$\mathrm{w}_{1+\infty}$ Algebra with a Cosmological Constant and the Celestial Sphere

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It is shown that there exists a simple deformed version of Strominger's infinite-dimensional $w_{1+\infty}$ algebra of soft graviton symmetries, which we conjecture to arise in spacetimes with a nonvanishing cosmological constant. The deformed algebra contains a subalgebra generating $SO(1,4)$ or $SO(2,3)$ symmetry groups of dS_4 or AdS_4, depending on the sign of the cosmological constant. The transformation properties of soft gauge symmetry currents under the deformed $\mathrm{w}_{1+\infty}$ are also discussed.

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Introduction.—The conservation laws reflect the symmetries of nature and provide a key to understanding the physical Universe. What was less appreciated until few years ago is the importance of a rather specialized area of quantum field theory and gravity devoted to studying the physical processes involving "soft" particles, with very low energies. The zero energy limit of the scattering amplitudes involving soft gauge bosons, gravitons, and other particles are described by "soft theorems" [1]. The long wavelengths of soft particles allow probing the large scale structure of the Universe, particularly the past and future asymptotic infinities. Hence, as shown by Strominger and collaborators [2–6], soft theorems are closely related to the conservation laws and symmetries. Actually, every single (known) soft theorem in asymptotically flat spacetime has been associated with an infinite number of symmetries of the celestial sphere at null infinity. These include Poincaré and extended Bondi-Metzner-Sachs symmetries. [7–9]. The connection between soft theorems and asymptotic symmetries has laid the foundations for the celestial holography program, which aims at describing four-dimensional physics in terms of a two-dimensional conformal field theory on the celestial sphere (CCFT) [10].

Two tears ago, Strominger performed a systematic study of the symmetries associated with soft particles carrying positive helicities [11]. He showed that all these symmetries are encompassed in an infinite-dimensional $\mathrm{w}_{1+\infty}$ algebra. $\mathrm{w}_{1+\infty}$ was extracted from the algebra of soft currents encoded in the operator product expansion (OPE) of celestial primary operators associated with gravitons and gauge bosons [12]. In CCFT, OPEs can be obtained from the collinear limits of (celestial) scattering amplitudes [13,14]. Strominger's results follow from tree-level amplitudes evaluated in flat spacetime with vanishing cosmological constant. (Various aspects of soft symmetry algebras are discussed in Refs. [15–37]).

In Friedman-Lemaître cosmology, the observed accelerated expansion of the Universe can be accounted for by a positive value of the cosmological constant $\Lambda \approx 10^{-52}\ \mathrm{m}^{-2}$ [38]. Hence, the Universe is not asymptotically flat—it is asymptotically de Sitter (dS), at least in the future. In this Letter, we construct an algebra similar to Strominger's $\mathrm{w}_{1+\infty}$, "deformed" by a nonvanishing cosmological constant. Instead of Poincaré, it contains a subalgebra generating the $SO(1,4)$ symmetry group of four-dimensional de Sitter spacetime. The modifications of $\mathrm{w}_{1+\infty}$ are obtained by analyzing OPEs associated with the collinear limits of gravitons and gauge bosons, now corrected by de Sitter curvature.

The idea.—The idea originates from the recent work of Alday, Hansen, and Silva, who computed the amplitude for the scattering of four gravitons on $\mathrm{AdS}_5 \times S_5$ [39–42]. Although no rigorous definition of the S matrix exists for nonasymptotically flat spacetimes like AdS, they formally used AdS/CFT correspondence [43] and expanded the amplitude in the inverse curvature radius $R^{-2} \propto \Lambda$. In the limit of $\Lambda \to 0$, known as the flat space limit of the AdS amplitudes [44–46], they obtained the well-known Virasoro-Shapiro amplitude. The subleading term is of order $\mathcal{O}(\Lambda)$. Instead of considering it as a part of a full-fledged AdS S-matrix element, we can consider it as a curvature-induced correction to the scattering amplitude in flat spacetime. For comparison, the proton-proton cross sections measured at the LHC also receive similar corrections although protons do not fly in from the cosmological horizon but come from a bottle of hydrogen stored in Meyrin, Switzerland. The subleading term has a very interesting property that becomes transparent after taking the string zero slope limit, while keeping fixed the gravitational coupling constant κ. Then, for the gravitons in the $(--++)$, i.e., in the maximally helicity violating helicity configuration,

$$A^{(1)}(s,t,u) = \kappa^2\Lambda \frac{\langle 12\rangle^6[43]}{\langle 13\rangle\langle 14\rangle\langle 23\rangle\langle 24\rangle\langle 34\rangle}\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right), \tag{1}$$

where s, t, u are the Mandelstam variables and for the momentum spinors, we used the notation of Ref. [47]. We are interested in the limit of collinear (lightlike) momenta p_3 and p_4, when $s\to 0$. For our purposes, it is convenient to parametrize momenta in terms of light-cone energies ω and complex coordinates z on the celestial sphere [10]. Then, for two arbitrary lightlike momenta p_i and p_j,

$$\langle ij\rangle = \sqrt{\omega_i\omega_j}(z_i - z_j), \qquad [ij] = \sqrt{\omega_i\omega_j}(\bar z_j - \bar z_i),$$
$$2p_ip_j = \omega_i\omega_j|z_i - z_j|^2. \tag{2}$$

In order to define the collinear limit, we complexify the momenta, so that z and $\bar z$ can be considered as independent complex variables [47]. The combined momentum of the collinear pair is defined as $P = p_3 + p_4$. In the collinear limit $z_3 \to z_4$ (while keeping $\bar z_3$ and $\bar z_4$ fixed) and $P^2 \to 0$. The leading collinear singularity has the form

$$A^{(1)}(s,t,u) \approx \kappa\Lambda \frac{\omega_P^2}{\omega_3^2\omega_4^2(z_3-z_4)^2}\left(\kappa\frac{\langle 12\rangle^6}{\langle 1P\rangle^2\langle 2P\rangle^2}\right), \tag{3}$$

therefore we obtain the three-graviton amplitude (enclosed in the brackets) times the collinear factor with a double pole $(z_3 - z_4)^{-2}$. This is a stronger collinear singularity than the single pole encountered in flat spacetime.

Graviton OPEs.—The leading term in the OPE of the primary CCFT operators $G^+_\Delta(z)$ with dimensions Δ, associated with the (positive helicity) gluons can be extracted from the amplitude (3) in the same way as in Ref. [13]. We obtain

$$G^+_{\Delta_3}(z_3,\bar z_3)G^+_{\Delta_4}(z_4,\bar z_4) \sim \kappa\Lambda\frac{B(\Delta_3-2,\Delta_4-2)}{z_{34}^2} \times G^+_{\Delta_3+\Delta_4-2}(z_4,\bar z_4), \tag{4}$$

where $z_{34} = z_3 - z_4$. One can also extract a single pole term $\sim z_{34}^{-1}$, which is necessary for the symmetry of the operator product under $3 \leftrightarrow 4$. There is, however, a problem with this OPE. If one proceeds along the lines of Ref. [12], and extracts the algebra of soft currents associated with the graviton operators, it fails Jacobi identity. It is not difficult though, to find a slight modification of the OPE coefficients leading to consistent double and single pole singularities:

$$\begin{aligned} G^+_{\Delta_3}(z_3,\bar z_3)G^+_{\Delta_4}(z_4,\bar z_4) = {} & -\frac{\kappa}{2}\frac{\bar z_{34}}{z_{34}}B(\Delta_3-1,\Delta_4-1)G^+_{\Delta_3+\Delta_4}(z_4,\bar z_4) \\ & + \frac{\kappa\Lambda}{2}\frac{\Delta_3+\Delta_4}{z_{34}^2}B(\Delta_3-2,\Delta_4-2)G^+_{\Delta_3+\Delta_4-2}(z_4,\bar z_4) \\ & + \frac{\kappa\Lambda}{2}\frac{\Delta_3}{z_{34}}B(\Delta_3-2,\Delta_4-2)\partial G^+_{\Delta_3+\Delta_4-2}(z_4,\bar z_4), \end{aligned} \tag{5}$$

where, for completeness, we also included, in the first term, the contribution of the zero slope limit of the Virasoro-Shapiro amplitude in flat spacetime. The coefficient of the double pole term, see the second term on the rhs of Eq. (5), contains an extra factor of $(\Delta_3+\Delta_4)/2$ as compared to the collinear limit (4). We can only speculate that it is due to a modified form of the momentum conservation law in curved spacetime. Indeed, as shown below, it will change the commutation relations of the "momentum" operators in a way expected for a spacetime with constant curvature.

From OPEs to cosmological $\mathrm{w}_{1+\infty}$.—After including the antiholomorphic descendants in the OPE of Eq. (5), it acquires the form

$$\begin{aligned} G^+_{\Delta_3}(z_3,\bar z_3)G^+_{\Delta_4}(z_4,\bar z_4) = {} & -\frac{\kappa}{2}\frac{1}{z_{34}}\sum_{n=0}^{\infty}B(\Delta_3-1+n,\Delta_4-1)\frac{(\bar z_{34})^{n+1}}{n!}\bar\partial^n G^+_{\Delta_3+\Delta_4}(z_4,\bar z_4) \\ & + \frac{\kappa\Lambda}{2}\frac{\Delta_3+\Delta_4}{z_{34}^2}\sum_{n=0}^{\infty}B(\Delta_3-2+n,\Delta_4-2)\frac{(\bar z_{34})^n}{n!}\bar\partial^n G^+_{\Delta_3+\Delta_4-2}(z_4,\bar z_4) \\ & + \frac{\kappa\Lambda}{2}\frac{\Delta_3}{z_{34}}\sum_{n=0}^{\infty}B(\Delta_3-2+n,\Delta_4-2)\frac{(\bar z_{34})^n}{n!}\partial\bar\partial^n G^+_{\Delta_3+\Delta_4-2}(z_4,\bar z_4). \end{aligned} \tag{6}$$

Next, we define the conformally soft graviton operators,

$$H^k = \lim_{\epsilon\to 0}\epsilon G^+_{k+\epsilon}, \quad k = 2, 1, 0, -1, \ldots, \tag{7}$$

with conformal weights $\{h, \bar{h}\} = \{(k+2)/2, (k-2)/2\}$. We represent them as truncated antiholomorphic series,

$$H^k(z, \bar{z}) = \sum_{n=\frac{k-2}{2}}^{\frac{2-k}{2}} \frac{H^k_n(z)}{\bar{z}^{n+\frac{k-2}{2}}}, \tag{8}$$

and further expand the holomorphic coefficients,

$$H^k_n(z) = \sum_{a=-\infty}^{\infty} \frac{H^k_{a,n}}{z^{a+\frac{k+2}{2}}}. \tag{9}$$

The OPE of Eq. (6) translates into the following algebra of soft currents:

$$[H^k_{a,m}, H^l_{b,n}] = -\frac{\kappa}{2}[n(2-k) - m(2-l)] \frac{\left(\frac{2-k}{2} - m + \frac{2-l}{2} - n - 1\right)!\left(\frac{2-k}{2} + m + \frac{2-l}{2} + n - 1\right)!}{\left(\frac{2-k}{2} - m\right)!\left(\frac{2-l}{2} - n\right)!\left(\frac{2-k}{2} + m\right)!\left(\frac{2-l}{2} + n\right)!} H^{k+l}_{a+b,m+n}$$
$$+ \frac{\kappa\Lambda}{2}(la - kb) \frac{\left(\frac{2-k}{2} - m + \frac{2-l}{2} - n\right)!\left(\frac{2-k}{2} + m + \frac{2-l}{2} + n\right)!}{\left(\frac{2-k}{2} - m\right)!\left(\frac{2-l}{2} - n\right)!\left(\frac{2-k}{2} + m\right)!\left(\frac{2-l}{2} + n\right)!} H^{k+l-2}_{a+b,m+n}. \tag{10}$$

To make connection with Strominger's $w_{1+\infty}$, we define

$$w^p_{a,m} = \frac{1}{\kappa}(p - m - 1)!(p + m - 1)! H^{-2p+4}_{a,m}, \tag{11}$$

where p run over the positive half integers

$$p = 1, \frac{3}{2}, 2, \frac{5}{2} \ldots, \tag{12}$$

and the condition of the truncated antiholomorphic mode expansion (8) turns into the following constraint on the indices labeled by m:

$$1 - p \le m \le p - 1. \tag{13}$$

The indices a, associated with the holomorphic modes, are integer for integer p and half-integer for half-integer p, similar to m, but their range is not restricted,

$$a = -\infty, \ldots, p - 1, p, p + 1, \ldots, \infty. \tag{14}$$

The algebra (10), written in terms of w generators, becomes

$$[w^p_{a,m}, w^q_{b,n}] = [m(q-1) - n(p-1)] w^{p+q-2}_{a+b,m+n}$$
$$- \Lambda[a(q-2) - b(p-2)] w^{p+q-1}_{a+b,m+n}. \tag{15}$$

It is easy to check that it satisfies Jacobi identity and closes within the range of indices given in Eqs. (12)–(14).

Properties of deformed algebra.—In order to understand the structure of the deformed $w_{1+\infty}$ algebra (15) and the role of the cosmological constant, we note that it contains a closed subalgebra of 10 generators: $w^1_{a,0}$ with $a = -1, 0, 1$, $w^{\frac{3}{2}}_{\pm\frac{1}{2},\pm\frac{1}{2}}$, and $w^2_{0,m}$ with $m = -1, 0, 1$. It reads

$$[w^1_{a,0}, w^2_{0,m}] = 0, \tag{16}$$

$$[w^1_{a,0}, w^1_{b,0}] = \Lambda(a - b) w^1_{a+b,0}, \tag{17}$$

$$[w^2_{0,m}, w^2_{0,n}] = (m - n) w^2_{0,m+n}, \tag{18}$$

$$[w^1_{a,0}, w^{\frac{3}{2}}_{k,l}] = \Lambda\left(\frac{a}{2} - k\right) w^{\frac{3}{2}}_{a+k,l}, \tag{19}$$

$$[w^2_{0,m}, w^{\frac{3}{2}}_{k,l}] = \left(\frac{m}{2} - l\right) w^{\frac{3}{2}}_{k,m+l}, \tag{20}$$

$$[w^{\frac{3}{2}}_{i,j}, w^{\frac{3}{2}}_{k,l}] = \frac{1}{2}(j - l) w^1_{i+k,j+l} + \frac{\Lambda}{2}(i - k) w^2_{i+k,j+l}. \tag{21}$$

In Strominger's $w_{1+\infty}$, $w^1_{a,0}$ are c-number operators. In the present algebra, however, they do not commute. To see it in a more transparent way, we define

$$w^1_{a,0} = \Lambda L_a, \qquad w^2_{0,m} = \bar{L}_m, \qquad w^{\frac{3}{2}}_{k,l} = P_{k,l}. \tag{22}$$

In terms of these operators, the above algebra reads

$$[L_a, \bar{L}_m] = 0, \tag{23}$$

$$[L_a, L_b] = (a-b)L_{a+b}, \tag{24}$$

$$[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}, \tag{25}$$

$$[L_a, P_{k,l}] = \left(\frac{a}{2} - k\right)P_{a+k,l}, \tag{26}$$

$$[\bar{L}_m, P_{k,l}] = \left(\frac{m}{2} - l\right)P_{k,m+l}, \tag{27}$$

$$[P_{i,j}, P_{k,l}] = \Lambda j \delta_{j,-l} L_{i+k} + \Lambda i \delta_{i,-k} \bar{L}_{j+l}. \tag{28}$$

In the limit of $\Lambda = 0$, it is the Poincaré subalgebra of extended Bondi-Metzner-Sachs symmetry, with four translations $P_\mu, \mu = 0, 1, 2, 3$, defined by

$$\begin{aligned} P_{-\frac{1}{2},-\frac{1}{2}} &= P_0 + P_3, \qquad P_{-\frac{1}{2},\frac{1}{2}} = P_1 - iP_2, \\ P_{\frac{1}{2},-\frac{1}{2}} &= P_1 + iP_2, \qquad P_{\frac{1}{2},\frac{1}{2}} = P_0 - P_3, \end{aligned} \tag{29}$$

and with six Virasoro operators $L_{1,0,1}, \bar{L}_{-1,0,1}$ related to the Lorentz generators $M_{\mu\nu} = -M_{\nu\mu}$ in the following way [48,49]:

$$M_{23} + iM_{10} = -L_{-1} + L_1, \quad -M_{23} + iM_{10} = -\bar{L}_{-1} + \bar{L}_1, \tag{30}$$

$$M_{20} + iM_{13} = -L_{-1} - L_1, \quad -M_{20} + iM_{13} = -\bar{L}_{-1} - \bar{L}_1, \tag{31}$$

$$M_{21} + iM_{03} = -2L_0, \qquad -M_{21} + iM_{03} = -2\bar{L}_0. \tag{32}$$

In the presence of a nonvanishing cosmological constant, however, translations do not commute and the Poincaré algebra is deformed, depending on the sign of Λ, to the algebra generating $SO(1,4)$ or $SO(2,3)$ symmetry groups of dS_4 or AdS_4, respectively. To see this, we define

$$P_\mu = \frac{\sqrt{|\Lambda|}}{2} M_{\mu 4}. \tag{33}$$

Then the algebra of Eqs. (23)–(28) can be written as

$$[M_{\mu\nu}, M_{\rho\lambda}] = i(\eta_{\mu\rho}M_{\nu\lambda} + \eta_{\nu\lambda}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\lambda} - \eta_{\mu\lambda}M_{\nu\rho}), \tag{34}$$

with

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1, -\mathrm{sign}(\Lambda)). \tag{35}$$

The cosmological $w_{1+\infty}$ algebra of Eq. (15) provides an infinite-dimensional extension of dS_4 and AdS_4 symmetries.

Gauge theory.—We started this discussion from the collinear limit of curvature corrections to the graviton scattering amplitudes [39–42]. It would be very interesting to study gauge theories coupled to gravity in a similar way, in particular curvature corrections to graviton-gauge boson interactions in Einstein-Yang-Mills theory. Here, we also expect a double pole in the OPE of the positive helicity graviton operator $G^+_\Delta(z, \bar{z})$ and the gauge boson operator $O^{+d}_\Delta(z, \bar{z})$, where d labels the group index. It would modify the corresponding OPE in the following way:

$$\begin{aligned} G^+_{\Delta_1}(z_1, \bar{z}_1) O^{+d}_{\Delta_2}(z_2, \bar{z}_2) = &-\frac{\kappa}{2}\frac{\bar{z}_{12}}{z_{12}} B(\Delta_1 - 1, \Delta_2) O^{+d}_{\Delta_1+\Delta_2}(z_2, \bar{z}_2) + \frac{\kappa\Lambda}{2}\frac{\Delta_1 + \Delta_2 - 1}{z_{12}^2} B(\Delta_1 - 2, \Delta_2 - 1) O^{+d}_{\Delta_1+\Delta_2-2}(z_2, \bar{z}_2) \\ &+ \frac{\kappa\Lambda}{2}\frac{\Delta_1}{z_{12}} B(\Delta_1 - 2, \Delta_2 - 1) \partial O^{+d}_{\Delta_1+\Delta_2-2}(z_2, \bar{z}_2). \end{aligned} \tag{36}$$

To see what is the corresponding deformation of the symmetry algebra, we define the conformally soft gluon operators,

$$R^{k,d} = \lim_{\epsilon\to 0} \epsilon O^{+d}_{k+\epsilon}, \qquad k = 1, 0, -1, \ldots \tag{37}$$

with conformal weights $\{h, \bar{h}\} = \{(k+1)/2, (k-1)/2\}$. We represent them as a truncated holomorphic series,

$$R^{k,d}(z, \bar{z}) = \sum_{n=\frac{k-1}{2}}^{\frac{1-k}{2}} \frac{R^{k,d}_n(z)}{\bar{z}^{n+\frac{k-1}{2}}}, \tag{38}$$

and further expand the holomorphic coefficients,

$$R^{k,d}_n(z) = \sum_{a=-\infty}^{\infty} \frac{R^{k,d}_{a,n}}{z^{a+\frac{k+1}{2}}}. \tag{39}$$

The algebra of the soft currents following from the OPE (36) has the form

$$[H^{k}_{a,m}, R^{l,d}_{b,n}] = -\frac{\kappa}{2}[n(2-k) - m(1-l)]\frac{\left(\frac{2-k}{2} - m + \frac{1-l}{2} - n - 1\right)!\left(\frac{2-k}{2} + m + \frac{1-l}{2} + n - 1\right)!}{\left(\frac{2-k}{2} - m\right)!\left(\frac{1-l}{2} - n\right)!\left(\frac{2-k}{2} + m\right)!\left(\frac{1-l}{2} + n\right)!}R^{k+l,d}_{a+b,m+n}$$
$$+ \frac{\kappa\Lambda}{2}((l-1)a - kb)\frac{\left(\frac{2-k}{2} - m + \frac{1-l}{2} - n\right)!\left(\frac{2-k}{2} + m + \frac{1-l}{2} + n\right)!}{\left(\frac{2-k}{2} - m\right)!\left(\frac{1-l}{2} - n\right)!\left(\frac{2-k}{2} + m\right)!\left(\frac{1-l}{2} + n\right)!}R^{k+l-2,d}_{a+b,m+n}. \tag{40}$$

Upon the redefinition written in Eq. (11) and

$$S^{q,d}_{a,m} = (q-m-1)!(q+m-1)!R^{3-2q,d}_{a,m}, \tag{41}$$

the commutators (40) become

$$\left[w^{p}_{a,m}, S^{q,d}_{b,n}\right] = [m(q-1) - n(p-1)]S^{p+q-2,d}_{a+b,m+n} - \Lambda[a(q-1) - b(p-2)]S^{p+q-1,d}_{a+b,m+n}. \tag{42}$$

While gravitational interactions are affected by curvature, we do not expect corrections to pure Yang-Mills theory, therefore the S algebra of soft gauge currents should remain in its original form [11],

$$\left[S^{p,d}_{a,m}, S^{q,e}_{b,n}\right] = -if^{deg}S^{p+q-1,g}_{a+b,m+n}. \tag{43}$$

The soft currents $S^{1,d}_{0,0}$ satisfy

$$\left[S^{1,d}_{0,0}, S^{1,e}_{0,0}\right] = -if^{\mathrm{deg}}S^{1,g}_{0,0}. \tag{44}$$

and generate global gauge transformations. From Eq. (42), it follows that

$$\left[L_a, S^{1,d}_{0,0}\right] = \left[\bar{L}_m, S^{1,d}_{0,0}\right] = \left[P_{k,l}, S^{1,d}_{0,0}\right] = 0. \tag{45}$$

Although we do not have a solid argument supporting the cosmological deformation written in Eq. (42), it is easy to check that the full symmetry algebra of Einstein-Yang-Mills systems, written in Eqs. (15), (42), and (43), satisfies all Jacobi identities.

Discussion.—In this Letter, we proposed a cosmological deformation of $w_{1+\infty}$ algebra, by including commutator terms proportional to the cosmological constant. As a result, the Poincaré subalgebra was replaced by the symmetry algebra of dS or AdS, depending on the sign of the cosmological constant. It is striking that this deformation was extracted from the singularity structure of the graviton scattering amplitudes in spacetime with constant curvature, although a slight modification of the corresponding graviton OPEs was necessary to ensure a self-consistent algebra. It would be very interesting to uncover a deeper reason for this modification. A precise connection to the construction of Alday, Hansen, and Silva [39–42] remains to be understood. In particular, their higher-order curvature corrections contain higher-order poles in Mandelstam variables. The physical interpretation of these poles is not clear.

The results of Alday, Hansen, and Silva [39–42] rely on the interpretation of CFT correlation functions as AdS scattering amplitudes. In this context, the kinematic (Mandelstam) variables are introduced by hand, by using a prescription proposed by Penedones [44] (based on the observations made by Mack in Refs. [50,51]) for relating them to conformally invariant cross ratios. While this prescription works in the $\Lambda \to 0$ limit, it is possible that some modifications are necessary beyond the leading order. There is no doubt that AdS scattering amplitudes must be consistent with the $SO(2,3)$ subalgebra of the cosmological $w_{1+\infty}$, therefore Penedones' prescription should be reexamined from this symmetry perspective. As for the dS case, we have no scattering data—in this work, we naively extrapolated from $\Lambda < 0$ to $\Lambda > 0$. We hope that our proposal will lead to some new insights into the long-standing problem of constructing dS scattering amplitudes.

Recently, it has been shown that celestial holography and Carrollian holography are linked [52–54] by modified Mellin transforms [55,56]. It would be interesting to check if this can help in understanding the origin of the proposed algebra from a different perspective. One can show, however, that at the leading singularity order, there is no difference between the OPE coefficients obtained by using the modified and standard Mellin transforms. As the interplay between celestial and Carrollian holography is being actively investigated; see, e.g., Refs. [52–54,57–63], there are many interesting questions that can be pursued in this direction.

It would be also interesting to see how the cosmological $w_{1+\infty}$ algebra is related to the asymptotic symmetries of dS spacetime, discussed in Refs. [64–71]. Furthermore, one might utilize the methods developed in [72] to find a realization of the cosmological $w_{1+\infty}$ algebra on the gravitational phase space.

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