The Left Digit Effect in an Unbounded Number Line Task

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Abstract

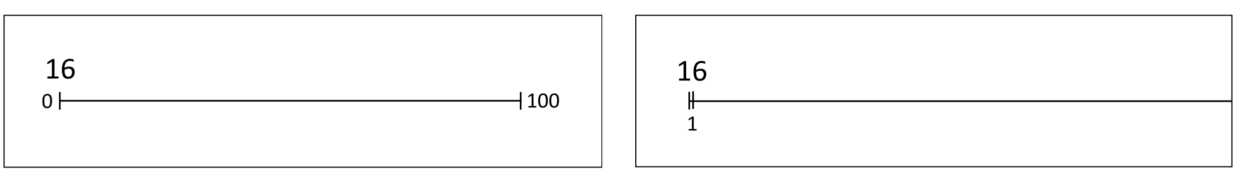
The left digit effect in number line estimation refers to the phenomenon where numerals with similar magnitudes but different leftmost digits (e.g., 19 and 22) are estimated to be farther apart on a number line than is warranted. The effect has been studied using a bounded number line task, a task in which a line is bounded by two endpoints (e.g., 0 and 100), and where one must indicate the correct location of a target numeral on the line. The goal of the present work is to investigate the left digit effect in an unbounded number line task, a task that involves using the size of one unit to determine a target numeral's location, and that elicits strategies different from those used in the bounded number line task. In a preregistered study, participants ($N = 58$ college students) completed four blocks of 38 trials each of an unbounded number line task, with target numerals ranging between 0 and 100. We found a medium and statistically reliable left digit effect ($d = 0.70$). The study offers further evidence that the effect is not driven by response strategies specific to the bounded number line task. We discuss other possible sources of the effect including conversion of symbols to magnitudes in these and other contexts.

Number line estimation tasks are simple-to-administer tasks that have been widely used to study number skills. In the most common version, the *bounded number line task* (see Figure 1a), one is presented with a horizontal number line with labeled bounded endpoints (e.g., 0 and 100; or 0 and 1000) and asked to estimate the location of a given target numeral on the line (e.g., Barth & Paladino, 2011; Siegler & Opfer, 2003). In another version, called the *unbounded number line estimation task* (Cohen & Blanc-Goldhammer, 2011; see Reinert & Moeller, 2021, for review), one is presented with a number but is shown the width of one unit on the line rather than the line's endpoints and asked to estimate the location of the target numeral (see Figure 1b). Measures of overall performance, which typically reflect the difference between placements and correct locations, are used to evaluate whether task performance is related to other number competencies. Such measures predict a wide range of mathematical skills in children (see Schneider et al., 2018, for review) and numeracy skills in adults (e.g., Patalano et al., 2020; Schley & Peters, 2014), making number line estimation tasks valuable tools for assessing and training number skills. The present work focuses on number line estimation in adults.

Figure 1

Schematics of (a) Bounded and (b) Unbounded Number Line Tasks

(a) (b)



Note. The task is to click with a mouse on the location on the line where the target numeral belongs. A vertical line then appears at the clicked location.

Modeling of placements in number line estimation

Placements in number line estimation tasks were once treated as a direct reflection of one's psychological representation of magnitude (e.g., Booth & Siegler, 2006; Dehaene et al., 2008). They are now more often thought to reflect both systematic error in magnitude estimation, called *magnitude estimation bias*, and specific strategies used to perform the task (e.g., Barth & Paladino, 2011; Cohen et al., 2018; Cohen & Sarnecka, 2014; Rouder & Geary, 2014; Slusser et al., 2013; but see Opfer et al., 2016). Magnitude estimation bias (specifically, bias in translating between numerals and line lengths) has been modeled using a power function (see Figure 2a; e.g., Cohen & Blanc-Goldhammer, 2011; Slusser & Barth, 2017). However, for the bounded number line, the actual placement pattern for adults is more often S-shaped, as in Figure 2b (*one-cycle* curve; e.g., Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011). This pattern is thought to arise because people use a strategy of *proportion judgment* to perform the task (e.g., judging 82 as a proportion of the space between the 0 and 100 endpoints) with biased estimates of magnitude (see Cohen et al., 2018, for full discussion; see Spence, 1990, for proof; see also Hollands & Dyre, 2000).[1] More complex patterns (e.g., *two-cycle* curve in Figure 2b) arise when additional reference points are used (such as the midpoint of the line). In proportion judgment models, magnitude estimation bias cannot be directly observed but is instead inferred from placement data (specifically, the bias parameter β is estimated when the curve in Figure 2b is fit to placement data), and is typically positively accelerating (i.e., as target numeral increases, overestimation increases; Barth & Paladino, 2011; Cohen et al., 2018; Patalano et al., 2023).[2]

[1] The strategy used to perform the task is also often referred to as a *subtraction-division* strategy because either of these arithmetic skills can be used to place the target in relation to the two boundary values.
[2] One might expect this bias to be negatively accelerating, especially based on some past work with children (e.g., Siegler & Opfer, 2003). However, this is not what is typically found for adults once task strategy is considered and experimental bias is reduced (e.g., Cohen & Blanc-Goldhammer, 2011; see Cohen & Ray, 2020 for discussion).

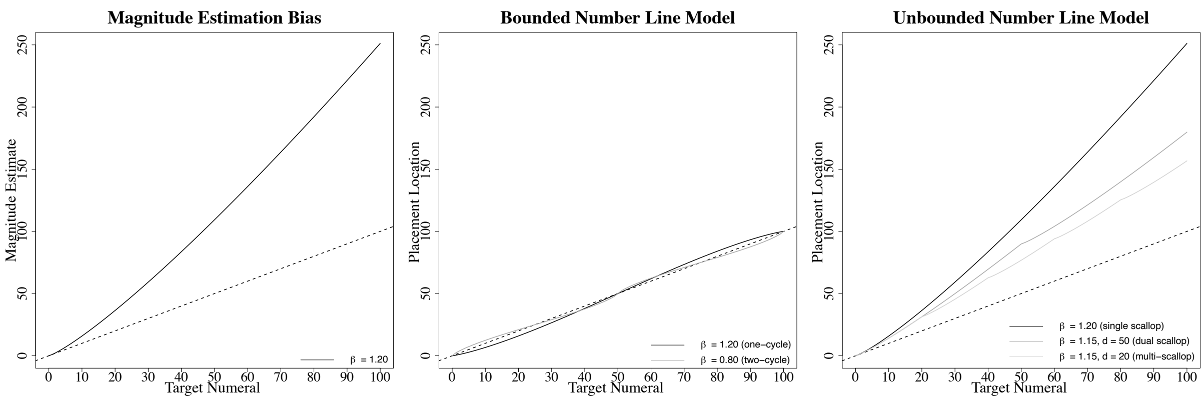
For the unbounded number line task, placement has been modeled as magnitude estimation bias combined with use of a different task strategy than what is used for the bounded number line. Placements have been shown to generally follow a convex curve as in Figure 2c (called a *single scallop* curve; Cohen & Blanc-Goldhammer, 2011; Cohen et al., 2018; Reinert et al., 2019). The pattern is thought to arise when biased estimates of magnitude are placed directly (a *direct* strategy), rather than being used in proportion judgment. One or more additional scallops (small displacements in the curve) can arise when an *additive* strategy is also used where one estimates part of the magnitude and then adds the remaining part. For example, one might place 34 by first estimating the location of 10 and then estimating the length of 24 additional units (*dual scallop* curve).[3] Or, one might instead estimate the location of 10, iterate this three times, and then estimate 4 additional units (*multi-scallop* curve; see Cohen et al., 2018, for details). Because these strategy-related influences on placements are more minimal than those on the bounded number line task, the unbounded task is sometimes considered a more direct measure of magnitude estimation bias (see Reinert & Moeller, 2021, for review; but see Kim & Opfer, 2017). Overall, given very different patterns of placements across tasks (Figure 2b vs. 2c), bounded and unbounded number line tasks appear to be performed quite differently.

[3] Put differently, this means estimating the length of 10 units, then estimating the length of 24 units, and then adding the second length to the end of the first length. It does not mean iterating by single units.

Figure 2

Examples of (a) Modeled Bias in Magnitude Estimation, (b) Bounded Number Line Placement Pattern, and (c) Unbounded Number Line Placement Pattern

(a) (b) (c)



Note. The solid curve in 1a is described by $y = x^{\beta}$, which is convex when $\beta > 1$, the latter reflecting accelerating magnitude estimation bias. The solid curves in 1b reflect two common placement patterns (for adults) on the bounded number line task: one-cycle ($y = x^{\beta}/(x^{\beta} + (100 - x)^{\beta}) * 100$), and two-cycle (see Barth & Paladino, 2011, for equation). The solid curves in 1c reflect the three most common placement patterns on the unbounded number line task: single scallop ($y = x^{\beta}$), dual scallop ($y = x^{\beta}$ when $x < d$, and $y = d^{\beta} + (x - d)^{\beta}$ when $x > d$), and multi-scallop (y = truncate (x/d, 0) * d^{β} + (x modulo $d)^{\beta}$; see Cohen et al., 2018, for details).[4] The dashed line on each graph is the identity line (reflects perfect accuracy).

[4] All parameter estimates reflect known typical values except *d*. It is not known what value *d* might take for a 0-100 number line, so we selected plausible values. (It has been found that $d \approx 10.6$ for dual and multi-scallop patterns on a 0-25 number line, Cohen & Blanc-Goldhammer, 2011, but we suspect *d* may be larger for the 0-100 range.)

The left digit effect in number line estimation

Implicit in most research on number line estimation is that placements are a function of the overall magnitudes of target numerals (e.g., Cohen & Blanc-Goldhammer, 2011; Siegler & Opfer, 2003; Slusser & Barth, 2017). For example, 59 and 61 are predicted to have similar placements because the numerals are close in overall magnitude. However, it has also been revealed that the individual digits that comprise numerals influence placements beyond their expected contribution to overall magnitude. Using a 0-1000 bounded number line, Lai et al. (2018) found that numerals on either side of hundreds boundaries are placed farther apart than is correct (e.g., 298 is placed too far to the left of where 301 is placed), but that this is not the case across fifties boundaries, where the leftmost digit does not change (e.g., 248 is placed appropriately close to 251). The effect size is large (*d*s ≈ 1.00 in adults; Kayton et al., 2022; Lai et al., 2018), and most individuals show a pattern consistent with the effect (e.g., 89% in Williams et al., 2022a).

This *left digit effect* has now also been observed on a 0-100 bounded number line, where numbers on either side of tens boundaries (e.g., 19 and 21) are placed farther apart than is correct (*d*s ≈ 0.50 and ~70% of individuals show the pattern; e.g., Patalano et al., 2023; Williams et al., 2022b). In addition, the effect arises with descending (e.g., 1000-0) number ranges (Williams et al., 2022a), and when there is feedback and motivational incentive to be accurate (Kayton et al., 2022; Williams et al., 2022a). Further, although the left digit effect is typically assessed using pairs of numerals around boundaries, it can be well characterized as an overweighting of leftmost digits more generally. Specifically, in the context of the 0-100 bounded number line, it has been found that placements of numerals with the same leftmost digit are compressed towards their lower digit boundary (e.g., placements of 21-29 are compressed towards the placement of 20; 31-

39 are compressed towards 30, etc.; Patalano et al., 2023). The presence of the left digit effect is important in that it suggests that placement is not only a function of a numeral's overall magnitude, but also depends on individual digits.

Thus far, the left digit effect has only been tested using the bounded number line. The goal of the present work is to test whether the left digit effect also emerges in the unbounded number line task. As we described, relative to the bounded task, the unbounded number line task is thought by many to draw on different strategies that require different skills; it elicits a different pattern of performance and it is often considered a more direct measure of magnitude estimation bias. Therefore, it is a useful context for testing the generality of the left digit effect and for gaining clues as to the source of the overweighting of leftmost digits, which is not yet known. If the left digit effect depends on the proportion judgment strategy used in the bounded number line task, it should not emerge in the unbounded context. If it emerges across a variety of tasks that evoke diverse strategies, it should emerge in the unbounded number line task as well. Evidence of a left digit effect in the unbounded task would suggest that rather than being associated with use of a proportion judgment strategy, the effect may have a source that is common across tasks, such as bias in the translation of numerals to magnitudes (as proposed by, e.g., Thomas & Morwitz, 2005, 2009; Patalano et al., 2023). Knowing whether there is a left digit effect in the unbounded task is also important for assessing, interpreting, and modeling performance, and for predicting the types of everyday contexts in which a left digit effect is likely to emerge.

Method

Participants

Participants were 58 undergraduates (31 women, 26 men, 1 undisclosed) who received course credit or monetary compensation for their participation. The size of the sample was sufficient to detect an effect size of $d = 0.38$ ($\alpha = .05$; power = .80), less than or equal to the $d \approx$ 0.38 – 0.62 from past 0-100 bounded number line studies (Williams et al., 2022b). The study was approved by the Wesleyan University Institutional Review Board. Methods and planned analyses were preregistered at: https://aspredicted.org/VNB_QZK.

Procedure

All participants completed four blocks (38 trials each) of an unbounded number line task with target numerals ranging between 0 and 100 presented in a different random order for each block and participant. The study was administered through lab.js computer software (lab.js. org; Henninger et al., 2019) in a lab setting. The task (as in Figure 1b) was to click with the mouse at the location on the line corresponding to the presented numeral given the size of one unit. The width of one unit on the number line was varied across trials (8, 10, 12, or 14 pixels) so that widths were used with similar frequency. (See Performance Measures section for how unit widths were matched across directly compared trials.) Following a response, mouse click location (and also response time) was recorded, and a "Next" button appeared centered at the bottom of the screen to advance to the next trial. A 500 ms blank screen separated trials. Participants were encouraged to take short breaks between blocks. Participants were not informed in advance of the target number range (to ensure that the task could not readily be treated as a bounded number line task), and they were not able to adjust initial placements.

Stimuli

Towards assessing the left digit effect, we used stimuli that included pairs of numerals on either side of tens boundaries (boundaries 20, 30, 40, 50, 60, 70, 80, and 90)[5] because they have

[5] As in past work (e.g., Williams et al., 2022b), the boundary of '10' was not used in left digit analyses, in case single-digit target numerals (numerals below the boundary) are evaluated differently than two-digit ones.

different leftmost digits (e.g., 19/22). We compared placements of these numerals to similarly spaced pairs of numerals that do not cross a left digit boundary (e.g., 24/27). Specifically each of four blocks contained the same 38 target numerals: 16 boundary targets (8 pairs of numerals that crossed tens boundaries and were three units apart: 19/22, 28/31, 39/42, 48/51, 58/61, 68/71, 79/82, 89/92), 18 nonboundary controls (9 pairs of numerals that crossed fives boundaries and were three units apart: 13/16, 24/27, 33/36, 44/47, 53/56, 63/66, 74/77, 84/87, 93/96), and 4 filler (4, 7, 9, 12; i.e., pairs containing single-digit values). All pairing was for the purposes of analyses only; numerals were presented individually and in a random order.

Display Specifications

The computer used was a 27-in. iMac (with screen resolution of 5120 x 2880 px; 218 dpi). The size of the canvas, a rectangle with a black border in the center of the screen, in which stimuli were displayed was 5011 x 2771 px. On each trial, a target numeral (e.g., '47'; 0.2 in. tall) was centered 0.4 in. above the left endpoint of a black horizontal line, as shown in Figure 1b. The horizontal line began 1.5 in. from the left side of the canvas and extended to the end of the canvas on the right side (a total of 21.4 in. or 4665 px). A vertical line 0.12 in. long and 0.02 in. (4 px) wide at the left boundary indicated "0" and another vertical line of the same size either 8, 10, 12, or 14 px (~0.04 – 0.06 in.) to the right of 0 indicated the width of one unit.[6] Directly below the rightmost vertical line was the label "1" indicating one unit. (The location of 100 on the line would be approximately one-fourth to one-third of the distance across the screen from the left side to the right.) Once a participant clicked on the number line to indicate their response, a black vertical line appeared in the selected location. The location on the line was recorded as a

[6] Using the formula of Cohen and Ray (2020), we calculated that a canvas of the size we used (5011 x 2771 px) with a starting point at 327 px, unit lines 4 px wide, and a unit width of a maximum of 14 px, could accommodate a 0-108 number line task (i.e., larger than the 0-100 line used here), assuming accelerating bias of $\beta = 1.2$.

value between 0 and 4665 px. This value was then divided by the number of pixels per unit for that trial to get the number corresponding to the selected location on the line.

Performance Measures

Overall accuracy error. Overall accuracy error was assessed using *percent absolute error (PAE),* a commonly used measure calculated as |*placement – correct location*| / 100 * 100, averaged over all trials (see, e.g., Booth & Siegler, 2006). PAE was computed for each block individually (using all 38 trials), and overall (by averaging the PAE across the four blocks). PAE was used to assess general performance by participants on the task.

Left digit effect. We adapted a previously used measure of the left digit effect to the present context. For the 0-100 bounded number line, the left digit effect is assessed using a *tens difference score* computed as *(placement of above-boundary target – placement of below-boundary target) – true difference between targets,* averaged across tens boundaries (e.g., Vaidya et al., 2022; Williams et al., 2022b). A score significantly greater than 0 indicates a left digit effect because it means that targets like 49 and 51 are placed farther apart then the true distance (e.g., of 2 units). However, recall that there is a positively accelerating placement bias for the unbounded line that is unrelated to the left digit effect. The positively accelerating placement bias signifies that as one moves to the right on the line, numerals are placed farther apart (as reflected in the curves in Figure 2c). Consequently, the distance between the actual placements of any two targets will always be larger than the true distance (e.g. 59 and 61 will be placed more than 2 units apart) and will increase as target magnitude increases (e.g., 59 and 61 will be placed farther apart than 54 and 56, but less far apart than 64 and 66). In order to isolate any left digit effect, the equation for the tens difference score must be constructed by subtracting out differences in placements due to this accelerating bias.

To do this, we computed the tens difference score here as: *(placement of upper boundary target – placement of lower boundary target) – (placement of upper nonboundary target – placement of lower nonboundary target)*. The equation was computed in Block 1 using the nonboundary pair falling below each boundary pair, and in Block 2 using the nonboundary pair above each boundary pair. Unit width was held constant across the eight trials used in the computations for each boundary. The scores from the two blocks were then averaged to get a complete tens difference score for each boundary. We used the combination of above-boundary and below-boundary controls because together they provide the best estimate of accelerating bias for boundary pairs. We did not use both controls from the same block (an obvious alternative approach) because, if we had, we would not have been able to vary unit width at all in a block (because each nonboundary pair serves as a control for multiple boundary pairs). Blocks 3 and 4 provided a second complete tens difference score for each boundary, which was averaged with the score from Blocks 1 and 2, then averaged across boundaries, to create a robust measure of the left digit effect. See Supplementary materials for individual trial details.

To spell out our computational steps, we did the following with each participant's data. First, we computed a tens difference score for each boundary in each block. We then averaged together the scores for Blocks 1 and 2 to get one *composite tens difference score* for each boundary, and did the same for Blocks 3 and 4 to get a second one. If a participant was missing any of the scores used in a computation up to this point, the boundary was simply excluded from further computations. We next averaged the two composite scores per boundary to get a single *overall tens difference score* for each boundary. Here, because the composite scores were already complete scores, if one composite score was missing, the other one was used alone as the overall tens difference score. Finally, the overall tens difference scores were averaged across boundaries to get one average overall tens difference score per participant. This latter measure was our key measure, and a score greater than zero was taken as evidence of a left digit effect.

Results

Exclusions

The exclusion criteria were preregistered, and were similar to those used in past work with the bounded number line task (Lai et al., 2018; Patalano et al., 2022). A participant's response to a target was excluded from all tens difference score calculations if it differed by more than two standard deviations from the mean response to that target for that block (2.12% of trials were excluded). A participant was excluded entirely from all analyses if more than three boundaries were missing for use in the computation of the average overall tens difference score (n = 1), or if the correlation between their placements and the target values was less than r =.50 (n =1) in any block. A total of 56 participants (of the original 58) were in the final data.

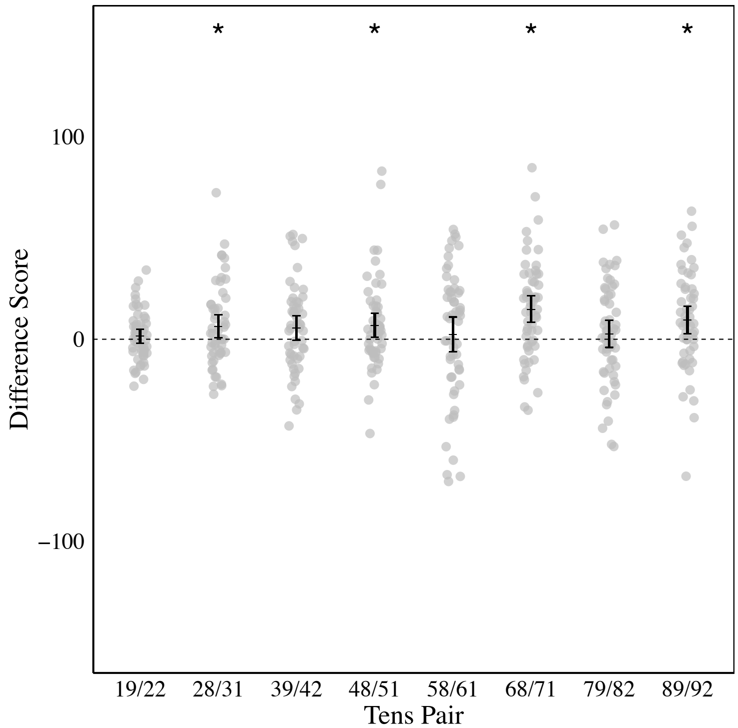
Preregistered Analyses

All t-tests were two-tailed. PAE had a M = 95% (SD = 43, *range* = 11.07 – 163.31). This means that placements were on average about 95 units of width away from the correct location (e.g., 16 might be placed at the location on the line where 111 belongs). Although the focus was not on modeling and we did not have the density of targets to test multi-scalloped patterns of Cohen and Blanc-Goldhammer (2011), we did use the simple $y = x^{\beta}$ (a single scallop) to assess accelerating bias in placement and found $\beta = 1.25$ ($R^2 = .98$), similar to Cohen and Blanc-Goldhammer's work (where $\beta = 1.11$). This is a valuable replication given that only a few studies using unbounded number lines have been sufficiently long to accommodate escalating overplacement (instead, responses had to be compressed into a small space, and thus did not well reflect true bias; see Cohen & Ray, 2020; Reinert & Moeller, 2021, for details).

The average overall tens difference score had a $M = 6.38$ ($SD = 9.83$, *range* = –13.67 – 31.82) and was significantly greater than 0, $t(55) = 4.859$, $SE = 1.31$, $p < .001$, $d = 0.65$, 95% *CI*[3.75, 9.01], consistent with a left digit effect. The majority of participants (42 out of 56; 75%) had an overall tens difference score greater than 0 (by binomial test, $p < .001$). Both the effect size and the percentage of individuals with scores above 0 were consistent with past work with the 0-100 bounded line (*d*s = 0.38 – 0.62 and ~70% in Williams et al., 2022b). We also computed the tens difference score for each tens pair individually, and found that all individual scores were in the predicted direction with several reaching statistical significance (similar to past work; e.g., Patalano et al., 2023). Those reaching statistical significance were at alternating tens boundaries (30s, 50s, etc.; see Figure 3). We do not have an explanation for this possible pattern, but note it for future consideration. There were no gender differences for the average overall tens difference score or PAE ($|t|$s < 1, *p*s > .162). There was also no reliable correlation between average overall tens difference score and PAE, $r(54) = .15$, $p = .266$.

Figure 3

Average Tens Difference Scores by Target Pair



*$p < .05$, two-tailed. *Note.* Bars reflect 95% confidence intervals. Tens difference scores were in the predicted direction (> 0) for all eight pairs, consistent with a left digit effect.

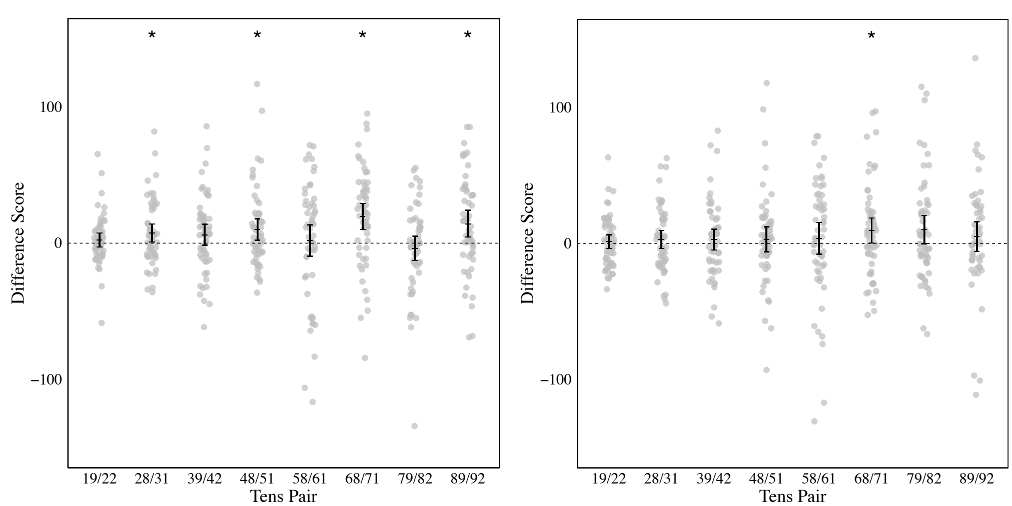
Exploratory Analyses

We considered whether there was evidence of a left digit effect for each composite tens difference score taken separately. The first composite score (Blocks 1 and 2; $M = 7.35$, $SD = 13.91$) and the second composite score (Blocks 3 and 4; $M = 4.83$, $SD = 11.47$) were both greater than 0 (ts > 3, ps < .004, ds = 0.42 – 0.53), consistent with the left digit effect, and did not differ reliably from one another, $t(54) = 1.07$, $SE = 2.39$, $p = .288$. We also considered whether there was evidence of a left digit effect for *below-boundary* blocks (Blocks 1 and 3; where the control targets were below-boundary targets) and *above-boundary* blocks (Blocks 2 and 4; where the control targets were above-boundary targets). The scores for the below-boundary blocks (*M* = 7.31, *SD* = 11.93) and the above-boundary blocks (*M* = 5.02, *SD* = 13.16) were both greater than 0 (*t*s > 2.5, *p*s < .05, *d*s = 0.38 – 0.61), and did not differ reliably from one another, *t*(55) = 1.07, *SE* = 2.15, *p* = .290, although the means were larger for the below-boundary blocks both overall and at individual boundaries (see Figure 4). The pattern is as expected in that the scores were somewhat greater when below-boundary controls were used but the positive tens difference score existed even when above-boundary controls were used and thus the positive score cannot be attributed only to accelerating overplacement.

Figure 4

Average Tens Difference Scores by Target Pair Using (a) Below-Boundary Pairs Versus (b) Above-Boundary Pairs As Controls

(a) (b)



*p < .05, two-tailed. *Note.* Bars reflect 95% confidence intervals. Below-boundary difference scores were in the predicted direction (> 0) for 7/8 pairs, and above-boundary difference scores were in the predicted direction (> 0) for 8/8 pairs, both consistent with a left digit effect.

Discussion

We found a medium left digit effect for adults on a 0-100 unbounded number line, with the pattern in the predicted direction for boundaries across the line and for the majority of participants. There have been several studies showing a robust left digit effect on various versions of the bounded number line (e.g., Kayton et al., 2022; Lai et al., 2018; Williams et al., 2022b), but this is the first evidence of a left digit effect on an unbounded number line task (with $d \approx 0.70$ here, as with the bounded line). Because the unbounded task elicits different strategies than the bounded one (Cohen & Blanc-Goldhammer, 2011; Reinert & Moeller, 2021; Reinert et al., 2019; but see Siegler & Opfer, 2003), the presence of the left digit effect on the unbounded number line task suggests that the left digit effect is not likely the result of strategies specific to the bounded number line estimation task and may instead be driven by mental processes common to both tasks, including those more directly related to translation of numerals to magnitudes. We consider possible sources shortly.

The study is also valuable in providing a method for measuring the left digit effect in the context of the unbounded number line. For past work on the bounded 0-100 line, the left digit effect was assessed by comparing placements of paired targets to the actual difference between the numerals. In the present work using the unbounded line, we instead subtracted placements for similarly spaced control targets that did not cross a boundary, in order to account for accelerating overplacement. While it is not possible to have controls that are precisely matched to boundary pairs, the present method of averaging above- and below- boundary controls offers a reasonable strategy for approximating the difference that should be expected for any boundary pair if there is no left digit effect. That said, based on the present findings, there is no evidence that the effect depends on averaging controls, as the effect emerged even when we used only the above-boundary controls (in Blocks 2 and 4) that worked against our hypothesis.

One possible source of the left digit effect consistent with the present findings is an overweighting of the leftmost digit when numerals are translated into magnitudes (e.g., Thomas & Morwitz, 2005, 2009). A recent model of number-to-quantity conversion (Dotan & Dehaene, 2012, 2020; see also McCloskey, 1992; McCloskey et al., 1986) in which digits are represented individually is useful for considering how overweighting might arise. In the model, digits are multiplied by place units (following base ten rules) to produce place values (e.g., 25 = (2*10) + (5*1)), and the latter are then merged into a whole number quantity. One way overweighting could arise is from use of imprecise place weights (e.g., if the 1 for the ones place were replaced with .8).[7] Such a possibility would be consistent with findings that place value information can be implicitly acquired (Yuan et al., 2020) and is frequently accessed automatically (e.g., García-Orza et al., 2017; Kallai & Tzelgov, 2012; Nuerk et al., 2015). Overweighting could also arise during the integration of place values into a whole number quantity, perhaps due to allocation of greater attention to most informative values (e.g., Lacetera et al, 2012). Recently, a proportion judgment model of bounded number line estimation was expanded to accommodate imprecise weighting of digits (Patalano et al., 2023). It fit placement data better than the original model, and predicted a left digit effect. In light of the present findings, it will be important to develop and test a similarly expanded version of the direct/additive model in future work.

One might ask whether task strategy could also explain the left digit effect. That is, could it be that the underweighting of the ones place in a multi-digit numeral is the result of a specific strategy used to perform number line tasks? We can think of one strategy that could give rise to a

[7] In this example, as with some proposed models, it is the underweighting of the rightward digit that leads to the relative overweighting of the leftmost digit (rather than a direct overweighting of the leftmost digit).

left digit effect. Specifically, we believe a left digit effect would emerge if one used a direct strategy to place the tens value of the target numeral and then added the value of the remainer. So, for 25, one would place 20 directly and add 5; for 38, one would place 30 directly and add 8, and so forth. Such a strategy could produce a large difference in placements surrounding tens boundaries, relative to placements around, for example, fives boundaries. There is no reported evidence that people use this specific strategy on the unbounded line (i.e., a strategy in which *d* changes from trial to trial), and little consideration of whether additive strategies might be used on the bounded line in concert with proportion judgment (e.g., judging the location of 38 by identifying 25 through proportion judgment and then adding 13). Nonetheless, a strategy-based account remains a possibility for further consideration. Future work on diversity of strategy use within a trial, across trials, and across participants, could further our understanding of whether there is a relationship between strategy use and the size of one's left digit effect.

Whether the source of the left digit effect observed here is specific to number line estimation tasks or reflects a domain-general mental process remains an open question. We know of no domain-general models of numeral-to-magnitude translation that suggest differential weighting of digits. The left digit effect has, however, been observed in many complex judgment tasks very dissimilar from number line estimation such as judging the health content of foods based on nutritional labels (Choi et al., 2019), and making medical treatment decisions based in part on patient age (Olenski et al., 2020; see Patalano et al., 2022, for a review). While some of these studies use response formats similar to a number line, others used purely symbolic formats (e.g., rating college applicants by directly typing numbers between 0 and 100; Patalano et al., 2022). There are also several studies that suggest a link between the left digit effect in number line estimation and the encoding of numerals more generally. For example, one study found that speakers of Dutch, a language with an inversion property (e.g., 41 is "one and forty"), did not show a left digit effect on a number line task (Savelkouls et al., 2020). Another study reported a correlation between the left digit effect and reading skills in adults (but not math skills; Williams et al., 2020). In the future, it will be important to better understand how the effect is related across contexts towards addressing whether sources of the effect might be domain general.

Open Practices Statement

Preregistration

The study was preregistered at < link to be added upon publication>.

Availability of data and materials

The data are available online through Open Science Framework (OSF) as <link to be added>.

Code availability

No code is available for this study.

Other Declarations

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Conflicts of interest/Competing interests

The authors declare that they have no conflicts or competing interests.

Ethics approval

The study was approved by Wesleyan University's Institutional Review Board and conducted in accordance with the ethical standards in the 1964 Declaration of Helsinki and its amendments.

Consent to participate

Informed written consent was obtained from each participant prior to participation.

Consent for publication

At the time consent to participate was obtained, participants were informed about future publication of the findings. No further consent to publish was obtained.

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Supplementary Materials

Stimuli Details

Table A is a complete list of all individual stimuli used in the study.

Table A

Individual Stimuli Used in the Study

	Target Type		Unit Width			
Target Numeral	Blocks 1 & 3	Blocks 2 & 4	Block 1	Block 2	Block 3	Block 4
4	Filler 1	Filler 1	10	8	12	8
7	Filler 2	Filler 2	10	14	12	14
9	Filler 3	Filler 3	10	10	12	12
12	Filler 4	Filler 4	10	10	12	12
13	Nonboundary 1	Filler 5	12	10	8	12
16	Nonboundary 1	Filler 6	12	10	8	12
19	Boundary 1	Boundary 1	12	12	8	8
22	Boundary 1	Boundary 1	12	12	8	8
24	Nonboundary 2	Nonboundary 1	8	12	14	8
27	Nonboundary 2	Nonboundary 1	8	12	14	8
28	Boundary 2	Boundary 2	8	8	14	14
31	Boundary 2	Boundary 2	8	8	14	14
33	Nonboundary 3	Nonboundary 2	14	8	10	14
36	Nonboundary 3	Nonboundary 2	14	8	10	14
39	Boundary 3	Boundary 3	14	14	10	10
42	Boundary 3	Boundary 3	14	14	10	10
44	Nonboundary 4	Nonboundary 3	10	14	12	10
47	Nonboundary 4	Nonboundary 3	10	14	12	10
48	Boundary 4	Boundary 4	10	10	12	12
51	Boundary 4	Boundary 4	10	10	12	12
53	Nonboundary 5	Nonboundary 4	8	10	10	12
56	Nonboundary 5	Nonboundary 4	8	10	10	12
58	Boundary 5	Boundary 5	8	8	10	10
61	Boundary 5	Boundary 5	8	8	10	10
63	Nonboundary 6	Nonboundary 5	10	8	12	10
66	Nonboundary 6	Nonboundary 5	10	8	12	10
68	Boundary 6	Boundary 6	10	10	12	12
71	Boundary 6	Boundary 6	10	10	12	12
74	Nonboundary 7	Nonboundary 6	14	10	8	12
77	Nonboundary 7	Nonboundary 6	14	10	8	12
79	Boundary 7	Boundary 7	14	14	8	8

82	Boundary 7	Boundary 7	14	14	8	8
84	Nonboundary 8	Nonboundary 7	12	14	14	8
87	Nonboundary 8	Nonboundary 7	12	14	14	8
89	Boundary 8	Boundary 8	12	12	14	14
92	Boundary 8	Boundary 8	12	12	14	14
93	Filler 5	Nonboundary 8	8	12	8	14
96	Filler 6	Nonboundary 8	14	12	14	14

Analyses by Block

Table B shows average tens difference scores and PAE for individual blocks. All four average tens difference scores were descriptively greater than 0; the differences were statistically significant for Blocks 1 and 4, but not Blocks 2 and 3. The four block-level average tens difference scores differed from one another ($F(3, 162) = 2.87$, $MSE = 260.16$, $p = .038$, $\eta_p^2 = .05$): scores were greater for Blocks 1 and 4 than for Blocks 2 and 3 (quadratic function: $F(1, 54) = 7.235$, $MSE = 310.76$, $p = .009$, $\eta_p^2 = .12$). In contrast, PAE did not differ across blocks ($F(3, 165) = 1.06$, $MSE = 0.03$, $p = .367$). There is no obvious explanation for the differences across blocks, as Blocks 1 and 4 were not collectively different from Block 2 and 3 (e.g., with regard to type of control pairs). However, trial order has been shown to affect the magnitude of the left digit effect (Kayton et al., 2022) and so may have contributed to variability across blocks.

Table B

Descriptive Data by Block

			Average tens difference score		**PAE (%)**	
			M	*SD*	*M*	*SD*
Composite 1	Block 1	HB-LB_B-HN-LN_B	11.14*	17.99	98.00	44.99
	Block 2	HB-LB_A-HN-LN_A	4.06	19.28	94.97	44.96

Composite 2	Block 3	HB-LB_B-HN-LN_B	3.33	15.41	94.71	45.36
	Block 4	HB-LB_A-HN-LN_A	6.26*	14.67	92.21	46.31

*$p < .005$. $N = 56$ except for Block 1 (where $N = 55$ because one participant had no average tens difference score for this block). HB = High Boundary (the higher numeral of a boundary pair); LB = Low Boundary; HN = High Nonboundary; LN = Low Nonboundary. Subscripts A = above-boundary and B = below-boundary refer to the location of the control pair relative to the boundary pair in that block.