulation by introducing the above valid inequalities is pivotal in the solution process. The computational benefits of the valid inequalities are evaluated in Section 7.3.

7. Numerical experiments

In this section, numerical results are presented to verify the effectiveness of the proposed SP models for PBUC. We compare the performance of the proposed MINLP models with the conventional PBUC model (MILP) without DDU and demand elasticity under a variety of WTP functions and operation scenarios. Section 7.1 describes the testing environment, the data, and the several IEEE benchmark test systems used in our analysis of the numerical experiments. Section 7.2 presents the parameterization of the WTP functions with the proposed data-driven learning approach. Section 7.3 is devoted to the computational efficiency tests to verify the scalability and computational tractability of our approach. Section 7.4 discusses in detail the test scenarios employed to (i) numerically demonstrate the performance of the proposed analytics under a variety of conditions, (ii) analyze their performance compared to the traditional PBUC practices with no elasticity considerations and to price the value of DDU considerations, and (iii) evaluate different WTP functions and the corresponding PBUC decisions. We also interpret the results and provide insights into the numerical tests and observations.

7.1. Data and testing environment

7.1.1. Test system description and data

In this study, the performance of the proposed models is tested on four different IEEE benchmark systems, including the IEEE 6-bus, IEEE 24-bus, IEEE 39-bus, and IEEE 118-bus test systems. Table 1 presents the general information of the four test systems, while the system configuration and detailed data of the generation cost coefficients, generating unit parameters, fixed and elastic demand at each time period, etc. are provided in Dehghanian (2021). The proposed model is generic enough to accommodate any proportion between elastic and inelastic demand. In our numerical analyses, we assume that 20% of the total demand in the studied systems is elastic (price-responsive) and the remaining 80% is fixed (inelastic).

We consider that the price for the fixed demand and the market prices for the spinning and non-spinning reserves at each time period are known (forecasted) parameters in all test systems, the data on which are provided in Dehghanian (2021). This assumption is reasonable as the focused PBUC problem is intended to be performed by GENCOs. Additionally, market operators and participants typically provide forecasts and historical pricing data for planning and trading purposes. These forecasts are often based on historical market behavior and current supply and demand conditions. A test case on the impact of uncertainties in the day-ahead price for energy is introduced in Supplementary Material I. The GENCO of interest, that manages all generating units in each test system, aims at maximizing its profit from the energy products. The GENCO utilizes the proposed PBUC models and participates in a day-ahead market, where other GENCOs submit their bids. The PBUC scheduling time horizon is set to 24 h in all tests. All optimization problems are coded with AMPL. The quadratic integer problems are solved with the `Gurobi 9.0.3` solver while the nonlinear and nonquadratic optimization problems are solved with the `Baron 20.10.16` solver on a PC with an Intel Core i7-6700 CPU 3.40 GHz processor and 32 GB memory.

7.1.2. WTP parameter learning and characterization: Data and assumptions

In order to characterize the proposed WTP functions, the data-driven learning approach presented in Section 5 is applied. Recall that the parameters to be specified include the price elasticity (τ) for the linear (4) and exponential (5) WTP functions and τ and ν for the logit (6) WTP function. Historical data on the price set by the GENCO and the corresponding WTP response of the elastic demand (in terms of price acceptance probability) are hence needed to train the data-driven learning models. Since such information could decode and reveal the strategic pricing and bidding strategies of the profit-seeking GENCOs in the electricity market, such data are typically confidential and not available to the public, as it otherwise could sacrifice the GENCO's competitiveness in the electricity market. Constrained by the unavailability (to the authors) of such data needed to fine-tune the WTP parameters, our approach is accordingly – and without loss of generality – geared toward the generation of a synthetic dataset that is

Table 1
IEEE benchmark test systems used in numerical experiments.

Test system	# Buses	# Generating units	# Transmission lines	Total capacity (MW)	Maximum demand (MW)
IEEE 6-Bus	6	3	7	360	260
IEEE 24-Bus	24	12	39	3375	2650.5
IEEE 39-Bus	39	10	47	1990	1455
IEEE 118-Bus	118	19	185	6859	5746

Table 2
WTP functions: Estimated parameters values and R^2.

WTP functions	τ	ν	R^2
Linear – **L1**	0.0117	NA	0.941
Exponential – **L2**	0.0197	NA	0.894
Logit – **L3**	0.0967	4.83	0.986

informed by real-world data. Accordingly, we have utilized data on the day-ahead electricity market prices of the PJM electricity market during the 1-year period of 12/01/2019–11/30/2020 (PJM Data Miner 2, 2021) taken in place of the day-ahead price set by the GENCO. We have generated three different datasets by sampling from the distribution corresponding to the relationships (linear, exponential, logit) between price and WTP probability. For each WTP function, we use 200 price data points (of the PJM market data) equally spaced between $18 to $77. For each price point, the WTP probability is calculated using the proposed WTP function (Definition 3) plus a randomly generated noise that follows a normal distribution. The fitted functions are estimated with the models presented in Proposition 4. From the 365 data points retrieved, 292 samples (80%) are utilized for training and 73 samples (20%) are used for testing the results of the learned model.

Note that the synthetic data generation approach is taken here solely to fill in the confidential data unavailability gap and to demonstrate the applicability of the proposed learning methods to estimate a WTP function for a given dataset that a GENCO could have access to. However, GENCOs could perform exploratory data analysis and employ their historical data to select the right choice of the WTP function, and dynamically learn and fine-tune the WTP parameters.

7.2. Learning parameters of WTP functions

In this section, we describe the results associated with the learning-driven parameter estimation of the WTP functions and the solution of the associated nonlinear least-square optimization models presented in Proposition 4. As above-explained, since real-life historical data are not available, we use synthetic data to conduct our numerical experiments and in particular to learn the parameter values of each WTP function through the solution of the least-square optimization problems **L1**, **L2**, and **L3**. The convex problem **L1** is solved with `Gurobi` in 0.05 s. The nonconvex problems **L2** and **L3** are solved with `BARON`. For **L2**, `BARON` finds the optimal solution in 1.72 s, while for **L3**, a high-quality feasible solution (i.e., small optimality gap) is found in 0.14 seconds.

To evaluate the result, we use the R^2 coefficient of determination which ranges from 0 to 1 and measures how much variance in the dependent variable Y can be explained by the independent variable π. Table 2 reports the estimates and corresponding R^2 values. In addition, Figures 1–3 (in Supplementary Material G) display the“synthetically” observed WTP probabilities (red dots) and those estimated with the learning models **L1**, **L2**, and **L3** (blue lines).

The values of the R^2 coefficients displayed in Table 2 and Figures 1–3 attest that the fitted WTP probabilities are extremely close to the observed ones and, each estimated WTP function is able to capture very well the overall trend in the dataset. The estimated values given in Table 2 are used to conduct the numerical tests in the next subsections.

Table 3
CPU Time for IEEE 6-, 24-, 39-, and 118-Bus test system.

Test System	Average CPU time over six instances (Seconds)					
	Linear WTP		Exponential WTP		Logit WTP	
	M1	SM1	RM2	SRM2	M3	SRM3
IEEE 6-Bus	0.12	0.10	0.09	0.10	0.09	0.11
IEEE 24-Bus	3.27	2.38	3.00	2.35	2.95	2.43
IEEE 39-Bus	43.39	18.32	42.93	20.45	44.57	20.15
IEEE 118-Bus	1.51	1.11	1.69	1.14	1.79	1.16

7.3. Computational efficiency

To investigate the computational tractability of the proposed approach, we conduct a battery of tests for the three proposed PBUC models with respectively linear, exponential, and logit WTP functions and with two formulations, i.e., with valid inequalities (**SM1**, **SRM2**, **SRM3**) and without (**M1**, **RM2**, **RM3**) — therefore six formulations. We consider five test power systems (IEEE 6-Bus, 24-Bus, 39-Bus, 118-Bus test systems as well as the Illinois 200-Bus power network) and generate for each six problem instances, giving us a total of 30 instances. We solve the 30 problem instances with six formulations and analyze below the results obtained for the solutions of the 180 resulting optimization problems. For each problem, we allow one hour of CPU time and use one single thread. As shown below, the problem instances for the 200-Bus power system are very challenging to solve. Therefore, we decompose the analysis of the results into two parts.

Table 3 reports the average CPU time per instance type for the IEEE 6-, 24-, 39-, and 118-Bus test systems. All instances for each WTP function and formulation (with or without valid inequalities) are solved in less than 50 s. The average solution time for the IEEE 6-, 24-, and 118-Bus instances is marginal and varies between 1 and 2 s with or without valid inequalities. Adding the valid inequalities decreases the solution time for the IEEE 24- and 118-Bus instances. The added value stemming from the valid inequalities is most apparent for the IEEE 39-Bus instances. Over these 18 instances, the valid inequalities reduce the solution time by 55%.

The benefits of the valid inequalities are even more visible for the most complex instances of the 200-Bus test system. Table 4 reports the (lower approximation of the) solution time with the two approaches, i.e., with and without valid inequalities. We use the term “approximation” since when optimality could not be proven in one hour, we report “3600” i.e., the total time allowed.

The use of valid inequalities permits solving to optimality twelve of the eighteen problem instances for the 200-Bus test system. The average optimality gap over the six instances not solved to optimality is 0.058%. When valid inequalities are not used, only five instances can be solved to optimality. In these five instances, the speedup provided is of the order of 2.3 as the average solution with and without valid inequalities is 1154 and 2660 s, respectively. For the thirteen other instances, we notice that, by using the valid inequalities, we can systematically find better feasible integer solutions as well as better bounds (when optimality is not proven).

In general, the valid inequalities considerably reduce the solution time and enable finding the optimal solutions within the one-hour limit for problems that were otherwise not solvable. This shows the scalability and efficiency of the proposed framework for modeling

Table 4
Computational results for different problem instances on the Illinois 200-bus test system.

Instance	CPU time (Seconds)					
	Linear WTP		Exponential WTP		Logit WTP	
	M1	SM1	RM2	SRM2	RM3	SRM3
Instance 0	3600	1908	3600	1806	3600	1795
Instance 1	3155	2063	3600	2674	3000	2473
Instance 2	3600	3600	3600	3600	3600	3600
Instance 3	2321	318	2392	510	2434	407
Instance 4	3600	2544	3600	2672	3600	2102
Instance 5	3600	3600	3600	3600	3600	3600

DDU in the WTP the price set by GENCOs. Although the models are extremely complex in their original forms (MINLP problems with non-convex continuous relaxations), our approach solves efficiently large instances. Indeed, the PBUC models with DDU in the WTP can be solved as quickly, if not quicker, as instances of the same size formulated with the traditional deterministic PBUC model (Supplementary Material E) which does not account for elasticity and uncertainty. The computational results are invariant for the three functions (linear, exponential, and logit), which underlines the wide applicability and robustness of our approach. The ability to handle a variety of WTP functions gives the latitude to decision-makers to account for variant price sensitivity and risk attitude.

7.4. Industry insights

We analyze the performance of the PBUC-DDU models via several IEEE benchmark test systems:

- Test Case I (Section 7.4.1) through which the proposed PBUC-DDU models with different WTP functions are solved where the total demand served by the GENCO is composed of fixed and elastic proportions. The results demonstrate the value for the GENCO to account for the demand elasticity and the uncertainty as to whether its price set for the elastic demand is favored in the market when compared to the traditional deterministic PBUC setting.
- Test Case II (Section 7.4.2) through which the impact of elastic demand volume on the performance of the proposed PBUC-DDU models is evaluated.
- Test Case III (Section 7.4.3) through which the impact of price elasticity of the demand on the performance of the proposed PBUC-DDU models is evaluated.

7.4.1. Test case I: Value quantification of PBUC-DDU models

With the growing proportion of flexible price-responsive demand in the electric power industry, the traditional, deterministic (Supplementary Material E) PBUC model may lead to an inaccurate estimation of the GENCO's profit, primarily due to its simplifying assumption that the entire demand is fixed and will be sold at a given price. We postulate that there is value to be gained if GENCOs consider the demand elasticity in pricing and use a WTP-based pricing approach for the elastic demand in the PBUC context. Next, we carry out two analyses to ascertain this assumption.

(1) Value of Decision – Dependent Solutions(V – DDS) : We refer to the added value of using a WTP pricing model for the elastic demand as the *value of the decision-dependent solution* **V-DDS** and propose a formal approach to quantify it. We calculate the added value (i.e., additional profit) of endogenizing the decision and the likelihood that buyers will accept to pay the set price. Accordingly, we compare the proposed PBUC-DDU models that account for demand elasticity with two variants of the deterministic PBUC model (Supplementary Material E) that assumes no elasticity in the demand and that buyers will pay the price set by the GENCOs. More precisely, the price for the demand considered to be elastic in the deterministic PBUC models is set to the:

- Price ρ_t applied to the fixed (non-elastic) demand in the first variant called **D-PBUC-N**. This is justified since GENCOs would price the demand indifferently given the assumption that the entire demand is fixed. This can be viewed as a risk-neutral attitude of GENCO.
- Highest admissible price u_t in the second variant **D-PBUC-T**. This relies on the assumption that there is no elasticity and it thus indicates, from a profit maximization perspective, to set the price as high as possible. This can be viewed as a risk-taker attitude of GENCO.

We proceed as follows to quantify the value of modeling DDU when part of the demand is elastic:

1. Solve the PBUC models **M1**, **RM2**, and **RM3** with DDU in which the demand elasticity is effectively accounted for via the exponential, and logit WTP functions.
   Let Z_1^*, Z_2^*, and Z_3^* be the optimal value of **M1**, **RM2**, and **RM3**, respectively, and $(\mathbf{x}^*, \pi^*)$, $(\mathbf{x}^*, \pi^*, \omega^*)$, and $(\mathbf{x}^*, \pi^*, \upsilon^*)$ be the optimal solution of **M1**, **RM2**, and **RM3**, respectively.
2. Consider the deterministic PBUC models **D-PBUC-N** and **D-PBUC-T** where the elastic demand is set as inelastic. In $(\mathbf{x}^*, \pi^*)$, $(\mathbf{x}^*, \pi^*, \omega^*)$, and $(\mathbf{x}^*, \pi^*, \upsilon^*)$ of **M1**, **RM2**, and **RM3**:
   - Replace $\pi_t, t \in \mathbf{T}$ by: (i) ρ_t and (ii) u_t.
   - Calculate the objective values of the three PBUC-DDU models **M1**, **RM2**, and **RM3**. Denote the resulting values of their objective as: W_i^j with the index $i = 1, 2, 3$ designating the type (**M1**, **RM2**, **RM3**) of PBUC-DDU model and the superscript $j = N, T$ identifying the two variants (**D-PBUC-N**, **D-PBUC-T**) of the deterministic PBUC model.
3. Calculate the value of the decision-dependent solution which quantifies the profit increase brought by accounting for the elasticity of the demand and the DDU regarding whether the buyers will accept to pay the price set by the GENCO:

$$\text{V-DDS}(\pi) = Z_i^* - W_i^j \ , \ i = 1, 2, 3 \ , j = N, T \tag{27}$$

Table 5 reports the results of this analysis for the first risk-neutral variant **D-PBUC-N** of the deterministic approach in which the same price ρ_t is applied to the entire demand. For each test instance, Z_i^* is larger than W_i^N, which confirms the need to account for DDU and demand elasticity. Taking the IEEE 24-Bus test system as an example, the daily additional expected profits, represented by **V-DDS**, amount to $9,480, $2,859, $26,489 for the linear, exponential, and logit WTP functions, respectively, which is equivalent to $3.46, $1.04 and $9.66 millions per year.

Table 6 summarizes the same results for the risk-taker variant **D-PBUC-T** of the deterministic approach in which the price for the elastic demand is set to the alternative largest admissible price. Similar to Table 5, GENCO's daily expected profits are larger when DDU and demand elasticity are properly accounted for (i.e, $Z_i^* > W_i^N$ and $Z_i^* > W_i^T$ for each i). For the IEEE 24-Bus test system, the daily expected profits **V-DDS** reach $19,770, $2,499, $47,017 for the linear, exponential, and logistic WTP functions, respectively, amounting to $7.21, $0.91 and $17.16 millions per year.

The results show that, in the case of elasticity in the demand, not accounting for DDU has negative impacts on the profit and leads to models whose results cannot be trusted. The **V-DDS** analysis shows that GENCOs can increase their expected profits by incorporating DDU in contrast to the standard PBUC models that assume no uncertainty and no demand elasticity

(2) Observed Acceptance (WTP) Probabilities and Elastic Demand Revenues with Deterministic Pricing Schemes : Another way to assess

Table 5
Value of decision-dependent solutions (V-DDS): D-PBUC-N formulation.

Test system	Z_1^*	W_1^N	V-DDS	Z_2^*	W_2^N	V-DDS	Z_3^*	W_3^N	V-DDS
IEEE 6-Bus	154,545	153,593	953	152,877	152,594	283	158,283	155,630	2,653
IEEE 24-Bus	1,720,470	1,710,990	9,480	1,703,589	1,700,730	2,859	1,758,407	1,731,918	26,489
IEEE 39-Bus	931,782	926,510	5,272	922,313	920,591	1,722	953,073	938,600	14,473
IEEE 118-Bus	3,458,520	3,440,074	18,446	3,428,113	3,422,399	5,714	3,526,046	3,476,822	49,224

Table 6
Value of decision-dependent solutions (V-DDS): D-PBUC-T formulation.

Test system	Z_1^*	W_1^T	V-DDS	Z_2^*	W_2^T	V-DDS	Z_3^*	W_3^T	V-DDS
IEEE 6-Bus	154,545	152,535	2,010	152,877	152,621	256	158,283	153,557	4,726
IEEE 24-Bus	1,720,470	1,700,700	19,770	1,703,589	1,701,090	2,499	1,758,407	1,711,390	47,017
IEEE 39-Bus	931,782	921,443	10,339	922,313	921,007	1,306	953,073	928,429	24,644
IEEE 118-Bus	3,458,520	3,421,350	37,170	3,428,113	3,423,260	4,853	3,526,046	3,442,800	83,246

Table 7
Price, probability of acceptance, and elastic demand revenues with WTP pricing models.

WTP function	Average price	Probability of acceptance	Elastic demand revenue
Linear	$41.96	50.91%	$12804.37
Exponential	$41.46	44.92%	$11213.58
Logit	$44.02	60.85%	$15658.16

Table 8
Price, probability of acceptance, and elastic demand revenues with deterministic pricing models.

Deterministic	Average	Probability of acceptance			Elastic demand revenue		
pricing model	price	Linear	Exponential	Logit	Linear	Exponential	Logit
D-PBUC-N	$41.46	47.40%	43.02%	57.51%	$11851.74	$11004.74	$13888.56
D-PBUC-T	$44.02	39.52%	38.18%	46.40%	$10793.65	$11051.84	$11881.05

the value of incorporating a WTP function in the PBUC context is to compute the data-driven WTP probability (and the resulting revenue) corresponding to the price of the elastic demand set by the two deterministic pricing schemes **D-PBUC-N** and **D-PBUC-T** introduced above. We proceed as follows:

1. We solve the optimization problems for the two deterministic pricing schemes **D-PBUC-N** and **D-PBUC-T** that do not include a WTP function.
2. Based on the data used in this study, we calculate with the three proposed WTP functions the acceptance probability of the price charged for the elastic part of the demand with the **D-PBUC-N** and **D-PBUC-T** models.
3. We compare the acceptance (WTP) probability and the price obtained with the **D-PBUC-N** and **D-PBUC-T** models to those obtained with the three WTP functions and models.
4. We calculate the revenues that would be generated with the **D-PBUC-N** and **D-PBUC-T** models based on the data-driven estimate of the acceptance (WTP) probabilities and compare the resulting revenues with those obtained with the three WTP pricing models.

We provide the results of the above procedure for the IEEE 6-bus test system. Table 7 reports the average price (over 24 periods) charged for the elastic portion of the demand, the WTP probability, and the revenue generated by the elastic portion of the demand with the WTP models.

Table 8 reports the average price (over the 24 periods) charged for the elastic portion of the demand, with the deterministic pricing models **D-PBUC-N** and **D-PBUC-T**, as well as the data-driven estimated WTP probabilities and generated revenues corresponding to these models.

Obviously, the comparison between the **D-PBUC-N** and **D-PBUC-T** models shows that the average price is larger and the average WTP probability is lower (since monotone decreasing in price) with **D-PBUC-T**. However, the difference in the three types of WTP probabilities varies quite significantly – ranging from 4.84% for the exponential WTP function, 7.88% for the linear WTP function, to 11.11% for the logit WTP function – which also translates into markedly different revenues generated from the elastic portion of the demand.

For the linear and logit WTP function, the larger price of the **D-PBUC-T** is not sufficient to compensate for the lower WTP probability and the **D-PBUC-N** model gives respectively 9.80% and 16.90% additional elastic demand revenues. Otherwise, for the exponential WTP function, the price difference between the **D-PBUC-T** and **D-PBUC-N** models is just sufficient to make the **D-PBUC-T** more beneficial (i.e., 0.43% additional revenue from elastic demand).

The last part of the analysis highlights the benefits of incorporating the WTP function in the PBUC model. Using the data-driven estimate of the WTP probabilities corresponding to the deterministic pricing models **D-PBUC-N** and **D-PBUC-T**, we compare their resulting revenues for the elastic portion of the demand with those obtained with the three proposed PBUC models with decision-dependent uncertain WTP probabilities. While the difference between the deterministic pricing approaches and the WTP pricing approaches is limited for the exponential WTP function (i.e., 1.90% and 1.46% additional benefits with WTP model as compared to the **D-PBUC-N** and **D-PBUC-T** models, respectively), the additional revenues with the WTP pricing approach are significant for the linear WTP function (8.04% and 11.63% additional benefits with respect to **D-PBUC-N** and **D-PBUC-T**) and particularly exacerbated with the logit WTP function, culminating to 12.74% and 31.79% additional benefits with respect to **D-PBUC-N** and **D-PBUC-T**.

7.4.2. Test case II: Impact of elastic demand volume

Here, we aim to analyze the performance of the proposed PBUC-DDU analytics under variations in the elastic demand volume in the system and investigate the decisions suggested by various WTP functions in the PBUC-DDU problem. We analyze five scenarios in which the elastic demand volume has varied from a base level in the studied networks (the data of which is provided in Dehghanian, 2021) by factors of −20% (S1), -10% (S2), 0% (S3), +10% (S4), and +20% (S5). Table 9 (in Supplementary Material F) reports the GENCO's profit

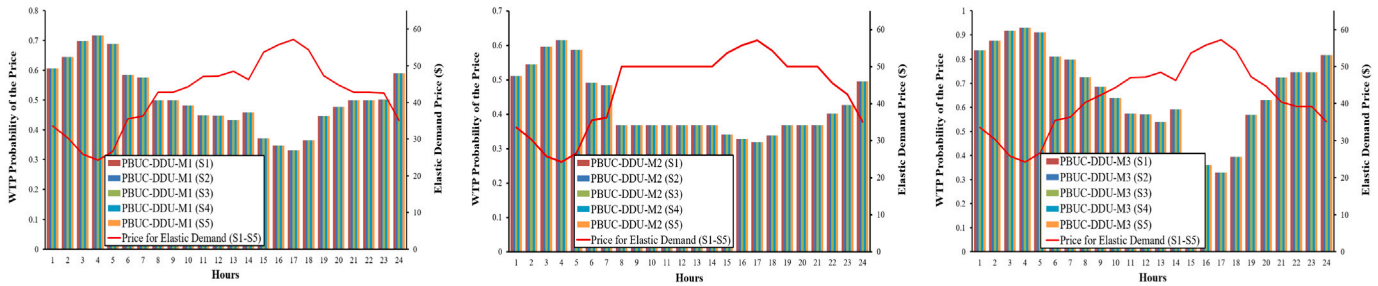

Fig. 2. Hourly WTP Probability of Price under Different Proportions of Elastic Demand and Different PBUC-DDU Models: IEEE 24-Bus System.

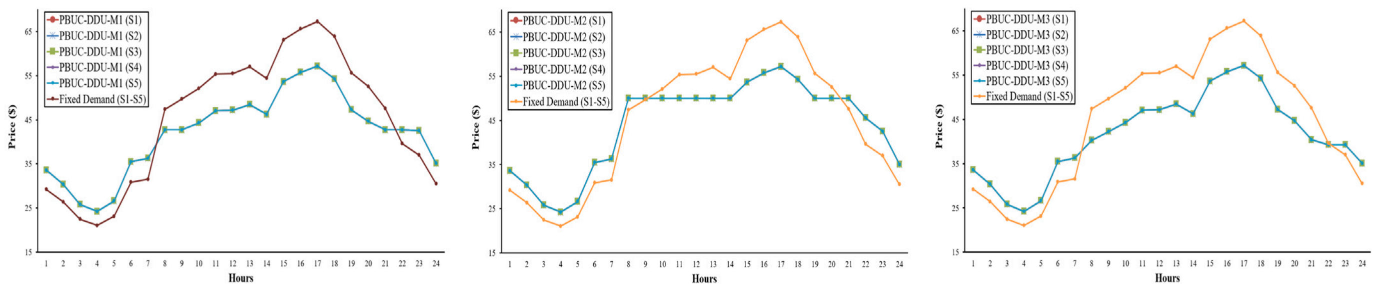

Fig. 3. Hourly Price Signals for Fixed and Elastic Demands under Different Elastic Demand Proportions and PBUC-DDU Models: IEEE 24-Bus System.

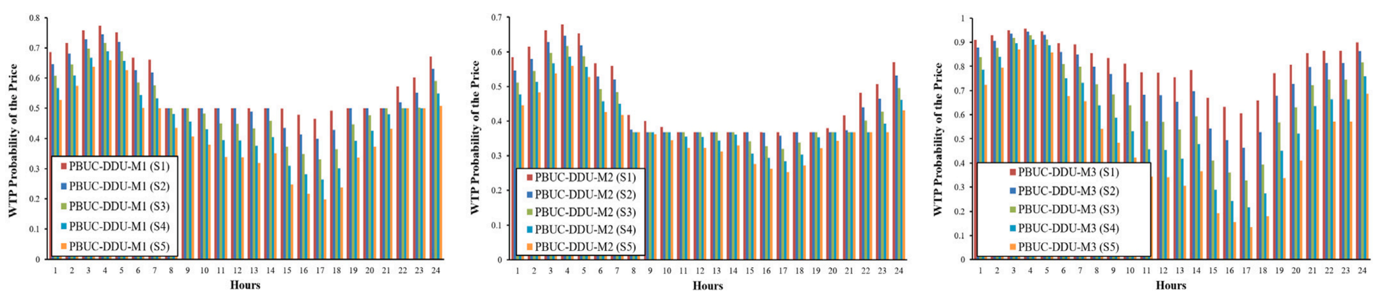

Fig. 4. Hourly WTP Probability of Price Set for Elastic Demand under Different Price Elasticity Levels and PBUC-DDU Models: IEEE 24-Bus System.

(objective function) achieved when the proposed PBUC-DDU models are applied to each of the studied test systems.

Across all test systems, one key observation prevails: as the elastic demand volume increases (from S1 to S5), the GENCO's estimated profit from PBUC models decreases. Further investigating this observation, one can see that as the elastic demand volume increases (e.g., from S1 to S2), the cost increments (i.e, last term in (2a)) for generating the additional volume surpasses the revenue increase (i.e., first term in (2a)). It is worth noting that the profit decrease consecutive to a larger demand volume is a unique characteristic of elastic demand. The elastic demand is the only portion of the demand (versus the fixed portion) that can be declined in the market while the GENCOs still need to pay the generating cost regardless of the market response.

We focus on the IEEE 24-Bus test system, and analyze further the observations made when the elastic proportion of the demand varies. Fig. 2 illustrates the WTP probability of the price set for the elastic demand in the studied scenarios and when different WTP functions are applied. The results demonstrate that the WTP probability is, as expected, higher in off-peak hours (e.g., hours 1–7, 21–24) and gets lower as the price increases (e.g., hours 15–19). One can also observe from Fig. 2 that the price acceptance (WTP) probability at each hour is invariant and not correlated with the changes in the volume of the elastic demand, i.e., as the proportion of the elastic demand in the system increases (from S1 to S5), the WTP probability remains unchanged.

Fig. 3 provides some insights into the hourly price signals for the elastic and fixed portions of the demand under different scenarios of elastic demand volume (S1–S5) and different PBUC-DDU models. The results presented in Fig. 3 illustrate that the price signals for the fixed and elastic demands remain constant at each hour as the volume of the elastic demand changes in the studied scenarios. Additionally, one can observe that the hourly price signals generally follow a similar trend when different PBUC-DDU models are applied. The price for the fixed demand is generally higher than that of the elastic demand during the peak hours while the price set for the elastic demand exceeds that of the fixed demand during off-peak hours.

7.4.3. Test case III: Impact of price elasticity of the demand

Here, we aim to analyze the performance of the proposed PBUC-DDU analytics under variations in the price elasticity of the demand and investigate the effect of the various WTP functions on the final decisions. We analyze five scenarios in which the price elasticity of the demand changes, from a base value presented in Section 7.2, by -20% (S1), -10% (S2), 0% (S3), +10% (S4), and +20% (S5). Table 10 (in Supplementary Material F) demonstrates the GENCO's profit (objective function) achieved when the proposed PBUC-DDU models are applied to each of the studied test systems. Across all tested power systems, one can observe that as the price elasticity of the demand increases (from S1 to S5), GENCO's estimated profit achieved from the PBUC models decreases.

Again, we focus on the IEEE 24-Bus test system and provide a detailed analysis of the observations made when the price elasticity of demand varies. Fig. 4 illustrates the acceptance probability of the price set for the elastic demand in the studied scenarios among the three WTP models. The results demonstrate that the WTP probability is, as expected, higher during off-peak hours when a lower price is set for the elastic demand. Fig. 4 shows that the WTP probability at each hour generally decreases as the price elasticity increases and the minimum WTP probability is typically achieved in S5 which features the highest price elasticity (+20%).

Figures 8–10 (in Supplementary Material G) illustrate the hourly price signals decided for the elastic and fixed portions of the demand under different scenarios of price elasticity levels (S1–S5) and the PBUC-DDU models with three WTP functions. When the linear and exponential WTP models are used, results show that the hourly price for elastic demand changes as the price elasticity levels vary. Different from the other two models, the logit WTP model has price signals for elastic and fixed portions of the demand that are not influenced by the changes in the price elasticity levels (S1–S5) — see Figure 10. As expected, the price for fixed demand is generally higher than that of the elastic demand during peak hours. Figures 8–10 (in Supplementary Material G) provide a closer look at the correlations of the hourly WTP probability and the price signals set for the elastic demand as the price elasticity increases and when the PBUC-DDU models are used.

8. Conclusion

This study proposes new stochastic PBUC optimization problem formulations that effectively capture the demand elasticity and the corresponding DDU in the WTP the price set for energy and can be used for pricing decisions by GENCOs. The resulting PBUC-DDU problems, originally taking the form of stochastic non-convex MINLP models, are converted into equivalent convex deterministic reformulations through a concavification approach. The continuous relaxations of the reformulations are tightened with valid inequalities. Parameterizing the WTP functions via a learning framework and introducing a new concept of the value of the decision-dependent solution, we test the performance and computational tractability of the suggested PBUC-DDU models on different test systems and under a variety of scenarios. Extensive numerical results demonstrate that (i) taking into account the participation of elastic demand in the PBUC models and whether they accept the price set by the GENCOs would enable a strategic revenue management practice that affects the GENCO's decisions regarding pricing and the schedule of the generating units during peak/off-peak hours and the estimated profits, (ii) different PBUC models proposed to account for the WTP response of buyers result in different profit estimations, capturing different risk attitudes of the decision maker, (iii) the valid inequalities considerably strengthen the continuous relaxation of the models and speed up their solutions, and (iv) the suggested approach in quantifying the value of DDU considerations about the WTP provides insights for the GENCO to account for buyers' responses and to select the right PBUC-DDU model and pricing scheme. Future research could explore risk-averse models for GENCO's decision-making and the use of state-of-the-art machine learning techniques for constraint learning in the context of the proposed optimization problems.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ejor.2023.12.006.

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