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ABSTRACT

Optical Fabry–Perot cavity with a movable mirror is a paradigmatic optomechanical system. While usually the mirror is supported by a mechanical spring, it has been shown that it is possible to keep one of the mirrors in a stable equilibrium purely by optical levitation without any mechanical support. In this work, we expand previous studies of the nonlinear dynamics of such a system by demonstrating a possibility for mechanical parametric instability and the emergence of the "phonon laser" phenomenon.

I. INTRODUCTION

The idea that cavity optomechanical phenomena can be observed in a cavity formed by optically levitating mirrors has been proposed and realized by several researchers.[1–3] The authors of Ref. 4 derived main equations describing the motion of the center of mass of a levitated mirror and studied the stability of its dynamics. They found that even if the system possesses a stable equilibrium in the quasi-static approximation, the oscillations around this equilibrium become unstable if one takes into account corrections to the quasi-stationary approximation. These corrections result in optically induced amplification of mechanical motion and run-away instability. In this paper, we show that the mechanical dissipation of the mirror, which is present due to the air resistance and can be controlled, stabilizes the nonlinear dynamics of the mirror resulting in a multimode phonon lasing-like behavior.[5–13] In this note, we present the results of our analysis of the periodic oscillations of the levitating mirror in this regime.

Equations of motion for optical and mechanical degrees of freedom can be written down as

$$M\frac{d^2x}{dt^2} + \gamma_m M\frac{dx}{dt} = -Mg - \hbar\frac{d\omega_c}{dx}a^\dagger a, \tag{1}$$

$$\frac{da}{dt} + \left(-i(\omega_L - \omega_c(x)) + \frac{\pi^2}{\mathcal{F}}\frac{c}{x}\right)a = \pi\sqrt{\frac{2c}{\mathcal{F}x}}a_{in}, \tag{2}$$

where, in Eq. (1), $x, M,$ and γ_m are coordinate, mass, and mechanical damping coefficients of the levitated mirror; g is the acceleration of gravity; ω_c is the optical resonance frequency dependent on the mirror coordinate x,

$$\omega_c = \frac{\pi N}{x}, \tag{3}$$

(N is the order of the optical resonance); and a is the amplitude of the optical mode. In addition, in Eq. (2), parameter ω_L is the frequency of the driving laser; $\mathcal{F}$ is the cavity finesse; c is the speed of light; and the term $\pi^2 c/\mathcal{F}x$ represents the cavity decay rate, while a_{in} describes the amplitude of the driving field normalized such that

$$\hbar\omega_L\left\langle a_{in}^\dagger a_{in}\right\rangle = P_{in}, \tag{4}$$

where P_{in} is the input power. Unlike standard optomechanical models,[14,15] Eq. (1) lacks a mechanical spring force so that the oscillations of the mirror occur solely due to the so-called "optical spring" effect.[16] It shall be noted that while mechanical spring is linear and instantaneous, the "optical spring" is nonlinear with respect to the mirror's displacement and is also characterized by a time delay determined by the lifetime of the optical cavity mode.

If the input power P_{in} exceeds the critical value $P_{cr} = Mgc\pi^2/(2\mathcal{F})$, the levitated mirror can be in the stable equilibrium in the position with coordinate x_{eq} defined as

$$x_{eq} \approx x_L + \xi\sqrt{\frac{P_{in}}{P_{cr}} - 1}, \tag{5}$$

where

$$x_L = \frac{Nc\pi}{\omega_L} = \frac{N\lambda_L}{2} \tag{6}$$

and parameter

$$\xi = \frac{c\pi^2}{\mathcal{F}\omega_L} = \frac{\pi\lambda_L}{2\mathcal{F}} \tag{7}$$

characterizes the linewidth of the cavity resonance expressed in terms of the wavelength of the laser λ_L rather than in terms of the frequency. The corresponding equilibrium field amplitude is given by

$$a_{eq} = \pi\sqrt{2\frac{c}{\mathcal{F}x}}\frac{a_{in}}{-i\left(\omega_L - \frac{Nc\pi}{x_{eq}}\right) + \frac{\pi^2 c}{\mathcal{F}x_{eq}}}. \tag{8}$$

It is convenient to rewrite the equations of motion in terms of real and imaginary parts of the relative deviation of the field amplitude from its equilibrium value a_{eq}, $w = \mathrm{Re}[(a - a_{eq})/a_{eq}]$ and $y = \mathrm{Im}[(a - a_{eq})/a_{eq}]$, and dimensionless mechanical displacement expressed in terms of the cavity linewidth ξ, $u = (x - x_{eq})/\xi$,

$$\frac{d^2u}{d\tau^2} + 2\eta\frac{du}{d\tau} - 2w = w^2 + y^2, \tag{9}$$

$$\epsilon\frac{dw}{d\tau} + w + ry = -uy, \tag{10}$$

$$\epsilon\frac{dy}{d\tau} - u + y - rw = wu. \tag{11}$$

Here, τ is the dimensionless time defined as $\tau = t\Omega_{\max}$, where

$$\Omega_{\max} = \sqrt{\frac{g}{\xi}} \tag{12}$$

is the maximum mechanical frequency of linear oscillations of the mirror, $2\eta = \gamma_m/\Omega_{\max}$ is the dimensionless mechanical damping parameter, and r is the dimensionless detuning from the equilibrium position of the mirror x_{eq} defined as

$$r = \frac{x_{eq} - x_L}{\xi} = \sqrt{\frac{P_{in}}{P_{cr}} - 1}.$$

Parameter r also serves as a measure of input power and is the main parameter controlling the behavior of the system. Equations (9)–(11) are similar to equations derived in Ref. 4 with one significant difference: Eqs. (9)–(11) contain the term proportional to the mirror's velocity $du/d\tau$, which is responsible for mechanical damping. In the absence of this term, the authors of Ref. 4 correctly predicted that optomechanical interaction results in amplification and the loss of stability of mechanical oscillations. However, as we will show below, the mechanical damping stabilizes the nonlinear dynamics of the mirror resulting in a behavior similar to phonon lasing.[8,10–12,17,18]

II. NONLINEAR DYNAMICS OF THE MIRROR IN THE PRESENCE OF MECHANICAL DISSIPATION

The cavity dynamics is controlled by parameter ϵ, $\epsilon = Q_c(\omega_{\max}/\omega_L)$, where $Q_c = x_{eq}/\xi \approx x_L/\xi \gg 1$ is the cavity's quality factor. The terms explicitly containing small parameter Q_c^{-1} in Eqs. (9)–(11) have been neglected. With $\omega_L \sim 10^{14}$ Hz, $\mathcal{F} \sim 10^4$, and resonance order $N = 200$, we can estimate $\xi \sim 10^{-10}$ m, $\omega_{\max} \sim 10^5$ Hz, and $Q_c \sim 10^6$, which yields $\epsilon \sim 10^{-3}$. Therefore, it is justified to analyze the system of Eqs. (9)–(11) using a perturbation expansion in small parameter ϵ. The same approach was used in Ref. 4 and earlier in Ref. 15. The zero-order solution results in the well-known quasi-stationary approximation, where the optical amplitude is assumed to follow the mechanical displacement. In terms of the variables used in this work, it is written as

$$y = \frac{u}{1 + (r + u)^2},$$

$$w = -\frac{u(r + u)}{1 + (r + u)^2},$$

$$\frac{d^2u}{d\tau^2} + 2\eta\frac{du}{d\tau} + \frac{ru}{1 + (r + u)^2} = 0. \tag{13}$$

In the absence of the damping, the mechanical motion in this approximation can be characterized by potential

$$U = \frac{g}{x_L}\left[x + \frac{P_{in}}{P_{cr}}\xi\arctan\left[\frac{x_L - x}{\xi}\right]\right],$$

as shown in Fig. 1 for several values of the detuning r. In the linear approximation, we have harmonic oscillations with frequency,

$$\Omega_M^2 = 2\Omega_{\max}\frac{r}{1 + r^2} = 2\Omega_{\max}\frac{P_{cr}}{P_{in}}\sqrt{\frac{P_{in}}{P_{cr}} - 1}, \tag{14}$$

where the maximum frequency corresponds to $P_{in} = 2P_{cr}$. These oscillations arise solely due to the "optical spring" effect as no mechanical springs are present in the system. The first-order correction in ϵ introduces an optical amplification, and the nonlinear dynamics of the mirror in this approximation is described by the following equation:

$$\frac{d^2u}{d\tau^2} + 2\eta_{eff}\frac{du}{d\tau} + \frac{u(u + 2r)}{1 + (r + u)^2} = 0, \tag{15}$$

where the effective dissipation/gain parameter η_{eff} is given by

$$\eta_{eff} = \eta - 2\epsilon\frac{(1 + r^2)(r + u)}{\left[1 + (r + u)^2\right]^3}. \tag{16}$$

If $\eta = 0$, this parameter is always negative and the optomechanical interaction results in an unsaturated mechanical gain, rendering the

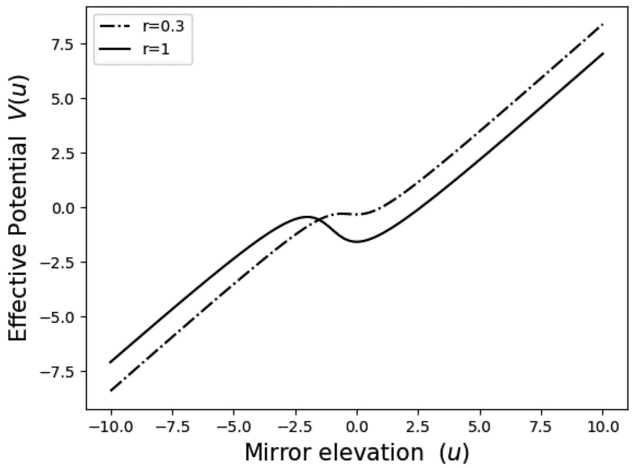

FIG. 1. Optomechanical potential for different values of the dimensionless detuning (input power parameter) r. Obviously, a larger r results in a deeper potential well.

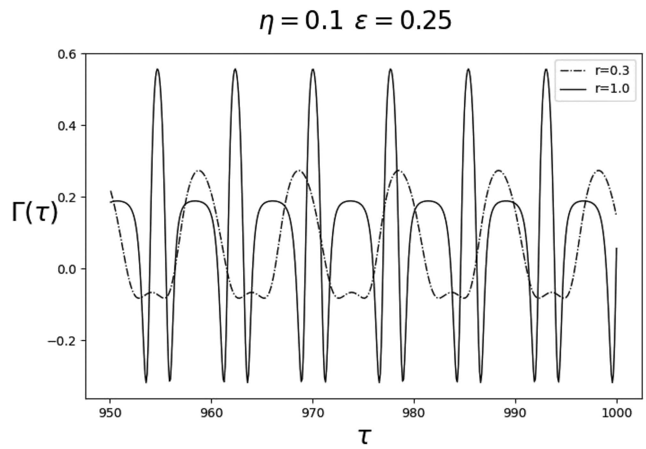

FIG. 3. Time-dependence of the effective nonlinear gain parameter for two different values of detuning (power) parameter r. One can see that the effective gain does not saturate to zero but keeps oscillating resulting in the energy flowing from light to mechanical degrees of freedom when the gain is negative and in the opposite direction when it is positive.

system unstable.[4] However, if $\eta \neq 0$, the strength of the initial linear amplification occurring when the linear gain parameter

$$\eta_{eff}^{(lin)} \approx \eta - 2\epsilon \frac{r}{\left(1+r^2\right)^2} \tag{17}$$

becomes negative is limited by nonlinear terms, and one can expect an initial growth of the mechanical amplitude to saturate and the system to settle in stable oscillations. The range of parameters allowing for amplification to happen is determined by the following inequality:[19]

$$\frac{\eta}{2\epsilon} < \frac{r}{\left(1+r^2\right)^2}. \tag{18}$$

The function on the right-hand side of this inequality has a maximum value of $3\sqrt{3}/16$ at $r^2 = 1/3$ so that the amplification regime is possible only if $\eta/2\epsilon$ does not exceed this value. For each value of $\eta/2\epsilon < 3\sqrt{3}/16$, the amplification region is limited by $r_{\min} < r < r_{\max}$, where $r_{\min}$ and $r_{\max}$ are the solutions to the following equation:

$$\frac{\eta}{2\epsilon} = \frac{r}{\left(1+r^2\right)^2}. \tag{19}$$

Not surprisingly, for r close to one of the boundaries, when amplification is weak, the time it takes for the system to reach the steady-state oscillations is longer than for r closer to the center of the amplification region. This point is illustrated in Fig. 2, which shows the saturation of the initial amplification as the nonlinear terms in Eq. (15) stabilize the effective gain for different values of r. One can clearly see the tendency for the increased time to steady-state oscillations for values of r closer to the boundaries of the amplification

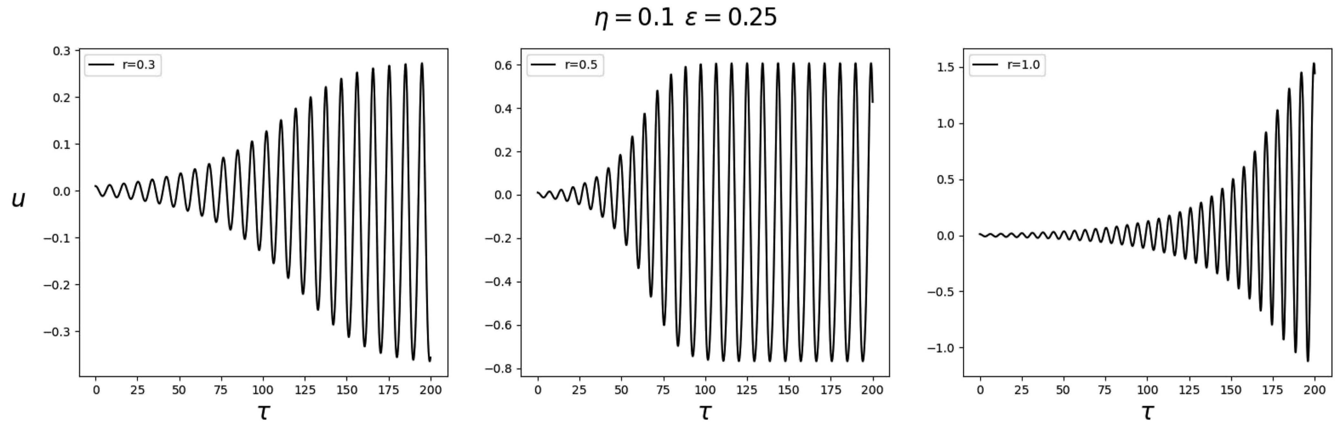

FIG. 2. Mechanical displacement u as a function of time for different values of r. The values of mechanical damping η and the cavity dynamical parameter ε are 0.1 and 0.25, respectively.

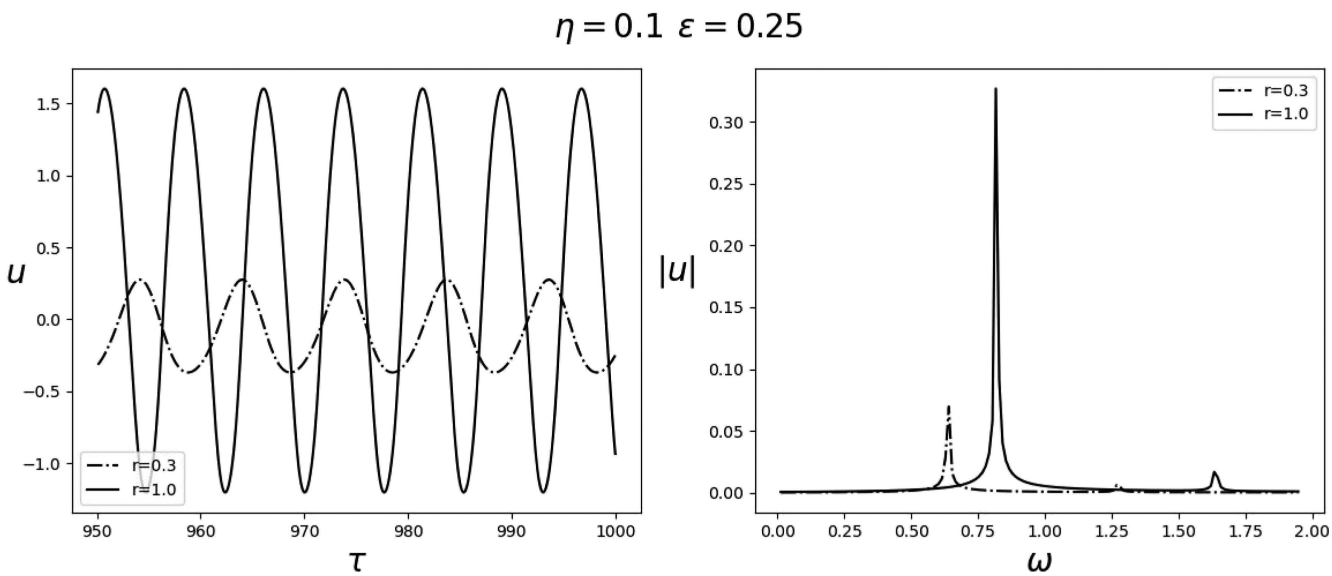

FIG. 4. Mirror oscillations in the steady-state regime for different values of the detuning and the corresponding Fourier spectra.

region, which for the chosen values of parameters $\eta/\epsilon =$ is between $r_{\min} = 0.194$ and $r_{\max} = 1.298$.

The observed nonlinear stabilization of oscillations is reminiscent of the population inversion saturation in regular lasers, but there is also a significant difference. In a simplest single-mode laser, the population inversion saturates to a constant value such that the effective gain in the steady lasing regime remains zero and the steady-state oscillations are harmonic. In the case considered in this work, the situation is more complicated—the effective gain oscillates around zero as shown in Fig. 3.

As a result of these oscillations, the steady-state regime of the mirror's oscillations is not monochromatic, and the degree of deviation from purely harmonic behavior depends on the input power via detuning parameter r.

This trend is illustrated in Fig. 4 presenting the time dependence of the mirror's displacement for two different values of r. Oscillations with larger amplitude correspond to a deeper potential well (see Fig. 1), which is better approximated by a quadratic behavior, and, therefore, are more harmonic than the oscillations with smaller amplitude: the former are characterized by at least three clearly discernible harmonics, while the latter's spectrum consists of one main frequency with a weak contribution from another harmonic. A similar trend is also seen in oscillations of the effective gain parameter shown in Fig. 3. The increase in the depth of the potential well with increasing r also explains the corresponding increase in the amplitude of the oscillations as seen in Fig. 5.

To further illustrate the nature of the nonlinear oscillations of the mirror, we have constructed the phase trajectories of the oscillations by plotting velocity $du/d\tau$ vs displacement u, as shown in Fig. 6. The circular and elliptic phase trajectories in the right panel are indicative of weakly anharmonic oscillations, while more complicated shapes in the left panel correspond to the stronger anharmonicity.

Finally, we present the phase portraits of our oscillating mirror presenting multiple phase trajectories (Fig. 7, left). One can clearly see two types of phase trajectories: those forming limiting cycles at the center of the figure and corresponding to stable oscillations, and

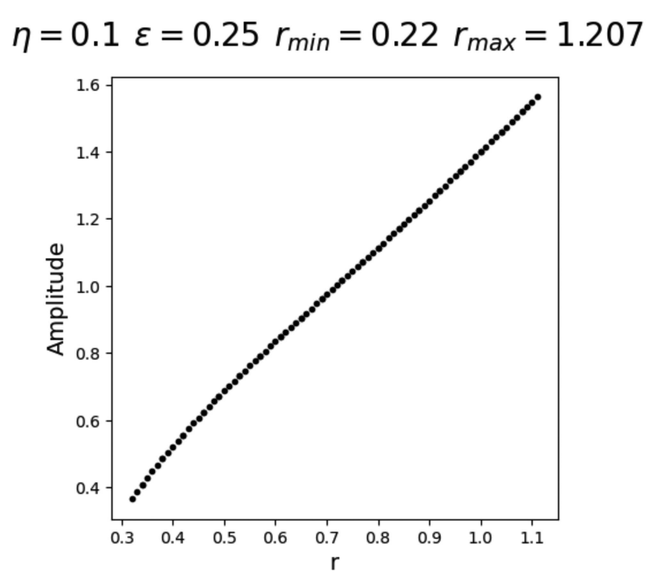

FIG. 5. The amplitude vs detuning.

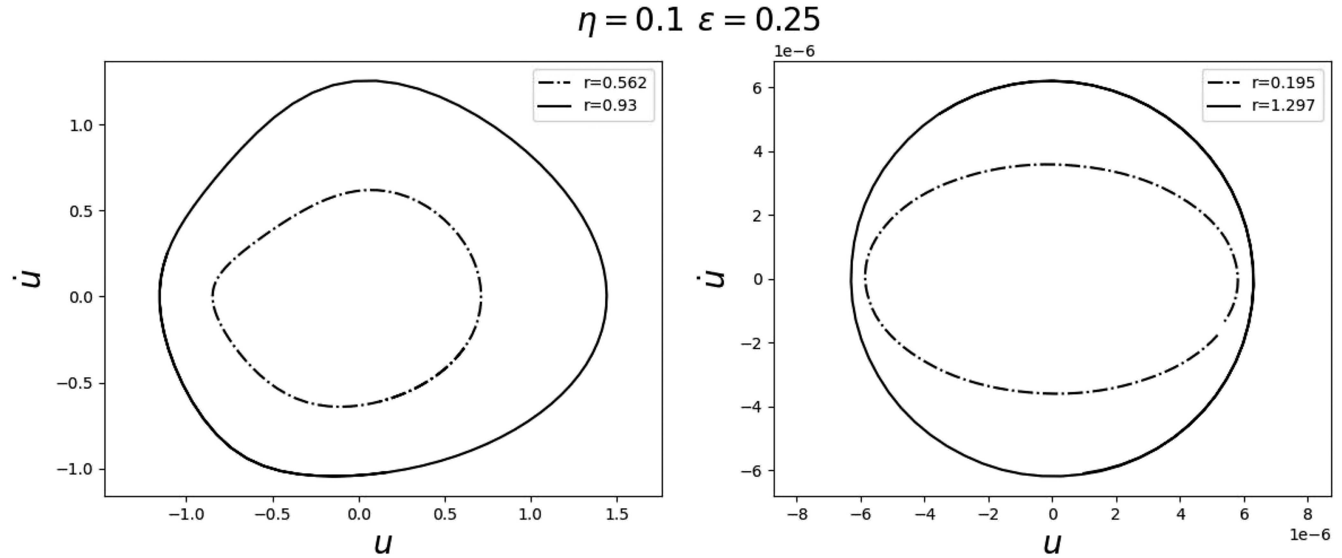

FIG. 6. Phase trajectories of the oscillating mirror in the steady-state regime for strong (left) and weak (right) nonlinear regimes.

those that move the system away from the limiting cycle corresponding to unstable motion of the mirror. This figure is complimented by a plot on the right showing the separation of the phase space in two regions. The region marked by dots corresponds to initial conditions resulting in limiting cycle type oscillations, and those marked by the arrows represent initial conditions resulting in unstable motion of the mirror. A remarkable feature revealed by this plot is that there exist initial conditions placing the mirror outside of the potential

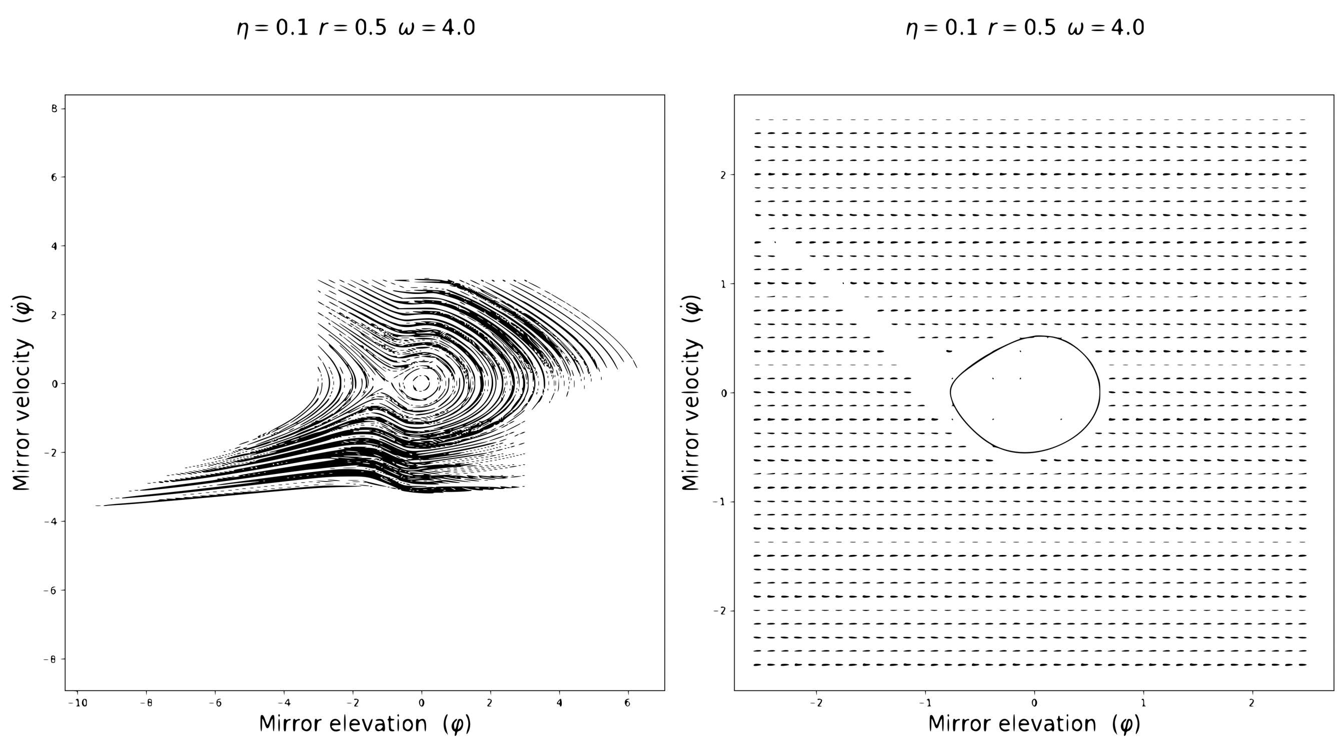

FIG. 7. (Left) The phase portrait of mechanical oscillations. The stable oscillations are seen in this figure as limiting cycles, approached by the phase trajectories. (Right) The separation of the phase space into regions of stable limiting cycles, and unstable regions.

well (Fig. 1) but still resulting in stable oscillations in the steady-state regime (points outside of the central region marked by a closed phase trajectory).

III. CONCLUSION

In this work, we analyzed the motion of the center of mass of a levitated mirror in the presence of mechanical damping. We showed that the damping stabilizes the nonlinear dynamics of the mirror resulting in a behavior reminiscent of phonon lasing. However, unlike the simplest lasing dynamics, the time dependence of the mirror's displacement in the steady state, in our case, is anharmonic with the effective gain parameter oscillating around rather than being pinned to zero.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Satyam Shekhar Jha: Data curation (lead); Formal analysis (equal); Software (lead); Visualization (lead). **Tal Carmon**: Conceptualization (supporting); Formal analysis (equal); Project administration (lead); Supervision (supporting); Validation (equal); Writing – original draft (supporting); Writing – review & editing (supporting). **Fan Cheng**: Conceptualization (supporting); Data curation (supporting); Investigation (supporting); Visualization (supporting); Writing – original draft (supporting); Writing – review & editing (supporting). **Lev Deych**: Conceptualization (lead); Formal analysis (lead); Project administration (equal); Supervision (lead); Validation (equal); Writing – original draft (lead); Writing – review & editing (lead).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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[19] We note that for realistic values of the parameters and small enough η, Eq. (18) is consistent with condition $\epsilon \ll 1$.