The Need for Equitable Coordination in Multi-agent Power Systems

Yuhan Du, Javad Mohammadi *University of Texas at Austin* Austin, TX, USA {yuhandu, javadm}@utexas.edu

Abstract— Increasing penetration of distributed energy resources is fueling the evolution of our centralized electric grid to a multi-agent system. System-level performance of multi-agent networks greatly depends on the communication and computational capabilities of nodes (customers). The equitable representation of customers with limited access to communication bandwidth (e.g., caused by sporadic internet access) or computational power (e.g., due to the age of their device) is not well-understood. To this end, this paper investigates equity in the context of multi-agent power systems and showcases the adverse impacts of overlooking struggling nodes. The case studies leverage the Consensus + Innovations approach to simulate the behavior of a multi-agent power system.

Index Terms—distributed optimization, energy aggregation, energy equity, energy justice, multi-agent systems

Nomenclature

| i | Agent index |
|---|---|
| N_N | Number of power system nodes (agents) |
| A | Set of agents in the power system |
| $c_{1,i}, c_{2,i}$ | Generators cost function parameters |
| P_{G_i} | Electric output of agent i |
| P_{L_i} | Electric demand of agent i |
| $\underline{P}_{G_i}, \overline{P}_{G_i}$ | Agent i's minimum/maximum output limit |
| $\Omega_{\underline{B}}$ | Set of generators reaching minimum generation |
| | limit in the energy aggregation |
| $\Omega_{\overline{B}}$ | Set of generators reaching maximum generation |
| _ | limit in the energy aggregation |
| $\begin{matrix} \lambda_i^t \\ P_{G_i}^t \\ \lambda_i^* \end{matrix}$ | Lagrangian multiplier of agent i at iteration t |
| $P_{G_i}^t$ | Electric output of agent i at iteration t |
| λ_i^* | Lagrangian multiplier of agent i at the optimal |
| | point |
| $(P_{G_i}^0)^*$ | Optimal electric output of agent i at previous |
| v | iteration |
| $P_{L_i}^0 \\ \beta_i^{t+1}, \alpha_i^{t+1}$ | Electric demand of agent i at previous iteration |
| $\beta_i^{t+1}, \alpha_i^{t+1}$ | Tuning parameters of agent i at iteration $t+1$ |
| Ω_i | Set of agent <i>i</i> 's neighbors |
| N_m | Maximum number of iterations for Consensus + |
| | Innovations process |
| f | Objective function value |
| f^* | Optimal objective function value |

I. INTRODUCTION

The evolution of the electric power grid is driven by connectivity and autonomy. In its broad sense, autonomy refers to independence in decision-making, inference, energy production, or storage. Wide-spread connectivity is the result of continuous digitalization [1] [2]. These transitions result in

increased distributedness and collected data volume [3]. Distributed information processing methods lend themselves well to processing vast amounts of heterogeneous and distributed data shared among multiple entities. Unlike today's centralized configuration, distributed methods fit well to facilitate information fusion across multi-agent systems [4]. These multi-agent methods' scalability, privacy preservation, and robustness have drawn much research interest [5]. In the framework of networked infrastructures, an agent is broadly defined as a node or a set of nodes that can execute computation or communication tasks. Multi-agent optimization methods establish a collaborative framework among agents to solve problems through local computations and communications. [2] These methods aim to reduce computational complexity while preserving privacy. [6]

Energy system transition makes addressing energy equity issues even more challenging [7]. Energy equity (Energy justice) aims to attain equity in both the social and economic participation in the energy system, particularly stressing the concerns of marginalized groups to make energy more accessible, affordable, clean, and fair-coordinated for all groups [8]. Put differently, central aspects of energy equity such as energy costs [9], air pollution [10], and assets costs [11] need to be addressed in the context of an evolving multientity complex system. Vulnerability to increased energy cost can be considered as an immediate impact on low-income customers (agents) with a limited budget to adopt energy efficiency solutions (e.g., smart meters, roof-top solars) [9]. According to authors in [12], merely 5.8% of total roof-top PV installed is owned by low-income households. Households with roof-top solar enjoy reduced energy bills and are less exposed to distribution grid tariffs [7] [8]. This disparity means the benefits are concentrated on wealthy households (agents) instead of equally distributed [13]. Even worse, the lack of access to clean energy might cause air pollution in their living areas. In addition, vulnerable agents suffer from disconnection problems [14]. In a multi-agent system, communication between agents is crucial to the performance of distributed optimization methods [15]. Sporadic and weak connectivity can widen the energy equity gap and further impact marginalized agents of future multi-agent energy systems. This paper intends to draw readers' attention to the impact of (computation and communication) access inequities on system-level coordination.

Recent works have addressed energy equity problems from different aspects. The author in [8] raises several energy justice-related challenges that power system engineers need to address to pursue energy equity, including equity in electricity planning, system operation and control, distributed energy resources coordination, electricity rate, and demand response program design, and reducing bias in data-driven algorithms. The authors in [7] study challenges for energy equity in multi-agent systems at three levels: intra-agent, inter-agent, and interactions between agents and the grid operator agent. From an economic perspective, [16] and [17] study the equity impacts of electricity tariffs and the influence of energy equity on the prices of agricultural commodities, respectively. Moreover, [18] mathematically models the cost-effectiveness of an equitable transition from fossil fuels to clean energy sources. Author in [10] assessed current energy justice programs in the US and found out that most of the programs addressing the equity problems are managed by non-profit organizations and vary in implementation strategies. Similarly, the author in [19] investigated how energy justice is pursued in renewable energy communities (RECs) from 71 European cases and concluded that some RECs actively contribute to energy justice by providing necessary resources to vulnerable groups. However, they need support and regulations from national legislation for better services. The relationships between energy equity and race [9] and gender [20] have been reviewed by researchers as well. While there has been extensive literature work on different aspects, to the authors' best knowledge, the inequity challenges of marginalized agents (e.g., with sporadic communication access) in the context of system-wide aggregation strategies are not broadly modeled and evaluated [8].

Specifically, this paper studies how marginalized agents with unstable communication will be represented in system-level results of multi-agent problem-solving. To this end, we will build on our models on the foundation of our extensive prior works on distributed consensus-based optimization methods [4] [6]. Specifically, we will leverage our pioneering work on the Consensus + Innovations method to solve optimization problems in multi-agent collaborative setups [4]. In this setup, the Consensus term enforces agreement among agents, while the Innovations term ensures that local constraints are satisfied. This approach has been widely adopted for solving Economic Dispatch (ED) [4], Security Constrained Optimal Power Flow [15], among many other optimization problems. Although the discussions are presented for solving the aggregation problem, our findings apply understanding energy equity in the context of various multi-agent energy optimization problems.

The rest of the paper is structured as follows. Section II includes the mathematical models of the aggregation problem and *Consensus* + *Innovations* approach and disruption modeling. Simulation settings and results are demonstrated in Section III. This section will use the IEEE 118-bus system and compare the results from four different scenarios with various communication line instabilities in normal and contingency times. Finally, Section IV concludes this paper.

II. MATHEMATICAL MODELS

This section introduces the mathematical models of the energy aggregation problem and the *Consensus* + *Innovations* multi-agent solution approach.

A. Energy Aggregation Problem

We adopt a multi-agent view of the electric network, where each agent (power system node) denotes individual electric demand, generation, or a combination of both. The electric connections in this setup enable power transfer between nodes and constitute the links of the power network. Given these preliminary definitions, the aggregation problem seeks to minimize the system-level energy cost while persevering the supply-demand balance and satisfying the physical limitations node's assets.

The system-level cost is modeled as a quadratic function of energy generation cost, and the centralized aggregation optimization problem is formulated as,

$$\min_{P_{G_i}} \sum_{i=1}^{N_N} C_i(P_{G_i}) = \sum_{i=1}^{N_N} (c_{1,i} P_{G_i}^2 + c_{2,i} P_{G_i})$$
 (1)

s.t.
$$\sum_{i=1}^{N_N} P_{G_i} = \sum_{i=1}^{N_N} P_{L_i}$$
 (2)

$$\underline{P}_{G_i} \le P_{G_i} \le \overline{P}_{G_i} \tag{3}$$

The Lagrange multiplier of (2) is regarded as the price signal (λ) , which is the same among agents that have not reached limits of (3). To solve (1)-(3), we remove the inequality constraints on generation to simplify the problem. The Lagrangian function of the above optimization problem reduces to,

$$\mathcal{L} = \sum_{i=1}^{N_N} C_i(P_{G_i}) + \lambda \cdot \left(\sum_{i=1}^{N_N} P_{L_i} - \sum_{i=1}^{N_N} P_{G_i}\right)$$
(4)

Deriving the first-order optimality conditions gives,

$$\frac{\partial \mathcal{L}}{\partial P_{G_i}} = 2c_{1,i}P_{G_i} + c_{2,i} - \lambda = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{N_N} P_{L_i} - \sum_{i=1}^{N_N} P_{G_i} = 0 \tag{6}$$

Assume that λ^* and $P_{G_i}^*$ give the solution to (5) and (6). Thus,

$$\lambda^* = \left(\sum_{i=1}^{N_N} \frac{1}{2c_{1,i}}\right)^{-1} \left(\sum_{i=1}^{N_N} P_{L_i} + \sum_{i=1}^{N_N} \frac{c_{2,i}}{2c_{1,i}}\right) \tag{7}$$

The above λ^* describes the Lagrange multiplier for nodes with non-biding inequality constraints. The following equations represent a generalization of it.

$$2c_{1,i}P_{G_i}^* + c_{2,i} - \lambda^* = 0, i \notin \Omega_{\overline{B}} \cup \Omega_B$$
(8)

$$P_{G_i}^* = \overline{P}_{G_i}, i \in \Omega_{\overline{B}} \tag{9}$$

$$P_{G_i}^* = \underline{P}_{G_i}, i \in \Omega_{\underline{B}} \tag{10}$$

Here, $\Omega_{\overline{B}} \cup \Omega_{\underline{B}}$ refers to the set of non-binding variables. Thus, (7) can be updated as

$$\lambda^* = \left(\sum_{i \notin \Omega_{\overline{B}} \cup \Omega_{\underline{B}}} \frac{1}{2c_{1,i}}\right)^{-1} \left[\sum_{i=1}^{N_N} P_{L_i} - \sum_{i \in \Omega_{\overline{B}}} \overline{P}_{G_i} - \sum_{i \in \Omega_{\underline{B}}} \underline{P}_{G_i} + \sum_{i \notin \Omega_{\overline{B}} \cup \Omega_{\underline{B}}} \left(\frac{c_{2,i}}{2c_{1,i}} - \underline{P}_{G_i}\right)\right]$$
(11)

Thus, we can analytically find λ^* and $P_{G_i}^*$ for a centralized aggregation problem by (8)-(11).

B. Consensus + Innovations Approach

The *Consensus* + *Innovations* approach aims to find solutions for the energy aggregation problem in a fully distributed manner [4]. In this iterative approach, agents collaborate to solve the energy aggregation problem (1)-(3) through local computations and communication with neighboring agents. Consequently, we first make local copies of the "consensus variable" (i.e., λ) and allocate each copy to an agent (power grid node). Then the *Consensus* term in the algorithm enforces consensus among all agents while the *Innovations* term ensures that local constraints are satisfied. Agents cooperate to make an agreement on local copies of λ while obtaining the optimal value for the nodal generation to preserve the supply and demand balance locally. In a nutshell, this process aims to solve the optimality conditions of the underlying optimization problem (energy aggregation) in a fully distributed fashion.

Based on our prior work [4], we update the optimization variables for this energy aggregation problem iteratively as follows.

$$\lambda_i^{t+1} = \lambda_i^t - \beta_i^{t+1} \sum_{j \in \Omega_i} (\lambda_i^t - \lambda_j^t) - \alpha_i^{t+1} (P_{G_i}^t - P_{L_i})$$
 (12)

$$P_{G_{i}}^{t+1} = \begin{cases} \frac{\lambda_{i}^{t+1} - c_{2,i}}{2c_{1,i}}, & 0 \le \frac{\lambda_{i}^{t+1} - c_{2,i}}{2c_{1,i}} \le \overline{P}_{G_{i}} \\ \overline{P}_{G_{i}}, & \frac{\lambda_{i}^{t+1} - c_{2,i}}{2c_{1,i}} > \overline{P}_{G_{i}} \\ 0, & \frac{\lambda_{i}^{t+1} - c_{2,i}}{2c_{1,i}} < 0 \end{cases}$$
(13)

Our prior works show that the updates in (12) (13) converge to the optimal solution as long as the communication topology forms a connected graph and α and β are properly tuned [4], [15]. Further, if we have knowledge about the previous instances of solving the underlying optimization problem, we can adjust (12) to,

$$\lambda_i^{t+1} = \lambda_i^t - \beta_i^{t+1} \sum_{j \in \Omega_i} (\lambda_i^t - \lambda_j^t) - \alpha_i^{t+1} (P_{G_i}^t - (P_{G_i}^0)^* - P_{L_i} + P_{L_i}^0)$$
 (14)

In each iteration, each agent i performs the updates locally and shares the updated values of λ_i with the neighboring agents until the consensus on λ is reached and optimality conditions of (1)-(3) are fulfilled. Mathematically, we define the convergence criterion as the difference between the local λ_i and optimal λ^* . This value is compared against a predefined threshold (ε) so that,

$$|\lambda_i - \lambda^*| \le \varepsilon, \forall i \in A \tag{15}$$

C. Disruption Modeling

In our multi-agent cyber-physical infrastructure, each node is an agent. In addition to the physical power lines connecting agents, there are communication lines supporting communication between agents. This paper assumes that the communication topology is the same as the physical connections (i.e., physics-based communication), which means communication only occurs between electrically connected nodes.

The promise of this paper is to examine the need for equitable aggregation and we evaluate our primary hypothesis by considering two scenarios for struggling agents (nodes): (i) lack of access to reliable communication and (ii) sporadic communication in the face of physical disruptions. As described in Section II-B, agents need to exchange local λ in each iteration to reach convergence. Under unreliable communication, each agent i cannot receive the latest λ_i from neighboring agents in Ω_i . Instead, agents may use the last communicated λ value (prior to the disruption) in upcoming iterations. During physical disruptions, agent i may lose communication and physical connections with some neighbors. Put differently, the set of neighboring agents Ω_i will be a function of iterations, i.e., $\Omega_i(k)$. The Consensus + Innovations update (outlined in (14)) skips the communication between disrupted physical lines.

III. SIMULATIONS & RESULTS

A. Test Setup and Initializations

We evaluated our algorithm using the IEEE 118 bus (node) test system [21]. Without loss of generality, we randomly choose three agents as struggling agents (nodes): agent 5, agent 35, and agent 95. We assume these agents experience disruption in communication with other agents. However, these disruptions do not lead to islanding. The disrupted connections are between agents 5 and 3, 5 and 4, 5 and 6, 5 and 8, 35 and 36, 95 and 96. We simulate the disruptions by breaking them off at a specific iteration during *Consensus + Innovations* update process.

Table I presents four scenarios of interest. Scenarios 1 and 2 are unreliable communication without disruption of physical power lines. The disruption in communication restores before *Consensus* + *Innovations* updates end (Scenario 1) or stays disconnected before *Consensus* + *Innovations* updates terminate (Scenario 2). In both scenarios, *Consensus* + *Innovations* uses the last communicated value before the disruption. For Scenarios 3 and 4, we have physical disruptions in addition to sporadic communication. We assume that physical disruption stays unresolved until the end of *Consensus* + *Innovations* process. Thus, *Consensus* + *Innovations* process skips the communication between disrupted physical lines. For comparison, we propose Scenario 0 (no disruption) as a baseline. The convergence performance of Scenario 0 is shown in Fig. 1.

We will use a high-quality starting point to initialize λ and P_G . We set the convergence threshold $\varepsilon=0.05$ in (15), and the maximum number of iterations $N_m=600$. Our tests are

TABLE I FOUR SCENARIOS OF CONNECTION DISRUPTIONS.

| Communication Line | Physical Power Line | |
|-----------------------------|---------------------|------------|
| Communication Line | No Disruption | Disruption |
| Disrupted and Recovered | Scenario 1 | Scenario 3 |
| Disrupted but Not Recovered | Scenario 2 | Scenario 4 |

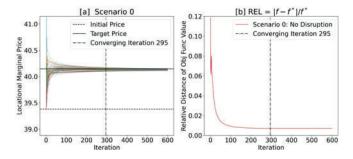


Fig. 1. Convergence performance for Scenario 0 (no disruption). The vertical dashed line indicates the iteration that the convergence criterion is met. The horizontal dashed line shows the initial value (initial price). The horizontal solid line shows the optimal value. [a] shows λ of all agents throughout iterations. [b] presents the relative distance of the objective function value $REL = |f - f^*|/f^*.$

performed using PyCharm (Version 2022.3.2) platform with Python 3.6 environment on a MacBook Pro (Intel, 2020).

B. Testing Results

1) Lack of Access to Reliable Communication: In this case, communication lines associated with struggling agents are disrupted but physical power lines stay intact. We disconnect communication lines at the 20th iteration for Scenario 1 and 2 and reconnect them at the 400th iteration only for Scenario

To display the convergence process, we illustrate the evolution of optimization variables and convergence metrics throughout iterations in Fig. 2 and 3. Scenario 1 takes 555 iterations to converge and Scenario 2 fails to converge. As it can be observed from Fig. 2 [b] and 3 [b], there is a large error and divergence between Scenario 0 and Scenarios 1 and 2. This is due to using the outdated communicated value to compensate for sporadic communication during Consensus + *Innovations* updates. This misleads the consensus away from the correct value. It is only when the communication recovers that the error decreases again (Fig. 2 [b]) . Similar results for communication lines disconnected at the 50th iteration and reconnected at the 150th iteration are shown in Table II. But the disruption is less detrimental as it starts later and terminates earlier.

TABLE II NUMBER OF ITERATIONS UNTIL CONVERGENCE FOR FIVE SCENARIOS UNDER TWO SETS OF DISRUPTION AND RECOVERY ITERATIONS.

| | Disruption 50, | Disruption 20, |
|------------|----------------|----------------|
| | Recovery 150 | Recovery 400 |
| Scenario 0 | 295 | 295 |
| Scenario 1 | 305 | 555 |
| Scenario 2 | 387 | N/A |
| Scenario 3 | 295 | 314 |
| Scenario 4 | 315 | 314 |

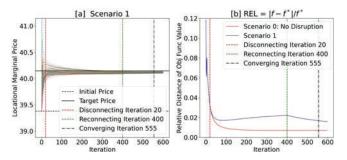


Fig. 2. Convergence performance for Scenario 1. The vertical dashed lines (from left to right) indicate the iteration when communication lines are disrupted, the iteration when communication lines recover, and the iteration when the convergence criterion is met. The horizontal dashed line shows the initial value (initial price). The horizontal solid line shows the optimal value (target price). [a] shows λ of all agents throughout iterations. [b] presents the relative distance of the objective function value $REL = |f - f^*|/f^*$, with the red curve for Scenario 0 and the blue curve for Scenario 1.

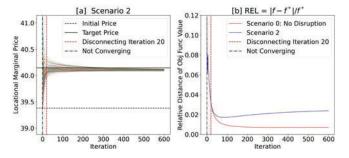


Fig. 3. Convergence performance for Scenario 2. The vertical dashed lines (from left to right) indicate the convergence criterion is not met and the iteration when communication lines are disrupted. The horizontal dashed line shows the initial value (initial price). The horizontal solid line shows the optimal value. [a] shows λ of all agents throughout iterations. [b] presents the relative distance of the objective function value $REL = |f - f^*|/f^*$, with the red curve for Scenario 0 and the blue curve for Scenario 2.

2) Sporadic Communication in the Face of Physical Disruptions: In this case, physical power lines associated with struggling agents are disconnected and cannot recover before Consensus + Innovations updates end. In Scenario 3, the communication recovers before Consensus + Innovations updates end. For Scenario 4, the communication stays disconnected. We disconnect a few communication lines at the 20th iteration for Scenarios 3 and 4 and reconnect them at the 400th iteration only for Scenario 3.

We depict the convergence process of optimization variables and convergence metrics in Fig. 4 and 5. It could be observed from [a] that both scenarios converge to the optimal value. In [b], the converge behavior of both scenarios matches scenario 0. Note both Scenarios take 314 iterations to converge. The converging iteration is the same as they reach convergence before the reconnecting iteration. The convergence speed is close to Scenario 0, which is 295. Considering results for communication lines disconnected at the 50th iteration and reconnected at the 150th iteration in Table II, Scenario 3 is almost unaffected by disruptions. Therefore, we consider skipping the communication of disconnected lines during communication disruption a promising direction for further research to safeguard energy equity.

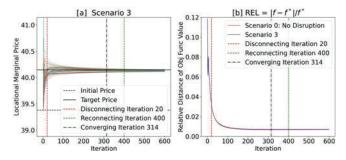


Fig. 4. Convergence performance for Scenario 3. The vertical dashed lines (from left to right) indicate the iteration when communication lines and physical power lines are disrupted, the iteration when the convergence criterion is met, and the iteration when communication lines recover. The horizontal dashed line shows the initial value (initial price). The horizontal solid line shows the optimal value. [a] shows λ of all agents throughout iterations. [b] presents the relative distance of the objective function value $REL = |f - f^*|/f^*$, with the red curve for Scenario 0 and the blue curve for Scenario 3.

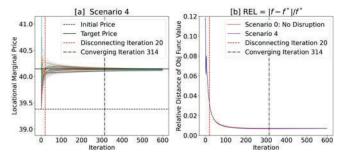


Fig. 5. Convergence performance for Scenario 4. The vertical dashed lines (from left to right) indicate the iteration when communication lines and physical power lines are disrupted and the iteration when the convergence criterion is met. The horizontal dashed line shows the initial value (initial price). The horizontal solid line shows the optimal value. [a] shows λ of all agents throughout iterations. [b] presents the relative distance of the objective function value $REL = |f - f^*|/f^*$, with the red curve for Scenario 0 and the blue curve for Scenario 4.

IV. CONCLUSION

This paper investigates energy equity in the context of multiagent power systems and demonstrates the adverse impacts of overlooking struggling nodes. The case studies utilize the multi-agent *Consensus + Innovations* approach to simulate the behavior of a multi-agent power system under two cases: lack of access to reliable communication and lack of access to reliable communication in the face of physical disruptions. Our simulation results show that connection issues with struggling nodes (agents) can lead to system-level divergence. Put differently, ignoring struggling agents (that are facing inequitable access) can result in system-level disruptions.

Current multi-agent systems lack the awareness of energy equity issues and usually overlook the connection problems between struggling nodes. We highlight this drawback by simulating four scenarios for possible communication and physical power line connection disruptions using IEEE 118 node test system. We will perform extensive simulations on a large-scale system with randomly selected struggling nodes to infer the significance of their connection problems. In addition, further strategies should be taken to reduce the effect of struggling nodes and ensure equitable coordination.

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