

A new self-adaptive mission aborting policy for systems operating in uncertain random shock environment

Gregory Levitin ^{a,b}, Liudong Xing ^c, Yuanshun Dai ^{a,*}

^a School of Computing and Artificial Intelligence, Southwest Jiaotong University, China

^b NOGA, Israel Independent System Operator, Israel

^c University of Massachusetts, Dartmouth, MA, 02747, USA



ARTICLE INFO

Keywords:

Mission abort
Random shocks
Inter-shock interval
System survivability

ABSTRACT

For systems operating in random shock environment, mission aborting policies based on the number of shocks experienced during a certain time interval are common and have received intensive attention. However, the number of shocks-based aborting policy (NSAP) is not effective when the shock rate during the primary mission is uncertain because it cannot adapt itself to the shock rate. This paper puts forward a new, self-adaptive inter-shock interval-based aborting policy (ISIAP), which determines the inter-shock interval within which the primary mission should be aborted upon the next shock based on the previous inter-shock interval. A probabilistic approach is suggested for assessing the mission success probability (MSP) and the system survival probability (SSP) under a given ISIAP. The optimal ISIAP problem is formulated and solved with the objective to maximize the expected MSP while meeting certain requirement on the SSP. A detailed case study of an unmanned aerial vehicle performing a payload delivery mission is provided to demonstrate the proposed mission aborting model and compare the effectiveness of the proposed ISIAP and the conventional NSAP. Impacts of the SSP requirement and the shock rate parameter on the mission performance metrics and optimal solutions are also examined in the case study.

1. Introduction

Mission aborting in the case of certain deterioration condition occurring is an effective means to alleviate the risk of valuable system losses in diverse critical applications. For example, unmanned aerial vehicles (UAV) can be exposed to electromagnetic interference from high voltage power lines, cell phone towers, large metal structures and other sources [1,2], which usually causes deterioration or damage to the UAV or its key components [3,4]. Hence, when the UAV has undergone and survived a certain number of interferences, it should terminate the planned mission (e.g., reconnaissance, target strike, rescue, goods delivery) and return to the nearest landing site or the base to avoid the asset loss. Other examples can be found in chemical reactor [5,6], aerospace [7,8], battlefield [9], healthcare [10], marine [11], transportation [12], etc.

A key design problem for implementing the mission aborting is to determine the specific deterioration condition of triggering the mission abort, referred to as the aborting policy (AP). For the UAV example above, the number of interferences experienced before the mission

termination defines the AP. If this number is too low, the mission may be aborted too early, unnecessarily reducing the probability that the planned mission can be accomplished successfully, termed as mission success probability (MSP). On the other hand, if the number is too high, the mission may be aborted too late, increasing the risk of losing the system and incurring low probability that the system performing the mission can survive, termed as system survival probability (SSP). The AP must be optimized to strike a balance between MSP and SSP. Different types of optimization problems can be formulated, for example, maximizing MSP subject to SSP meeting a certain level, maximizing SSP subject to MSP meeting a certain level, or minimizing a metric (e.g., expected cost) that is a function of MSP and SSP [13,14].

While the research on APs can be traced back to 1970s [15,16], it has received significant attention from the reliability community only since around 2018 [17]. APs based on different condition parameters have been modeled as listed below. The types of systems studied in those AP researches are also exemplified in the list.

* Corresponding author.

E-mail address: 1125105129@qq.com (Y. Dai).

Acronyms	
DP	destination position
ISIAP	inter-shock interval-based aborting policy
MSP	mission success probability
NSAP	number of shocks-based aborting policy
PM	primary mission
RP	rescue procedure
SSP	system survival probability
UAV	unmanned aerial vehicle
HPP	homogeneous Poisson process
<i>Notation</i>	
τ	duration of PM
T_i	random arrival time of the i th shock
U_i	random time interval between the i th and the $i-1$ -th shocks
u_i	realization of U_i
$\varphi(t)$	required duration of RP activated at time t from the beginning of the mission
Λ	shocks rate during PM
$\Lambda_{\min}, \Lambda_{\max}$	minimum and maximum possible values of shock rate
α	ratio between shock rates during PM and RP
ξ_i	time after the $i-1$ -th shock during which occurrence of i th shock triggers PM abort
m	maximum allowed number of shocks after which the PM can be aborted
$P(t, i, \rho)$	occurrence probability of i shocks in $[0, t]$ given that the shock rate is ρ
$q(i)$	probability that a system survives the i th shock
Ω	system survival probability upon the first shock
ω	shock resistance deterioration factor
$R(\xi, m)$	MSP under the ISIAP ξ, m
$S(\xi, m)$	SSP under the ISIAP ξ, m
$E(\xi, m, \Lambda_{\max}, \Lambda_{\min})$	expected MSP under ISIAP ξ, m and shock rate range $[\Lambda_{\min}, \Lambda_{\max}]$
S^*	required SSP level

- The number of malfunctioned or damaged units: k -out-of- n : G systems [18], k -out-of- n : F balanced systems [19], warm standby systems [20], UAVs [21,22].
- System degradation level: phased-mission systems [23], multi-state systems [24], safety-critical systems [25], UAVs [26].
- The amount of work accomplished: heterogeneous warm standby systems [13], standby systems with propagated failures [27], standby systems with state-dependent loading [28], standby systems with maintenance [29].
- Operation time elapsed from the beginning of the mission or the system age: self-healing systems [14], UAVs [26].
- Duration of defective state: safety-critical mission systems [30].
- The system balance degree [31].

In addition, for systems operating in random shock environment, the number of shocks experienced has been used as a key condition parameter for defining the AP. For example, the number of shocks-based aborting policy (NSAP) was modeled and optimized for single-component systems [10], systems with random rescue time [32], multi-state systems with inspections [33], drone-truck systems [34], multi-state repairable systems [35], multi-task systems [2], concurrent multi-attempt mission systems using kamikaze components [36], and consecutive multi-attempt mission systems with the common abort command [37,38]. The NSAP presumes the primary mission aborting when a predefined number of shocks occur during a certain time interval since the beginning of the mission. Such policy is not effective when the shock rate during the primary mission is uncertain because the NSAP policy cannot adapt itself to the shock rate.

This work expands the horizons in the AP research by putting forward a new inter-shock interval-based aborting policy (ISIAP) for systems operating in uncertain random shock environment. The ISIAP possesses the self-adaptation feature as it determines the inter-shock interval within which the primary mission should be aborted upon the next shock based on the previous inter-shock interval. Under the proposed self-adaptive AP model, this work makes further contributions to the body of knowledge on the AP research as listed below:

- 1) Developing a probabilistic approach of assessing the MSP and SSP under a given ISIAP.
- 2) Formulating and solving the optimal ISIAP problem to maximize the expected MSP while meeting certain requirement on the SSP.

- 3) Conducting a detailed case study of a UAV payload delivery mission system to demonstrate the proposed model and compare the effectiveness of the proposed ISIAP and the conventional NSAP.
- 4) Investigating impacts of the SSP requirement and the shock rate parameter on the mission performance metrics (MSP and SSP) and optimal solutions.

The structure for the rest of the paper is: Section 2 depicts the ISIAP model and formulates the optimal ISIAP problem. Section 3 presents the probabilistic approach of assessing MSP and SSP under a given ISIAP. Section 4 conducts the case study. Section 5 concludes the work and points out future research directions.

2. Problem formulation

The system's goal is to accomplish a primary mission (PM), which requires the system to operate during time τ in a random environment modeled by a homogeneous Poisson process (HPP) of shocks with uncertain rate. Let $T_1 < T_2 < \dots < T_i$ denote random shock arrival times. To complete the PM, the system must survive all shocks occurring during time τ .

The system deteriorates more as the number of shocks it survives increases, leading to larger risks of system failure and loss. Thus, to reduce the probability of the system loss, the PM may be aborted before its completion. The PM abortion leads to the failure of the mission and is immediately followed by the activation and execution of a rescue procedure (RP). The required time to complete the RP $\varphi(t)$ depends on the activation time of the RP since the beginning of the mission. During the RP, the system is exposed to an HPP of shocks with rate $\alpha\Lambda$, where α is a shock rate ratio between the PM and the RP. The realizations of RP depend on the specific mission and system. For example, when a computational system performs a data processing task, the RP activation presumes switching the processor from the data processing task to encoding the data and transferring it to a safe storage to avoid an unauthorized access. The time needed to save the data depends on the amount of produced/processed data at the moment when the processing task is aborted. When a UAV performs a delivery mission, the RP activation presumes immediate change of the UAV route from the destination point to the closest emergency landing position with changing the UAV's flight altitude to reduce the electromagnetic interference rate. The distance to the closest landing position and, therefore, the time needed to complete the RP depends on the UAV's location at the time of the mission abort.

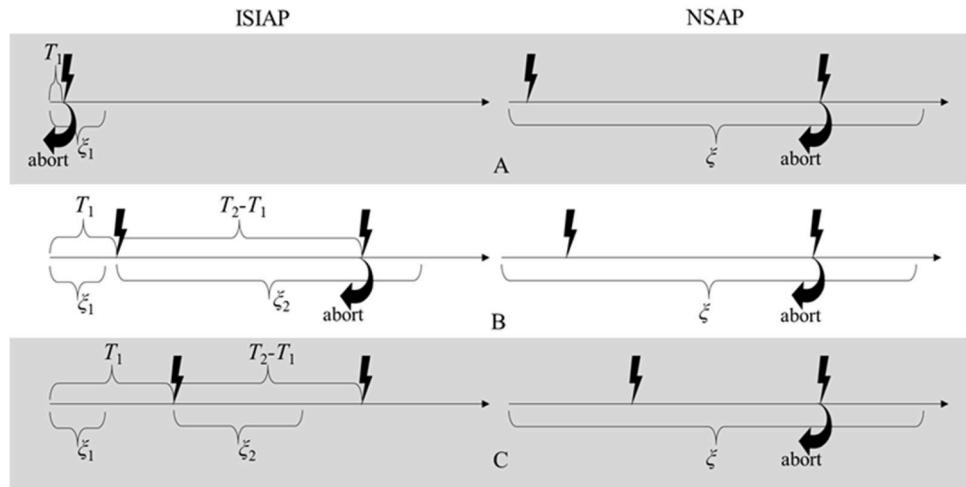


Fig. 1. Examples of PM outcomes for ISIAP and NSAP with $m = 2$.

The inter-shock interval-based aborting policy (ISIAP) presumes the PM aborting if for $i > 1$ the time interval between the i th and $i-1$ -th shock, i.e., $T_i - T_{i-1}$ is shorter than the value of $\xi_i(T_{i-1}, T_{i-2})$, where $\xi_i(t)$ is a pre-specified function. By definition, $T_0 \equiv 0$ corresponds to the beginning of the mission and ξ_1 is a fixed time that determines an interval during which the PM is aborted upon arrival of the first shock. If according to the ISIAP, the PM is not aborted after m shocks, it is never aborted. Indeed, if the system experiences and survives m shocks with the given inter-shock intervals, it is reasonable to assume that it is close to the PM termination and let it complete the mission. Thus, the ISIAP is determined by the values of m , ξ_1 and m -1 functions, $\xi_2(t), \dots, \xi_m(t)$. Observe that for $m = 1$, the ISIAP and the NSAP are identical because the mission is aborted upon the occurrence of the first shock under both of these policies.

To explain the adaptivity of the ISIAP, consider simple examples of a PM's outcomes for the ISIAP with $m = 2$ and decreasing function $\xi_2(t)$ presented in Fig. 1.

If the first shock arrives at time earlier than ξ_1 from the mission beginning, the PM is aborted because the early shock arrival evidences about the high shock rate (case A). When the first shock arrives at time later than ξ_1 from the mission beginning, it can be assumed that the shock rate is low. To check this assumption, the system sets the time threshold $\xi_2(T_1)$ during which no shocks should occur and continues the PM (case B). The second shock occurs at time shorter than $\xi_2(T_1)$ since the first shock. The system decides that the assumption about the low shock rate is wrong and aborts the PM. In case C where the first shock arrives later than in case B, the assumption that the shock rate is low is straightened. The system can set a lower time threshold $\xi_2(T_1)$ for which the average time between shock arrivals remains low. As the second shock arrives at time later than $T_1 + \xi_2(T_1)$, the system does not abort the PM. On the contrary, under NSAP the system always aborts the PM upon the occurrence of the second shock without respect to the inter-shock interval duration, which makes NSAP non-adaptive to the shock rate variations.

The mission fails if the system either aborts the PM or is lost during performing the PM. The system survives if it does not fail during either the PM or the RP phase. Two metrics characterize the mission accomplishment: the MSP R , i.e., the probability that the system completes the PM; and the SSP S , i.e., the probability that the system either completes the PM or aborts the PM and completes the subsequent RP.

When the system operates in uncertain environment, the exact shock rate Λ is unknown and only an interval $[\Lambda_{min}, \Lambda_{max}]$ to which this rate belongs can be estimated. In this case, assuming that the shock rate is uniformly distributed in the interval $[\Lambda_{min}, \Lambda_{max}]$, one can estimate the expected MSP as

$$E(\xi, m, \Lambda_{max}, \Lambda_{min}) = \frac{\int_{\Lambda_{min}}^{\Lambda_{max}} R(\xi, m, \Lambda) d\Lambda}{\Lambda_{max} - \Lambda_{min}} \quad (1)$$

The optimal ISIAP $[m, \xi = \{\xi_1, \xi_2(t), \dots, \xi_m(t)\}]$ should be found that maximizes the expected MSP while providing a required SSP level S^* for the worst case of maximal shock rate, which is formulated as

$$E(\xi, m, \Lambda_{max}, \Lambda_{min}) \rightarrow \max \text{ s.t. } S(\xi, m, \Lambda_{max}) \geq S^*. \quad (2)$$

While this work focuses on the solution to (2), other formulations of the optimization problem balancing MSP and SSP, like maximizing the SSP while meeting a required MSP level may be similarly solved.

The following assumptions are made in this model:

- 1) All the shocks are observable;
- 2) The mission aborting decision can be made immediately after the occurrence of shocks;
- 3) All the shocks have the same severity (influence on the system loss probability);
- 4) The system survives if it completes the PM (no RP is needed after the PM completion);
- 5) The shock arrivals obey the homogeneous Poisson process;
- 6) The shock rate is uniformly distributed in the interval between its minimum and maximum values.

3. Deriving the MSP and SSP for a given ISIAP

3.1. System survivability as function of number of experienced shocks

According to the model of [39], the loss probability of a system upon the occurrence of a shock increases when the number of survived shocks increases. Let $q(i)$ represent the probability that the system survives the i th shock, and $q(0) \equiv 1$. A common model used for computing $q(i)$ is

$$q(i) = \Omega \omega(i) \text{ for } i > 0, \quad (3)$$

where Ω is the survival probability of the system under the first shock. $\omega(i)$ is referred to as the shock resistance deterioration factor, which is a decreasing function of its argument with $\omega(0) = 1$. For example, $\omega(i) = \omega^{i-1}$, $0 < \omega < 1$. Thus, the system survival probability upon the occurrence of each shock reduces as the number of survived shocks occurring in interval $[0, t)$ increases. The probability that the system can survive I shocks can be evaluated as

$$\prod_{i=0}^{I-1} q(i) = \Omega^I \omega^{\frac{I(I-1)}{2}} \quad (4)$$

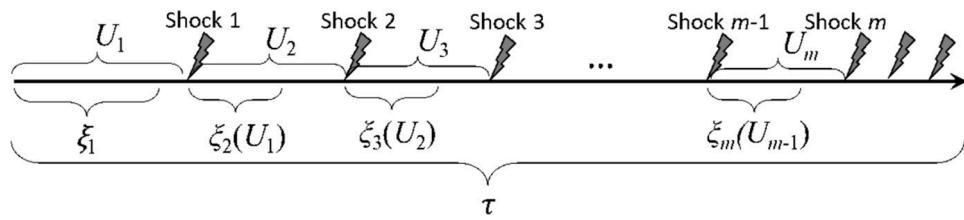
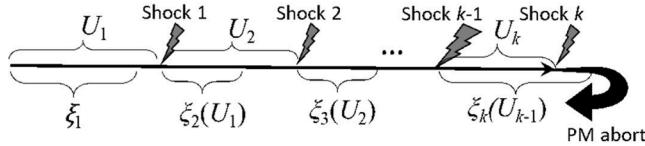


Fig. 2. Example of shock occurrence when the mission is not aborted.

Fig. 3. Example when the mission is aborted after the k -th shock.

3.2. MSP

Let $P(t, i, \rho)$ represent the probability that i shocks occur to the system during time t under the shock rate of ρ . As shock arrivals obey the HPP, $P(t, i, \rho)$ is computed as

$$P(t, i, \rho) = e^{-\rho t} \frac{(\rho t)^i}{i!}, \text{ for } i = 0, 1, 2, \dots \quad (5)$$

where $\rho = \Lambda$ for the PM and $\rho = \alpha\Lambda$ for the RP. If $t < 0$ $P(t, i, \rho) = 0$ for any i and ρ by definition.

The probability that the first shock from the HPP with rate ρ occurs in time interval $[t, t+dt]$ since some event is $P(t, 0, \rho)dt$, where dt is infinitesimal.

Let $U_i = T_i - T_{i-1}$ be the random time interval between the i th and i -th shock ($U_1 = T_1$ because $T_0 = 0$) and u_i be a realization of U_i .

The system completes the PM with probability 1 if no shocks occur during time τ . In this case, the MSP is $r_0 = P(\tau, 0, \Lambda) = e^{-\Lambda\tau}$.

The system completes the PM with probability $q(1)$ if only one shock occurs during the PM and this shock does not cause the mission abort, which happens when $\xi_1 < U_1 \leq \tau$, i.e. when no shocks occur in the interval $[0, \xi_1]$ and one shock occurs in the interval $[\xi_1, \tau]$. In this case, the MSP can be obtained as

$$r_1 = q(1)P(\xi_1, 0, \Lambda) P(\tau - \xi_1, 1, \Lambda) = q(1)\Lambda e^{-\Lambda\tau} (\tau - \xi_1). \quad (6)$$

The system completes the PM with probability $q(1)q(2)$ if only two shocks occur during the PM and these shocks do not cause the mission abort, which happens when

$$\xi_1 < U_1 \leq \tau, U_2 > \xi_2(U_1) \text{ and } T_2 = U_1 + U_2 \leq \tau. \quad (7)$$

The probability that $\xi_1 < U_1 \leq \tau$ is

$$\Lambda \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) du_1. \quad (8)$$

The conditions (7) hold when no shocks occur during time $\xi_2(U_1)$ after the first shock and exactly one shock occurs during the remaining PM time $\tau - U_1 - \xi_2(U_1)$. Thus, the conditional probability that only two shocks occur during the PM and these shocks cause no abort given that $U_1 = u_1$ is

$$P(\xi_2(u_1), 0, \Lambda)P(\tau - u_1 - \xi_2(u_1), 1, \Lambda). \quad (9)$$

Notice that if $u_1 + \xi_2(u_1) > \tau$ (i.e., the PM cannot continue after the second shock), (9) takes the value of 0 because the first parameter of its second term is negative.

The probability that the system completes the PM after experiencing two shocks can be obtained as

$$\begin{aligned} r_2 &= q(1)q(2)\Lambda \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda)P(\xi_2(u_1), 0, \Lambda)P(\tau - u_1 - \xi_2(u_1), 1, \Lambda) du_1 \\ &= q(1)q(2)\Lambda^2 e^{-\Lambda\tau} \int_{\xi_1}^{\tau} (\tau - u_1 - \xi_2(u_1)) du_1 \end{aligned} \quad (10)$$

Now consider the case where the system completes the PM after experiencing exactly $k < m$ shocks. The system survives these shocks

Table 1
Best obtained NSAP solutions for different values of S^* .

S^*	ξ_1/τ	E	S
0.87	0.001	0.928	0.877
0.88	0.025	0.916	0.881
0.89	0.085	0.886	0.891
0.90	0.157	0.851	0.901
0.91	0.229	0.818	0.910

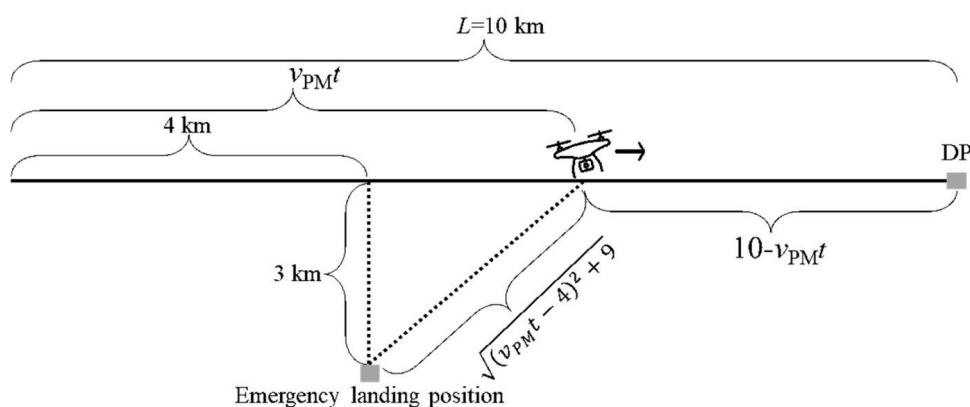


Fig. 4. UAV mission.

Table 2

Best obtained ISIAP solutions with $m = 2$ and $\xi_2(u_1) = \max(0, a_2 + b_2 u_1)$ and different values of S^* .

S^*	ξ_1 / τ	a_2 / τ	b_2	E	S
0.87	0	0.006	-0.027	0.930	0.870
0.88	0.001	0.018	-0.006	0.924	0.880
0.89	0.001	0.456	-0.999	0.900	0.890
0.90	0.073	0.528	-0.999	0.865	0.900
0.91	0.169	0.588	-0.999	0.824	0.910

Table 3

Best obtained ISIAP solutions with $m = 2$ and $\xi_2(u_1) = a_2 \exp(b_2 u_1)$ and different values of S^* .

S^*	ξ_1 / τ	a_2 / τ	b_2	E	S
0.87	0	0.001	-55.0	0.930	0.875
0.88	0.001	0.006	-44.9	0.927	0.881
0.89	0.025	0.486	-43.3	0.900	0.890
0.90	0.073	0.582	-23.1	0.864	0.900
0.91	0.193	1.056	-43.3	0.825	0.910

with probability $\prod_{i=1}^k q(i)$ and it does not abort the mission if after any i th shock, no additional shock occurs during time $\xi_i(u_{i-1})$. The last k -th shock occurs in time interval $[\sum_{i=1}^{k-1} U_i + \xi_k(U_{k-1}), \tau]$. The condition that the k -th shock causing no abort can occur during the PM is

$$T_k = \sum_{i=1}^{k-1} U_i + \xi_k(U_{k-1}) < \tau. \quad (11)$$

The probability that the system completes the PM after experiencing $k < m$ shocks can be obtained as

$$r_k = \prod_{i=1}^k q(i) \Lambda^{k-1} \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) \int_{u_1 + \xi_2(u_1)}^{\tau} P(u_2, 0, \Lambda) \dots \int_{u_{k-2} + \xi_{k-1}(u_{k-2})}^{\tau} P(\xi_k(u_{k-1}), 0, \Lambda) \times P\left(\tau - \sum_{i=1}^{k-1} u_i - \xi_k(u_{k-1}), 1, \Lambda\right) du_1 du_2 \dots du_{k-1}. \quad (12)$$

According to the considered ISIAP, after surviving m shocks during the PM, the system never aborts the PM upon experiencing any number of additional shocks that occur in time interval $[\sum_{i=1}^{m-1} U_i + \xi_m(U_{m-1}), \tau]$ (see Fig. 2). If h shocks occur in this final interval, the system survives with probability $\prod_{i=1}^{m+h-1} q(i)$. Thus, the MSP in the case where the system experiences at least m shocks takes the form

$$r_m = \Lambda^{m-1} \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) \int_{u_1 + \xi_2(t_1)}^{\tau} P(u_2, 0, \Lambda) \dots \int_{u_{m-2} + \xi_{m-1}(u_{m-2})}^{\tau} P(\xi_m(u_{m-1}), 0, \Lambda) \times \sum_{h=1}^{\infty} P\left(\tau - \sum_{i=1}^{m-1} u_i - \xi_m(u_{m-1}), h, \Lambda\right) \prod_{i=1}^{m+h-1} q(i) du_1 du_2 \dots du_{m-1} \quad (13)$$

As the events of the PM completions after different numbers of shocks are mutually exclusive, the overall MSP can be obtained as

$$R = \sum_{i=0}^m r_i. \quad (14)$$

3.3. SSP

The system aborts the mission after the first shock if this shock occurs at time $T_1 = U_1 \leq \xi_1$ from the mission beginning. In this case, the system survives the aborted mission if it survives one shock during the PM and

all the shocks during the RP, which takes time $\varphi(U_1)$. Thus, the probability of system survival when it aborts the mission after the first shock is

$$s_1 = \Lambda \int_0^{\xi_1} P(u_1, 0, \Lambda) \sum_{h=0}^{\infty} P(\varphi(u_1), h, \alpha\Lambda) \prod_{i=0}^{h+1} q(i) du_1. \quad (15)$$

The system aborts the mission after the second shock if the first shock occurs at time $\xi_1 < U_1 = T_1 < \tau$, which happens with probability $\Lambda \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) du_1$ and the time interval between the first and the second shock is less than $\xi_2(U_1)$, i.e., the second shock occurs at time $\min(\tau - U_1, \xi_2(U_1))$ since the first shock. The conditional probability that the second shock occurs within this time interval given that $U_1 = u_1$ is

$$\Lambda \int_0^{\min(\tau - u_1, \xi_2(u_1))} P(u_2, 0, \Lambda) du_2. \quad (16)$$

The system survives the aborted mission if it survives two shocks in the PM and all the shocks in the RP, which takes time $\varphi(U_1 + U_2)$. The probability of this event is

$$s_2 = \Lambda^2 \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) \int_0^{\min(\tau - u_1, \xi_2(u_1))} P(u_2, 0, \Lambda) \times \sum_{h=0}^{\infty} P(\varphi(u_1 + u_2), h, \alpha\Lambda) \prod_{i=0}^{h+2} q(i) du_1 du_2. \quad (17)$$

Generalizing, we consider the situation when the system aborts the mission after the k -th shock (see Fig. 3), which happens when the first shock occurs at time $\xi_1 < u_1 < \tau$, the time between the j -th and the $j-1$ -th shock for any $j < k$ shock obeys the inequality

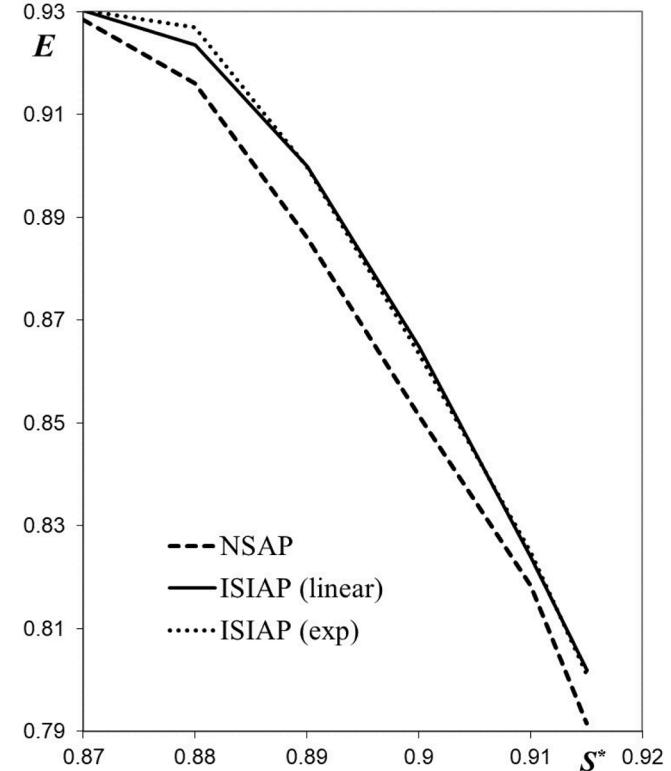


Fig. 5. Expected MSP for NSAP and ISIAP as functions of the desired worst-case SSP level S^* .

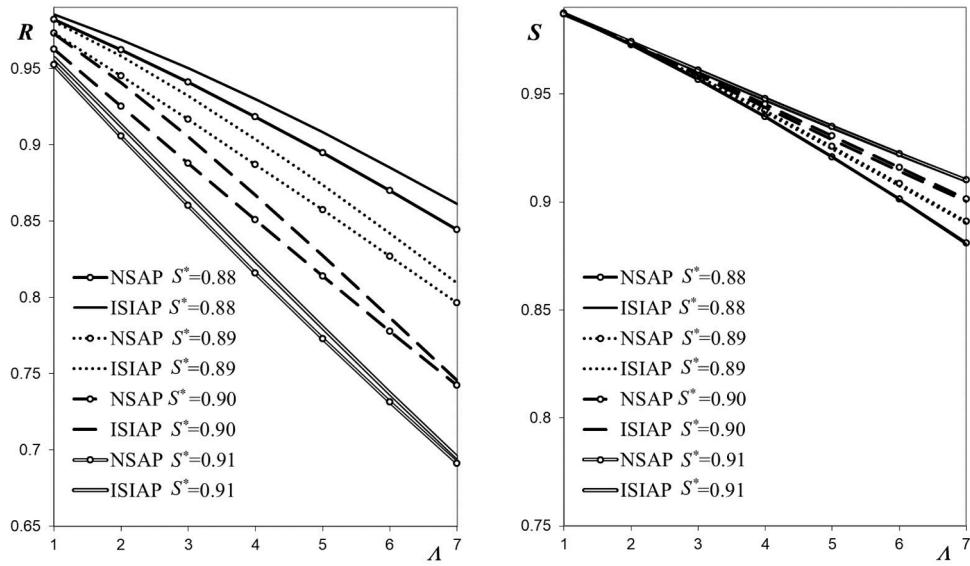


Fig. 6. MSP and SSP corresponding to the best NSAP and ISIAP (with exponential function $\xi_2(u_1)$) as functions of the shock rate Λ .

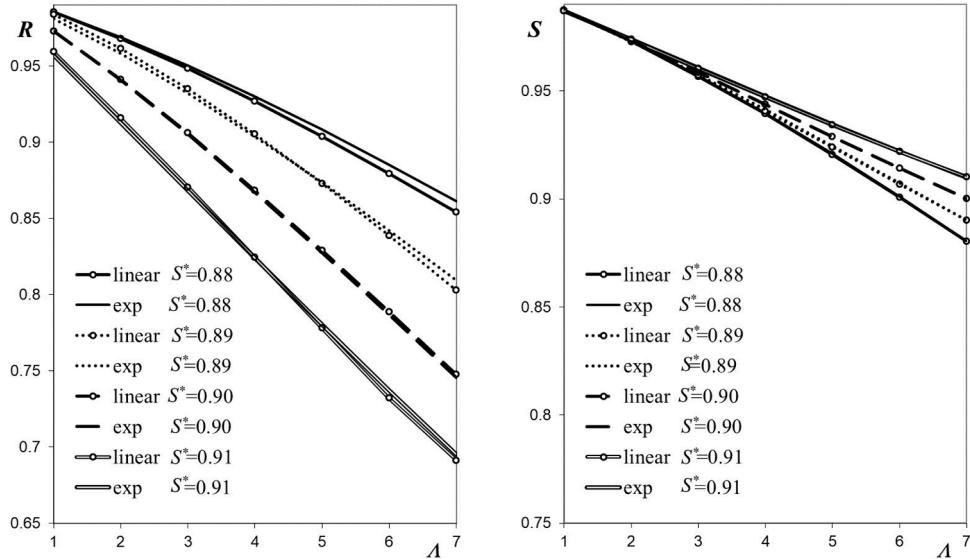


Fig. 7. MSP and SSP corresponding to the best ISIAP with linear and exponential function $\xi_2(u_1)$ for different shock rates Λ .

$$\xi_j(u_{j-1}) < u_j < \tau - \sum_{i=1}^{j-1} u_i \quad (18)$$

and the time between the k -th and the $k-1$ -th shock obeys the inequality

$$u_k < \min\left(\tau - \sum_{i=1}^{k-1} u_i, \xi_k(u_{k-1})\right). \quad (19)$$

Thus, the probability of mission aborting after the k -th shock is

$$\Lambda^k \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) \int_{\xi_2(u_1)}^{\tau-u_1} P(u_2, 0, \Lambda) \cdots \int_{\xi_j(u_{j-1})}^{\tau-\sum_{i=1}^{j-1} u_i} P(u_j, 0, \Lambda) \cdots \min\left(\tau - \sum_{i=1}^{k-1} u_i, \xi_k(u_{k-1})\right) \times \int_0^{\min\left(\tau - \sum_{i=1}^{k-1} u_i, \xi_k(u_{k-1})\right)} P(u_k, 0, \Lambda) du_1 du_2 \cdots du_k. \quad (20)$$

The system survives the mission aborted upon the k -th shock if it

survives k shocks in the PM and all the shocks in the RP that takes time $\varphi(\sum_{i=1}^k u_i)$. The probability of this event is

$$s_k = \Lambda^k \int_{\xi_1}^{\tau} P(u_1, 0, \Lambda) \int_{\xi_2(u_1)}^{\tau-u_1} P(u_2, 0, \Lambda) \cdots \int_{\xi_j(u_{j-1})}^{\tau-\sum_{i=1}^{j-1} u_i} P(u_j, 0, \Lambda) \cdots \min\left(\tau - \sum_{i=1}^{k-1} u_i, \xi_k(u_{k-1})\right) \times \int_0^{\min\left(\tau - \sum_{i=1}^{k-1} u_i, \xi_k(u_{k-1})\right)} P(u_k, 0, \Lambda) \sum_{h=0}^{\infty} P\left(\varphi\left(\sum_{i=1}^k u_i\right), h, \alpha\Lambda\right) \prod_{i=0}^{k+h} q(i) du_1 du_2 \cdots du_m \quad (21)$$

The system survives the mission either when it completes the PM or if it aborts the mission after $k \leq m$ shocks and completes the corresponding RP. As these survival cases are mutually exclusive, the total SSP can be obtained as

$$S = R + \sum_{k=1}^m s_k. \quad (22)$$

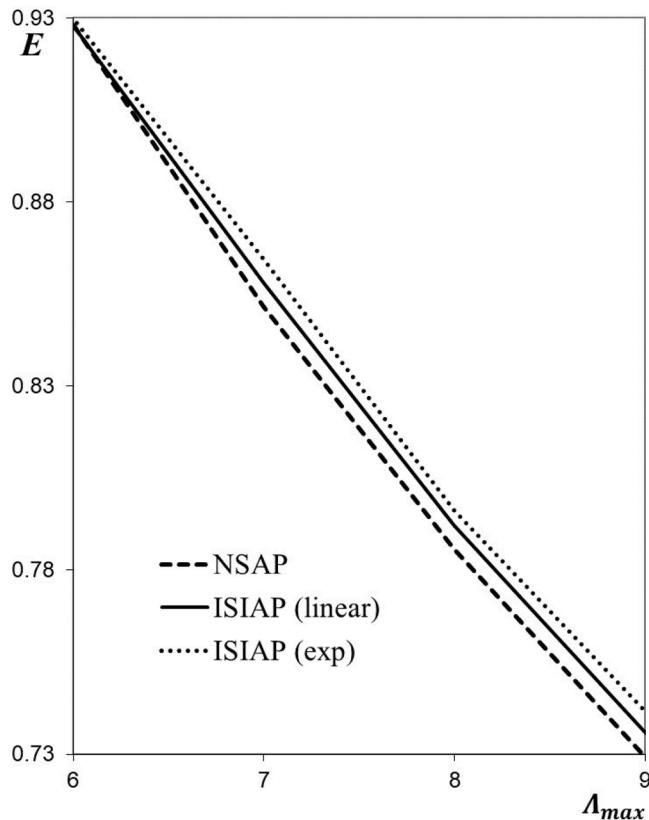


Fig. 8. Expected MSP for NSAP and ISIAP as functions of the upper limit of the shock rate Λ_{max} for desired worst-case SSP level $S^*=0.9$.

4. UAV case study

Consider a UAV that must accomplish a payload delivery mission (PM). To complete the PM, the UAV must cover a distance $L = 10$ km to a destination position (DP). During the flight to the DP, the UAV keeps speed $v_{PM}=60$ km/h and remains at an altitude which allows carrying the payload [40]. At this altitude, the UAV is exposed to shocks caused by low range electromagnetic interference. The shocks may destroy the control equipment of the UAV and cause its crash/loss. The number of

shocks arrivals during flying to the DP obeys the HPP with uncertain rate Λ belonging to the interval $[1.0, 7.0]$ h⁻¹. The interference filter that protects the UAV deteriorates as the number of experienced shocks increases due to overheating, thus causing the decrease of its resistance to shocks. Such deterioration is considered using Eq. (4) with $\Omega = 0.92$, $\omega = 0.9$. The PM flight duration is $\tau = L/v_{PM} = 0.1667$ h = 10 min.

To reduce the risk of the UAV loss, the PM can be aborted according to a chosen ISIAP. If the PM is aborted at time t from its beginning, the UAV drops the payload, rises to the altitude where the shock rate is 0.4Λ , and flies with speed $v_{RP} = 50$ km/h either to the DP or to an emergency landing position (see Fig. 4), choosing the closest between them. Thus, if the PM is aborted at time t since the beginning of the mission when the UAV has covered the distance $v_{PM}t$, the time of flying to the closest landing position is

$$\varphi(t) = \frac{\min\left(L - v_{PM}t, \sqrt{(v_{PM}t - 4)^2 + 9}\right)}{v_{PM}}.$$

Any functions $\xi_j(u_{j-1})$ with adjustable parameters can be used in the suggested ISIAP. In this work two functions have been checked for defining the ISIAP: linear function

Table 4
Best obtained NSAP solutions for $S^*=0.9$ and different values of ω .

ω	ξ_1/τ	E	S
0.86	0.229	0.815	0.901
0.88	0.193	0.833	0.901
0.90	0.157	0.851	0.901
0.92	0.109	0.877	0.901
0.94	0.049	0.910	0.901
0.96	0.001	0.940	0.903

Table 5
Best obtained solutions for $S^*=0.9$, ISIAP with $m = 2$, $\xi_2(u_1) = \max(0, a_2 + b_2u_1)$ and different values of ω .

ω	ξ_1/τ	a_2/τ	b_2	E	S
0.86	0.170	0.602	-0.93	0.817	0.900
0.88	0.110	0.663	-0.99	0.835	0.900
0.90	0.110	0.301	-0.51	0.858	0.900
0.92	0.050	0.241	-0.33	0.885	0.900
0.94	0.049	0.060	-0.09	0.910	0.901
0.96	0.001	0.060	-0.08	0.940	0.903

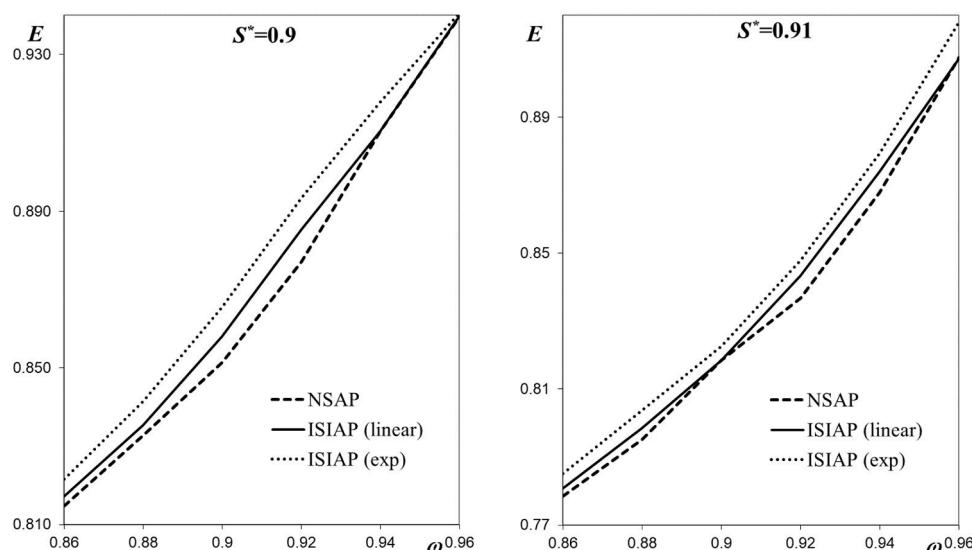


Fig. 9. Expected MSP for NSAP and ISIAP as functions of the shock resistance deterioration parameter ω for desired worst-case SSP levels $S^*=0.9$ and $S^*=0.91$.

Table 6

Best obtained ISIAP solutions for $S^*=0.9$, ISIAP with $m = 2$, $\xi_2(u_1) = a_2 \exp(b_2 u_1)$ and different values of ω .

ω	ξ_1/τ	a_2/τ	b_2	E	S
0.86	0.17	2.892	-49.5	0.822	0.900
0.88	0.17	1.988	-116	0.841	0.900
0.90	0.11	4.157	-111	0.866	0.900
0.92	0.05	1.747	-99	0.893	0.900
0.94	0.03	0.241	-115	0.918	0.900
0.96	0.00	0.060	-120	0.941	0.902

$$\xi_j(u_{j-1}) = \max(0, a_j + b_j u_{j-1})$$

and exponential function

$$\xi_j(u_{j-1}) = a_j \exp(b_j u_{j-1}).$$

For both functions, $2m-1$ parameters $\xi_1, a_2, b_2, \dots, a_m, b_m$ should be found as solutions of the optimization problem (2) to determine the ISIAP ξ, m .

The parameter b_j should always take negative values. Indeed, with an increase in the inter-shock time u_{j-1} , it is reasonable to accept for the rest of the PM a riskier aborting policy and allow PM continuation when the next inter-shock interval is shorter (see Fig. 1).

The best NSAP obtained for the considered mission presumes aborting the PM after the first shock if it occurs within time ξ_1 since the mission beginning. Thus, the best NSAP coincides with ISIAP with $m = 1$. Tables 1–3 present the comparison of the solutions obtained for NSAP with $m = 1$ and two ISIAPs with $m = 2$ (with linear and exponential functions $\xi_2(u_1)$) for different values of the desired worst-case SSP. Fig. 5 presents the values of the expected MSP for the best obtained aborting policies as functions of the desired worst-case SSP level S^* for uncertain Λ uniformly distributed in the interval $[1.0, 7.0]$. The difference of expected MSP for $m = 2$ and $m > 2$ is negligible.

It can be seen that under the ISIAP, the first shock causes PM abort during shorter period than under the NSAP. However, unlike the NSAP, the ISIAP allows aborting the PM after the second shock. The interval during which the PM can be aborted upon the second shock depends on the occurrence time of the first shock, which provides the ISIAP with self-adaptation to the shock rate and results in greater efficiency than NSAP.

For $S^* = 0.87$ under ISIAP, the PM is never aborted after the first shock ($\xi_1 = 0$) and the time after the first shock during which it should be aborted upon the second shock decreases with an increase in the occurrence time of the first shock.

It can be seen that both linear and exponential ISIAPs outperform the NSAP, and the exponential ISIAP provides the greatest expected MSP, though the results of the linear and exponential ISIAPs are very close.

Fig. 6 compares the MSP and SSP corresponding to the best obtained NSAP and ISIAP with the exponential function $\xi_2(u_1)$ presented in Tables 1 and 3 under different fixed shock rates. It can be seen that the self-adapted ISIAP provides greater MSP than the NSAP practically without reducing the SSP.

Fig. 7 presents the comparison of the MSP and SSP obtained for the best ISIAPs with linear and exponential functions $\xi_2(u_1)$ for solutions obtained in Tables 2 and 3 and their dependence on the shock rate. For low values of the shock rate Λ , the ISIAP with the linear function slightly outperforms the ISIAP with the exponential function. On the contrary, for high values of the shock rate, the ISIAP with the exponential function provides slightly better results.

Fig. 8 presents the comparison of the expected MSP for NSAP and ISIAP as functions of the upper limit of the shock rate Λ_{max} when $\Lambda_{min} = 1$ remains fixed for the desired worst-case SSP level $S^* = 0.9$. It can be seen that the difference between the expected MSP obtained under ISIAP and NSAP increases when the uncertainty of shock rate increases due to

the adaptive feature of ISIAP. The exponential ISIAP provides the greatest expected MSP.

To analyze the influence of the system shock resistance deterioration parameter ω on the optimal expected MSP, we solve the constrained optimization problem (2) for different values of parameter ω when the desired worst-case value of the SSP remains fixed.

Fig. 9 presents the values of the expected MSP for the best obtained aborting policies as functions of the parameter ω for desired worst-case SSP levels $S^* = 0.9$ and $S^* = 0.91$. Tables 4–6 present the comparison of the solutions obtained for NSAP with $m = 1$ and two ISIAPs with $m = 2$ for $S^* = 0.9$ and different values of ω .

Intuitively, when the shock resistance deterioration factor increases (deterioration decreases and the system becomes less sensitive to number of experienced shocks), the expected MSP increases. Both the NSAP and ISIAP become riskier allowing PM aborting upon the first shock during shorter time (ξ_1 decreases). The interval between the first and the second shocks in which PM is aborted under ISIAP also tends to decrease. The exponential ISIAP provides the greatest expected MSP.

5. Conclusion and future research directions

This work advances the state of the art in the AP research by proposing a new self-adaptive ISIAP model for systems operating in uncertain random shock environment. A probabilistic approach is suggested for evaluating the mission performance metrics of MSP and SSP under any chosen ISIAP. Based on the MSP and SSP evaluation, the optimal ISIAP problem is further formulated and solved to maximize the expected MSP while meeting a required SSP level. As demonstrated by the case study of a UAV payload delivery mission system, the proposed ISIAP outperforms the conventional NSAP, providing higher expected MSP. The dependence of MSP and SSP on the shock rates for the best obtained ISIAP with two different functions is studied. It is intuitive that both MSP and SSP decrease as the shock rate increases. It is also revealed that under fixed low shock rates, the ISIAP with the linear function performs better while under fixed high shock rates, the ISIAP with the exponential function provides better results. When the value of shock rate is uncertain the ISIAP with the exponential function provides greater expected MSP under constrained worst-case SSP.

In the proposed model, the mission task may be attempted only once. It is possible to extend the model that allows the task to be re-attempted following a successful RP [22,35] and explore attempt-dependent ISIAP to enhance the MSP.

CRediT authorship contribution statement

Gregory Levitin: Writing – original draft, Software, Methodology, Conceptualization. **Liudong Xing:** Writing – original draft, Formal analysis. **Yuanshun Dai:** Project administration, Data curation.

Declaration of competing interest

There is no conflict of interests associated with this paper.

Data availability

No data was used for the research described in the article.

Acknowledgement

The work of L. Xing was partially the National Science Foundation under Grant No. 2302094.

References

- [1] Xing L, Johnson BW. Reliability theory and practice for unmanned aerial vehicles. *IEEE Internet. Things. J.* 2023;10(4):3548–66. <https://doi.org/10.1109/JIOT.2022.3218491>.
- [2] Levitin G, Xing L, Dai Y. Optimal task sequencing and aborting in multi-attempt multi-task missions with a limited number of attempts. *Reliab. Eng. Syst. Saf.* 2023; 236:109309.
- [3] Kim SG, Lee E, Hong IP, Yook JG. Review of intentional electromagnetic interference on UAV sensor modules and experimental study. *Sensors. (Basel)* 2022;22(6):2384. <https://doi.org/10.3390/s22062384>.
- [4] Li X, Wang S, Li H, Zhou Y, Guo H. Electromagnetic interference of unmanned aerial vehicle in high voltage environment. *J. Phys. Conf. Ser.* 2023;2522(1). <https://doi.org/10.1088/1742-6596/2522/1/012034>.
- [5] Cheng G, Li L, Shangguan C, Yang N, Jiang B, Tao N. Optimal joint inspection and mission abort policy for a partially observable system. *Reliab. Eng. Syst. Saf.* 2023; 229:108870.
- [6] Levitin G, Xing L, Dai Y. Mission aborting in n-unit systems with work sharing. *IEEE Trans. Syst., Man, Cybern.* 2022;52(8):4875–86.
- [7] Dong T, Luo Q, Han C, Xu M. Parameterized design of abort trajectories with a lunar flyby for a crewed mission. *Adv. Space Res.* 2023;71(6):2550–65.
- [8] Ryan S. The difficulties with replacing crew launch abort systems with designed reliability. *J. Syst. Saf.* 2023;58(1):19–24. <https://doi.org/10.56094/jss.v58i1.216>.
- [9] Zhao X, Fan Y, Qiu Q, Chen K. Multi-criteria mission abort policy for systems subject to two-stage degradation process. *Eur. J. Oper. Res.* 2021;295(1):233–45.
- [10] Levitin G, Finkelstein M. Optimal mission abort policy for systems operating in a random environment. *Risk Anal.* 2018;38(4):795–803.
- [11] Thompson F, Guihen D. Review of mission planning for autonomous marine vehicle fleets. *J. Field. Robot.* 2019;36(2):333–54.
- [12] Mayrhofer M, Wächter M, Sachs G. Safety improvement issues for mission aborts of future space transportation systems. *ISA Trans.* 2006;45(1):127–40. [https://doi.org/10.1016/S0019-0578\(07\)60072-X](https://doi.org/10.1016/S0019-0578(07)60072-X).
- [13] Levitin G, Xing L, Dai Y. Mission abort policy in heterogeneous non-repairable 1-out-of-N warm standby systems. *IEEE Trans. Reliab.* 2018;67(1):342–54.
- [14] Qiu Q, Cui C, Wu B. Dynamic mission abort policy for systems operating in a controllable environment with self-healing mechanism. *Reliab. Eng. Syst. Saf.* 2020;203:107069.
- [15] Filene RJ, Daly WM. The reliability impact of mission abort strategies on redundant flight computer systems. *IEEE Trans. Comput.* 1974;C-23(7):739–43. <https://doi.org/10.1109/T-C.1974.224023>.
- [16] Hyle CT, Foggatt CE, Weber BD, Gerbranckt RJ, Diamant L. Abort planning for Apollo missions. In: The 8th Aerospace Sciences Meeting; 1970. <https://doi.org/10.2514/6.1970-94>.
- [17] Rodrigues A, Cavalcante C, Alberti A, Scarf P, Alotaibi N. Mathematical modelling of mission-abort policies: a review. *IMa J. Manag. Math.* 2023. <https://doi.org/10.1093/imaman/dpad005>.
- [18] Myers A. Probability of loss assessment of critical k-out-of-n: G systems having a mission abort policy. *IEEE Trans. Reliab.* 2009;58(4):694–701.
- [19] Wu C, Zhao X, Qiu Q, Sun J. Optimal mission abort policy for k-out-of-n: F balanced systems. *Reliab. Eng. Syst. Saf.* 2021;208:107398.
- [20] Zhao X, Liu H, Wu Y, Qiu Q. Joint optimization of mission abort and system structure considering dynamic tasks. *Reliab. Eng. Syst. Saf.* 2023;234:109128.
- [21] Zhao X, Lv Z, Qiu Q, Wu Y. Designing two-level rescue depot location and dynamic rescue policies for unmanned vehicles. *Reliab. Eng. Syst. Saf.* 2023;233:109119.
- [22] Zhao X, Dai Y, Qiu Q, Wu Y. Joint optimization of mission aborts and allocation of standby components considering mission loss. *Reliab. Eng. Syst. Saf.* 2022;225: 108612.
- [23] Liu B, Huang H, Deng Q. On optimal condition-based task termination policy for phased task systems. *Reliab. Eng. Syst. Saf.* 2022;221:108338.
- [24] Levitin G, Xing L, Dai Y. Optimal mission aborting in multistate systems with storage. *Reliab. Eng. Syst. Saf.* 2022;218:108086. Part A.
- [25] Zhao X, Li R, Cao S, Qiu Q. Joint modeling of loading and mission abort policies for systems operating in dynamic environments. *Reliab. Eng. Syst. Saf.* 2023;108948.
- [26] Liu L, Yang J. A dynamic mission abort policy for the swarm executing missions and its solution method by tailored deep reinforcement learning. *Reliab. Eng. Syst. Saf.* 2023;234:109149.
- [27] Levitin G, Xing L, Luo L. Influence of failure propagation on mission abort policy in heterogeneous warm standby systems. *Reliab. Eng. Syst. Saf.* 2019;183:29–38.
- [28] Levitin G, Xing L, Dai Y. Co-optimization of State Dependent Loading and Mission Abort Policy in Heterogeneous Warm Standby Systems. *Reliab. Eng. Syst. Saf.* 2018;172:151–8.
- [29] Levitin G, Xing L, Dai Y. Joint optimal mission aborting and replacement and maintenance scheduling in dual-unit standby systems. *Reliab. Eng. Syst. Saf.* 2021; 216:107921.
- [30] Levitin G, Xing L, Dai Y. Mission abort policy for systems with observable states of standby components. *Risk Anal.* 2020;40(10):1900–12.
- [31] Wang S, Zhao X, Tian Z, Zuo M. Optimal mission abort policy with multiple abort criteria for a balanced system with multi-state components. *Comput. Ind. Eng.* 2021;160:107544.
- [32] Levitin G, Xing L, Dai Y. Optimal aborting policy for shock exposed missions with random rescue time. *Reliab. Eng. Syst. Saf.* 2023;233:109094.
- [33] Levitin G, Finkelstein M, Xiang Y. Optimal inspections and mission abort policies for multistate systems. *Reliab. Eng. Syst. Saf.* 2021;214:107700.
- [34] Yan R, Zhu X, Zhu XN, Peng R. Optimal routes and aborting strategies of trucks and drones under random attacks. *Reliab. Eng. Syst. Saf.* 2022;222:108457.
- [35] Levitin G, Finkelstein M, Xiang Y. Optimal mission abort policies for repairable multistate systems performing multi-attempt mission. *Reliab. Eng. Syst. Saf.* 2021; 209:107497.
- [36] Levitin G, Xing L, Dai Y. Using kamikaze components in multi-attempt missions with abort option. *Reliab. Eng. Syst. Saf.* 2022;227:108745.
- [37] Levitin G, Xing L, Dai Y. Optimal task aborting policy and component activation delay in consecutive multi-attempt missions. *Reliab. Eng. Syst. Saf.* 2023;238: 109482.
- [38] Levitin G, Xing L, Dai Y. Optimizing component activation and operation aborting in missions with consecutive attempts and common abort command. *Reliab. Eng. Syst. Saf.* 2023 to submit.
- [39] Levitin G, Finkelstein M. Optimal mission abort policy for systems in a random environment with variable shock rate. *Reliab. Eng. Syst. Saf.* 2018;169:1–17.
- [40] NEXTECH (2024). High Speed drones for ISR/OODA, <https://nextech.online/high-speed-drones-for-isr-ooda/#>, assessed in April 2024.