

Revisiting the Water Permittivity: 0-50 GHz Measurements at Temperatures up to 50 °C

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Abstract—Measurements of the complex permittivity of water at specific frequencies and temperatures have been used to develop analytical models for the water permittivity in decades-old studies. This work provides an update to those models based on a set of recent permittivity measurements for distilled water at temperatures between 0 °C and 50 °C (from 273.15 K to 323.15 K) across the microwave frequencies from 0.1 GHz to 50 GHz. Measurements have been performed with the state-of-the-art equipment and a new water permittivity model has been presented as a function of frequency and temperature. It has been noted that the new permittivity model implies higher electromagnetic loss, especially at low frequencies and high temperatures which might have significant ramifications in various fields from environmental microwave remote sensing to biomedical and healthcare applications.

Keywords— water; permittivity; frequency; temperature.

I. INTRODUCTION

Water is one of the most abundant molecules on Earth with great significance. Its relative complex permittivity helps us to understand the microwave propagation in water or water-carrying media; thus, needs to be characterized as a function of frequency and temperature. Previous studies, often decades-old, have reported measurements of the static permittivity of water versus temperature [1], complex permittivity measurements across a range of microwave frequencies at a fixed temperature [2], and indirect measurements of the water permittivity versus frequency and temperature with non-uniform frequency and temperature intervals and resolutions [3]. On the other hand, a complete set of water permittivity measurements with the state-of-the-art equipment is missing in the literature. To fill this gap, this paper presents the relative complex permittivity of distilled water at frequencies up to 50 GHz and temperatures up to 50 °C. A revised analytical permittivity model is also given, and the differences between the current and the new model are discussed in the context of important practical applications.

II. MEASUREMENT SETUP AND CALIBRATION

A Keysight N9952B FieldFox 50 GHz Microwave Analyzer connected to a slim form coaxial probe through a 50 GHz flexible cable was used to measure the water complex permittivity. The probe immersed in water was placed in a TestEquity 1007H Temperature/Humidity

chamber which controlled the temperature, and Keysight N1500A Materials Measurement Suite has provided the real and imaginary parts of the relative permittivity values versus frequency. Measurements were performed at temperatures from 0 °C to 50 °C with 5 °C increments and at frequencies from 0.1 GHz to 50 GHz with 0.1 GHz resolution. At each temperature the system was recalibrated before taking measurements. Fig. 1 shows the measurement setup as a simplified diagram.

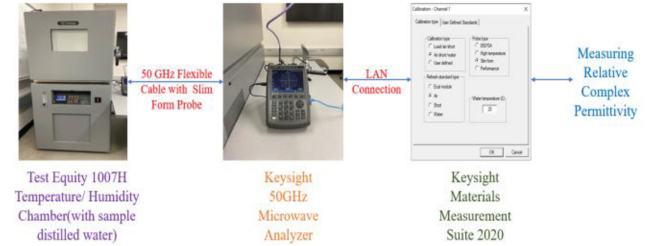


Figure 1. A simplistic view of the measurement setup used in this study to measure the relative complex permittivity of water.

III. RESULTS

The relative complex permittivity of water can be expressed as a function of frequency and temperature with the Debye model as follows:

$$\varepsilon(\omega, T_{\text{water}}) = \varepsilon_{\infty}(T_{\text{water}}) + \frac{\varepsilon_0(T_{\text{water}}) - \varepsilon_{\infty}(T_{\text{water}})}{1 + j\omega\tau(T_{\text{water}})} \quad (1)$$

where ω is the angular frequency, and $\varepsilon_0(T_{\text{water}})$, $\varepsilon_{\infty}(T_{\text{water}})$, and $\tau(T_{\text{water}})$ are the low frequency relative permittivity, extrapolated high frequency relative permittivity, and the relaxation time, respectively, which are all temperature dependent. Permittivity data measured by the microwave analyzer have been cleaned to remove the outliers detected as clear measurement errors and ε_0 , ε_{∞} , and τ in the Debye model have been extracted through data fitting minimizing the root-mean-square errors (RMSE). (2), (3), and (4) describe these three parameters as functions of water temperature, T_{water} , where all of them decrease as the temperature increases:

$$\varepsilon_{\infty}(T_{\text{water}}) = 8.592 - 0.04787 \times (T_{\text{water}}) \quad (2)$$

$$\varepsilon_0(T_{\text{water}}) = 10^{(1.9647 - 0.0025 \times (T_{\text{water}}))} \quad (3)$$

$$\tau(T_{water}) = 1.517 \times 10^{-11} \times e^{-0.01764 \times (T_{water})} \quad (4)$$

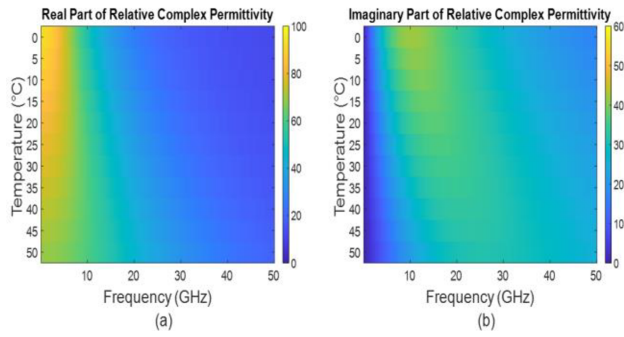


Figure 2. (a) Real and (imaginary) parts of the relative permittivity of water as functions of frequency and temperature as measured in this study.

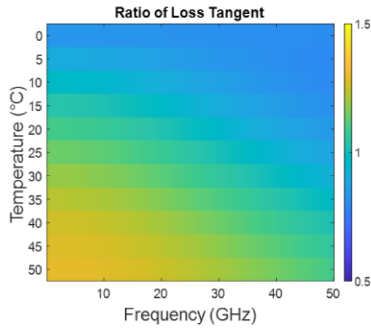


Figure 3. Ratio of the loss tangent values based on this study to those calculated with the permittivity models in the literature ($\delta_{new}/\delta_{literature}$) as a function of frequency and temperature.

Putting these expressions in (1), the real and imaginary parts of the relative complex permittivity of water, ϵ' and ϵ'' , respectively, have been calculated as functions of temperature and frequency as shown in Fig. 2. The dielectric constant ϵ' decreases as frequency increases at all temperatures but the rate of decrease is higher at lower temperatures. It is also important to note that ϵ' decreases with temperature at lower frequencies whereas this trend is reversed at higher frequencies. The dielectric loss factor ϵ'' , on the other hand, reaches its maximum values at mid-frequencies where the peak diminishes and slowly shifts towards higher frequencies as temperature increases. These measurements show overall similar trends with the current permittivity models available in the literature [3], but the actual permittivity values differ which can be seen by the differences in the loss tangent (δ) values. Fig. 3 demonstrates the ratio of the loss tangent values based on this study to those calculated with the permittivity models in the literature ($\delta_{new}/\delta_{literature}$) as a function of frequency and temperature. It can be seen that the loss tangent values based on our measurements are higher, especially at higher temperatures and lower frequencies.

IV. CONCLUSIONS AND DISCUSSION

Table I shows the effective electromagnetic penetration depths calculated at two sets of frequencies

and temperatures with the loss tangent values derived in this study and those based on the models available in the literature. The frequency and temperature values have been selected carefully to explore the consequences of the differences in real applications as L-band instruments operating at frequencies around 1.4 GHz are deployed in ocean remote sensing applications [4], and $\sim 35^\circ\text{C}$ human body is monitored via microwave radiometers with a 1.2 GHz frequency channel [5]. The table indicates $\sim 15\text{-}20\%$ decrease in the penetration depths in both cases which may impact retrieval of important parameters such as ocean salinity and human body temperature. More examples can be given for other microwave applications at different frequencies and temperatures, but it is clear that the revisions in the water permittivity model could lead to significant changes in the electrical properties of various media which may improve the accuracy of microwave sensing in various fields.

Future research will include increasing the temperature range and resolution in water permittivity measurements, as well as utilizing the measurement setup for electrical characterization of other materials of importance in microwave applications.

TABLE I. ELECTROMAGNETIC PENETRATION DEPTHS CALCULATED WITH PERMITTIVITY VALUES MEASURED IN THIS STUDY AND THOSE AVAILABLE IN THE LITERATURE

Frequency & Temperature	This Study	Literature
1.4 GHz, 25 °C	4.92 cm	5.68 cm
1.2 GHz, 35 °C	8.2 cm	10.09 cm

ACKNOWLEDGMENT

This material is based on the work supported by the National Science Foundation under Grant no. 2143592.

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