Seeing mathematics together: A comparative case study of youths and facilitators collaborating to learn mathematics in informal settings

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Availability of data and materials

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Abstract

Background: This comparative case study examined the use of math walks with middle grade youths and adult facilitators in an informal STEM learning space. Math walks are place-based walking tours where youths and facilitators critically examine and ask math-related questions about their environment.

Method: Drawing on situated theories of learning and frameworks for understanding group participation, we examined how facilitators constrained or supported youths' mathematical thinking as they participated in math walks at the local zoo.

Results: Using interaction and stance analysis, we identified, analyzed, and compared three contrasting cases: In the first case, the facilitator may have overly constrained youths' mathematical thinking by asking leading questions and not providing time for youths to discuss their personal interests. In the second case, the facilitator may have underly constrained youths' mathematical thinking by allowing youths to ask too many new questions without refining or developing any one specific question. In the third case, the facilitator supported mathematical thinking by praising youths' work, layering on mathematical terminology, and providing clear and actionable instructions for how youths could refine their mathematical questions.

Conclusions: Findings support efforts to understand how adult facilitators can support youths in seeing mathematics within and asking mathematical questions about the world around them.

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Math walks are place-based walking tours where people critically examine and ask math-related questions about everyday spaces or informal learning environments (Wang et al. 2021). During a math walk, people explore a space by walking, learn about mathematical concepts central to the design or function of the space, and ask new mathematical questions about the space. For example, youths on a math walk at a local zoo could observe the design of an animal enclosure and pose questions about the various heights of walls surrounding the animals (Authors., under review). The purpose of a math walk is to make explicit (and fun) the beautiful and important connections between mathematics and everyday life (English et al., 2010; Fessakis et al., 2018; Richardson, 2004; Wang et al., 2021). Our broader research investigates the affective and psychological benefits of engaging in math walks (Milton, et al., 2023; Wang & Walkington, 2023). We are interested in whether and how math walks improve students' interest in mathematics and attitudes towards mathematics. In this study, we take a closer look at the interactions between math walk facilitators and youths to better understand how people *reason* about mathematics while on a math walk. We motivate this work by describing three separate but related issues:

First, empirical research which examines how youths reason about mathematics in informal settings have typically focused on those settings which are not explicitly designed for learning. For example, Nasir (2000) examined students' mathematical practices involved in basketball, Saxe (1991) examined youths' mathematical reasoning in markets in northeastern Brazil, and Taylor (2009) examined students' mathematical abilities in the context of local liquor stores . Our research focuses exclusively on mathematical reasoning that takes place in informal settings which are designed for learning (i.e., museums). Informal learning environments are settings, beyond the traditional classroom, where learning occurs through everyday experiences, interactions, and activities, often characterized by flexibility, spontaneity, and learner-driven exploration (e.g., afterschool programs, museums, summer camps, and zoos; Pattison et al., 2017). Despite the growing popularity of informal learning environments (Mokros, 2006), it is challenging to examine how youths learn mathematics in these spaces (Pattison et al., 2017). Youths often do not recognize when they are thinking mathematically while exploring museums, zoos, or other community spaces. This is due in part to youths' highly contextualized understanding of mathematics as an activity that only takes place in school settings (Gyllenhall, 2006; Pattison et al., 2017). Many informal spaces also tend to have facilitators whose background is in the sciences (e.g., animal/plant science at outdoor spaces, physical/earth sciences at museums) rather than mathematics (National Research Council, 2005; Yackel & Cobb, 1996).

To remedy this, informal learning environments can recruit facilitators to support youths in seeing, discussing, or reasoning with mathematics (Nemirovsky et al., 2013; Pattison et al., 2016, 2017, 2018; Vandermaas-Peeler et al., 2015). This brings us to the second issue: even less is known about how facilitators in informal environments can best support meaningful mathematics learning. While facilitators may not have formal mathematical pedagogical training, they often have instructional experience or more general pedagogical training (Hmelo-Silver & Barrows, 2008) and, depending on the site, may have deep knowledge of the youths they serve. Yet, similar to youths, facilitators may have difficulties with recognizing mathematics in informal settings (Peck et al., 2022; Wang et al., 2021).

Complicating both issues, the learning sciences and mathematics education research have argued for an expanded view of what counts as mathematical reasoning (Abrahamson, 2019; Abrahamson et al., 2020; Nathan et al., 2019; Nathan & Walkington, 2017). Traditionally, mathematical reasoning has been viewed as a discrete and mental enterprise that involves working with systems of abstraction – i.e., the so-called “romantic view” of mathematics (Nathan, 2012). From this view, mathematics is a transparent and observable property of the world that is learned and experienced in the minds of individuals. Presently, researchers now view the human body and collaborative group as sites where rich mathematical reasoning occurs (Abrahamson & Trninic, 2015; Schansker & Binker-Ahsbahs, 2016). Sociocultural, embodied, and situated perspectives on mathematical reasoning require detailed attention to group dynamics, environment, proximity, gesture, and discourse (Abrahamson et al., 2020; Leung et al., 2013; Nathan et al., 2019; Walkington et al., 2018) to describe, understand, and ultimately develop mathematical reasoning. This ‘opens up’ what can be considered as mathematics to include: spatial and perceptual reasoning with actions, gestures, and simulations; the formation of conceptual metaphors from grounded experience; the observation of regularities and patterns from real “messy” environments; and practices of estimation, problem-solving, and justification. While we view this expansion as necessary and equitable, it may be difficult for facilitators and youths who are more familiar with mathematics as it has traditionally been conceived.

This research explores these three entangled challenges. We investigate small groups of facilitators and youths as they participated in math walks at the City Zoo (a pseudonym). The youths were from upper elementary or middle grade levels and the facilitators were either employees from the City Zoo or volunteer chaperones from the research team. During the math walks, facilitators and youths explored various exhibits at the zoo, watched videos which highlighted the connection between animals or exhibits and mathematical ideas, and created their own mathematical questions about some aspect of the zoo. Informed by Ragin and Becker's conceptualization of qualitative case study methodology (1992), we searched the data for moments when facilitators and youths collaborated to reason about mathematics. Grounded in an embodied and situated perspective on learning (e.g., Abrahamson et al., 2020; Greeno, 2006), we focused on two forms of mathematical reasoning: *seeing mathematics in informal spaces* and *asking mathematical questions about informal spaces.* By *seeing mathematics*, we are referring to the *process* of observing and interpreting everyday spaces using the languages and tools of mathematics. We consider this process to be collaborative and involve participants' bodies (e.g., gesture, physical location, and proximity), talk (e.g., between youths or between youths and facilitators), and aspects of the near environment (i.e., materials or animals at the zoo). By *asking mathematical questions*, we are referring to the *products* of the math walk. At the end of a math walk, youths selected a single photograph, asked a mathematical question about this photograph, and presented this question to their peers for consideration. Using techniques from interaction (Jordan & Henderson, 1995) and conversation analysis (Goodwin, 2007), we analyzed video recordings and transcripts of youth-facilitator collaboration to trace how mathematical reasoning unfolded. In this study, we narrate three contrasting cases (Schwartz & Bransford, 1998) which highlight how context and youth-facilitator dynamics shaped youths' ability to see mathematics and ask mathematical questions. The research question guiding this investigation was: *How can youths be supported (by facilitators) in "seeing" mathematics and asking mathematical questions in informal STEM learning environments?* In the next section, we describe our theoretical framework for tracing mathematical reasoning and refine our research question further.

Theoretical Framework

The mathematical activity that takes place during a math walk appears to be quite different than mathematical activity that unfolds in a (stereo)typical classroom. Youths on a math walk are constantly on-the-move (Marin et al., 2020), talking with each other and adult facilitators, watching short videos about mathematical concepts, taking their own photographs, and asking/refining their own mathematical questions. To understand mathematical reasoning in this dynamic and multi-modal environment, we ground ourselves in embodied (Abrahamson, 2019; Nathan, 2012; Goodwin, 2007) and situative (Brown, Collins, & Duguid, 1989; Hutchins; 1995; Greeno, 1998; 2006; Lave, 1988) perspectives on learning.

Embodied perspectives on learning view reasoning as a process that is distributed across an individual's mind and body (Abrahamson, 2019; Nathan, 2022; Goodwin, 2007). That is, people's gestures and movements are important modalities through which mathematical ideas can be expressed (Goodwin, 2018; Streeck, Goodwin, LeBaron, 2011) or conceptually developed and refined (Abrahamson et al., 2020; Walkington et al., 2018). Situative perspectives on mathematical learning view reasoning as a process which unfolds within activity systems: complex social and technical arrangements which involve people, tools, symbols, materials, and aspects of the physical environments (Brown, Collins, & Duguid, 1989; Greeno, 1998; 2006; Lave, 1988). From the situative perspective, mathematical ideas are 'found' within social interactions (e.g., asking questions, posing solutions, distributing work) and within material interactions (e.g., using a tool to measure, creating a symbol to represent a quantity).

These perspectives consider reasoning to be distributed beyond the representations within the mind (Greeno & van de Sande, 2007). Combining these perspectives, we view reasoning as a *public process* which unfolds across an individual's body and among interactions between multiple individuals and their environment. In the following sections, we explain how embodied and situative perspectives on learning allow us to observe and trace the mathematical reasoning which takes place on math walks. We begin by relating embodied and situative theories of learning to the broader pedagogical design undergirding our enactment of math walks. Then, we take a closer look at the actions and interactions which constitute mathematical reasoning as youths and facilitators participate in math walks.

Math Walks as an Informal Pedagogical Activity

A math walk (also known as a math trail) is an informal pedagogical activity where youths and facilitators go on a walk, learn about how mathematics appears in the world around them, and pose new mathematical questions (English et al., 2010; Richardson, 2004; Fessakis et al., 2018; Walkington et al., 2018). This approach draws upon philosophical commitments from place-based education, where local communities are considered critical sites for learning and active engagement within a community is both a process and product of learning (Gruenewald, 2003 Sobel, 2004). These opportunities for informal math learning (Pattison et al., 2017) can be powerful forms of outreach that allow people to see math in new and different ways.

Math walks can be conducted with little to no technological supports. For example, a facilitator could lead a math walk where they discuss the mathematical concepts relevant to the space and allow participants to pose new questions. However, our approach to math walks is mediated by technology in two novel ways: First, youths are provided a tablet with pre-created videos called *walk stops*. Walk stops are short videos, three to five minutes in length, which explain mathematical concepts relevant to various locations within an informal learning site. For example, while exploring the giraffe exhibit, youths could use their tablet to watch a walk stop video about the mathematical patterns underlying a giraffe's spots. In our broader research endeavor, walk stops are collaboratively created by members of the research team and educators from the informal learning sites. In the case of the City Zoo, two members from the research team worked with two educators from the City Zoo to design five walk stops at various animal exhibits. We view walk stops as a supplemental resource (in tandem with facilitators' site-specific expertise) for helping youths see mathematics within everyday settings (Authors, 2023; Wang & Walkington, 2021, 2023;). The video-based nature of the walk stops allows learners viewing them to see people and their bodies in real environments – walking around, talking, gesturing, and calling attention to different visual, haptic, auditory, and dynamic elements of new and familiar places in real time. The videos are intended to be viewed while physically standing at the site displayed in the video, so the learner can feel immersed in the embodied experience of the place. The videos can also be annotated to layer on mathematical representations – such as definitions on-screen or overlaying measurements on real environments – to augment the real world with useful mathematical information.

Second, youths are given the opportunity to create their own walk stops to share with the broader community. Using a tablet, youths pose new mathematical questions, take photos to accompany these questions, and annotate the photographs using photo-editing tools. Rather than answering the questions, youths are encouraged to imagine tools or strategies that might help others in the community answer their question. The walk stop creation process is inspired by authentic inquiry (Edelson et al., 1999), where learners first notice and wonder about their surroundings (Rumack & Huinker 2019), and then pose a question they have generated to explore.

Our enactment of math walks draws on research in mathematics education related to *problem-posing*, where students ask mathematical questions or create mathematical tasks. Problem posing enables facilitators to monitor students' mathematics learning (Silver, 1994). The degree of sophistication of youths' mathematical questions provides facilitators with insights into youths' reasoning abilities, creative thinking, and interests. (Cai & Leikin, 2020; Cai, et al., 2023; Cai and Hwang, 2023; Singer, Ellerton, & Cai, 2013). Meta-analyses suggest that engaging in problem-posing activities can enhance attitudes towards mathematics and mathematics learning (Wang et al., 2022). Learners pose problems most effectively when they have an authentic audience for their problems (Crespo, 2003), are given structure, examples, and support for their problem-posing activities (Walkington & Bernacki, 2015), when they collaborate with peers (Walkington & Hayata, 2017), when they are working with familiar contexts and objects (Bonotto, 2013; English, 1998), and when structured and unstructured approaches to problem-posing processes are combined (Wang et al., 2022). Although problem-posing has become a popular and important approach in the mathematics education literature, as it can promote productive struggle, research on how the nature of problem-posing tasks and the supports students receive can enhance problem-posing is lacking (Cai & Hwang, 2023; Walkington et al., in press).

Embodied and situative perspectives on learning also inform our enactment of math walks. Regarding embodied perspectives on learning, reasoning on a math walk is evidenced by how youths move their bodies around a space and gesture to or about entities within a space. For example, youths at the City Zoo can point to specific animals within an enclosure, gesture about the size or shape of the animal, and move to new locations to capture a different perspective on that animal. Similar to research on mathematical learning which explores how youth use their bodies to enact and explore mathematical ideas in formal learning environments (Abrahamson et al., 2020; Nathan, 2012; Nathan, 2017), gestures and body movements are sites for observing mathematical reasoning in informal learning environments.

Regarding situative perspectives on learning, reasoning on a math walk unfolds in complex activity systems involving youths, facilitators, tablets loaded with pre-created walk stop videos, exhibits at the zoo, and other aspects of the physical environment. For example, youths at the City Zoo explored exhibits, watched videos, took photos, and talked with facilitators to discuss their emerging mathematical questions or ideas. Each of the interactions – whether between two people or between people and materials (i.e., tablets, zoo exhibit information, animals) – have the potential to carry within them rich mathematical ideas. Although the youths we examine are beyond the walls of the classroom, we see these activity systems as similar to those which have been studied in formal mathematics learning environments (e.g., Greeno, 1997; Hall & Stevens, 1994; Stevens & Hall, 1998).

Mathematical Reasoning on a Math Walk

As described above, our enactment of math walks involves youths exploring a space with the support of facilitators, viewing videos about mathematics relevant to the space, and taking photographs to pose new mathematical questions. While embodied and situative theories of learning inform *where* we look to find mathematical reasoning, Goodwin's embodied participation framework informs *how* we trace mathematical reasoning across individual's bodies, between individuals within a group, and among individuals and their immediate environment (Goodwin, 2007; 2018).

Goodwin's work explores how small groups of people collaborate with tools and aspects of their immediate surroundings to reason about the world. For example, Goodwin has examined how archaeologists work together to view and describe soil strata (Goodwin, 2000), how scientists negotiate differences between color (Goodwin, 1997), how doctors collaboratively navigate surgery (Koschmann et al., 2007), and how parents and children work together (or work against one another) to complete math homework (Goodwin, 2007). What makes Goodwin's embodied participation framework useful for studying reasoning is its attention to small groups of people, body posture and gesture, and shared use of tools to make sense of immediate tasks. Two concepts within Goodwin's embodied participation framework inform our work: (a) environmentally coupled gestures; and (b) participation stances.

Environmentally Coupled Gestures.

Environmentally coupled gestures are any moments when an individual uses their body in concert with language to draw shared attention to some feature of the immediate environment (Goodwin, 2007). Goodwin (2007) explains that gestures are a "multimodal package of complementary meaning-making practices" (p. 56) comprised of language (e.g., talk or verbal expression), body movements (e.g., pointing or waving), and structures from the immediate environment (e.g., objects, entities, features of the environment). For example, when a child at the City Zoo points to a specific chimpanzee and exclaims "that one!", they are creating an environmentally coupled gesture which makes public their focus on a specific entity within the environment (Goodwin, 2003). Once public, other individuals within an activity system can share focus on and discuss this entity. By identifying and describing environmentally coupled gestures, we can understand *what* an individual person sees within their immediate environment and *how* they are reasoning about that entity. This constitutes the smallest grain size at which we view mathematical reasoning, and how we operationalize embodied perspectives on learning within our study.

Participation Stances.

If environmentally coupled gestures are a 'window' into how *individual*s attend to the immediate environment, participation stances are a window into how *groups* work together to share attention and negotiate activity. Goodwin understands that any turn-at-talk represents a stance which is publicly available for others in the environment (Goodwin, 2007; 2018). By stance, we mean a position that an individual takes with relation to other participants, the immediate environment, and the unfolding activity. Take for example the earlier description of a child pointing to a chimpanzee and exclaiming “that one!”. In this turn at talk, the individual child has created an environmentally coupled gesture which reveals their focus on a specific chimpanzee. We can also view this turn-at-talk as a public stance which lets other people in the group know which chimpanzee they are focused on. From there, other people in the group can pursue discussions about the specific chimpanzee. Another child could respond by asking “what *about* that one?”. This new stance is a request for more information about the chimpanzee from the original child. By focusing on participation stances, we can trace how people collaborate to negotiate shared attention about the immediate environment.

Goodwin provides a typology of stances which support researchers in tracing how participants reason together. Goodwin names five types of stances which can be enacted during a turn-at-talk or gesture: (a) instrumental stances; (b) epistemic stances; (c) cooperative stances; (d) affective stances; and (e) moral stances. A single turn-at-talk can express one or more of these stances. In Table 1 we briefly describe these stances, how they appear in interaction, and what they reveal about collaborative reasoning on math walks.

Table 1
Goodwin’s five interactional stances, definitions, and examples.

Stance	Definition	Example
Instrumental	Stances which draw people's attention towards entities in the environment (objects, materials, etc.).	Referring or pointing to an object (i.e., an animal) when posing a question
Epistemic	Stances that make claims about the identity, the nature (i.e. size, shape), or any other knowledge declarations about an object.	Discussing the height of a tree, naming an animal, or discussing relevant objects.

Cooperative	Stances which organize a person's body and attention toward others, materials, or the environment to sustain an activity	Involving other members in the group during a discussion.
Affective	Stances which convey emotions from one individual towards others, materials, or the environment.	Telling youths about a person's emotions, gesturing about emotions.
Moral	Stances which convey that a person is trustworthy(aligned with the goals of the group) or untrustworthy (misaligned with the goals of the group)	Telling other participants that someone should believe them.

Instrumental stances are any turns-at-talk which orient people to some aspect or entity within the immediate environment for further consideration. That is, instrumental stances are any moments when an individual points toward or draws attention to an object, symbol, tool, or idea. Instrumental stances are necessary because "in order to carry out relevant courses of action, participants must position themselves to see, feel, and in other ways perceive as clearly as possible, and in ways relevant to the activities in progress, both consequential structure in the environment that is focus of their attention, and each other" (Goodwin, 2007, p. 61). By examining the instrumental nature of a turn-at-talk reveals, researchers can see what aspects of the immediate environment individuals are attending to, and whether or not other people in the group take up or ignore these foci.

Epistemic stances are any turns-at-talk which position an individual as able to know about the surrounding world. In other words, epistemic stances are knowledge claims that people make in their talk, gesture, or interaction with the environment. For example, a child pointing to a chimpanzee and exclaiming "that one!" has instrumentally expressed interest towards a specific organism within the enclosure. If the child were to go on and say, "that chimpanzee is the fastest!", they have added an epistemic stance to their turn-at-talk by presenting a knowledge claim about the chimpanzee's speed. By focusing on epistemic qualities of individual's turns-at-talk, we can gather a better understanding of what individuals believe to be true about the aspects of the immediate environment they are examining. Furthermore, related to our interest in mathematical reasoning, we can see when youths begin to use mathematical vocabulary to describe aspects of the immediate environment.

Cooperative stances are any turns-at-talk which reveal whether and how participants in an unfolding conversation or activity are aligned with one another. Cooperative stances can be accomplished with the body (e.g., facing towards another individual, pointing towards the same object) or through talk (e.g., agreeing or disagreeing). Cooperative stances work to demonstrate "that by visibly orienting to both other participants and the environment that is the focus of their work, an actor is appropriately cooperating in the joint accomplishment of the activity in progress" (Goodwin, 2007, p. 61). We examine cooperative stances to understand whether and how youths and facilitators are working together or separately to mathematize their surrounding environment.

The final two stances are relevant to our work, but harder to isolate in our video recordings of interaction data. Affective stances are any turns-at-talk which reveal emotional orientations between participants involved in collaboration. Moral stances are any turns-at-talk which reveal or classify specific actors as trustworthy or untrustworthy. By trustworthy, we mean moments when one social actor is understood by others as having the group's best interest or goals in mind. Both stances appear in interaction through talk and gesture. For example, participants can express excitement or trust with their faces (e.g., smiling, nodding), hands (e.g., shaking hands), or dialogue (e.g., expressing excitement or trust with language). Affective and moral stances provide a way to see the emotional valence of cooperative activity. However, in our math walks at the city zoo, recording conditions made it difficult to ascertain the affective or moral qualities of individuals' turns-at-talk. We discuss this further in the methods section.

Goodwin's participation stances provide an analytical means for tracing how mathematical reasoning in an activity system is shaped through successive turns-at-talk. Furthermore, Goodwin's participation stances are how we operationalize situated perspectives on learning within our study.

Summary

We began this manuscript by describing our goal of understanding how youths and facilitators collaborate to reason about mathematics while participating in a math walk. We view *seeing mathematics* and *asking mathematical questions* as both embodied and situated enterprises. These processes unfold, within activity systems, as groups of people collaborate with each other, interact with tools, focus on aspects of the immediate environment, and ask questions. Equipped with Goodwin's concepts of environmentally coupled gesture and participation stances, we can trace: (a) what aspects of the City Zoo youths attend to; (b) whether and how youths relate mathematical ideas to the City Zoo; (c) youths' interactions with technology (i.e., tablet videos and photograph tools); and (d) youths' interactions with the facilitators. In the next section, we explore our methods for tracing these features of reasoning in interaction.

Methods

This empirical study is part of a broader, multi-year, multi-site project called the MathExplorer Project (a pseudonym). The MathExplorer project is a 5-year research partnership between a mid-sized private university in the U.S. Southwest, a community-based STEM non-profit, and nine informal learning sites across the region. The goals of the MathExplorer project are three-fold: 1) to develop a gamified mobile app that allows middle grade learners to create math walks at various informal learning sites, 2) research how math walks and the mobile app can be best designed to enhance youths outcomes related to mathematics, and 3) the establishment of an informal STEM learning research practice partnership (RPP), that forms a STEM ecosystem in the community.

During the first year of this project, we conducted research at three of the nine informal learning sites: a community center's afterschool program, a flight museum, and the City Zoo, ***without*** the mobile app for math walks being developed yet. This manuscript focuses on data collected from the City Zoo. The City Zoo is located in a large city in the Southwest U.S. with over 2000 animals representing over 400 species. This three-day camp took place during a Thanksgiving break camp.

The goal of the three-day camp was for youths to ask mathematical questions about exhibits at the zoo, capture and annotate photographs of the zoo that would relate to their mathematical questions and present these questions and annotated photographs to their peers. The youths came from a variety of schools in the local area, and many did not know each other before the camp. On the first day, the groups: (a) explored the zoo; (b) watched pre-created videos which highlighted mathematical ideas within certain zoo exhibits; (c) engaged in discussions with the adult facilitators about the students; and (d) completed worksheets to capture their ideas related to the videos. These videos were designed by the research team in collaboration with employees from the City Zoo. On the second day, once the youths completed the above tasks, they were better able to understand how math might be seen within the zoo - they explored the zoo again with an eye towards asking their own math-related questions. During this phase, youths revisited each animal exhibit, asked new mathematical questions, took photos with a tablet, and annotated their pictures with photo editing tools. Facilitators – either from the research team or employees from the zoo – collaborated with youths along the way for support.

On the final day of the camp, youths refined their mathematical questions and presented their questions and photographs to their peers.

Context and Participants

We partnered with informal educators from the City Zoo to design four math walk stops, in various locations within the zoo. Over the course of the three-day camp, the youths watched four MathExplorer videos about mathematical patterns within different animal exhibits: (a) video one explored mathematical walking patterns of quadrupeds, (b) video two explored the coat patterns of giraffes, (c) video three explored elephants' behavior patterns using ethograms, and (d) video four explored the nesting patterns of South African penguins. After each video, the youths were able to visit the animal exhibit to pose their own mathematical pattern questions. Additionally, the youths were able to visit other animal enclosures and exhibits, such as the chimpanzees, gorillas, and the reptile house, to pose *new* mathematical questions.

Seventeen middle grade youths enrolled in a three-day City Zoo camp engaged in these math walks activities. Participants were enrolled in 6^{th} grade (n=4), 7^{th} grade (n=8), or 8^{th} grade (n=6). Seven of the participants identified as girls and ten of the participants identified as boys. The participants racially or ethnically self-identified as Black/African American (n=4), Latino/a/x (n=4), White (n=7), and Mixed Race (n=2). The participants were organized into six groups (usually 3 youths per group) and paired with one adult facilitator. Facilitators received a 4-hour training about how to facilitate the math walk activity and which mathematical concepts were present in the videos for that site. This training involved discussions about logistics (i.e., the location of walk stops, the itinerary for the day), technical training (i.e., how the tablets work, where to find walk stop videos, how to take pictures with the tablet), and the 4-step processes for creating math walk stops (i.e., Notice, Question, Curate, Design). Further, the training addresses the mathematical principles related to each of the walk stop videos that were collaboratively made for the site. The discussion about mathematics was also designed to elicit facilitators' ideas about how mathematics might relate to various aspects of the City Zoo, and the facilitators were encouraged to think ahead about a broad range of mathematical connections that could be made. A member of the research team and a non-profit partner specializing in math walks brainstormed these connections with facilitators and provided some instruction about and demonstrations related to various mathematical terms and ideas (e.g., surface area, volume, frequency, and ratio). While this training was not intensive, it did provide a space for facilitators to ask questions about math walks and prepare to lead their own small group in a math walk.

Data Collection

Three forms of data were collected: (a) a pre- and post-survey; (b) video recordings of each group as they engaged in the math walk activities; (c) youth-created artifacts (worksheets and annotated photographs from the zoo). The pre- and post-survey were designed to measure youths' affective and psychological attitudes towards mathematics, before and after participating in a math walk. The video recordings were designed to capture the finer details of how small groups of youths engaged with pre-created walk stops, worksheets, facilitators, and aspects of the City Zoo. The worksheet was designed to support youths in summarizing what they learned from each walk stop video and prime the youths to ask new questions related to the City Zoo (See Appendix B).

This manuscript focuses exclusively on video recordings of the math walks and the youth-created artifacts. For findings related to the pre- and post-survey, see Milton et al. (Under Review). At the beginning of our study, we set out to record each of the six small groups for the entirety of the camp. This would have yielded a total of 18 video recordings (3 recordings for each of the six groups). However, we encountered several challenges related to recording in the City Zoo. First, Group 1's recording device was corrupted, resulting in the loss of all of group 1's data. Second, battery issues with the cameras caused researchers to selectively record certain times at the City Zoo over others. For example, time spent at an animal exhibit or watching a walk stop video were prioritized over movements between animal exhibits or down time. We recognize that this biases our data collection towards static moments (times when youths are stationed at a particular exhibit) over mobile moments (times when youths are moving between exhibits). Third, over the three-day camp, several youths were absent, contributing to an incomplete dataset for certain groups. With these limitations in mind, on day 1 we recorded groups 3 and 6, on day 2 we recorded groups 2, 4, and 6, and on day 3 we recorded groups 4 and 5. The resulting data corpus included 7 video recordings, each totaling about 30 minutes per recording, for a total of 3.5 hours of video footage, which is tailored to the scope and objectives of this study. The focused nature of our analysis allows for a detailed examination of how youths see and ask mathematical questions. Thus, despite its size, the corpus is sufficient to achieve the intended insights and outcomes of this project.

Data Analysis

This study draws upon Ragin and Becker's conceptualization of qualitative case study methodology (1992). Ragin and Becker view cases as products of social inquiry, constructed by the researcher to demonstrate important differences in social activity, rather than, pre-determined boundaries within the social world. In our work, we set out to identify different ways in which facilitators interacted with youths which either supported or constrained their abilities to see mathematics and ask mathematical questions. To identify these different facilitator-youth interactions, we proceeded through two analytical processes: interaction analysis and then conversation analysis.

Interaction Analysis

Our first pass at the data involved techniques from interaction analysis (Jordan & Henderson, 1995). While we incorporated techniques from traditional interaction analysis, our method was adapted to suit the specific needs of our project. First, we began by collaboratively viewing each video file and taking notes on any mathematical ideas that seemed to appear in youths or facilitators talk or gesture. This worked to familiarize ourselves with the video data as well as determine best practices for our next step, content logging.

Second, we created one content log for each video file. To create a content log, two members of the research team viewed a five-minute segment of the video, paused the video, and then wrote a short descriptive memo about what happened in the five-minute sequence. This memo included information about who was speaking, what body gestures or positions seemed relevant, and whether the youths interacted with technologies or facilitators. If a conversation or action was interrupted by pausing the video at five-minute intervals, the researchers would make sure to include prior context in the next five-minute interval and resulting descriptive memo. This process continued until each video file was watched completely and content logged. This resulted in a total of seven content logs.

Third, after content logs were created for each video, we read each content log and flagged instances when youths and facilitators discussed mathematical topics or asked mathematical questions together. At this point, we noticed that three of the seven recordings possessed different relationships between the facilitator and youths that seemed to relate to depth of mathematical discussions. In one of group 2's recordings, the facilitator maintained a high degree of control over the youths' activity. In one of group 6's recordings, the facilitator maintained a low degree of control over the youths' activity. In one of group 5's, the facilitator only periodically interacted with the youths, but did so in clear and concise manner. We decided to treat these three groups as emerging and contrasting cases (Ragin & Becker, 1992). We transcribed each of the three cases in its entirety. Each transcript captured talk, gesture, and aspects of the immediate environment that participants were referring to when participating in the math walk. Then, we proceeded with our second phase of analysis, conversation analysis.

Conversation Analysis

In our second pass at the data, we drew on Goodwin's (2007) concepts of environmentally coupled gestures and participation stances to trace how youths and facilitators coordinated shared attention to see mathematics within and ask mathematical questions about the City Zoo. Working with the transcript and video recording, we re-watched each of our three emerging cases. We paused the video after each turn-at-talk and wrote a short analytical memo which described any environmentally coupled gestures or participation stances that the speaker created through talk, body position, and gesture. As described earlier, a single turn-at-talk is a multimodal meaning package that contains a variety of gestures and stances. The outcome of this note-taking process was an annotated transcript which traced what aspects of the City Zoo a small group was focused on, how they spoke about (or gestured about) this aspect of the City Zoo (in mathematical or non-mathematical terms), and whether the youths' interactions with technology or facilitators supported or constrained these growing foci. Furthermore, this process allowed us to view, from a causal standpoint, whether and how facilitators' interactions with the youths supported or constrained their developing ideas about what is mathematical within the

City Zoo. In the next section, we re-narrate these three cases through the lens of Goodwin's embodied participation framework.

Findings

We present three cases which highlight different youth-facilitator interactions which supported or constrained youths mathematical reasoning and problem-posing activities. Case 1 (video recordings from group 2) involved a facilitator whose interactions with youths overly structured the youths' efforts to see mathematics related to animal walking patterns at the zoo. Case 2 (video recordings from group 6) involved a facilitator whose interactions with youths supported the youths' efforts to ask mathematical questions about Giraffes at the zoo but did not provide enough guidance to refine these questions. Case three (video recordings from group 5) involved a facilitator whose interactions with the youths supported the youths' efforts to ask mathematical questions about the position of the hot wire (a safety feature) in the Chimpanzee enclosure and provided enough guidance to help the youths refine this question as well as annotated photographs to support explaining this question.

Because each case spanned more than 30 minutes of video-data, we focused our analysis on the key moments of mathematical discussion between youths and adult facilitators. However, moments in between these mathematical discussions were key to how the mathematical discussions unfolded. Removing these 'in-between' moments would unduly simplify each event. Therefore, in retelling these cases, we use a mixture of narrative writing and stance analysis of selected transcripts. Narrative writing will help the reader follow the interaction as it unfolded, and conversation analysis of selected transcripts will highlight the key moments where adult and youths' interactions supported or constrained mathematical activity. After examining each case in detail, we close with a discussion about recommendations for future efforts to support youths in seeing mathematics and asking mathematical questions about everyday settings or informal learning environments. For a summary of all three cases, see table 9.

Case 1: Examining Animal Walking Patterns

Case one illustrated how, when facilitator interactions with youths are *overly structured*, youths become 'pigeon-holed' into focusing on a singular mathematical idea. By overly structured, we mean interactions where facilitators lead youths towards the facilitator's own conclusion about how mathematics might be seen within the City Zoo. When this happens, youths' efforts to collaboratively see mathematics within and ask mathematical questions about the City Zoo is prematurely cut short.

Case one followed three youths and a facilitator. The three youths were Elena, Bella, and Kiana (pseudonyms). All three participants identified as middle-school aged girls. Bella and Elena identified ethnically as Latina, and Kiana identified ethnically as African/Black American. The facilitator (Dev) also served as the video recorder; he was a member of the research team. Dev identified as a South Asian man.

This case examined two episodes of interaction which took place between Bella, Kiana, Elena, and Dev. Throughout the episodes, the participants watched various walk stop videos and attempted to craft their own mathematical questions. First, located in the City Zoo classroom, the participants watched a walk stop video about animal walking patterns and then discussed new questions they might have about the City Zoo. Second, located on a bench near the elephant exhibit, the participants watched a walk stop video about ethograms - a tool for quantifying and tracking animal behavior. Both episodes illustrate overly structured interactions between the facilitator and youths.

Episode 1: Determining the Walking Patterns of Animals

Episode one opened with Bella, Kiana, and Elena sitting on the ground around Dev. The youths had just finished watching a walk stop video about animal walking patterns and were eating a snack. Bella had a clipboard, worksheet, and a pencil in her hand. The clipboard and worksheet were designed so that the youths could summarize what they learned from the video and write down new questions they might have about the zoo. Dev decided to ask a series of questions to help youths summarize what mathematical content was revealed in the short walk stop video about animal walking patterns. Dev asked the students what the walk stop video was about, and Kiana responded immediately by simply saying “walking patterns”. This was either not the answer Dev was looking for, or, was overly simple. Dev paused for a moment to allow other students to answer, but nobody else answered. Then, a conversation unfolded where Dev would successively ask questions and evaluate youths’ answers until they reached the idea Dev had decided was the focus of the video (Table 2).

[INSERT TABLE 2 HERE]

After a few seconds of silence, Dev asked the youths “how else would you describe what’s being answered though” (line 2.01). This question accomplished three stances: Instrumentally, he drew youths' attention back to the video they had watched earlier. Cooperatively he faced the entire group, panning the camera back and forth to see all the youths. Epistemically he signaled that youths' initial summary of the short video as being about ‘animal walking patterns’ was not enough. Rather, he was interested in new ways of describing what the short video was about.

Kiana responded first, and said “like how the walking patterns work” (line 2.02). Kiana’s response is an epistemic clarification of her previous remark. Previously, she had said the walk stop video was about ‘animal walking patterns.’ Now, she clarified that the walk stop video is about how animal walking patterns *work*. We interpret her remark to be a clarification that the walk stop video is about the mechanism underlying animal walking patterns (i.e., how they work, not just what they are).

Dev did not respond to her epistemic clarification (how the walking patterns *work*), and instead posed a new question: “but we’re trying to find a pattern between which animals?” (line 2.03). We interpreted this new question as both an epistemic stance (redirecting what types of mathematical questions should be asked) and a new cooperative stance (continuing to ask youths to provide him answers - just not the one Kiana had provided). Elena responded to Dev by saying “all of them?” (Line 2.04). Elena’s response is cooperative in that it continues the line of inquiry Dev is putting forward, and epistemic in that it is searching for the correct way (according to Dev) for how to summarize the walk stop video.

What followed in the rest of the transcript is a back-and-forth line of questioning (lines 2.05-12) where Dev led the youths towards the ‘correct’ answer by affirming correct answers and ignoring incorrect answers. Dev first qualified that “it’s a complicated word,” then attempted to scaffold the posed question by asking for a non-scientific or “dumb-like definition of that word.” We acknowledge Dev’s effort to explain the scientific concept in accessible language and commend the facilitator for this attempt. Dev continued his instrumental stance by focusing on getting the group to the anticipated word. Dev took an epistemic stance through his use of scaffolded language to further clarify his position. Dev also continued his cooperative stance by opening his comment and question to the entire group. Dev eventually led the youths, through iterative questioning, to the scientific term ‘quadruped’ (line 2.16).

The discussion ended when Dev made two final statements. First, he made an epistemic stance by summarizing their conversation: “So it's trying to find a walking pattern between quadrupeds right - and then we, we figured out that they all walked the same across - every single animal." Then, he made an instrumental stance by instructing Bella to complete the worksheet section under question 1: "you can - you can just write that - what are walking patterns? Or how do you - how do four legged animals walk - right?" (Figure 1).

Figure 1

Group 3's Discussion Worksheet for the Walking Pattern Video

1. What did you find most interesting in this video?

1. The walking-patterns are the same.
2. The walking-patterns are intresting.
3.

2a. What's the specific question being addressed in this video?

1. How do four-legged animals walk?
2. What are walking patterns?
3. How quadrupeds walk?

quadrupeds

b. What's the major takeaway for you from this video about this place and the animals you can see right now - that you also watched in the video? (list all team member responses)

1. The triangle is the strongest base.
2. The triangle is the strongest shape.
3. My dog walks the same way an elephant does.

3. What other math pattern-related questions come to your mind based on what you notice right here and right now?

1.
2.
3. How are walking patterns different from running patterns?

Episode 2: Problem Posing about the Ethogram

Episode two began shortly after episode one when the participants walked from the City Zoo classroom to the elephant enclosure. Once at the elephant enclosure the three girls gathered around the tablet to watch a walk stop video about how zoologists use ethograms. Ethograms are tools used to categorize and quantify animal behaviors, to gain an understanding of how different species of animals spend their time within the zoo. Ethograms can be used to monitor animal behavior so that if a change occurs, zoologists can investigate and see if the animal is sick. After having watched the walk stop video, the girls began completing the worksheet to summarize the video (Figure 2). Kiana stepped away from the group and was not included in this episode. Similar to episode one, Dev began asking a series of questions to ensure the youths understood the mathematics behind the walk stop video about ethograms. The following conversation unfolded (Table 3).

[INSERT TABLE 3 HERE]

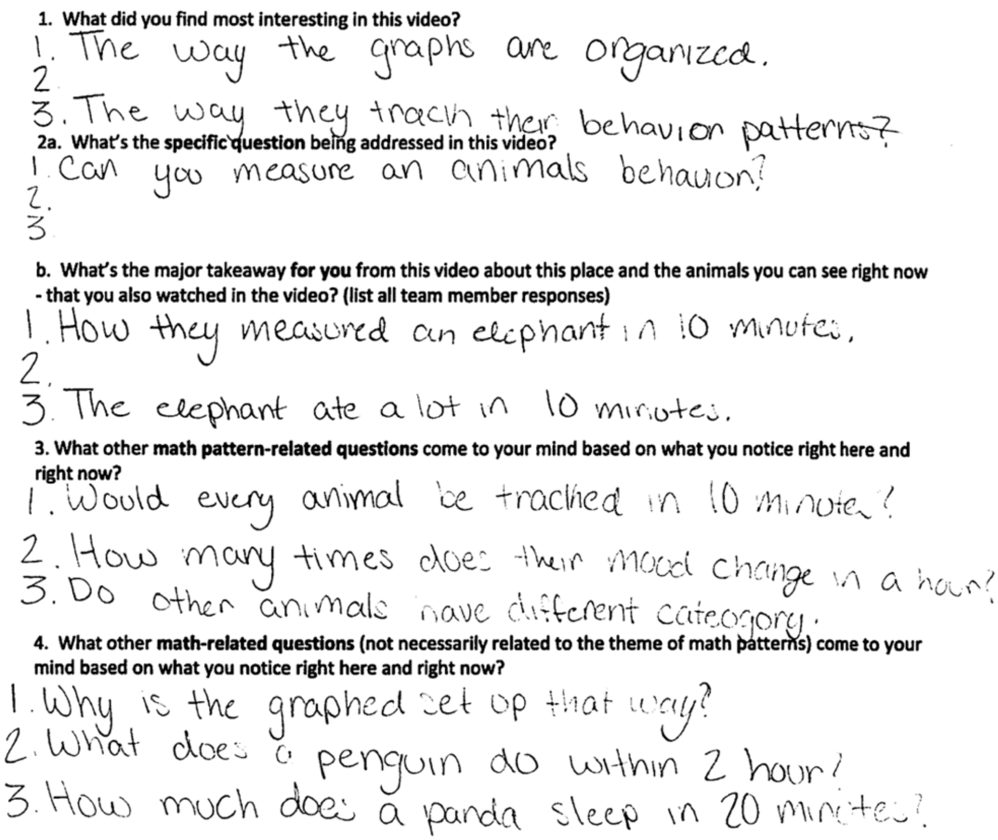
First, Dev posed the question, “what are the graphs telling you” (line 3.01). In this turn-at-talk, Dev accomplished three stances: Instrumentally, he referred the girls back to the video (which discussed how zoo staff used ethograms) rather than to the elephants in the enclosure just behind the fence. Epistemically, he shifted the topic of conversation in a new direction: summarizing the graphs from the video. Cooperatively, he asked Bella and Elena to answer his question. Bella took up Dev’s epistemic and cooperative stances by answering his question with “how they spend their time” (line 3.02-3). Elena agreed with Bella (line 3.03) and Dev affirmed both Elena and Bella’s responses (3.04).

With this idea named (the graphs are about how animals spend their time), Dev pressed further and continued to clarify the youths’ understanding of the graphs (line 3.06-19). Similar to episode one, Dev attempted to scaffold (epistemic stance) his questions, leading to what he deemed as an acceptable answer (line 3.08). We categorized this as a cooperative stance as he is asking the group to provide an answer. Bella and Elena take up Dev’s cooperative stance by answering his question (lines 3.09-10); however Bella’s epistemic stance qualifies that the word is “really hard to say,” while Elena’s response (epistemic stance) is aiming to find the *correct* synonym for the word “organized.”

Dev continued his line of back-and-forth questioning (lines 3.11-13) by having the group refer back to what they said about the graph. He took an instrumental stance by directly referring to the graphs from video they watched, an epistemic stance by attempting to rephrase the question to get the youths to provide his idea of an acceptable answer, and a cooperative stance by opening the question back up the group. Bella went on to reflect on what she saw in the video and answered by gesturing with her pencil and hands the axes of the graph (instrumental stance to the video and the pencil/hand she used to gesture). Bella embodied an epistemic stance by both *showing* and *describing* what she recalled form the graph (line 3.14).

Figure 2

Group 3's Discussion Worksheet for Animal Behavior Patterns



1. What did you find most interesting in this video?
1. The way the graphs are organized.
2
3. The way they track their behavior patterns?

2a. What's the specific question being addressed in this video?
1. Can you measure an animals behavior?
2.
3

b. What's the major takeaway for you from this video about this place and the animals you can see right now - that you also watched in the video? (list all team member responses)
1. How they measured an elephant in 10 minutes.
2.
3. The elephant ate a lot in 10 minutes.

3. What other math pattern-related questions come to your mind based on what you notice right here and right now?
1. Would every animal be trached in 10 minutes?
2. How many times does their mood change in a hour?
3. Do other animals have different categogory.

4. What other math-related questions (not necessarily related to the theme of math patterns) come to your mind based on what you notice right here and right now?
1. Why is the graphed set up that way?
2. What does a penguin do within 2 hour?
3. How much does a panda sleep in 20 minites?

Dev confirmed Bella's response (cooperative stance), but pushed the group to expand upon what was on the bottom axis of the graph (instrumental and epistemic stance). Elena provided a new answer related to Dev's question (line 3.16). Elena expanded upon Dev's epistemic stance and continued the instrumental stance by reflecting on the video and the graph.

To help get the group to an acceptable answer, Dev provided examples of the categories on the bottom of graph, which include "eating [and] sleeping." Dev stayed focused on his epistemic and instrumental stances by leading the group to his idea of a correct answer. Elena was not sure what Dev was wanting, so she decided to move onto the next question (line 3.20). Elena took an epistemic stance by moving away from Dev's line of questioning. Bella then questioned Elena's thought process (cooperative), confirmed they will be moving forward without finishing question 3 (see Figure 1), and took a new instrumental, epistemic, and cooperative stance by reading and asking the last question on the discussion worksheet. This video concluded with Kiana rejoining the group and finishing out the discussion worksheet about animal behavior patterns (See Figure 2). The remaining three minutes of the video demonstrated similar facilitator-led questioning; however, most of the discussions were not related to mathematics.

Summary of Case 1

Case one illustrated how Dev led, through successive questioning and evaluation of youths' responses, Bella, Kiana, and Elena towards a single mathematical concept. In episode one, Dev employed instrumental, cooperative, and epistemic stances to guide the youths in understanding animal walking patterns. He redirected the discussion, qualified the topic as complicated, and used scaffolded language to lead them to the scientific term 'quadruped.' Dev engaged the entire group in a cooperative stance, posing questions and encouraging diverse answers. The sequence concluded with Dev summarizing the discussion epistemically and instructing Bella to complete a related worksheet. In episode two, Dev adopted an instrumental stance by asking about ethogram graphs, guiding the group through related questions. He signaled an epistemic stance, changed the conversation to focus on video content, and engaged in a cooperative stance, affirming responses from Bella and Elena. The sequence concluded with

Dev's instrumental and epistemic stances, Elena diverging with an epistemic stance, and the group moving forward cooperatively. Overall, both sequences depict a dynamic interplay of instrumental, epistemic, and cooperative stances, with Dev guiding discussions based on video content and ethogram graphs.

Throughout this case, Dev commanded the direction of the mathematical thinking and mathematical problem posing, limiting the youths' ability to participate. Because of the facilitator-led discussions, the youths put the ideas and questions from the facilitator on their worksheet. There was a slight shift at the end of the second interaction sequence, only because Bella and Elena were not sure of Dev's prompts. Furthermore, there was a lack of opportunity for participants to use their body to bring in aspects of the local environment to their conversation. Most of the conversations related to the previously watched videos, rather than to the animal enclosures that youths were watching. We believe this may have overly structured youths' focus and reduced opportunities to see mathematics within the actual zoo exhibits.

Case 2: Examining Giraffe Anatomy and Behavior

Case two illustrates how, when facilitator interactions with youths are too *unstructured,* or not scaffolded enough, youths never focus on and refine a single mathematical idea. By unstructured, we mean interactions where the facilitator may value and encourage student participation, but never supports students in selecting a single mathematical idea to focus on and refine over the course of a math walk.. When this happens, youths' efforts to collaboratively see mathematics within and ask mathematical questions is never refined from original and divergent ideas of interest.

Case two follows three youths and one adult facilitator. The three youths were Kyle, Will, and Martin (pseudonyms). All three youths identified as middle-school aged boys. Kyle and Will both identified ethnically as mixed race, and Martin identified ethnically as Latino. The facilitator was a member of the research team, First Author (FA), who identified as a white male. This video is from the first day of the three-day camp.

This case examines two episodes of interaction between Kyle, Will, Martin, and FA. Throughout the episodes, the participants observed the giraffe habitat, discussed the giraffe walk stop video, and attempted to pose new mathematical and non-mathematical questions to their facilitator (FA). First, located along the fence of the giraffe enclosure, the participants summarized the video and asked new questions related to giraffe coat pattern. Second, also along the fence of the giraffe enclosure, the participants shifted to focus instead on how giraffes drink water and related mathematical questions. Both episodes illustrate how the unstructured conversations between the facilitator and youths yielded a lack of focus on any one specific mathematical idea.

Episode 1: Posing a Math Pattern Question

Episode one opened with Kyle, Will, and Martin standing along the fence at the giraffe exhibit with FA facing the camera towards them. After watching the short video about giraffe coat patterns, FA asks the students what the video was about, by directing the students to the discussion worksheet. In an attempt to get the youths to think about other pattern-related questions, FA redirects the students to the questions on the discussion worksheet. The conversation unfolded where FA would continually direct students to consider potential questions to pose, however instead of leading the students to develop a single question, FA enables the students to pose multiple questions based on their noticings throughout the giraffe exhibit (Table 4).

[INSERT TABLE 4 HERE]

The episode begins with FA taking an instrumental stance (line 4.01) by redirecting the youths to the worksheet (See Figure 3). He then starts both an epistemic and cooperative stance by having the youths consider other pattern-related questions they can pose, related to the giraffe enclosure. After FA made the initial epistemic and cooperative stance, Kyle took up FA's cooperative stance by providing an initial answer related to the shapes and sizes of the giraffe's patterns (line 4.02). Martin then shifted the epistemic stance by posing two separate questions comparing the patterns on females or measuring the patterns (line 4.02). After Martin provides an answer, FA cooperatively directed a question to Will to ensure he was participating in the discussion (line 4.03). Not feeling heard, Martin attempts to expand upon his initially posed epistemic stance, however he is still not able to construct a clearly posed question, according to FA (lines 4.06-7). FA does not take up Martin's cooperative stance, but further attempted to cooperatively engage Will in the conversation.

Will engaged in FA's cooperative stance by providing an answer, then posing his own question (line 4.08). However, although FA validated his response (line 4.09), FA reminded Will to consider the instrumental prompt from the worksheet (See Figure 3, Question 3). To cooperatively re-engage the group, FA reposed the question asked on the worksheet, "What pattern-related math questions can we think of?" (line 4.11). Martin cooperatively responded to FA's question by epistemically posing the beginning of a new comment (line 4.12). In line 4.13, FA validated Martin's response with a simple, "Okay." Will then jumped in with an attempt to provide a question that suits FA's idea of an acceptable pattern-related question; however, Will ends the start of this question with a pause (line 4.14). This allowed FA to repose the original instrumental and epistemic stances, however FA cooperatively called on Kyle to provide an answer (line 4.15). Will directly and cooperatively engaged in FA's epistemic stance by posing the start of a new question, however his question does not provide enough details to satisfy the activity (line 4.16; e.g., the ratio of what?). Kyle then provided an epistemic stance by beginning a question, “which giraffe has the most...”, however, he then trailed off and failed to conclude his posed question.

In the concluding portion of this interaction sequence (lines 4.18-22), Martin began discussing a potential solution, as to *how* to solve a math-related problem. As Martin described his example, FA validated his epistemic stances through confirming comments (e.g., lines 4.19 and 21), and in line 4.22, Martin ended his mathematized explanation of how to solve proportions to the group. After Martin ended his comments, Kyle posed a new question (epistemic stance) in an attempt to clarify what Martin was saying (line 4.23). This episode concluded with FA validating Kyle’s posed question (line 4.24), as an acceptable example of a canonical and traditional mathematics question within this activity system, according to FA, by instructing Martin (cooperative stance) to write it down on the worksheet (instrumental stance). The next episode from this case centers around the discussions about Question 4 in Figure 3.

Figure 3
Group 6’s Discussion Worksheet for Giraffe Patterns

1. What did you find most interesting in this video?

You can measure the Patterns of the giraffe.

2a. What's the specific question being addressed in this video?

Is there anything mathmatical About the Pattens on giraffes.

b. What's the major takeaway for you from this video about this place and the animals you can see right now - that you also watched in the video? (list all team member responses)

They have different Patterns and shapes on the body. The male is darker and Taller.

3. What other math pattern-related questions come to your mind based on what you notice right here and right now?

What is the difference of Patterns between two Giraffes.

4. What other math-related questions (not necessarily related to the theme of math patterns) come to your mind based on what you notice right here and right now?

The hegiht between male and female giraffe.
What's the Angle of A giraffe makes when it drinks.

Episode 2: Posing a Math Question about Giraffes Drinking Water

This episode (See Table 5 for the transcript) began shortly after episode one, as the group progressed through the worksheet. FA recalled an earlier comment made by one of the group members to answer the final question from Figure 3. Kyle then responded and was referring to a portion of the video they watched about the giraffe patterns (see Figure 4). Similar to episode one, FA encourages the students to pose mathematical questions as they relate to the giraffe exhibit, however, he continues to remain *unstructured* in his line of questioning (Table 5).

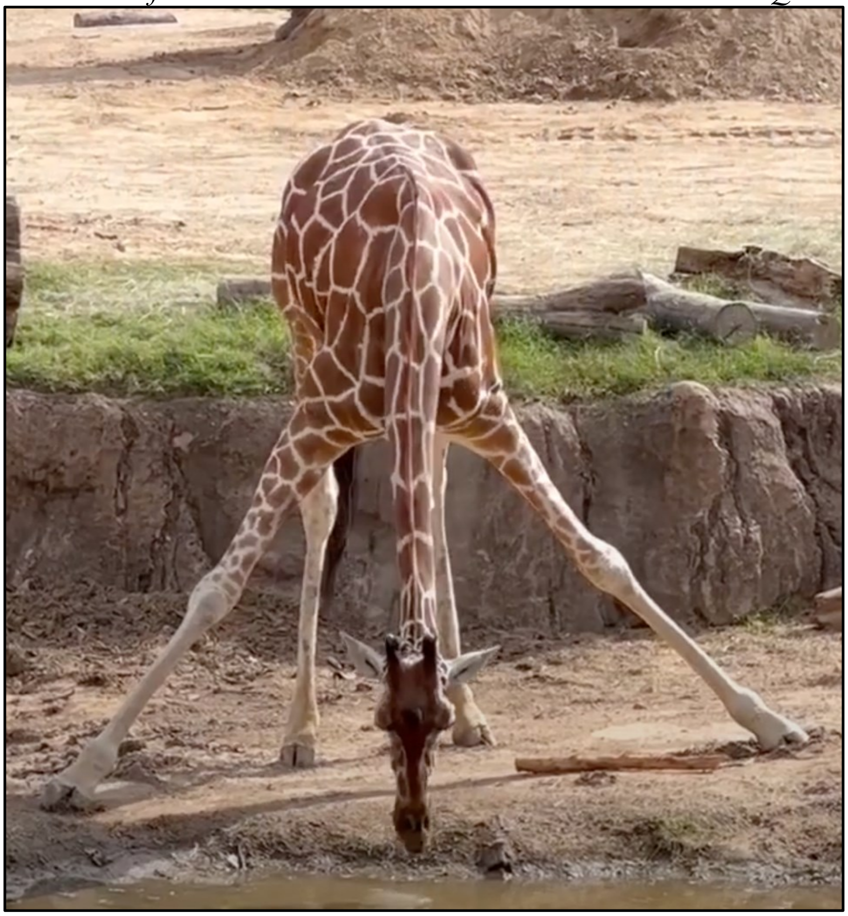
[INSERT TABLE 5 HERE]

FA begins this episode by recalling whether it was Will or Kyle that made a comment about how the giraffe drank water. FA's question was instrumental by referring to both a segment in the video, as well as the giraffes within the enclosure. His question also began an epistemic stance, for the youths to consider how to *mathematize* their observation. After Kyle affirmed that it was him (line 5.02), FA then explicitly opened the question up to the group in the following turn-of-talk (cooperative stance), as to how they "could frame it as a math question." We consider this an instrumental stance towards the video, the giraffes within the enclosure, and question 4 from the worksheet, and FA was continuing his epistemic stance by pushing for the mathematizing their observation as it was a part of the designed activity system.

Kyle began by posing a few questions (line 5.04) and gestured with his hands that the giraffe is making a triangle shape when it goes down to drink water. Kyle took an instrumental stance using an environmentally coupled gesture as it related to the position a giraffe as it drinks water, as well an epistemic stance by posing different questions as he sought to gain clarity on an acceptable mathematical position. FA then redirected Kyle (cooperative stance) to "think about a triangle," which sought to further advance FA's original epistemic stance by confirming Kyle's environmentally coupled gesture of the triangle. Will then posed a question, shifting the epistemic stance (line 5.08), which has already been answered by FA (cooperative stance). FA encouraged the youths to "take that a step further." Epistemically, FA is directly suggesting that this posed question is not an acceptable question and that it should be more rigorous. Will then went on to introduce a mathematical term, encouraged by FA (cooperative stance; lines 5.10-12). In this segment, instrumentally the speakers continued the line of questioning related to the worksheet, and epistemically, there is affirmation from FA to continue the use of mathematical terminology.

Figure 4

Screenshot from the Video Related to Case 2's Posed Math Question



FA then attempted to get the youths to pose an acceptable math-related question (epistemic stance; line 5.13) and instrumentally, FA redirected the youths to observe the giraffes to reconsider how the giraffes go about drinking water. Martin then chimed in by reading directly off of the worksheet (instrumental stance) to repose the original question (epistemic stance). FA continued his epistemic stance by encouraging the group (cooperative stance) to pose a question that is mathematical (line 5.15). Further, FA took an instrumental stance by generating an environmentally coupled gesture with his hands and the camera to mimic the giraffe taking a drink of water. After seeing this, Will and Kyle both resumed trying to think of the mathematical term that beings with "D." This is a continuation of the epistemic stance posed by Will, and is encouraged by FA. To push the youths to pose a mathematical related question, FA asked about "measuring" the triangle. This is an epistemic stance by suggesting a way to better mathematize the originally posed question, according to FA.

In line 5.26, FA took an epistemic stance that validated Kyle's response ("There you go, Absolutely"). Following this, FA made a rare moral stance by stating "That's a great one." This stance positioned FA himself as trustworthy in evaluating students' questions, as well as positioning the other questions as less rigorous (see line 5.09) or not mathematical enough (see line 5.03-6). As indicated in the passage, FA's stance assumes that the other posed questions were either less rigorous or not mathematical enough. By praising one question as "great," FA indirectly critiques the quality or relevance of the previous questions without directly disparaging them. This can be seen as a diplomatic way of expressing disagreement or preference while still maintaining a positive and constructive tone in the conversation. This episode concluded with FA taking an epistemic stance by restating the posed question, an instrumental stance in relation to the discussion worksheet and video, and a cooperative stance by telling Martin what to write (See Figure 4).

Summary of Case 2

Case two illustrated how Kyle, Will, and Martin worked with FA to consider their mathematical thinking and questioning, in an unstructured way. In the first episode, the discussion revolved around giraffe coat patterns. FA initially took an instrumental stance, redirecting the youths to the worksheet, and introduced an epistemic and cooperative stance by prompting them to consider additional pattern-related questions related to the giraffe enclosure. The subsequent analysis focused on shifts in epistemic and cooperative stances. Kyle engaged cooperatively by providing an initial answer, and Martin introduced new epistemic questions. FA attempted to engage Will cooperatively, but Martin struggled to construct a clear question. Will participated in FA's cooperative stance, posing a question, but FA emphasized the instrumental prompt from the worksheet. The episode involved cooperative re-engagement, with Martin and Kyle posing questions, and FA guiding the discussion with instrumental and epistemic stances.

In episode two, the discussion centered on posing a math question about giraffes drinking water. FA recalled an earlier comment from Kyle about giraffe drinking behavior. He started with an instrumental and epistemic stance by referring to the video and giraffes within the enclosure, encouraging the youths to mathematize their observation. Kyle posed questions and provided environmentally coupled gestures, taking an instrumental and epistemic stance. FA redirected Kyle cooperatively, encouraging him to think about a triangle. Will introduced a question that has already been answered, and FA pushed them to take it a step further. Will then introduced a mathematical term encouraged by FA's cooperative stance. FA attempted to get the youths to pose an acceptable math-related question, redirecting them to observe giraffes. Martin read off the worksheet to restate the question, and FA encouraged the group to pose a more mathematical question. Kyle and Will continue thinking, and FA suggested measuring the triangle, taking an epistemic stance. FA validated Kyle's response with a moral stance and concluded by restating the question, taking an instrumental stance related to the worksheet and video, and guiding Martin on what to write.

It is important to note that FA’s mathematical intentions are to elucidate a question related to patterns, while the group of youths lacked such a clear intention, which underscores the necessity for a deeper inquiry into the factors influencing problem posing dynamics and effective facilitation of problem-posing. This is imperative to discern whether these variations primarily arise from FA’s involvement, the youths’ purposes or intentions in posing the problem, or other external factors influencing the process. Initially, the youths posed a question related to the ratio of patterns between two different giraffes, then posed a question related to the angle a giraffe makes when it drinks water. This mathematical thinking and questioning involved calculations. Throughout the case, FA (a) praised youths thinking; (b) encouraged the use of mathematical terminology, and (c) was focused on having the youths pose multiple acceptable, canonical math questions, as opposed to refining a single question.

Case 3: Examining the Placement of the 'Hot Wire'

Case three illustrates what we considered to be the most successful interaction between a facilitator and youths. In this case, the facilitator: (a) praised the youths' ideas; (b) re-framed the youths' ideas with their own words *and* domain-specific mathematical terminology; and (c) provided clear and instrumental instructions for how youths could complete the activity. When this happens, youths are afforded the opportunity to present multiple initial mathematical questions about the City Zoo, select a single question to focus on, and refine that question through successive conversations with the facilitator.

Case three follows three youths and a facilitator. The three youths were Sofia, Matt, and Antonio (pseudonyms). Sofia identified as a middle-school aged girl and Matt and Antonio identified as middle-school aged boys. All three youths identified as ethnically Latino/a. The facilitator was a staff member from the City Zoo. In the transcript, we refer to her as the 'Zoo Teacher.' The Zoo Teacher identified as a white woman.

This case examines three episodes of interaction which took place between Sofia, Matt, Antonio, and the Zoo Teacher. In the case, the youths worked with the facilitator to progressively refine a question about the placement of the *hot wire*: an electrified safety line which circles the perimeter of the chimpanzee exhibit in the canopy of the trees. The hot wire is responsible for ensuring that chimpanzees do not leap from the trees above out of the exhibit. First, working along the fence line of the chimpanzee exhibit, the youths and facilitator decide between two possible mathematical questions to ask. Second, working on the ground next to the chimpanzee exhibit, the youths refine their mathematical question. Third, working again along the fence line, the youths finalize their mathematical question and create an annotated photograph to illustrate their question. All three episodes illustrate sequentially refined mathematical thinking engendered by actions of the youths and the facilitator.

Episode 1: Posing the Question about the Hot Wire

This episode opened with Matt, Sofia, and Antonio standing in a line against the fence, all facing towards the chimpanzees within the exhibit. All three youths were instrumentally oriented towards the chimpanzee exhibit. Immediately preceding this interaction sequence, the youths were epistemically focused on taking a photograph to match their initial mathematical question. This was evidenced by a short conversation, moments before, where the youths discussed their initial mathematical question: how many places does the chimpanzee's blanket show up in the enclosure across a single day?

The episode began when Antonio asked his group mates if they "need a picture of the chimps, Or the blanket?" (line 6.01). Instrumentally, Antonio drew attention to two objects within the zoo: the chimpanzees and the blanket. Epistemically, he drew a connection between their initial mathematical question (how many places does the blanket show up?) and these two objects as potential candidates to serve as photographic evidence of their mathematical question. Cooperatively, he invited his group mates into discussion as to whether either or both would suffice for evidence of their mathematical question. A short period of silence followed his question. Sofia broke the silence and suggested "we might need to pick a new question" (line 6.02). Instrumentally, Sofia shifted focus from the photograph to the initial mathematical question. Epistemically, Sofia evaluated the initial mathematical question as insufficient – but did not provide a reason why. We imagine that the initial mathematical question (how many places does the blanket show up?) may have been too mathematically simple. Cooperatively, Sofia's assertion that "we" might need a new question indicated that their group should take up this idea with her. Antonio immediately agreed with Sofia and all three youths physically moved away from the fence line and towards the ground: Matt knelt to write on the clipboard, Sofia knelt in front of Matt, and Antonio stood over both Matt and Sofia. We interpreted this as evidence of an instrumental, epistemic, and cooperative alignment: all three youths were instrumentally oriented towards the clipboard and epistemically/cooperatively focused on asking a new mathematical question.

Antonio made the first recommendation of a new question when he said, "how about the high - high wire (inaudible) how high the wire is?" (line 6.06). Instrumentally, Antonio brought a new object into focus for the group: the hot wire. Epistemically, Antonio posed a question about the hot wire that we interpreted to be evidence of nascent mathematical thinking. The group's original question, 'how many places does the blanket show up in a day' only required counting. Antonio's new question will require the group to find a way to measure an object that is beyond their physical capacity to measure – and could open meaningful mathematical conversations about how a person can measure an object that is too large to apprehend with a ruler or measuring tape. Cooperatively, Antonio posed his question with some hesitation – implying that he wanted his group's opinion on whether this would be a 'good' question. The video recording is inaudible for a few seconds. However, we can see that Matt began writing Antonio's question on the worksheet and Sofia began explaining the original and new question to the Zoo Teacher, who had just walked into the field of view of the camera. We interpreted this as evidence of an instrumental, epistemic, and cooperative alignment: all three youths were (again) instrumentally oriented towards the clipboard and epistemically/cooperatively focused on their new question about the height of the hot wire.

Table 6

Sofia, Matt, and Antonio posing their question about the hot wire

Time	Speaker	Talk [Gesture, Movement]
00:11	Antonio	So we need a picture of the chimps? Or the blanket? (inaudible) (5.0 seconds)
00:20	Sofia	(inaudible) we might have to pick a different question
00:21	Antonio	Yeah let's pick a different question
00:25	Sofia	Yeah lets pick a different one
00:33		[All three youths move away from fence. Matt is holding the clipboard and crouches down to start writing, Antonio is standing behind Matt talking, and Sofia is crouched in front of Matt giving him ideas]
00:37	Antonio	How about the high - high wire (inaudible) how high the wire is? [Matt is writing crouched on the ground]
00:46		[Sofia is speaking to the Zoo Teacher, explaining the group's original question and their new question about the hot wire. The teacher is standing in front of the group of youths and is looking down at Matt as he writes the question on the clipboard]
01:12	Zoo Teacher	That is actually a really interesting question to me – about the height of the hot wire. So you are going to get a picture of the hot wire and you are going to mark it up and use it as a sort of (inaudible). [The teacher continues to re-explain to the youths what they are going to do, the camera is facing Matt who is still crouched on the ground writing in his journal].

At this point, we see the first interaction with the adult facilitator: the Zoo Teacher. After Sofia has explained the group's new question, the Zoo Teacher made two remarks we saw as significant. First, the Zoo Teacher said, "This is actually really interesting to me" (line 6.08). Instrumentally, the Zoo Teacher responded directly to Antonio's new mathematical question. Epistemically, the Zoo Teacher evaluated this question as 'interesting.' We saw this as an important move on the part of the Zoo Teacher because it valued the youths' mathematical thinking. Second, the Zoo Teacher said, "So you are going to get a picture of the hot wire and you are going to mark it up" (line 6.08). Instrumentally, the Zoo Teacher is redirecting attention from the mathematical question (and the worksheet) back to the chimpanzee exhibit. Epistemically, the Zoo Teacher shifted focus from *asking mathematical questions about the zoo* back to *seeing mathematics within the zoo.* Cooperatively, she gave the youths a short and actionable instruction to return to the fence line, take a photo, and mark it up to show their new thinking. We saw this as an important direction from the Zoo Teacher because it was actionable and in response to the line of inquiry the youths had already developed.

This episode closed with the group of youths having solidified a final mathematical question: how high is the hot wire placed? The Zoo Teacher had confirmed that this was an interesting mathematical question and sent them off to capture and annotate a photograph to support this mathematical question. Following this interactional sequence, the youths spent about 4 minutes taking photos of the hot wire, editing the photos to make the hot wire and trees more apparent, and preparing to refine their mathematical question.

Episode 2: Refining the Question about the Hot Wire

This episode opened shortly after the group of youths had taken a photograph of the hot wire while standing along the fence line. The youths returned to the ground to write out their mathematical question. Matt had the clipboard and was writing down their question and ideas for how to answer it. Sofia was kneeling in front of him watching him write and Antonio was standing above the two of them. While this interaction sequence does not involve adult interaction, it was critical to show because it demonstrates how the youths refined their mathematical question about the hot wire.

Table 7

Sofia, Matt, and Antonio refining their question about the hot wire.

Line	Time	Speaker	Talk [Gesture, Movement]
7.01	07:02	Sofia	the hot wire is there because...how do they... do they determine it when the tree starts to get weaker?
7.02	07:30	Antonio	Isn't it to prevent them from jumping over? All the way up the tree?
7.03	07:33	Sofia	Oh yeah you mean like jumping out of it. If they climb too high and they jump to another tree (inaudible) climbing too high [Sofia gestures her hands like monkey jumping from one tree to another] and jumping from one tree to another.
7.04	08:05	Sofia	[Sofia forcefully claps hands together to make a point] So that helps us to determine how high they want to put it because we know how far they jump the most and they want to make it so that if they jump as far as they can they won't jump out.

The episode began when Sofia asked *why* the hot wire is placed where it is. She said, "the hot wire is there because…how do they? Do they determine it when the tree starts to get weaker? (line 7.01). Instrumentally, Sofia continued to orient to the height of the hot wire. However, epistemically we interpreted her remarks as a shift in mathematical focus. Initially, the Antonio's question asked, '*what* is height of the hot wire?'. While the answer to this question might be simple, youths would have to think deeply about different methods for measuring objects which are far away and too large to measure with hand-held measuring devices. Here, Sofia's question pivoted away from '*what* is the height of hot wire' to '*how* do they determine' the height of the hot wire. This new question was interested in the mathematical reasons behind *why* the hot wire is placed at the height it is placed. The new question would necessitate thinking deeply about multiple calculations that factor into the height of the hot wire. We considered this to be a more complex and refined mathematical question that built on Antonio's original question.

Antonio responded to Sofia by saying "isn't it to prevent them from jumping over" (line 7.03). Epistemically, he followed Sofia's shift in focus and offered a reason: it prevents chimpanzees from jumping over. Sofia took up Antonio's reasoning and related it to her idea further. Through gesture and dialogue, she posited a scenario where a chimpanzee could potentially escape by jumping from a tall tree over the enclosure fence (line 7.03). In this turn-at-talk, Sofia simulates a monkey's behavior and how the hot wire prevents the monkey from escaping. Following this gestured simulation, Sofia said, "So that helps us to determine how high they want to put it…" (line 7.04). Epistemically, this refined the initial mathematical question even further. Originally, Antonio's question only focused on the height of the hot wire. Sofia's refinement now considers a mathematical factor that related to the height of the hot wire: the behavior of chimpanzees to jump from a tree. Sofia posited that by measuring how far the chimp can jump from a tree, the zoo facilitators can determine how high to place the hot wire.

This episode closed with the youths refining their initial mathematical question. The youths have not solidified a focus on the hot wire, but rather than just measuring the height of the hot wire – they are interested in the mathematical calculations that factor in or relate to the placement of the hot wire. Sofia has posited one possible factor that could relate to the placement: the maximum length a chimpanzee can jump from a tree. In the next sequence, we show how the final facilitator interaction further clarified and refined youths' mathematical thinking.

Episode 3: Settling the Question about the Hot Wire

This episode opened when the facilitator came back over to the group of youths to check their progress on completing the activity for the day. All three youths were knelt in a triangle formation on the ground nearby a wooden pole. Matt held the clipboard and was transcribing the youths' final ideas about the hot wire onto the worksheet. Sofia held the tablet and was watching on as Matt wrote. Antonio sat behind both Matt and Sofia. The Zoo Teacher approached the youths from the right side, leaned over Sofia and Matt, and began to ask them about the development of their mathematical question. Matt explained that their group was interested in how the zoologist know how high to place the hot wire. Then, the Zoo Teacher helped the youths refine their mathematical thinking:

[INSERT TABLE 8 HERE]

This episode began with the Zoo Teacher clarifying the youths' mathematical question and asking how they might answer the question (line 8.01). Matt answered first and said, "you would look at the picture and see how high it is – see how high the chimpanzees can go" (line 8.02). Instrumentally, Matt oriented towards the photograph of the hot wire – rather than the hot wire in the enclosure itself. This orientation shift paralleled an epistemic shift: Matt (and his group mates) are using the photograph to imagine calculating the height of the hot wire and compare this with the height chimpanzees can climb and jump. However, this calculation was not explained. Sofia cooperated and extended Matt's response by saying "if you know how far a chimpanzee can jump you don't want to the trees if they are close to the outside to be too high so they can't jump out" (line 8.03). Here, Sofia crafted an epistemic stance which clarified what Matt had begun to state. Sofia posited two mathematical ideas that are in relation to the placement of the hot wire. First, there is the quantity that explains how far a chimpanzee can jump. Second, there is the location and the height of the tree. When these two factors are known, the zoologist can place the hot wire so that chimpanzees cannot jump out of the enclosure. Sofia laminated this point by using her body as a tree, and pantomimed as if a chimpanzee were to jump from her body towards the boundary of the enclosure.

At this point, the Zoo Teacher took a moment to understand what the youths were asking. In line 8.04, she asked the youths if they wanted to know the height of the hot wire *or* the reason it is placed at that height. In this turn-at-talk, the Zoo Teacher is clarifying the epistemic purpose of the youths' question. In line 8.05, Sofia confirmed that they were interested in the *reason* behind the wire placement. Matt, elaborated on Sofia's response, and explained what could happen if the hot wire were placed *too* high (line 8.06).

In the remainder of this episode, the Zoo Teacher praised the youths' mathematical question and layered on some new mathematical terminology to support the youths' thinking. First, the Zoo Teacher praised the youths and said, "Wow I think you actually just took it to another level" (line 8.09). Instrumentally, the Zoo Teacher oriented to the youth's work of changing their question multiple times throughout the Zoo exploration. Epistemically, she praised this transition from a simpler mathematical question to a more complex mathematical question. Second, the Zoo Teacher layered on mathematical terminology and said that their question "is about the relationship between all of the things the chimps move on and the distance from those things to one another" (line 8.09). Instrumentally, the Zoo Teacher oriented to the youths' final mathematical question. Epistemically, she restated the youths' question but added in some new mathematical terminology; specifically, *relationship* and *distance*. The Zoo Teacher closed this turn at talk by asking youths how they could capture these intricacies in an annotated photograph.

Sofia responded first and claimed that they could capture a photograph of a barbed wire fence (line 8.10). Antonio followed Sofia and responded that they could capture a photograph of the entire enclosure and "photograph the areas you don't want chimps to go" (line 8.11). The Zoo Teacher closed the interaction by giving the youths clear instrumental instructions on what to photograph and how to annotate the photograph – all rooted in youths' earlier remarks (line 8.12). She closed the interaction by giving Antonio the directive to take the photograph and sent the youths on their way. The statement is more about making a practical suggestion or recommendation rather than expressing a moral judgment. It proposes that Antonio should be the one to take the picture because the speaker believes he has a vision for it. Further, the phrase "Very cool" indicates the speaker's personal opinion about the idea of Antonio taking the picture. This opinion reflects the speaker's subjective assessment of the situation rather than a moral evaluation. The term "cool" typically conveys a sense of approval or enthusiasm, but it does not necessarily imply a moral stance.

Summary of Case 3

Case three illustrated how Matt, Sofia, and Antonio worked with the Zoo Teacher to refine mathematical questions about the placement of the hot wire. Initially, the youths asked a simple question about the number of times a blanket gets moved by chimpanzees. This mathematical question only involved counting. In the first episode, the youths, Matt, Sofia, and Antonio, initially focused on their mathematical question about the chimpanzee's blanket locations. Antonio suggested taking a photograph to answer their question, but Sofia deemed the initial question insufficient, prompting the group to consider a new one. Antonio proposed measuring the height of the hot wire, introducing a more complex mathematical aspect. The Zoo Teacher praised the question's interest and directed them to photograph and annotate the hot wire. This interaction involved instrumental, epistemic, and cooperative stances, leading to a refined mathematical question.

The second episode showcased the youths refining their question about the hot wire. Sofia shifted the focus from merely measuring the height to understanding how the zoo determines its placement based on chimpanzee behavior. Antonio suggested the wire prevents chimpanzees from jumping over, and Sofia expanded on this idea, introducing factors like the distance a chimpanzee can jump and the height of trees. The youths demonstrated instrumental and epistemic stances by exploring refined mathematical considerations for the hot wire placement.

In the third episode, the Zoo Teacher clarified the youths' question and they discussed using a photograph to address it. Sofia highlighted the need to consider the distance a chimpanzee can jump and the height of trees to determine hot wire placement. The Zoo Teacher praised their transition to a more complex question, introducing mathematical terminology like "relationship" and "distance." The youths received instructions on photographing and annotating, showcasing instrumental and epistemic stances. The interaction involved praise for the youths' mathematical thinking and guidance on capturing intricate details in the photograph.

Throughout the case, the Zoo Teacher: (a) praised youths thinking; (b) layered mathematical terminology onto the youths' existing questions; and (c) provided instrumental instructions that were actionable for youths. Because of these facilitator interactions, the youths were able to end with a rather sophisticated mathematical question. This question involved understanding how chimpanzee behavior (the distance a chimpanzee can leap from a tree), tree placement, and tree height – all factor together to contribute to the placement of the hot wire. A screenshot of the youths' final annotated photograph is provided below (Figure 5).

Figure 5
Sofia, Matt, and Antonio's final mathematical annotated photograph about the hot wire

Comparison of the Three Cases

This manuscript examined how youths learned mathematics in informal learning environments and how adult facilitators support or hinder youths learning of mathematics in these spaces. To foster youths learning at the City Zoo, we utilized math walks to engage and connect learners to mathematics and the real-world, through a place-based perspective (English et al., 2010; Fessakis et al., 2018; Richardson, 2004; Wang et al., 2021). We drew upon situated theories of learning (Brown et al., 1989; Greeno, 2006) which suggest that learning consists of participation structures specific to the setting (i.e., the City Zoo). To analytically map the tensions, we drew upon Goodwin's (2007) participation framework to analyze our selected interactional sequences (Jordan & Henderson, 1995), using Goodwin's five stances: instrumental, epistemic, cooperative, affective, and moral. After reviewing our data corpus, we identified three cases: (1) case one illustrated overly structured facilitator intervention, (2) case two illustrated facilitator intervention that was too unstructured, and (3) case three illustrated a more successful facilitator interaction (see Table 9).

Table 9
Summary of Stances by Case

Case 1	Case 2	Case 3
Epistemically and cooperatively led by the facilitator. The	Epistemically and cooperatively led by both the facilitator and	Epistemically and cooperatively led by the youths and supported

instrumental focus was on a single mathematical idea (the mathematization of quadrupeds).	youths. The instrumental focus was on a variety of mathematical ideas, but without refinement (giraffe posture, coat patterns, etc.)	by facilitator. The instrumental focus was on a single idea which was continually refined (the placement of the hot wire).

The broader impacts of this manuscript were to determine how adult facilitators facilitated seeing and asking mathematical questions. First, we describe the adult interactions which constrained youths' abilities to reason mathematically at the informal learning site. Second, we describe the adult interactions which fostered youths' abilities to reason mathematically at the informal learning site. Finally, we conclude with recommendations for future research into mathematical thinking and learning in informal education contexts.

What Constrains Seeing Mathematics and Asking Mathematical Questions?

In reviewing our findings, adults constrained mathematical reasoning in a variety of ways: (a) epistemically, adult facilitators imposed mathematical ideas and questions *for* the youths, (b) instrumentally, adults limited youths' interactions to only entities or objects that were necessary for the completion of the activity (i.e., the tablet and the worksheet), and (c) cooperatively, adult facilitators overly directed and led the mathematical discussions.

Epistemically, the adult facilitator in case one, Dev, consistently enforced the idea of only having one "right" answer. In both episodes, Dev imposed a back-and-forth question strategy to have the group of youths arrive at a desired term; for example, 'quadrupeds' in episode one. Similarly, in case two, FA also epistemically encouraged the youths to utilize mathematical concepts (i.e., dimensions, degree of angle, ratio). Epistemic stances are the knowledge claims introduced and defended by participants (Goodwin, 2007; Heritage, 2013). In cases one and two, the epistemic stances are primarily led by the facilitators, as a reproduction of a canonical mathematical classroom and FA guiding the youths toward an 'acceptable' canonical mathematical concept or question.

The instrumental stances for Dev and his youths in case one were limited to references to either the worksheet or the video that youths previously watched. The instrumental stances in case two primarily involved attention toward the worksheets and videos; there was also a greater instrumental stance to incorporate the giraffes within their exhibit. For greater participation, instrumental stances involve placing or drawing attention to entities necessary for joint meaning making (Goodwin, 2007; Philip et al., 2016). So, limiting the entities in cases one and two led to less opportunities for youths to make explicit connections to the mathematical concepts and to pose mathematical questions.

The mathematical discussion for case one was cooperatively led by Dev. Youths participated only by taking up Dev's cooperative stance and attempting to provide answers to his questions. When youths did not provide correct answers, they either did not receive a facilitator response (as in the case with Kiana) or were redirected with a new question (as with line 2.10). Although there were more youth-led discussions for case two, the cooperative stances for the mathematical discussions were still heavily influenced by FA. For example, case two episode two began with FA bringing up a comment made by Kyle, which he continued to question all three youths to consider *how* to mathematize their original comment. Cooperative stances are used to *share, construct,* and *sustain* ideas that contribute within the activity system (Goodwin, 2007; Hall & Stevens, 1994). By limiting *who* is sharing, and *how* they shared their ideas, the youths could begin to reject a posed cooperative stance, which is exhibited by Bella and Elena at the end of the second interactional sequence (Goodwin, 2007, 2018; Singer et al., 2013).

To summarize, facilitators constrained 'seeing mathematics' and 'asking mathematical questions by: (a) epistemically: promoting canonical mathematical ideas and questions, which limited the knowledge claims for the youths; (b) instrumentally: restricting the entities necessary for joint meaning making of mathematical concepts and questions; and (c) cooperatively: maintaining facilitator control over the direction of the conversation.

What Supports Seeing Mathematics and Asking Mathematical Questions?

In reviewing our findings, what supported the youths to see mathematics and ask mathematical questions, were: (a) epistemically: allowing youths to lead the mathematical discussions with both their dialogue (i.e., talk) and bodies (e.g., shared perspectives and pointing); (b) instrumentally: the youths' observations and noticings furthered their discussions, moving beyond the worksheets and videos, and towards actual entities within the City Zoo; and (c) cooperatively: with the positive support of the adult facilitators and interactions, the youths were able to primarily led their mathematical discussions and end with a rather sophisticated mathematical question.Epistemically, the youths from case three took charge of how they saw mathematics at the chimpanzee enclosure, which can be seen through their discussions in episodes one and two. To further advance their epistemic claims, the adult facilitator layered mathematical terminology onto the youths' existing questions. This approach from the adult facilitator differed from both cases one and two, where the adult facilitators imposed their own mathematical noticings or questions onto the youths (Rumack & Huinker, 2019).

The youths in case three instrumentally based their observations and discussions off of what they *saw* in the chimpanzee exhibit, more specifically, the hot wire surrounding the exhibit. Although the youths from case three were using the worksheet as a launching point for completing the activity, these youths moved beyond what they watched in the video to pose a unique mathematical question of their interest (Cai et al., 2023; Greeno, 2006; Streek et al., 2011). Additionally, the youths in case three instrumentally utilized their tablet, in a way that was not seen by the groups in cases one and two. The youths in case three were not only able to *pose* a unique mathematical question but take a photograph and annotate the photograph to depict their epistemic stances, to further advance their knowledge claim (Cai et al., 2023; Goodwin, 2007; Rumack & Huinker, 2019).

Cooperatively, the youths in case three involved each other in their discussions (Goodwin, 2007) to build upon their epistemic and instrumental stances. Further, in case three-episode three, the adult facilitator cooperatively gained an understanding of their questions and how they ended up there, praised the youths' mathematical thinking, and offered actionable instructions for how the youths could advance in the activity. Because of these facilitator interactions, the youths were able to end with a rather sophisticated mathematical question. Whereas, in cases one and two, the cooperative stances are primarily facilitated and led by the adult facilitators, which did not advance the youths' mathematical questioning, like it did for the youths in case three.

To summarize what fostered the youths to see mathematics and ask mathematical questions: (a) epistemically, the adult facilitator facilitated mathematical discussions by offering a way to include the desired mathematical terminology, without imposing it on the group; (b) instrumentally, the adult facilitator encouraged the use of both the youths' observations from the exhibit and the tablet to advance within the activity; and (c) the adult facilitator cooperatively praised the youths, then offered actionable advice to further advance within the activity, as well as to deepen their epistemic claims.

Implications, Future Directions, and Limitations

This study examined how facilitators can support mathematical reasoning and problem-posing in informal learning environments, through the exploration and creation of math walk stops. In this section, we will discuss three main implications of this work.

First, this study gives guidance as to how facilitators in informal learning environments can support students in *recognizing mathematics* in these environments and *posing mathematical problems* in these environments. Rather than being solely for youth supervision, adult facilitators can *see mathematics* alongside youths and provide epistemic, cooperative, or instrumental feedback to refine youths' *mathematical questions*. We address the lack of research on mathematics learning in informal environments (Pattison et al., 2017), and give guidance for how facilitators can support students in seeing and reasoning with mathematics (Nemirovsky et al., 2013; Pattison et al., 2016, 2017, 2018; Vandermaas-Peeler et al., 2015). In particular, we recommend that facilitators (1) layer mathematical ideas and terminology onto students' existing observations, (2) limit the emphasis on their own mathematical noticings and wonderings, and (3) allow youths to lead mathematical discussions, while offering actionable and specific feedback.

Second, this study underscores the significance of aligning the intentions behind problem-posing with the perspectives and goals of both facilitators and learners. When engaging in problem-posing practices, it is difficult to separate oneself from the limits of canonical forms of mathematics (e.g., Walkington & Hayata, 2017), and to be open to a full range of possibilities for what counts as mathematical questions. We address the lack of research on structures, designs, and supports for problem-posing (Cai & Hwang, 2023; Walkington et al., in press) and expand ideas about how problem-posing can draw on familiar contexts and artifacts (Bonotto, 2013; English, 1998). We also build off of recent quantitative research suggesting the combining structured and unstructured approaches to problem-posing may be most effective (Wang et al., 2022). We recommend when engaging in problem-posing in informal environments, that facilitators (1) do not let pre-conceived goals relating to what it means to do and learn canonical mathematics direct the activities, (2) support students in iteratively refining their mathematical questions, transitioning from informal noticings and wonderings to questions that increasingly layer on mathematical, quantitative, and spatial considerations, (3) carefully manage the amount of structure they provide to students in their problem-posing activities, drawing upon prior student thinking and work whenever possible, while also maintaining rigor.

Third, this study provides guidance on how experiences in informal learning environments can be a way to expand what counts as mathematics, through effective facilitation practices. In informal environments, the "product" students are working towards can be an annotated photo accompanied by a rich conversation about that photo, as shown in our third case, rather than achievement on a mathematics test. Students can see mathematics in the world around them in a video format when experiencing walk stops, seeing in the video bodies in motion experiencing mathematics in a sensory and perceptual manner. Students can move through spaces and use gestures to express and advance mathematical knowledge (Abrahamson et al., 2020; Nathan, 2023; Walkington et al., 2018), as we saw when Kyle made the triangle with his hands, when Sofia "became" the money, and when Bella points to a part of a graph that was previously in the space in front of her. We recommend that informal learning facilitators:

(1) Consider the physical spaces learners can occupy when engaging in problem-posing and informal mathematical reasoning, and consider the affordances and characteristics of those spaces. After considering these spaces, facilitators should design activities that maximize the use of physical environments to enhance learning. For example, create learning stations that encourage movement and interaction with physical objects to illustrate mathematical concepts.

(2) Pay close attention to students' gestures and also monitor the ways in which they can effectively use gestures to advance spatial and embodied mathematical ideas. Facilitators should actively incorporate gesture-based activities into their lessons, encouraging students to use their hands and bodies to represent mathematical ideas, and provide feedback on these gestures to reinforce learning..

(3) Avoid value judgments relating to what modality of mathematics (e.g., verbal versus drawn versus gestured) is most desirable or what mathematical foci (e.g., Western mathematics versus other mathematical traditions) are most important. Facilitators should embrace a diverse range of mathematical expressions and traditions in their teaching. This can be done by integrating non-Western mathematical examples and encouraging multiple forms of expression, ensuring all students feel their methods and cultural backgrounds are valued.

Future research is needed to examine the relationship between contextual aspects of various informal learning sites and the mathematical concepts and ideas that are relevant to these sites. For example, how might different informal learning sites (e.g., a nature preserve, an art museum, or an after-school program) lend themselves to different mathematical ideas? Furthermore, additional studies are needed to examine the mathematical and pedagogical training that adult facilitators could benefit from to support mathematical reasoning in informal learning environments.

This study has various limitations. First, we did not deeply investigate how the identities (e.g., racial, gender, or other social identities) impacted the forms of cooperation and participation between youths and adult facilitators. Furthermore, cross-site comparisons could yield new understandings about how to train adult facilitators more generally to support youths in *seeing mathematics* and *asking mathematical questions.* Finally, the findings from this study might not reflect the experiences of all participants in similar programs. Therefore, it's essential to be careful when applying generalizing these results to a wider range of informal learning environments.

Declarations

Availability of data and materials
The datasets generated during and/or analyzed during the current study are not publicly available to protect the participants identities but are available from the corresponding authors on reasonable request.

Competing interests
The authors declare that they have no competing interests.

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Appendix A: Tables

Table 2

Bella, Kiana, Elena, and Dev discussing the animal walking patterns walk stop video

Line	Time	Speaker	Talk [Gesture, Movement]
2.01	2:40	Dev	So, for this one you could - like how else would you describe what's being answered though? The video is about walking patterns but
2.02	2:52	Kiana	Like how the walking patterns work
2.03	2:56	Dev	But we're trying to find a pattern between which animals
2.04	2:57	Elena	All of them?
2.05	2:58	Dev	or not all animals, but what type of animals?
2.06	3:02	Elena	mammals
2.07	3:03	Bella	mammals
2.08	3:04	Dev	Mammals, but
2.09	3:05	Elena	it's called something I remember at the beginning.
2.10	3:07	Dev	I know it's a complicated word.
2.11	3:08	Elena	(It is)
2.12	3:11	Dev	(But) what is it? What's like a dumb like definition of that word?
2.13	3:14	Bella	Animals that walk on four legs
2.14	3:15	Dev	Yeah, yeah, there you go. four legged animals.
2.15	3:16	Kiana	Quad - quad - quadra
2.16	3:17	Dev	quadrupeds
2.17	3:18	Bella	That's what it was (*upward inflection*)

2.18	3:20	Dev	So, it's trying to find a walking pattern between quadrupeds right - and then we we figured out that they all walked the same across - every single animal - (1 sec pause) so that's, that's kind of the question we're answering. Right?
2.19	3:25	Bella	Yeah

Table 3

Dev, Bella, Elena, and Kiana discussing Animal Behaviors

Line	Time	Speaker	Talk [Gesture, Movement]
3.01	26:15	Dev	So, what are the graphs telling you?
3.02	26:19	Bella	How they spend their time?
3.03	26:20	Elena	Yeah, I was going to say that
3.04	26:21	Dev	How They spend their time right.
3.05	26:22	Bella	And the location
3.06	26:24	Dev	So, what's one? Yeah, so like, what? They told you what exactly what category of behavior, right? Like what they're doing? Also told you what was that word they use for like the percentage, like for which time they're doing what?
3.07	26:41	Elena	Oh, I forgot
3.08	26:42	Dev	Like, for this time, so for this 10-minute period, maybe two minutes they're doing they're eating? Two minutes, they're standing two minutes there by the tree. Right. So, it organizes right What's one way to say that in your own words
3.09	26:58	Bella	Umm, like I don't know like. It's really hard to say it.
3.10	27:07	Elena	I don’t know., like What's another word for organized
3.11	27:11	Dev	So, whatever. Just think about what the what the graphs told you like, what were the in the bar graph what were like what was on the bottom?
3.12	27:19	Bella	What was on the what?
3.13	27:20	Dev	What was on the bottom of the bar graph?
3.14	27:22	Bella	I only saw the sides. I only saw the time [Gesturing with pencil/hand that the time was on the side of the bar graph]
3.15	27:24	Dev	yeah. So, they were at the bottom
3.16	27:27	Elena	Oh, was it the percentage?
3.17	27:29	Dev	So, on the on the top is the percentage, right? Sure.
3.18	27:31	Elena	Oh, for those
3.19	27:33	Dev	Bottom is like eating sleeping so it would a
3.20	27:41	Elena	I'm just gonna move over to the next question
3.21	27:43	Bella	What are you talking about? (Pointing to the worksheet) (Inaudible, but the girls are discussing and pointing to the worksheet)
3.22	27:55	Bella	We’re just going to move on. (Reading off the worksheet) What is a major takeaway from the video?

Table 4
FA, Kyle, Will, and Martin discussing giraffe patterns

Line	Time	Speaker	Talk [Gesture, Movement]
4.01	20:01	FA	Alright, so what are some other math or pattern [pointing to the worksheet] related questions that come to mind when we're thinking about the giraffes?
4.02	20:11	Kyle	Shapes or, or how big they are?
4.03	20:17	Martin	Or you could, you know the shapes on the screen you could compare it to females to spaced out or how much they have. Or you could measure [pause]
4.04	20:28	FA	Will, what do you think?
4.05	20:41	Will	Um,
4.06	20:43	Martin	You could measure the patterns to the
4.07	20:45	FA	What do you think Will?
4.08	20:46	Will	The height. You could measure the height of?
4.09	20:48	FA	Remember, think of patterns, though
4.10	20:49	Will	Oh
4.11	20:50	FA	What pattern-related math questions can we think of?
4.12	20:57	Martin	If one has more, than the other
4.13	20:58	FA	OK
4.14	20:59	Will	What, what, ok let me see. What [pause]
4.15	21:07	FA	Kyle what can you think of for a pattern question?
4.16	21:09	Will	What ratio does this giraffe have and this giraffe have?
4.17	21:18	Kyle	Which giraffe has the most...
4.18	21:21	Martin	Or it could be proportions, so one giraffe has like 20, 20 shapes on it’s body
4.19	21:28	FA	Mm hmm [confirming Martin]

4.20	21:29	Martin	While the other has like 30
4.21	21:32	FA	Okay
4.22	21:33	Martin	And like, in total at the bottom, is like 35, its 35 and you multiple on the top, take it to 35 and then or half...
4.23	21:57	Kyle	What is the ratio of the patterns of this giraffe and this giraffe?
4.24	22:00	FA	Ok, you can put that [Martin begins writing down this answer on the worksheet]

Table 5

FA, Kyle, Will, and Martin Posing a Question About Giraffes

Line	Time	Speaker	Talk [Gesture, Movement]
5.01	22:15	FA	Kyle was it you, or Will was it you that was asking about the drinking?
5.02	22:20	Kyle	Oh, that was me
5.03	22:22	FA	So, think about how you could frame that as a math question. It doesn't have to be patterns?
5.04	22:30	Kyle	What is the height? Or the weight? What is the height? how high they whenever they go down? What's the height [drawing a triangle with his hands to show the height of a triangle]?
5.05	22:43	FA	So, think about a triangle.
5.06	22:45	Kyle	What's the height of that.
5.07	22:46	FA	Think about a triangle
5.08	22:51	Will	What's the shape they make when they go down?
5.09	22:54	FA	Ok, but, then how can you take that a step further?
5.10	23:03	Will	What's the diame [pause]
5.11	23:07	FA	Close
5.12	23:08	Will	The dia [pause]
5.13	23:10	FA	So, whenever they go down to drink [Pause and pans the video to the giraffe]
5.14	23:20	Martin	[Reading off the worksheet] What other math related questions can you think of?
5.15	23:23	FA	So, whenever they go down to drink, so they go [Gesturing with the camera/hands]
5.16	23:27	Will	it starts with a D, I think.
5.17	23:29	Kyle	What's the diametrical, or something?
5.18	23:32	FA	What about dimensions?
5.19	23:34	Kyle	Dimensions? What is the dimension? No, what's the shape they make when they go down?
5.20	23:39	FA	Okay
5.21	23:41	Kyle	What's the triangle?

5.22	23:44	FA	Okay. So, what about measuring that?
5.23	23:48	Kyle	90, oh, a 90?
5.24	23:52	FA	So...
5.25	23:53	Kyle	So, what degree angle?
5.26	23:56	FA	There you go. Absolutely. That's a great one.
5.27	24:05	FA	What’s the degree of angle that they make when they go down to drink water? [To Martin so that he can write it down – question 3 on the Worksheet].

Table 8

Sofia, Matt, Antonio, and the Zoo Teacher settling their question about the hot wire

Line	Time	Speaker	Talk [Gesture, Movement]
8.01	11:04	Zoo Teacher	Okay so you have a picture of the wire and so the question is about how high the wire needs to be put up [Sofia responds "Yeap!"]. So what do you think – in step five it is like what could you do to determine an answer to that? What could you do to figure that out?
8.02	11:19	Matt	You would look at the picture and see how high it is - see how high the chimpanzees can go.
8.03	11:31	Sofia	There are some factors that are in it too just like if you know how far a chimpanzee can jump you don't want to the trees if they are close to the outside to be too high so they can't jump out and yeah you don't want them to go past a certain point [Sofia is pointing between herself and away from her, then raises her hand up in a height gesture].
8.04	11:50	Zoo Teacher	Mm Hmm So you are you wanting to know where it is at? [Zoo Teacher points to the tablet where the photo is] or how to figure it out/
8.05	11:50	Sofia	Like how to determine exactly where it needs to be [Sofia slices hand into her palm]
8.06	11:59	Zoo Teacher	How do they know how to get it that high or the right height? [All of the youths say "yeahh"] Oh yeahh... And you [teacher points to Sofia] says we are going to know how chimps act.
8.07	12:09	Sofia	Yeah they are going to have to determine that
8.08	12:11	Matt	Because if they can go too high they can jump from tree to tree and if a tree is outside of the enclosure they will get out.
8.09	12:21	Zoo Teacher	So it is about all of the things that the chimps can move around on and their relationship to each other. Wow I think you actually just took it to another level. Your question about how high they know how to put the wire is about the relationship between all of the things the chimps move on and the distance from those things to one another. Right? How could you get a picture and ask that question?
8.10	12:52	Sofia	Well if we angled it properly we could get a shot of the barbed wire
8.11	12:59	Antonio	You could get a picture of the entire area and photoshop the areas you don't want chimps to go
8.12	13:01	Zoo Teacher	You could draw that on. You could use the highlighter to circle that or draw that on or point to that. And then you could have a way to measure the distance between all of those things. And then you might have an idea of the answer, right? Okay [the entire group stands up] I think you've got a vision? Do you want

	Antonio to take that picture I think he has a vision for that picture [Sofia hands the tablet to Antonio and he walks over to the enclosure to take a photo] Very cool.

Appendix B: Video Discussion Worksheet

Mathfinder Video Discussion Sheet

Use this worksheet at each of the 4 Mathfinder stops you visit in the Chapel, Welcome Center, Gym, and Garden. Spend about **10-12 minutes** at each stop.
Team Names: ____
Team jobs: Scribe, Timekeeper, Facilitator
Mathfinder video stop (circle which one you are at):

Welcome Center Gym Garden Chapel

1. What are your general impressions of this video content?

2a. What's the question being asked in this video?

b. What's the major takeaway **for you** from this video about this place that you are in right now? (list all team member responses)

3. What other math-related questions come to your mind at this location? (list all team member responses)

4. In what places on campus do you see a connection to the math strategies you watched in this particular video? Briefly describe the connection for each place you think of.