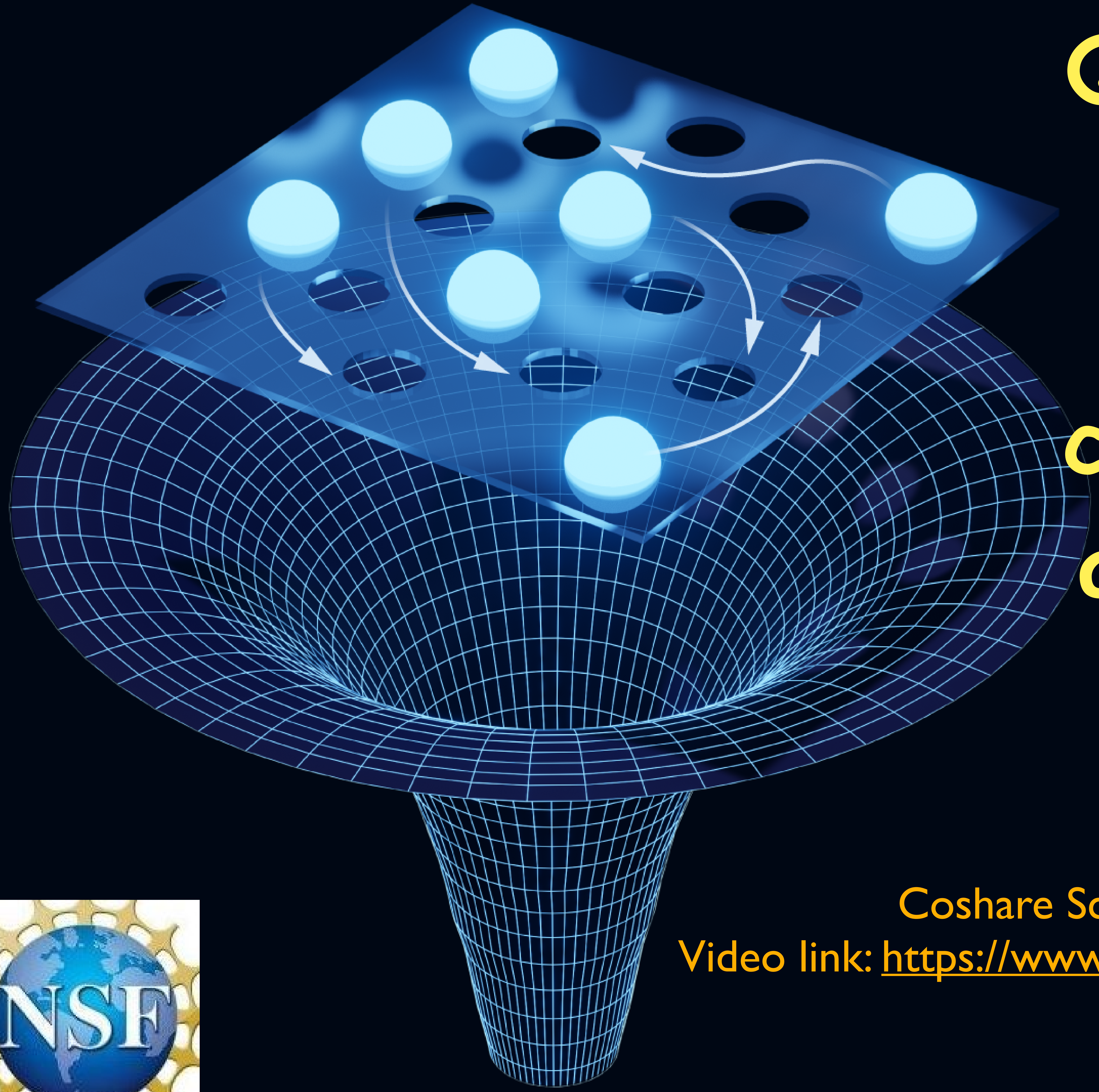


Quantum statistical mechanics of the SYK model, charged black holes and strange metals

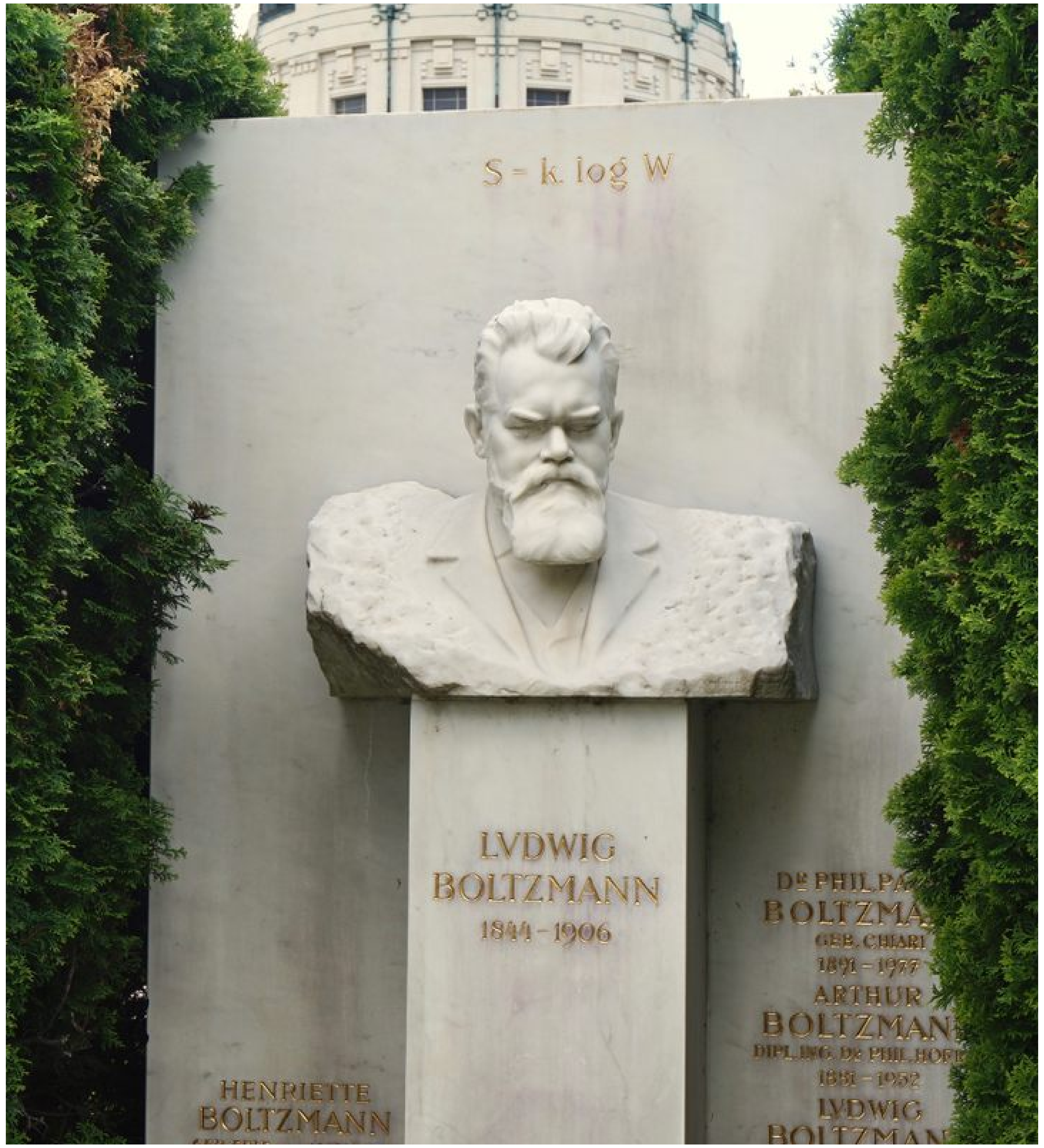
Review articles:
arXiv:2304.13744, 2305.01001

Coshare Science **01**, v4, 1-91 (2023)
Video link: https://www.cosharescience.com/videodetail/119

Subir Sachdev

PHYSICS
HARVARD

Foundations
by
Boltzmann

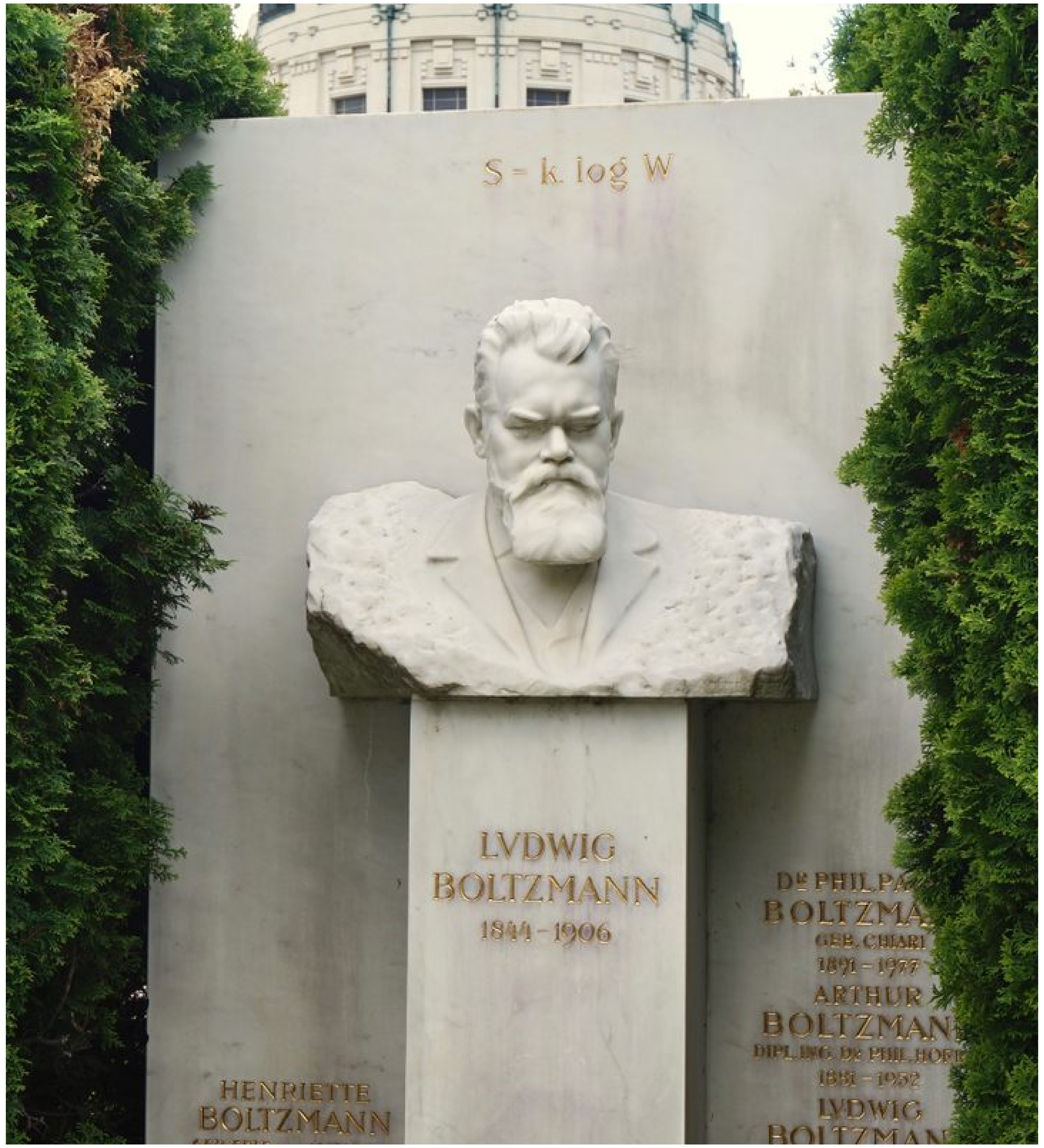


Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Statistical interpretation of entropy (1870)

$$S = k_B \log W$$

Density of quantum states $D(E) = \exp(S(E)/k_B)$



Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

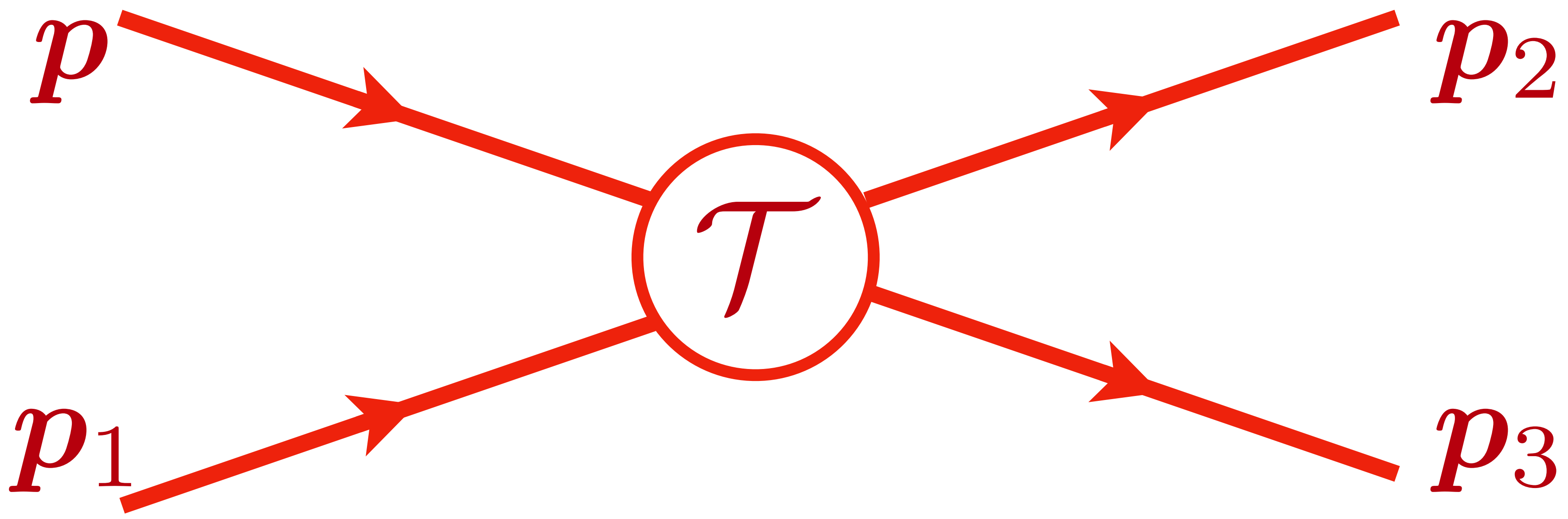
Boltzmann equation (1872)
Dilute classical gas

Molecular chaos: successive collisions are statistically independent

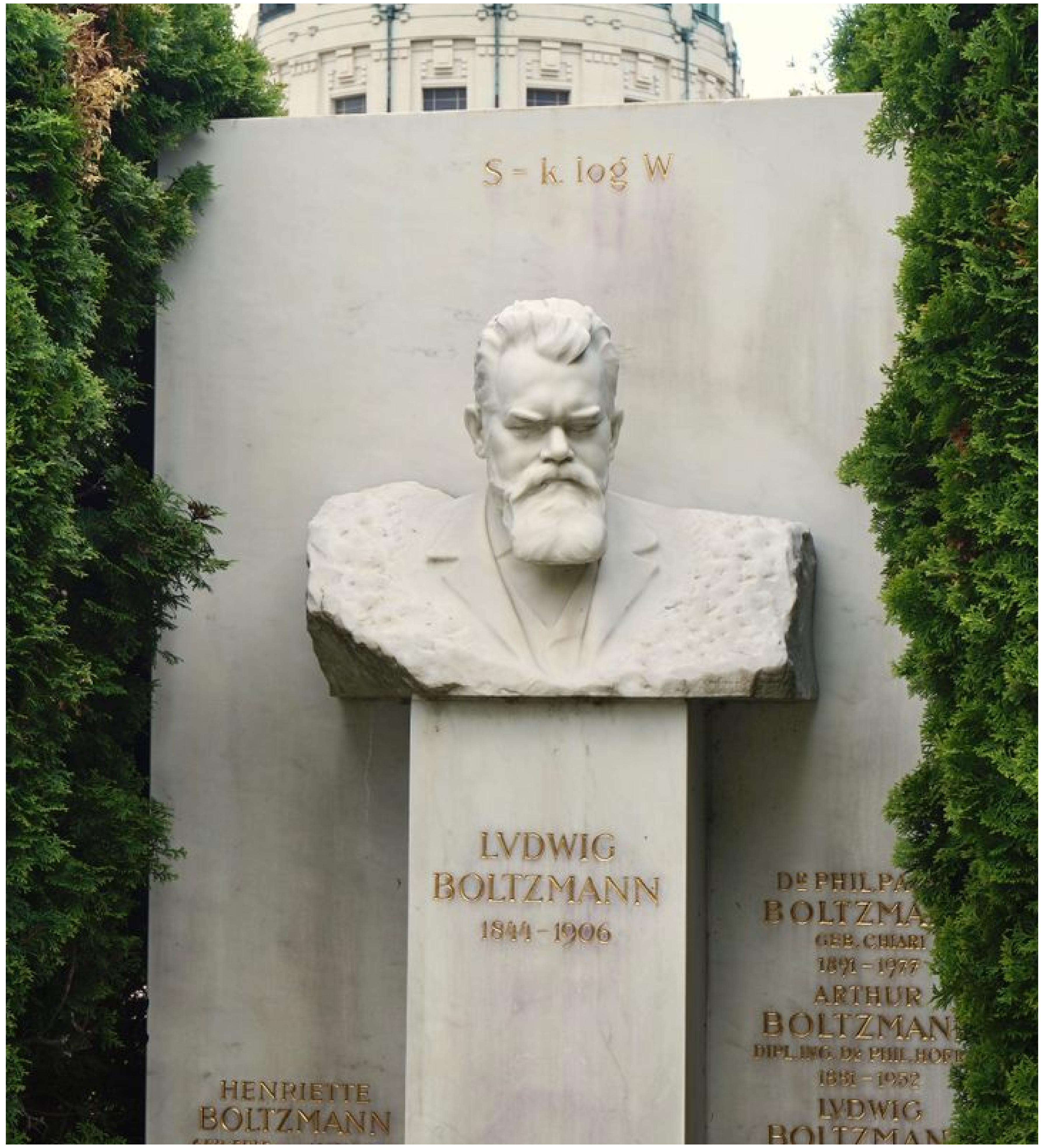
$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$

$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$

$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} \right]$$



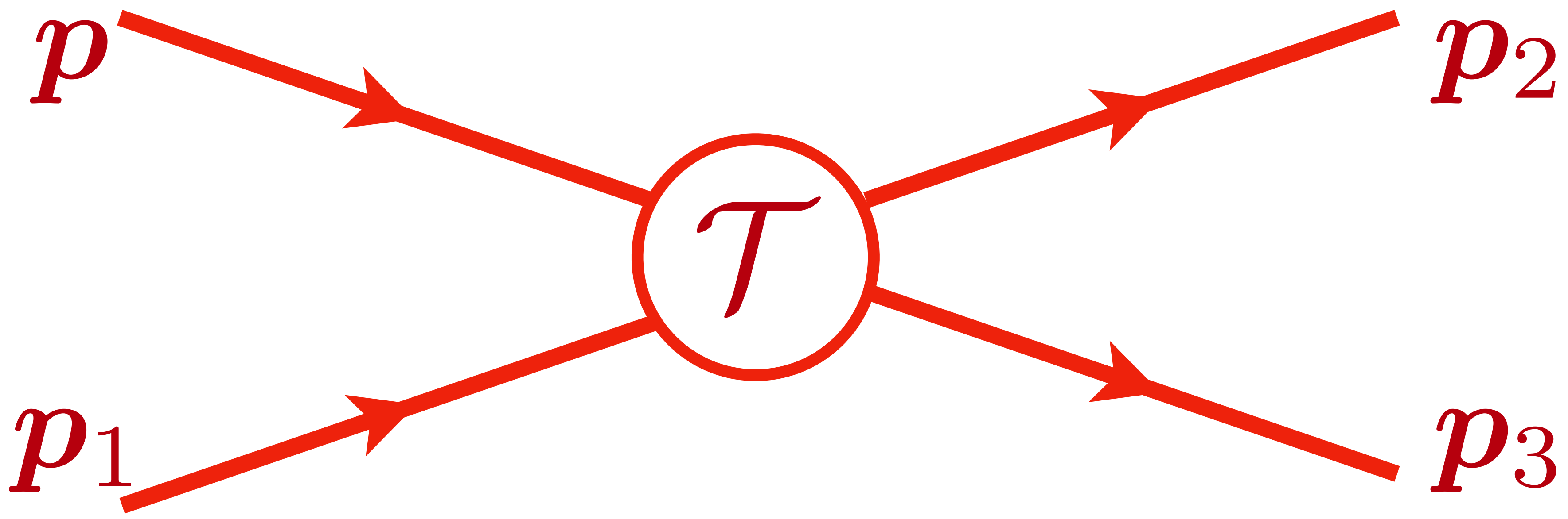
Quantum Boltzmann equation (Landau)
Dense gas of electrons



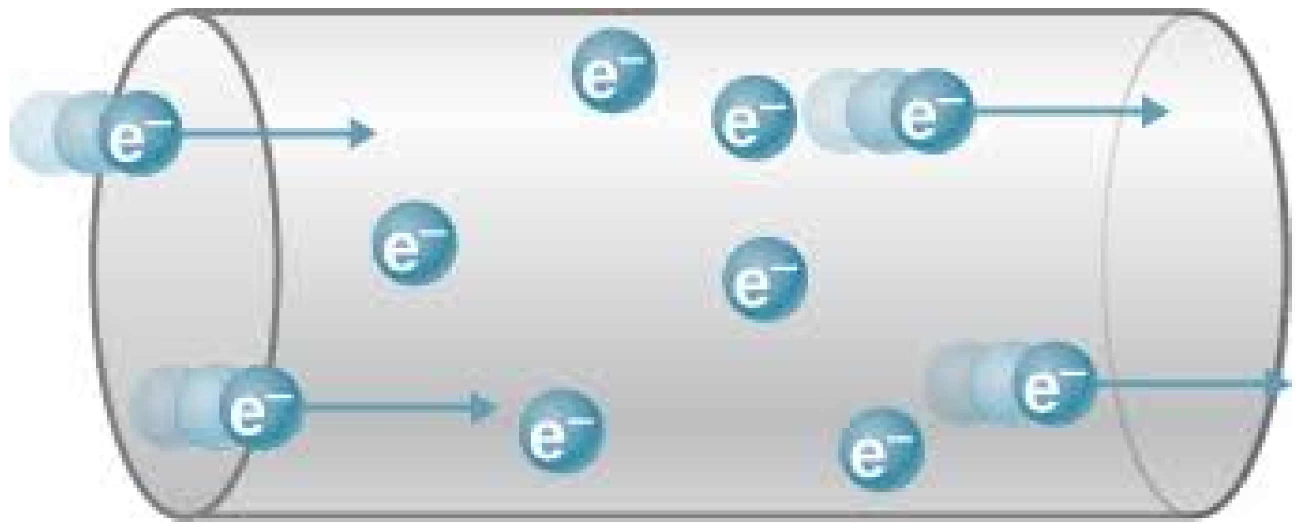
Ludwig Boltzmann
20 February 1844 - September 5, 1906
Vienna, Austria

Neglects quantum interference (entanglement)
between successive collisions

$$\frac{\partial f_{\boldsymbol{p}}}{\partial t} + \frac{\partial \varepsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}} \cdot \nabla_{\boldsymbol{r}} f_{\boldsymbol{p}} + \boldsymbol{F} \cdot \nabla_{\boldsymbol{p}} f_{\boldsymbol{p}} =$$
$$- 2\pi \int_{\boldsymbol{p}_{1,2,3}} |\mathcal{T}|^2 \delta(\varepsilon_{\boldsymbol{p}} + \varepsilon_{\boldsymbol{p}_1} - \varepsilon_{\boldsymbol{p}_2} - \varepsilon_{\boldsymbol{p}_3}) \delta(\boldsymbol{p} + \boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{p}_3)$$
$$\times \left[f_{\boldsymbol{p}} f_{\boldsymbol{p}_1} (1 - f_{\boldsymbol{p}_2})(1 - f_{\boldsymbol{p}_3}) - f_{\boldsymbol{p}_2} f_{\boldsymbol{p}_3} (1 - f_{\boldsymbol{p}})(1 - f_{\boldsymbol{p}_1}) \right]$$



Current flow with electrons in ordinary metals

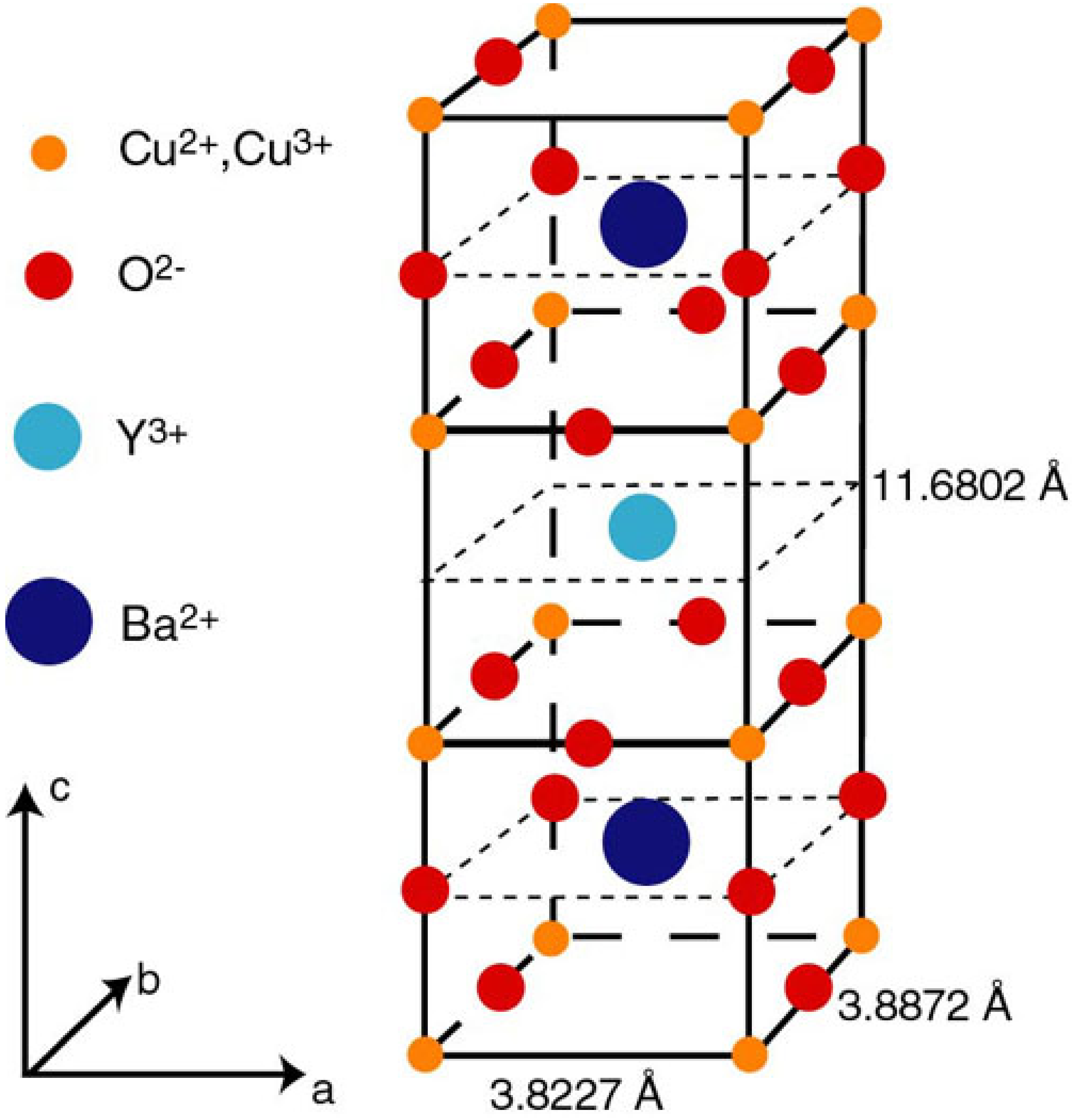


Flow of electrons described by Boltzmann equation $\Rightarrow$
typical scattering time $\tau \sim 1/T^2$, resistivity $\rho(T) = \rho(0) + AT^2$

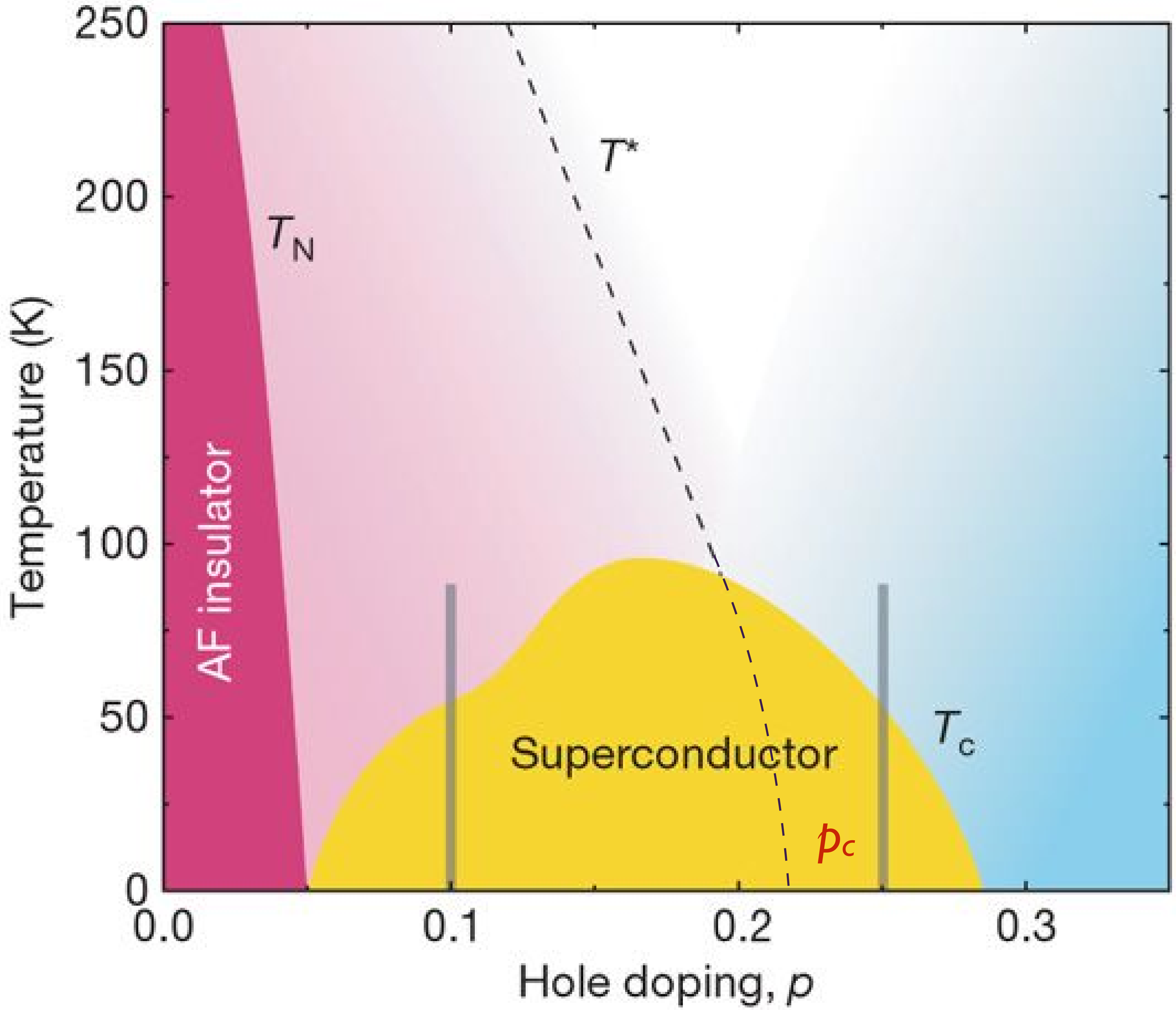
The time τ is much longer than a limiting 'Planckian time' $\dfrac{\hbar}{k_B T}$.

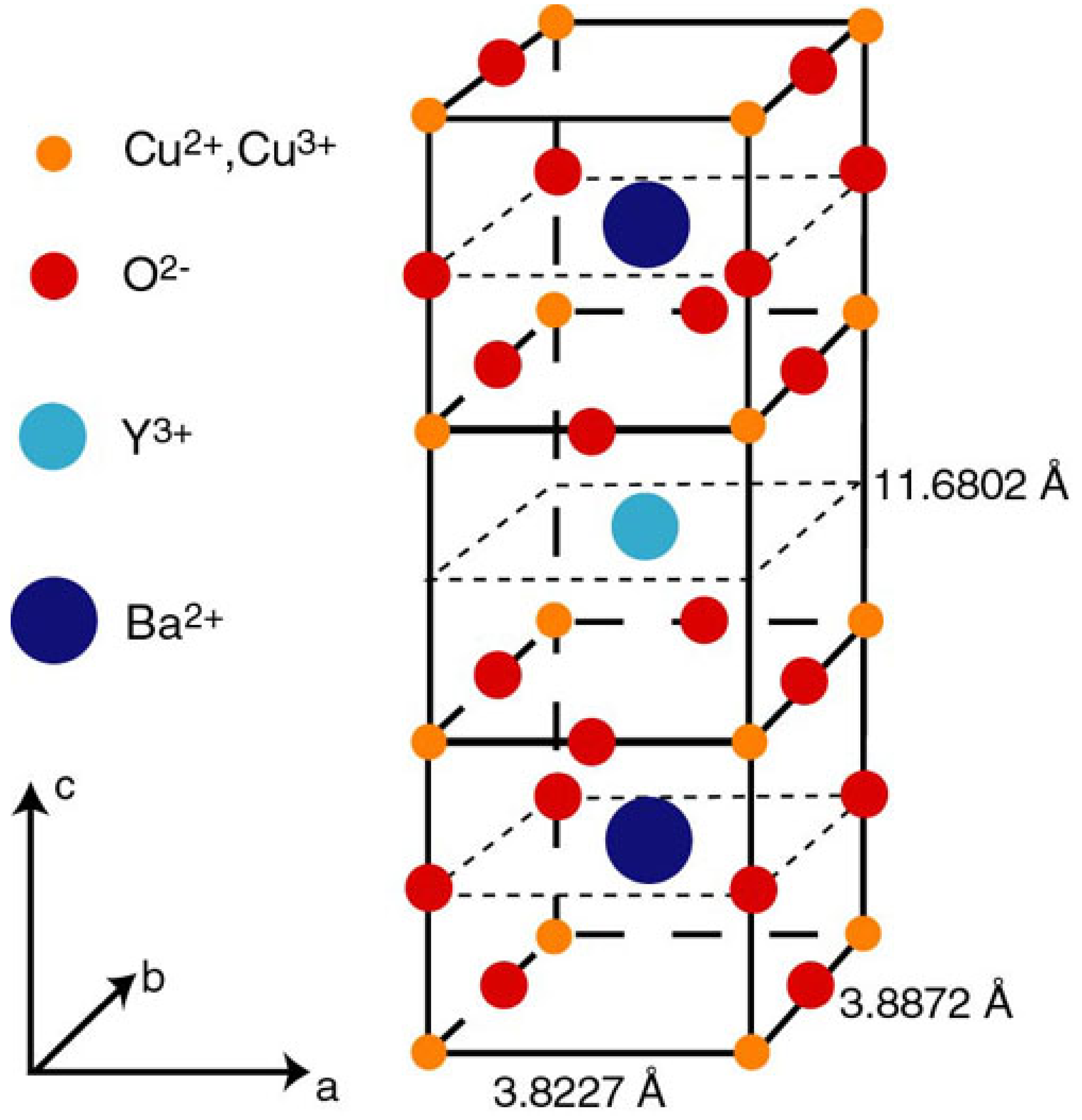
The long scattering time implies that individual electrons are well-defined.

Cuprate high temperature superconductors

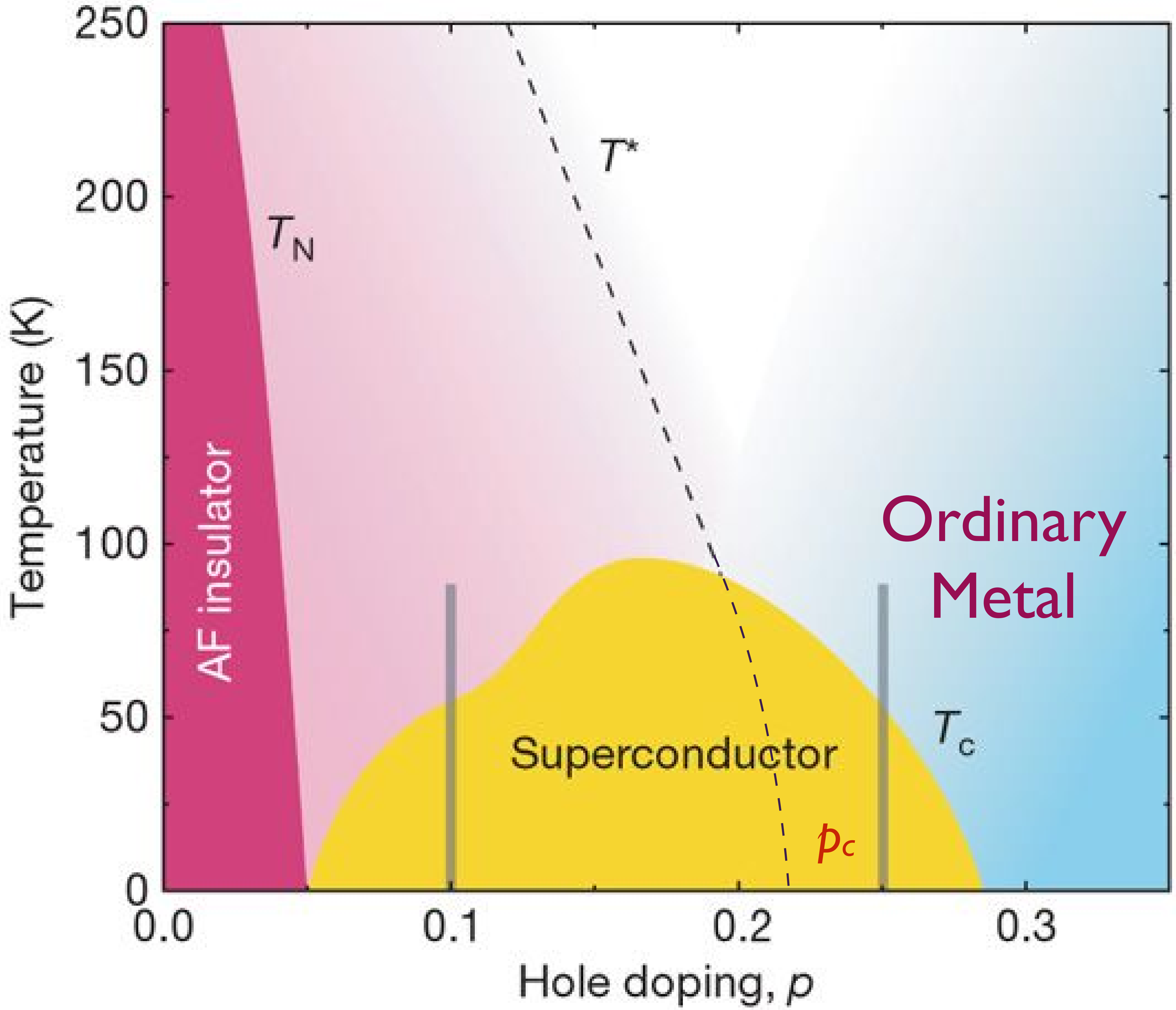


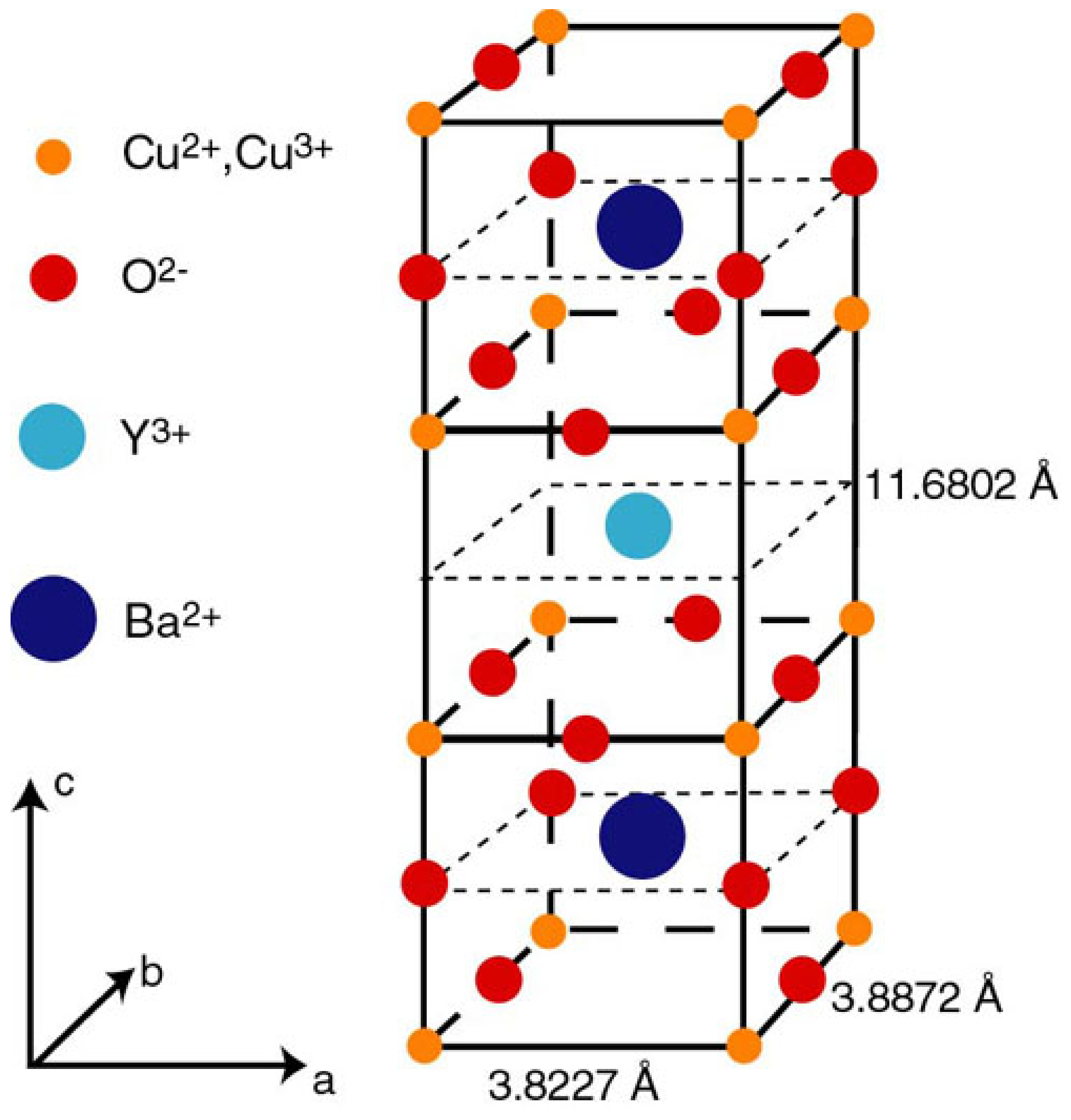
$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
O^{2-}
Y^{3+}
Ba^{2+}
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Ordinary Metal
Superconductor
T_c
p_c


Cu^{2+},Cu^{3+}
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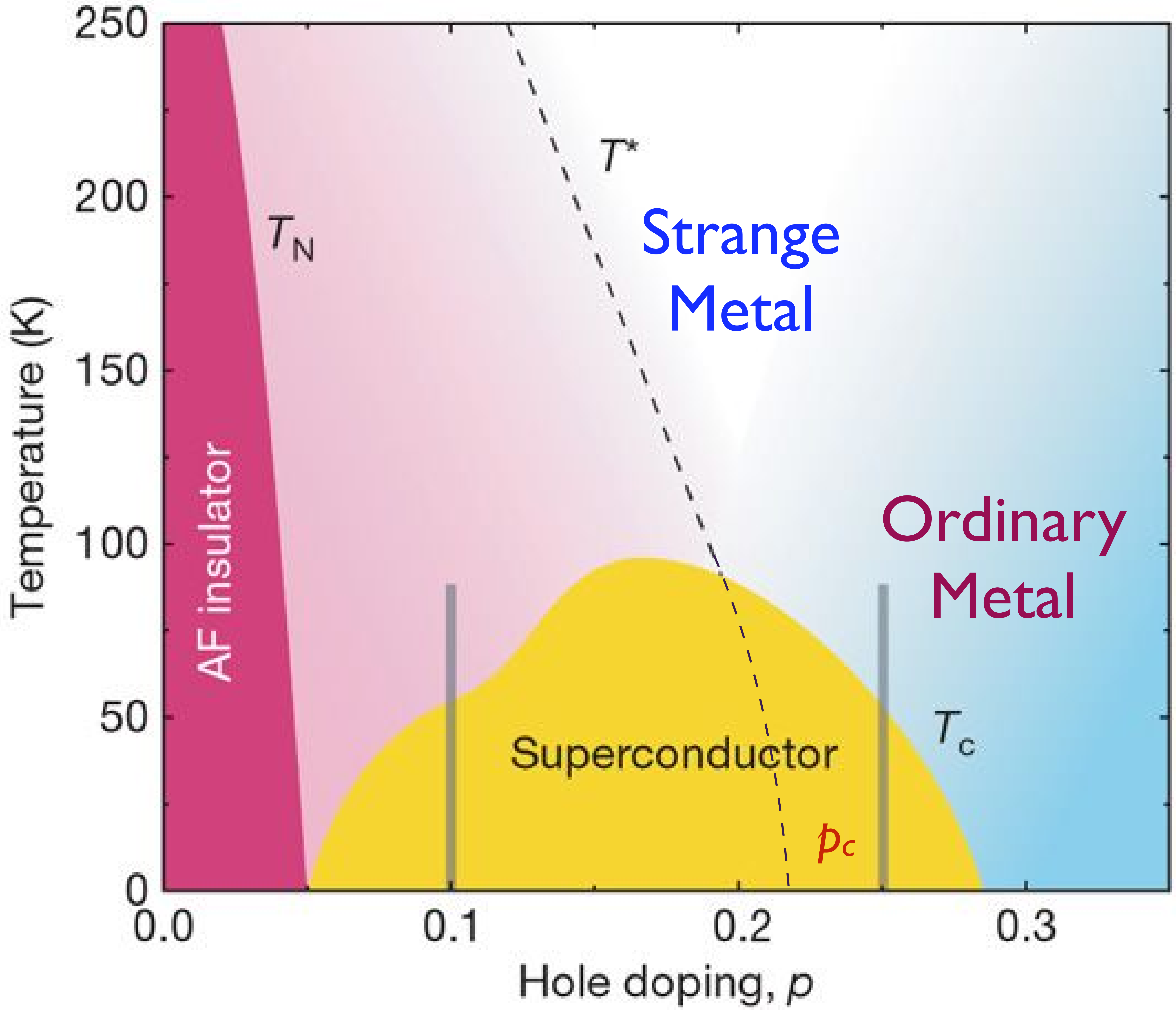

250
200
150
100
50
0
Temperature (K)
0.0
0.1
0.2
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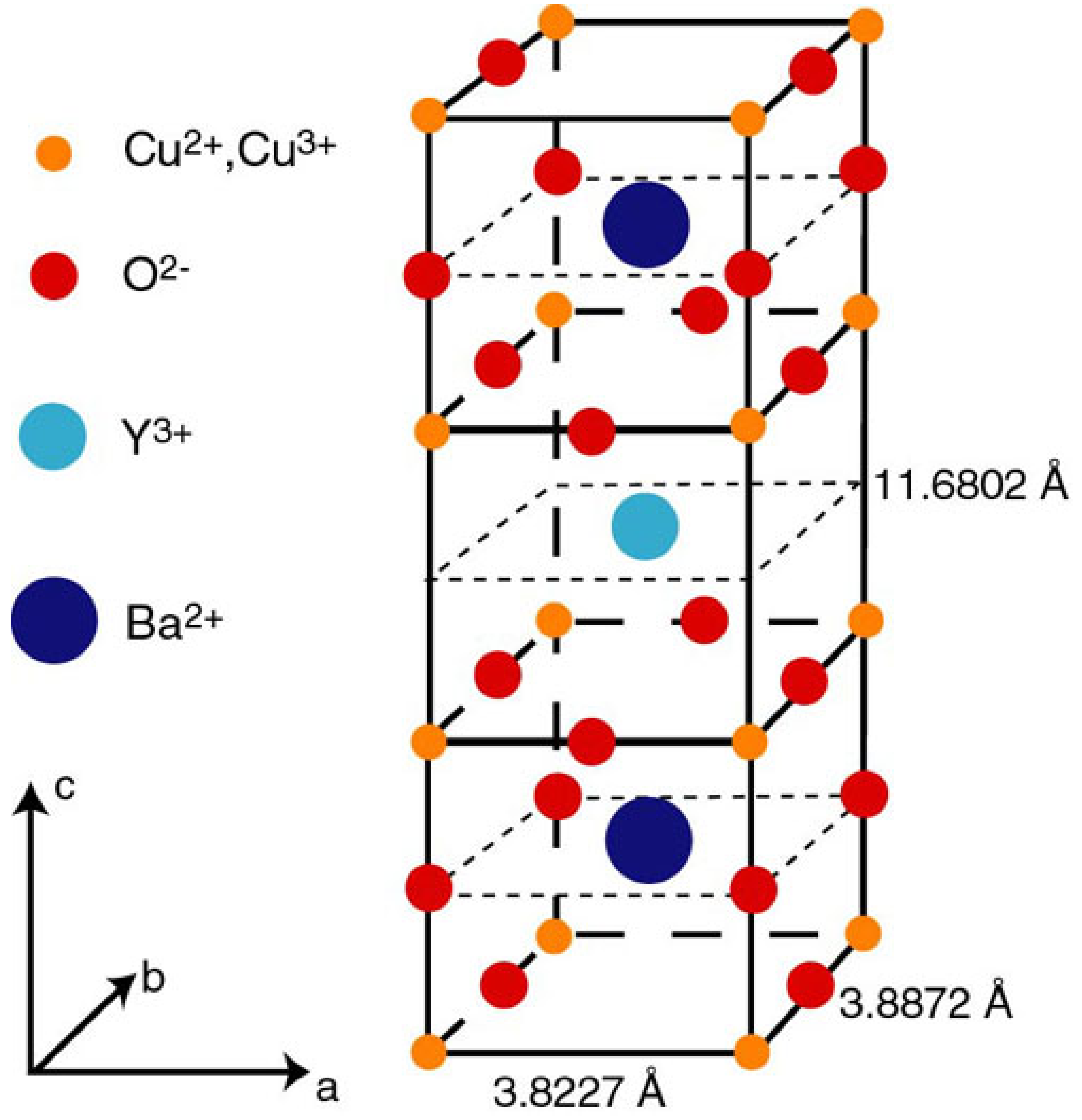
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
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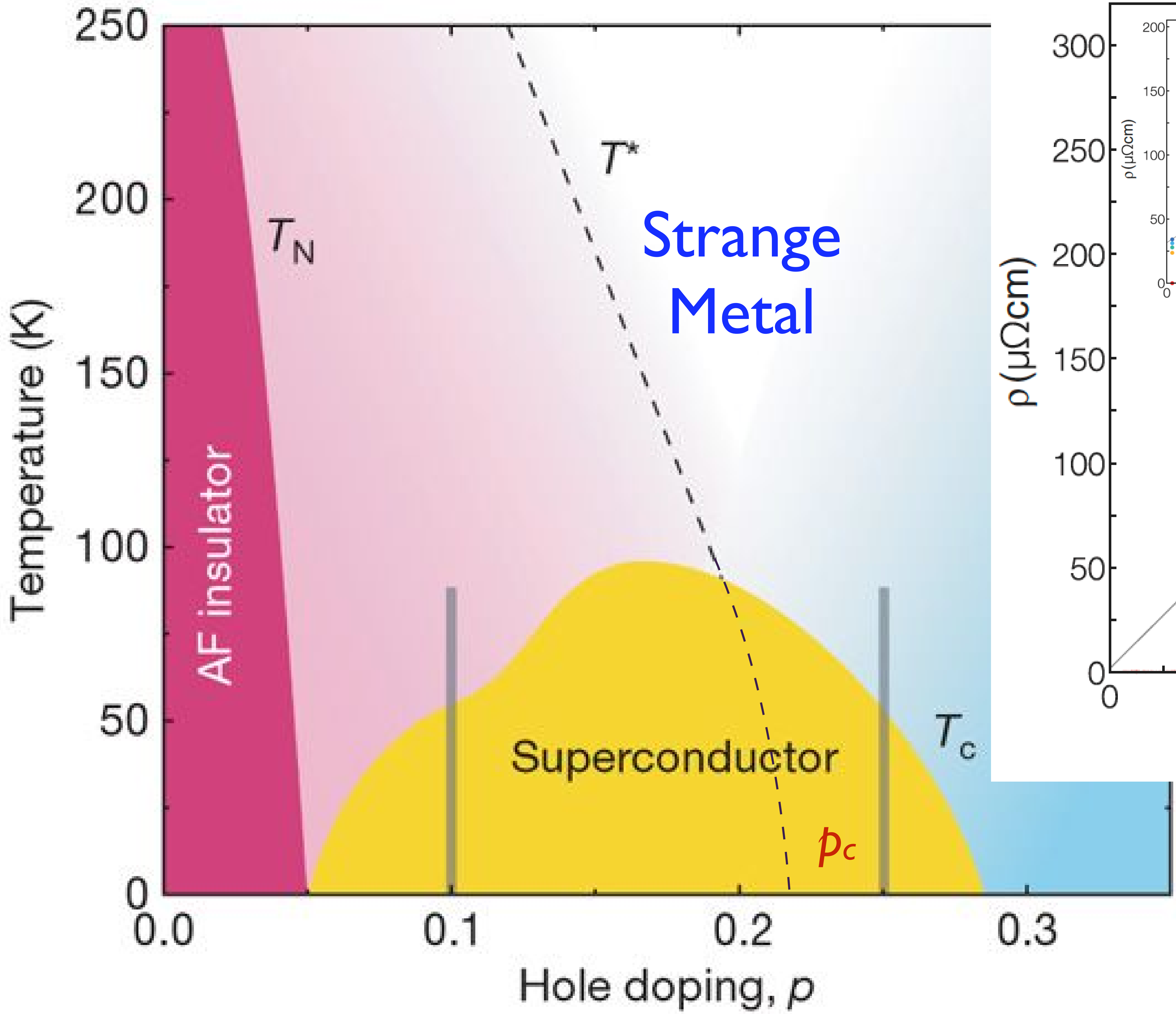
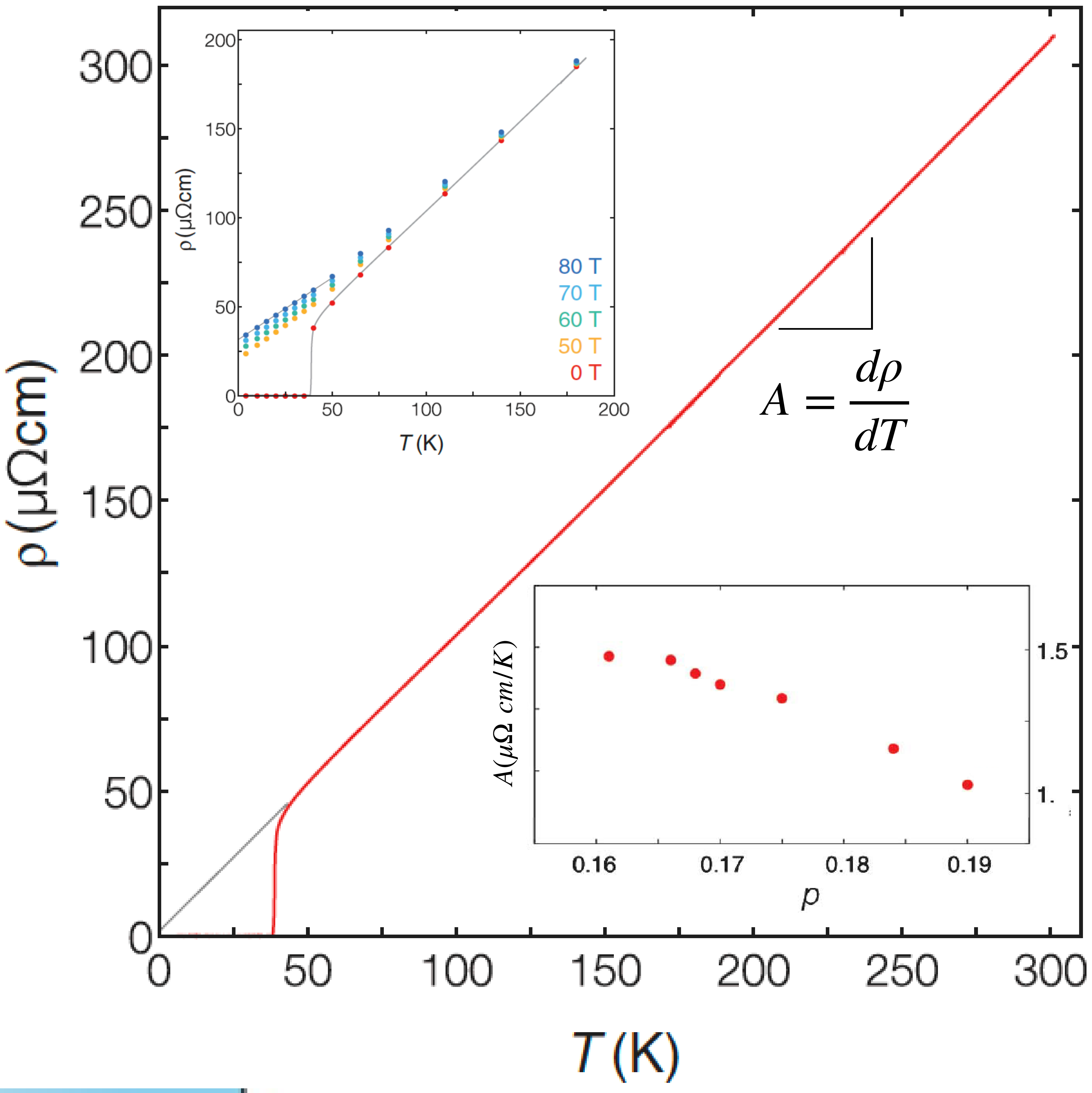
$YBa_2Cu_3O_{6+x}$



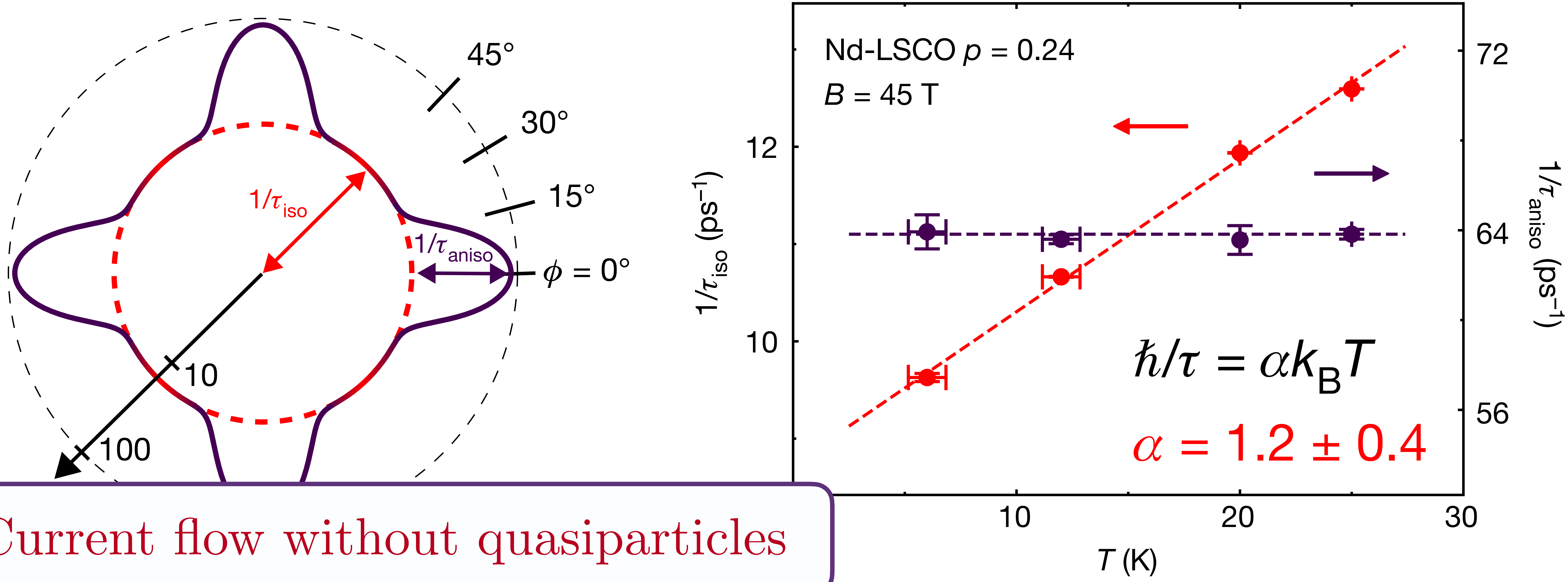


LSCO: Giraldo-Gallo et al.
Science **361**, 479 (2018)

Linear-in temperature resistivity from an isotropic Planckian scattering rate

Grissonnanche et al.
Nature **595**, 667-672 (2021)

G. Grissonnanche, Y. Fang, A. Legros, S. Verret, F. Laliberté, C. Collignon, J. Zhou, D. Graf, P. Goddard, L. Taillefer, B. J. Ramshaw



Current flow without quasiparticles

No Boltzmann-Landau quasiparticle description $\Rightarrow$
Many particle quantum entanglement
from quantum interference between "collisions"

Sachdev-Ye-Kitaev Model

The SYK model

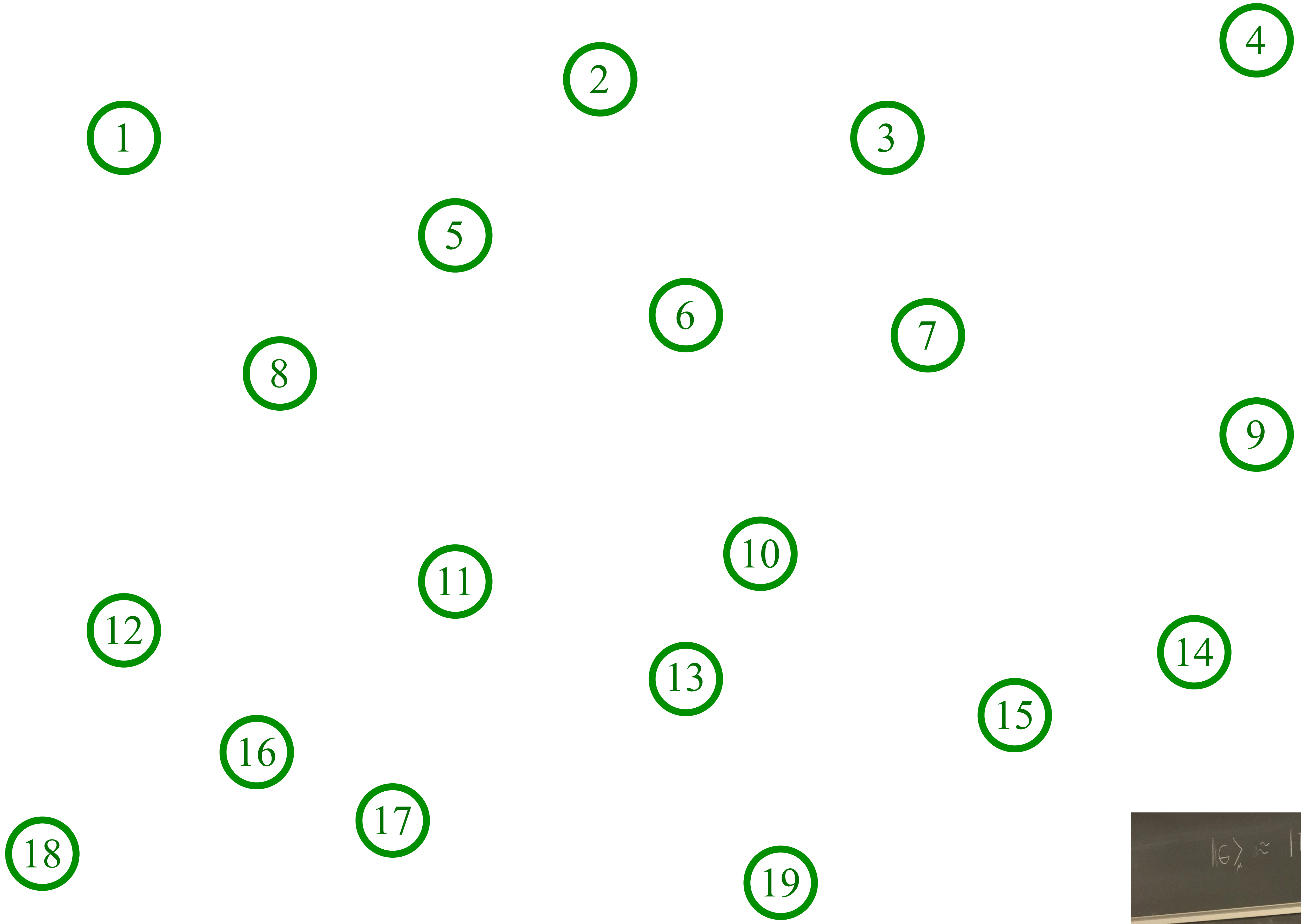
Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015) (unpublished)

A solvable model of multi-particle quantum entanglement.

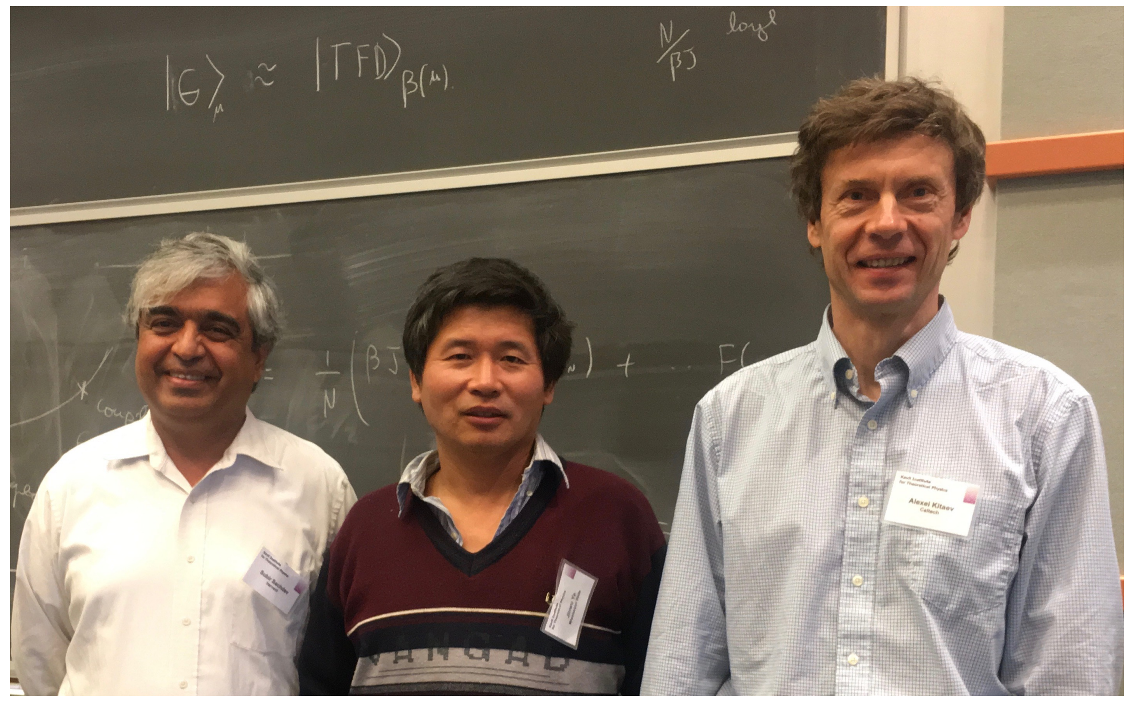
Yields a metal in which current is carried not by individual electrons, but by an entangled "quantum soup"

The SYK model

Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015) (unpublished)



Pick a set of random positions



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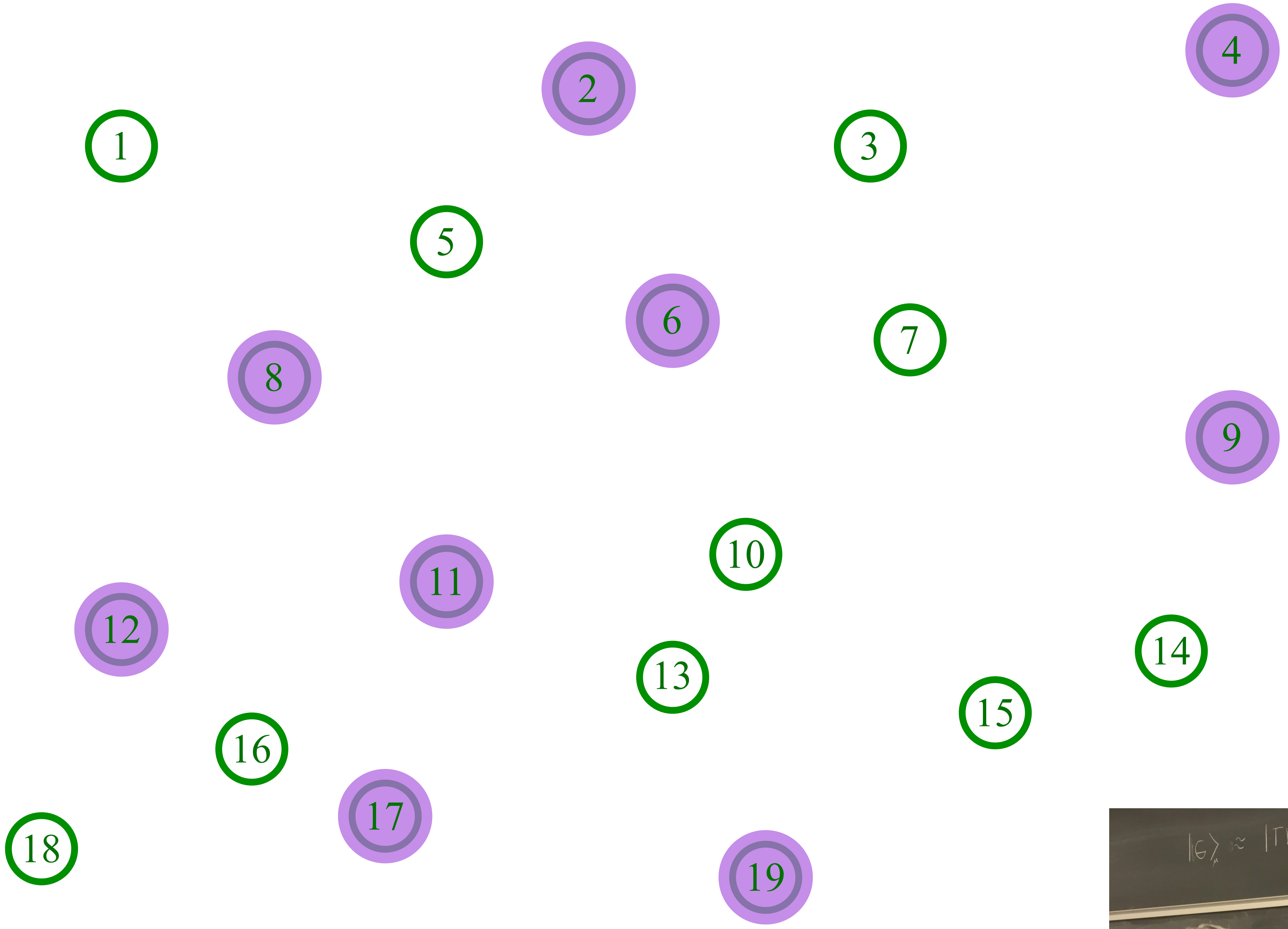
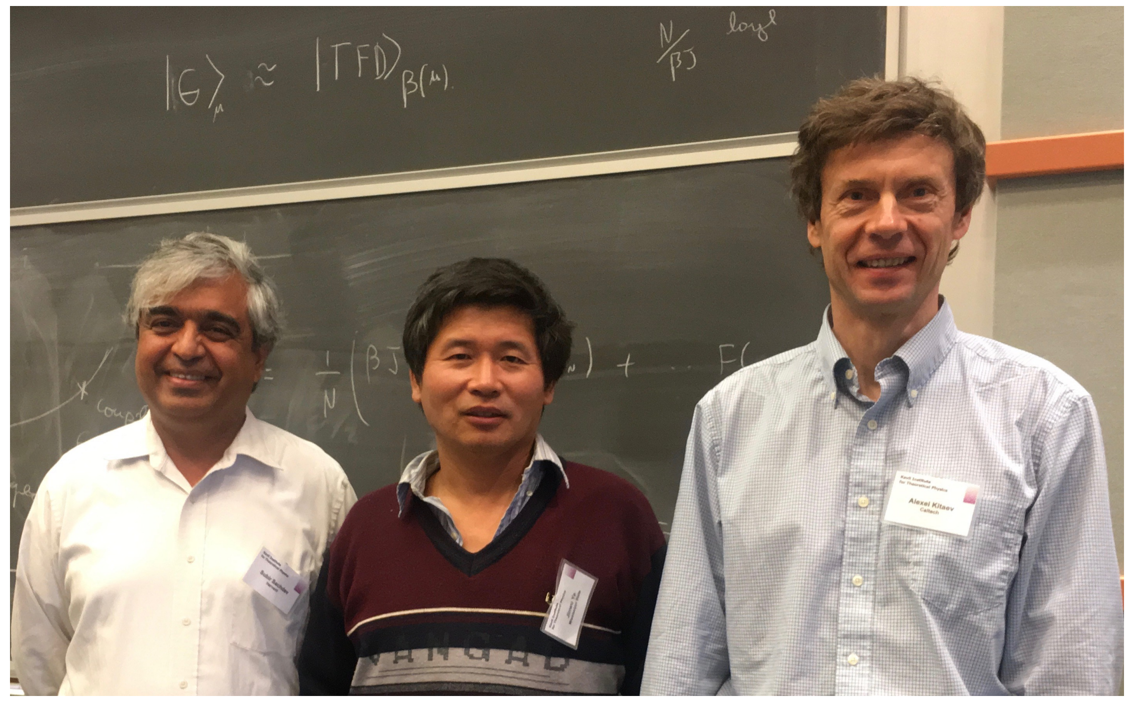


Place electrons randomly on some sites



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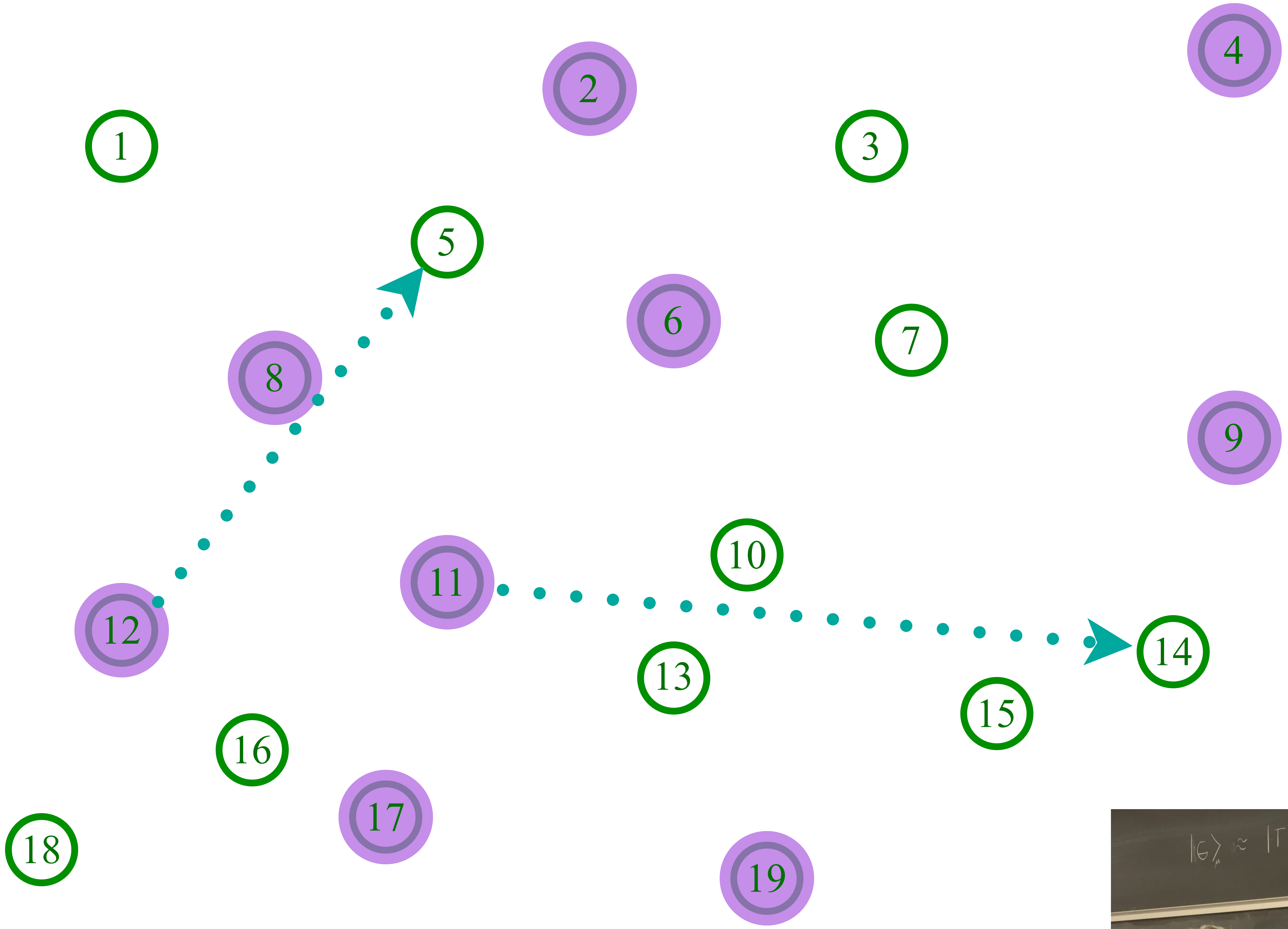
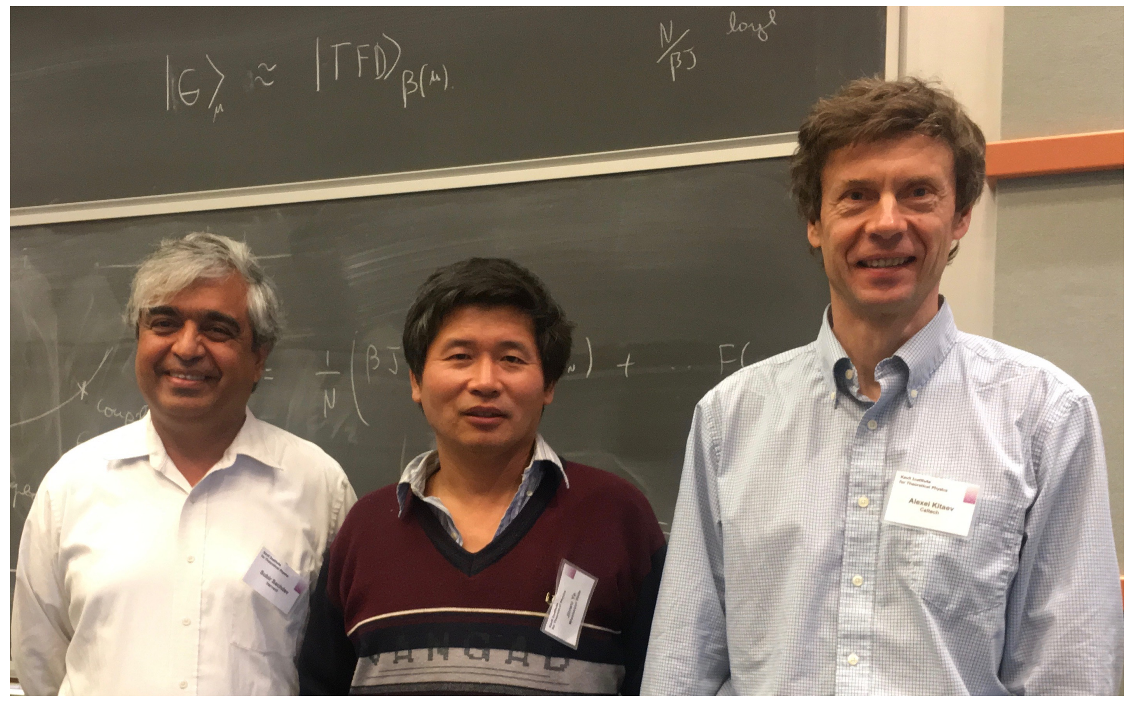
Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015)   (unpublished)

$$U_{11,12;5,14}$$



Place electrons randomly on some sites



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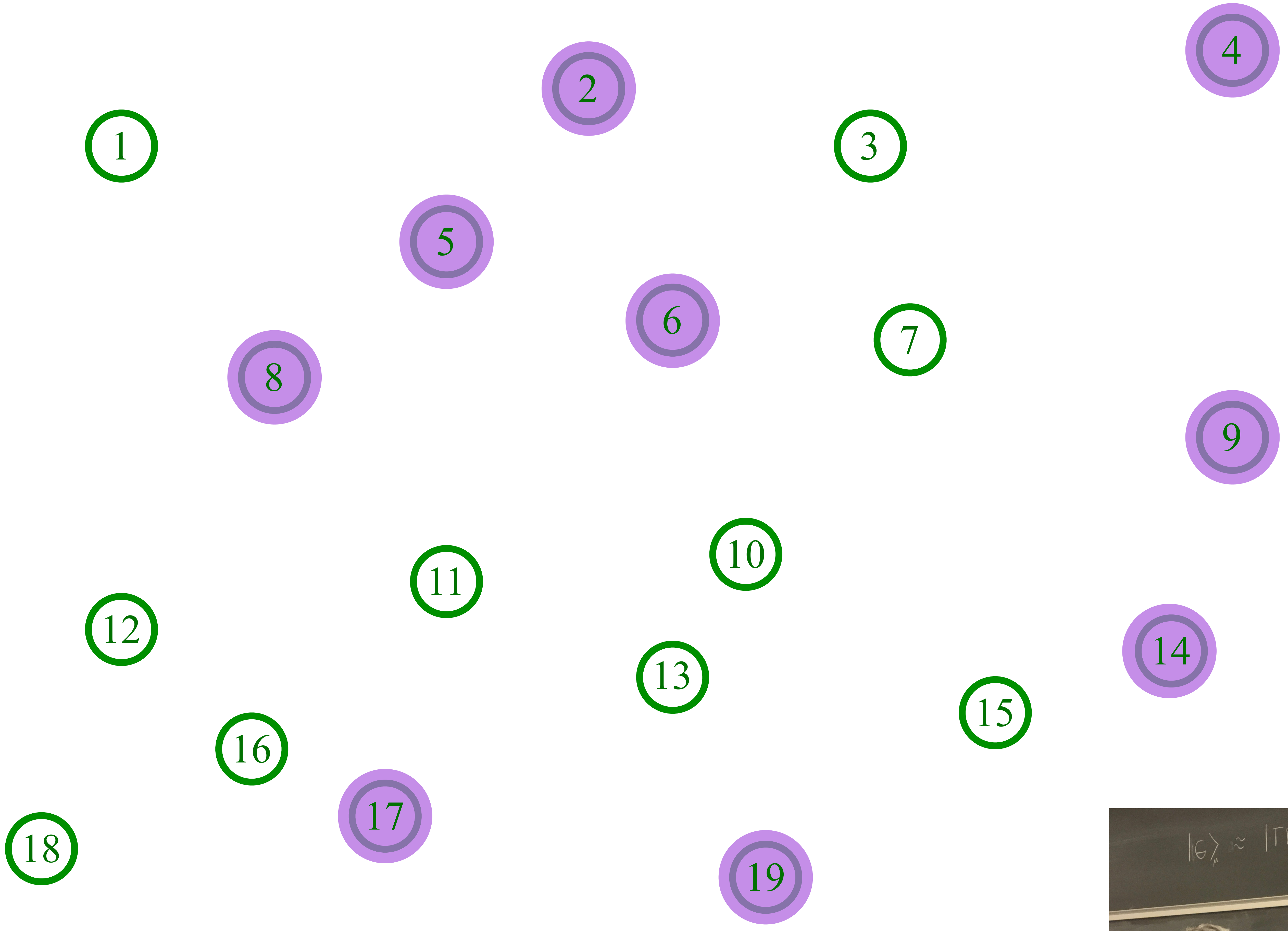
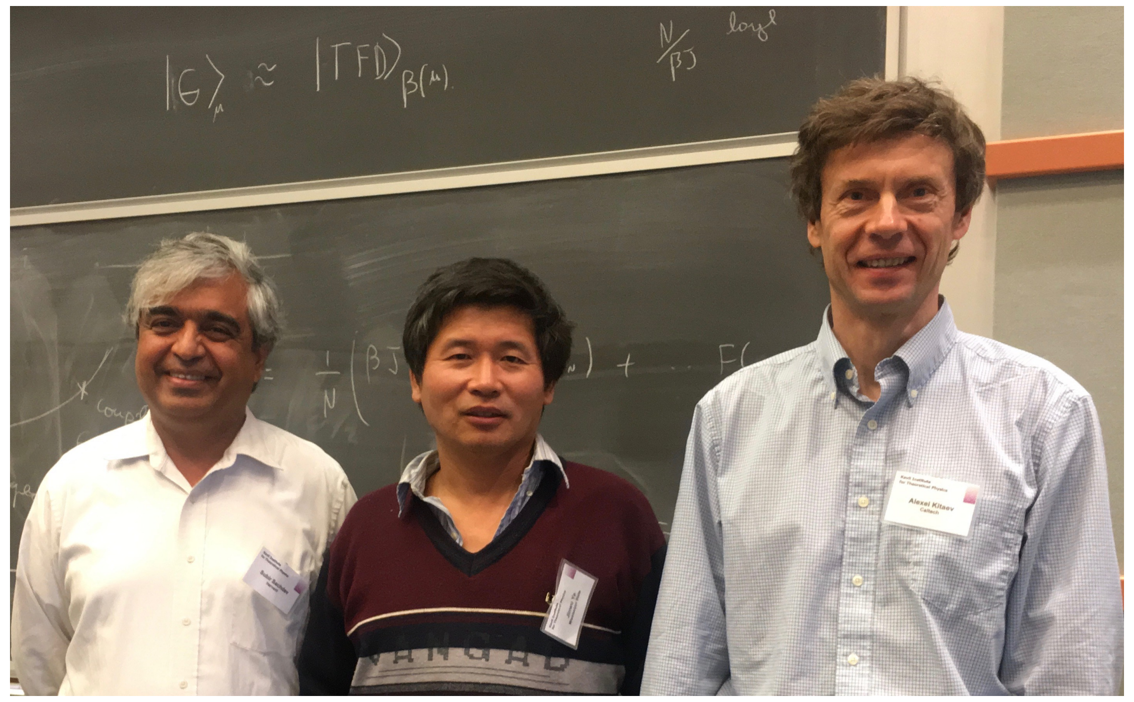


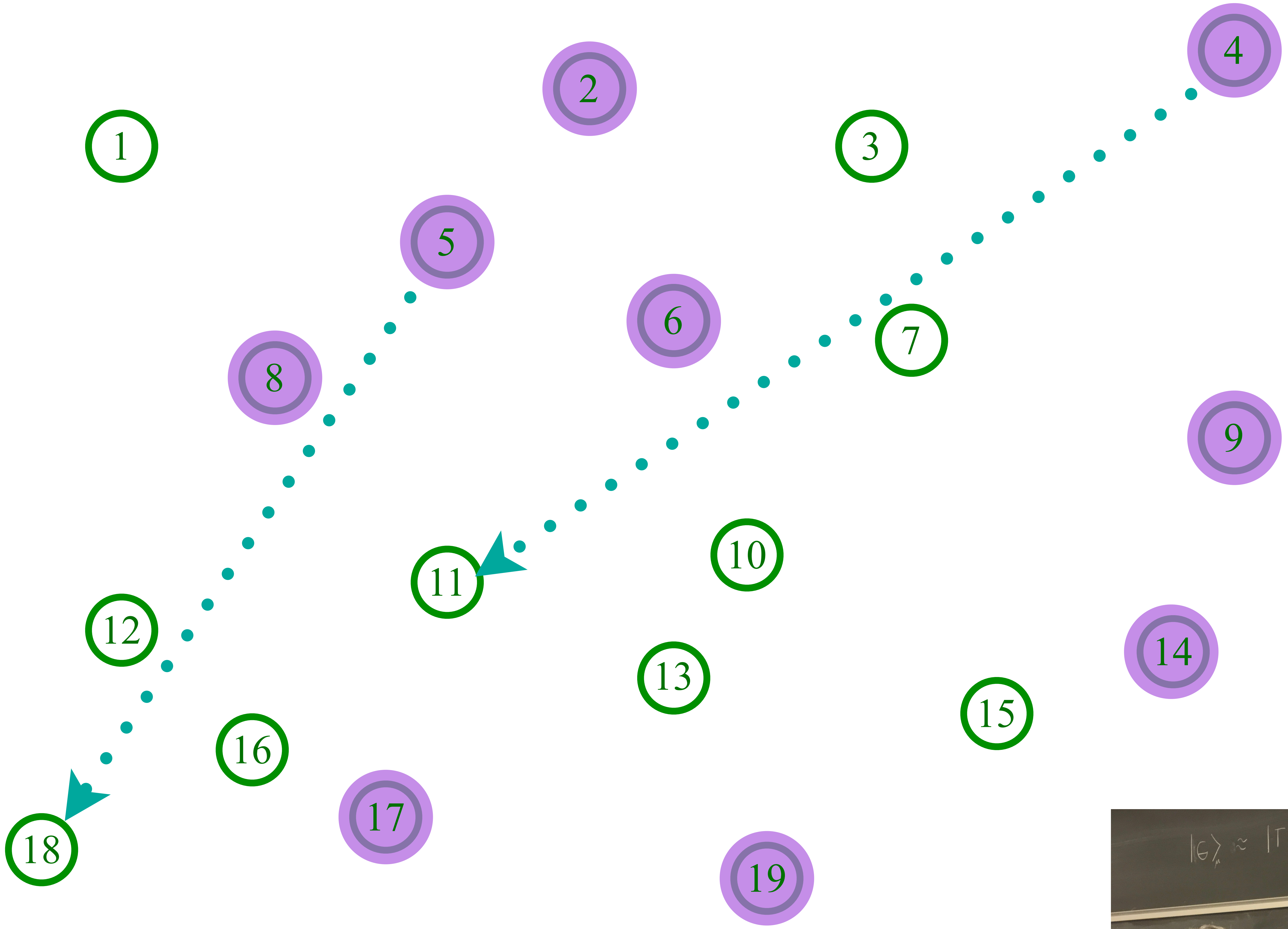
Entangle electrons pairwise randomly



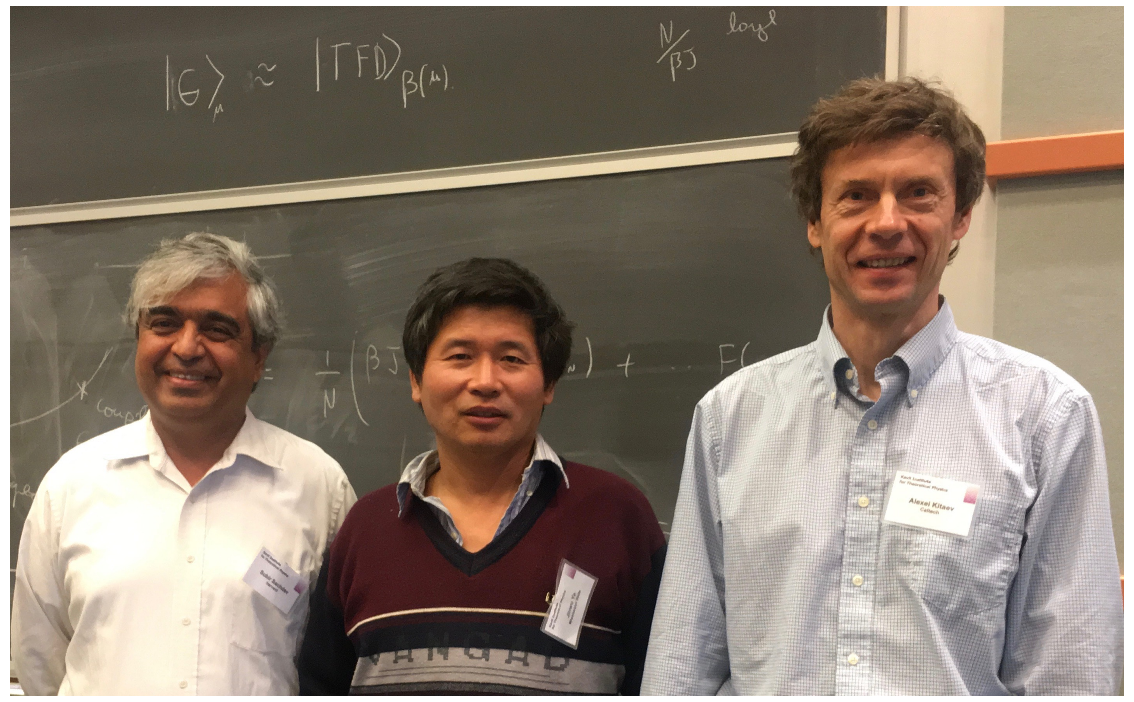
The SYK model

Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015) (unpublished)

$$U_{4,5;11,18}$$



Entangle electrons pairwise randomly



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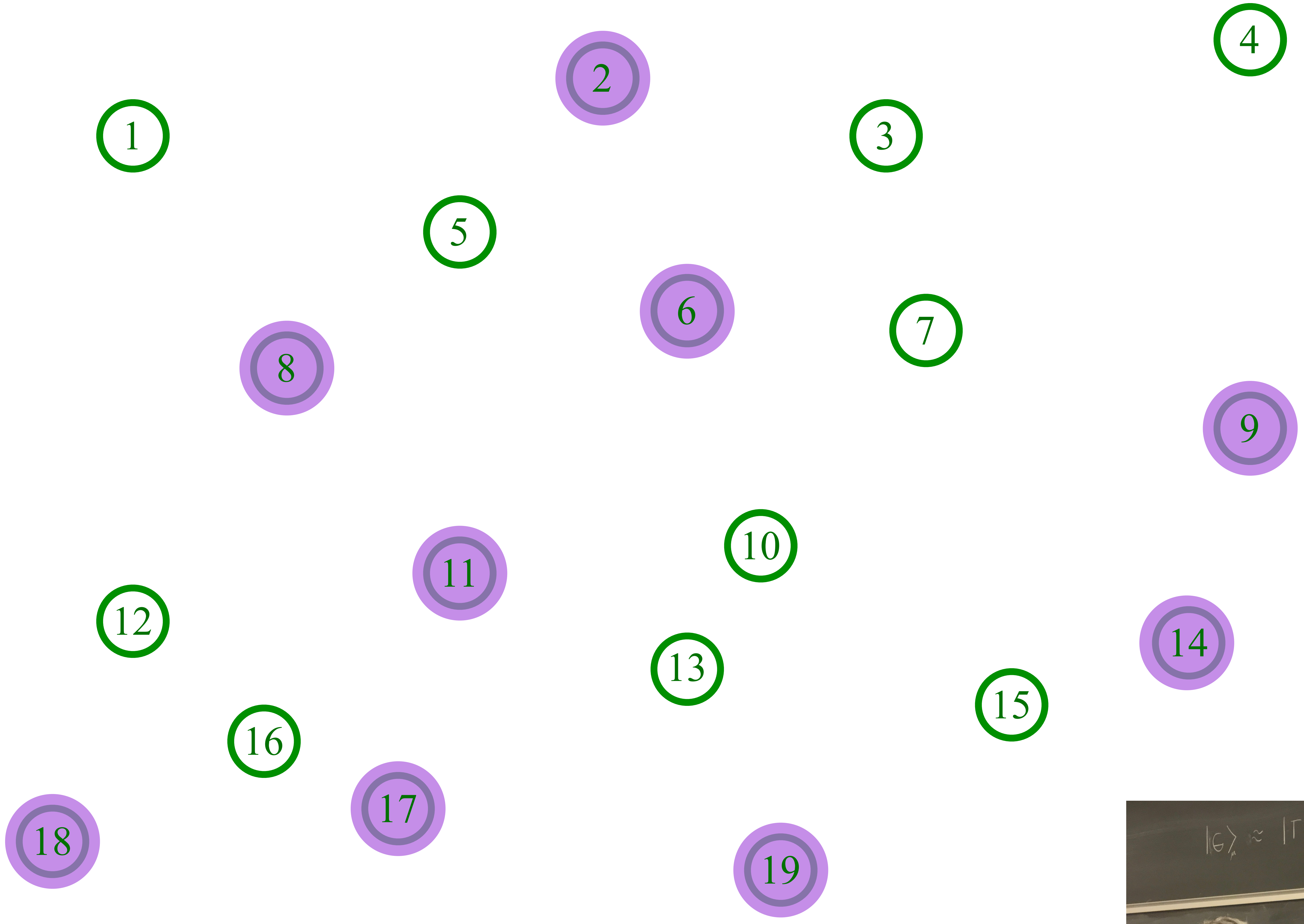

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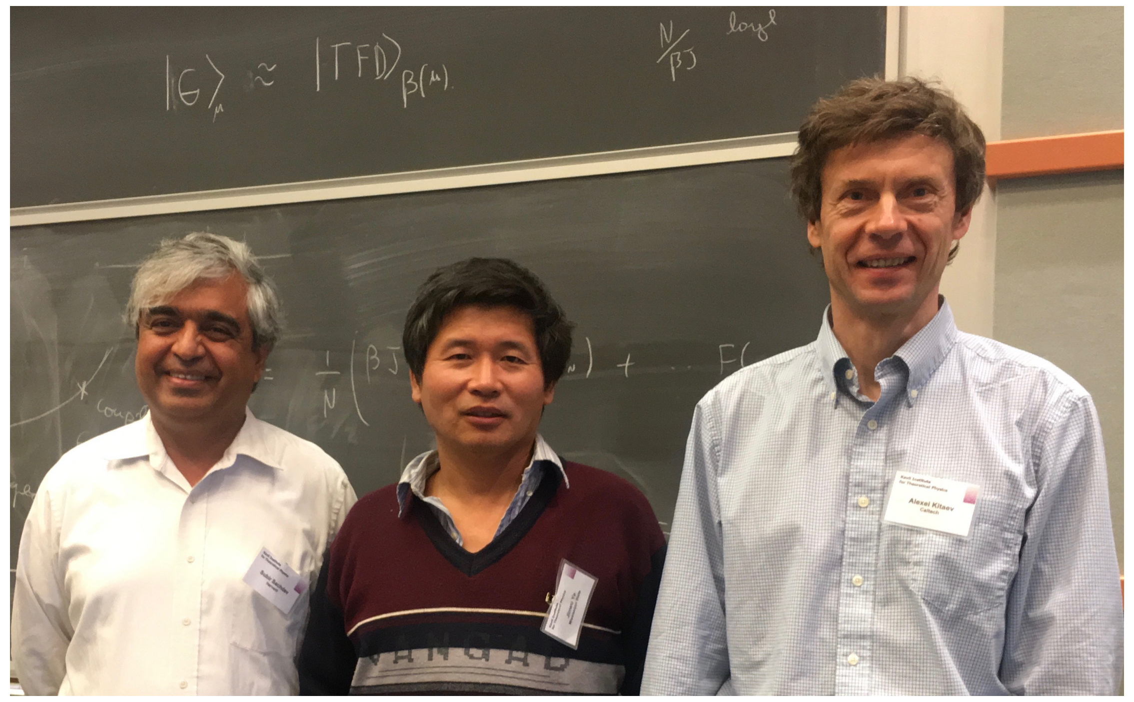

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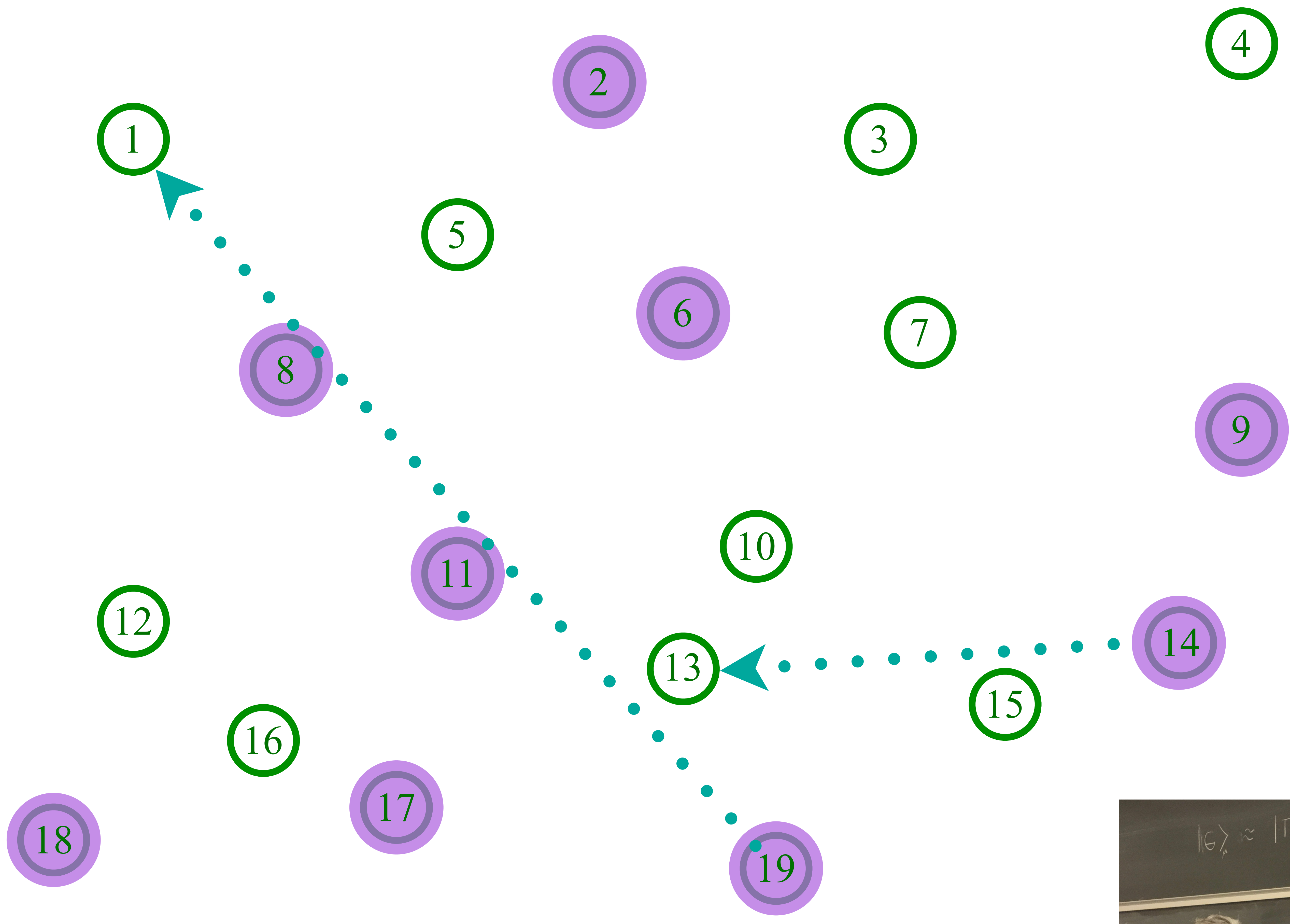

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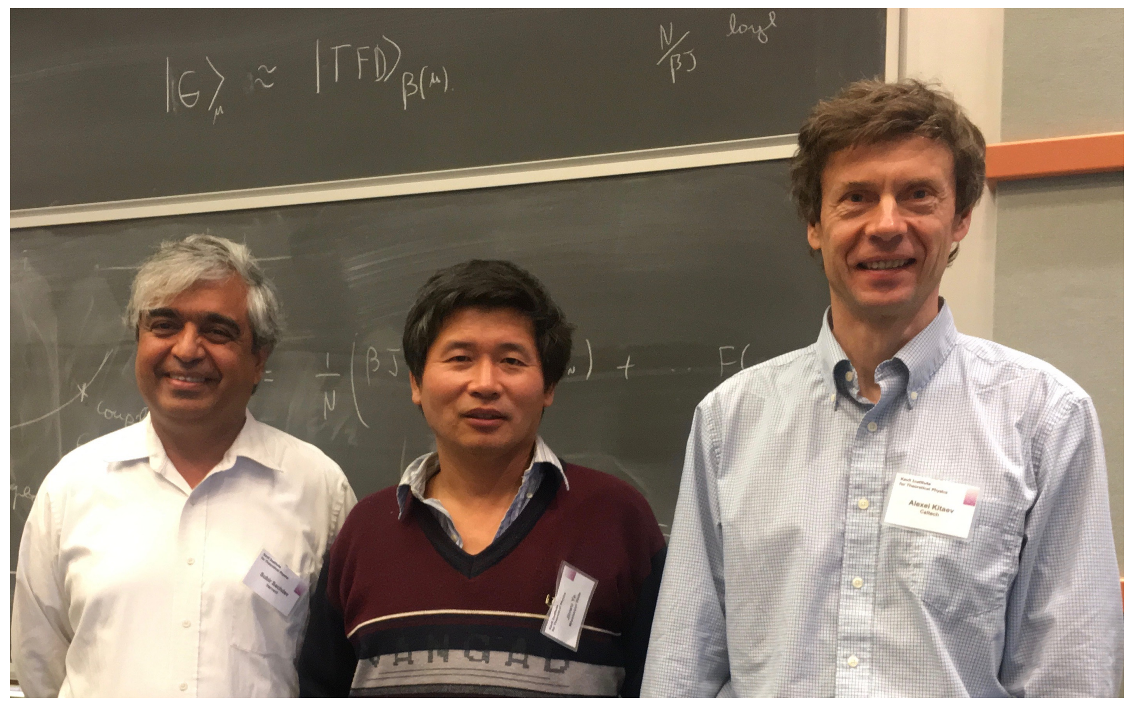

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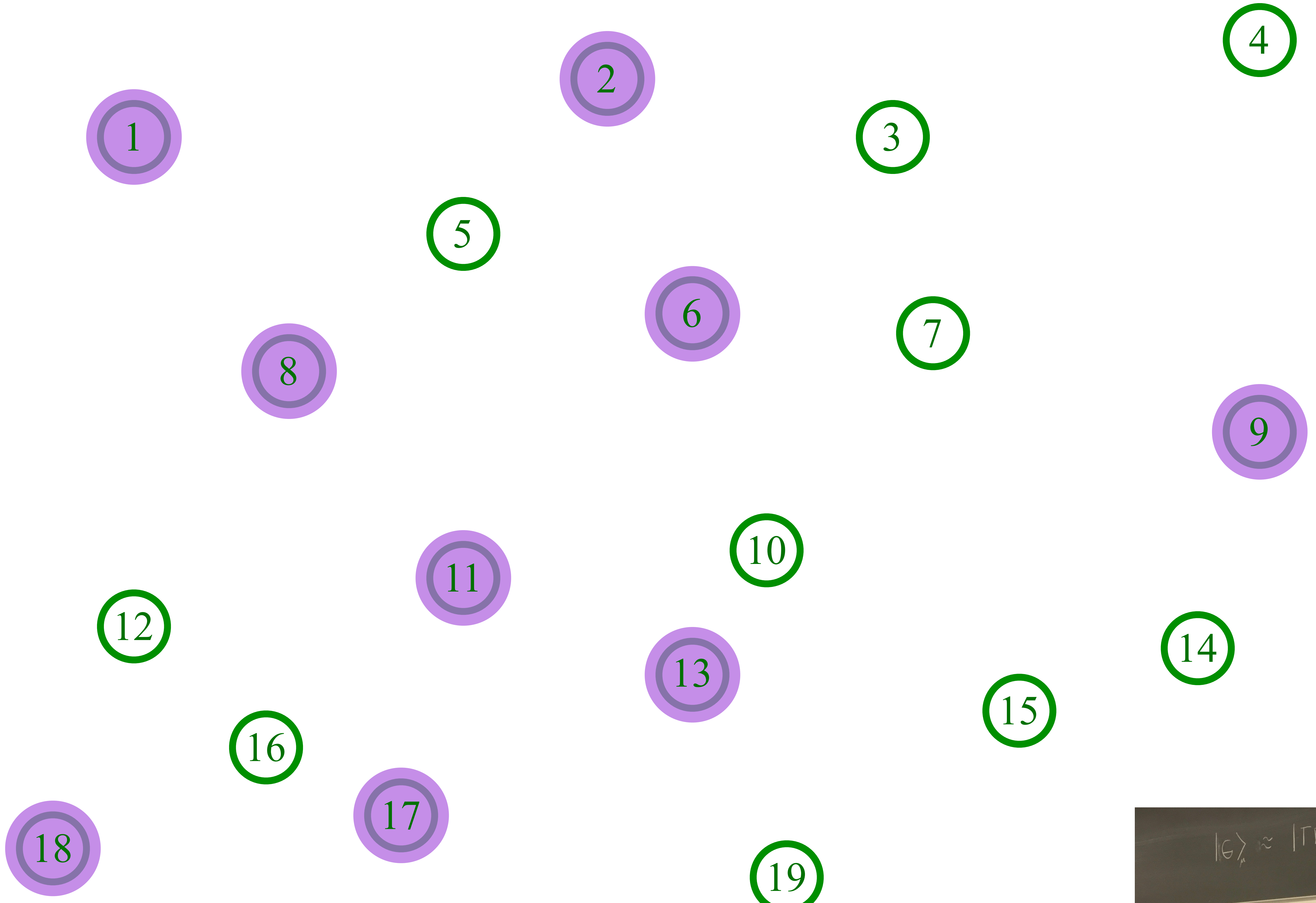
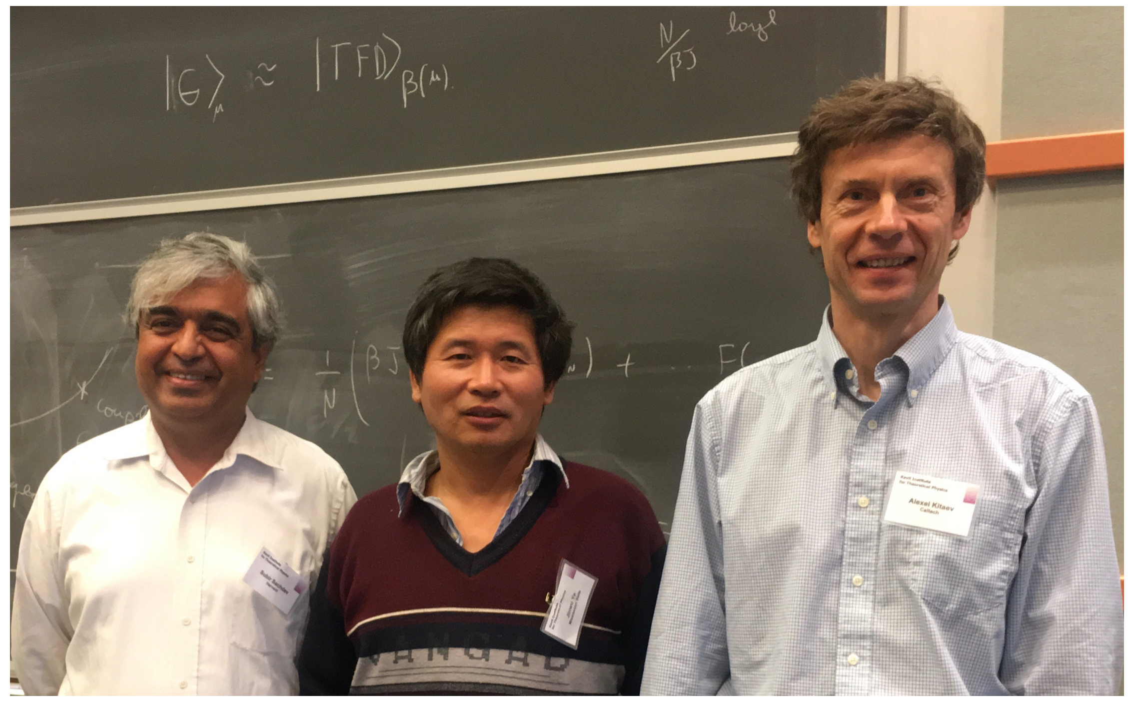


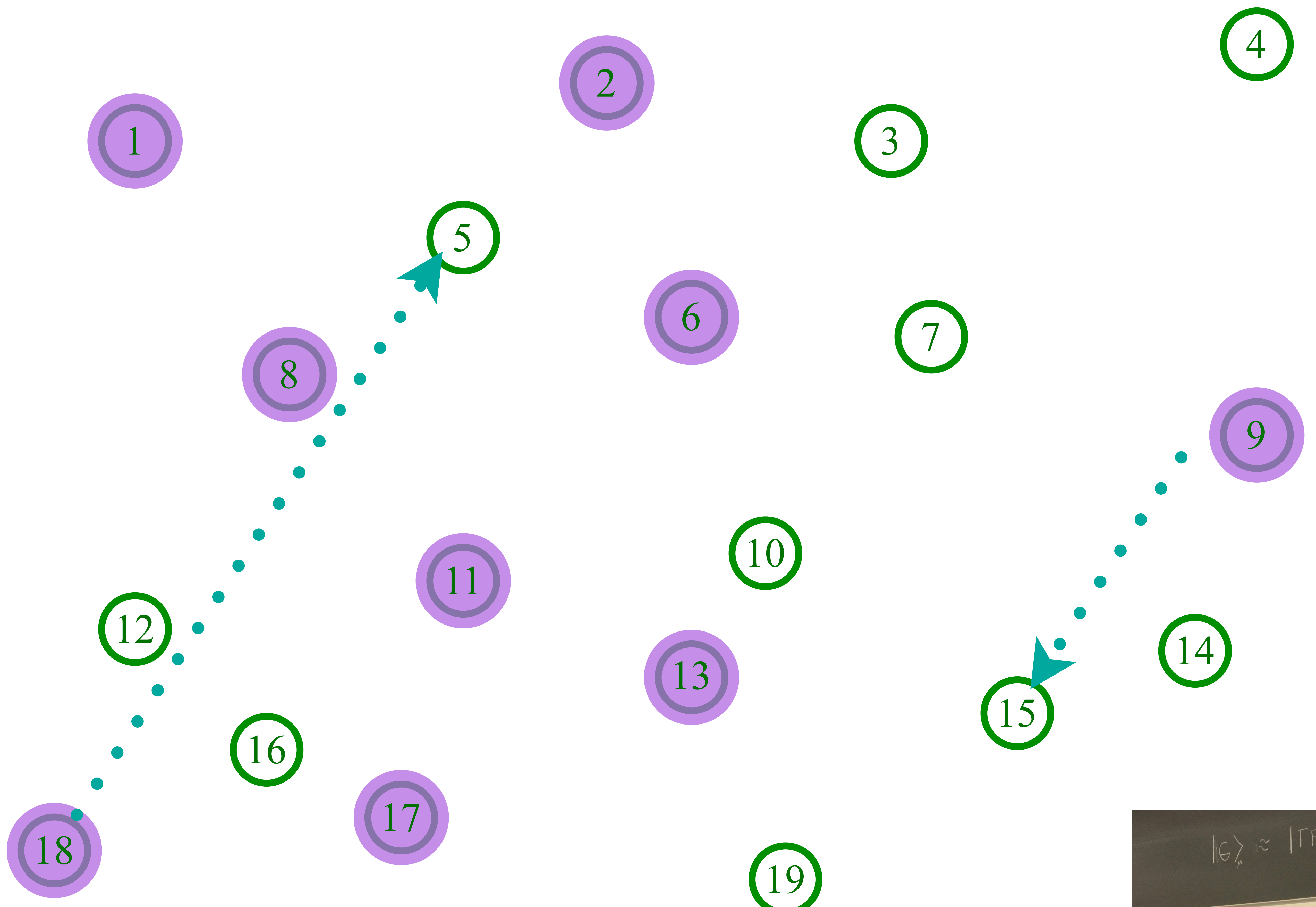
Entangle electrons pairwise randomly



The SYK model

Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015) (unpublished)

$$U_{9,18;5,15}$$



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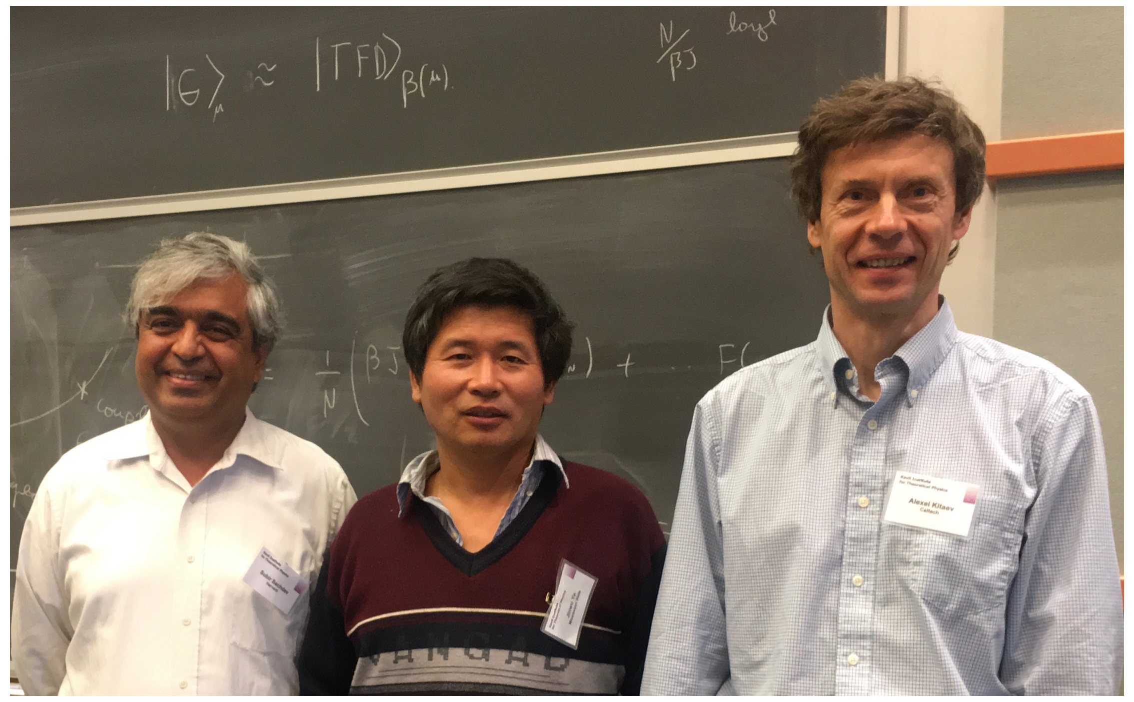

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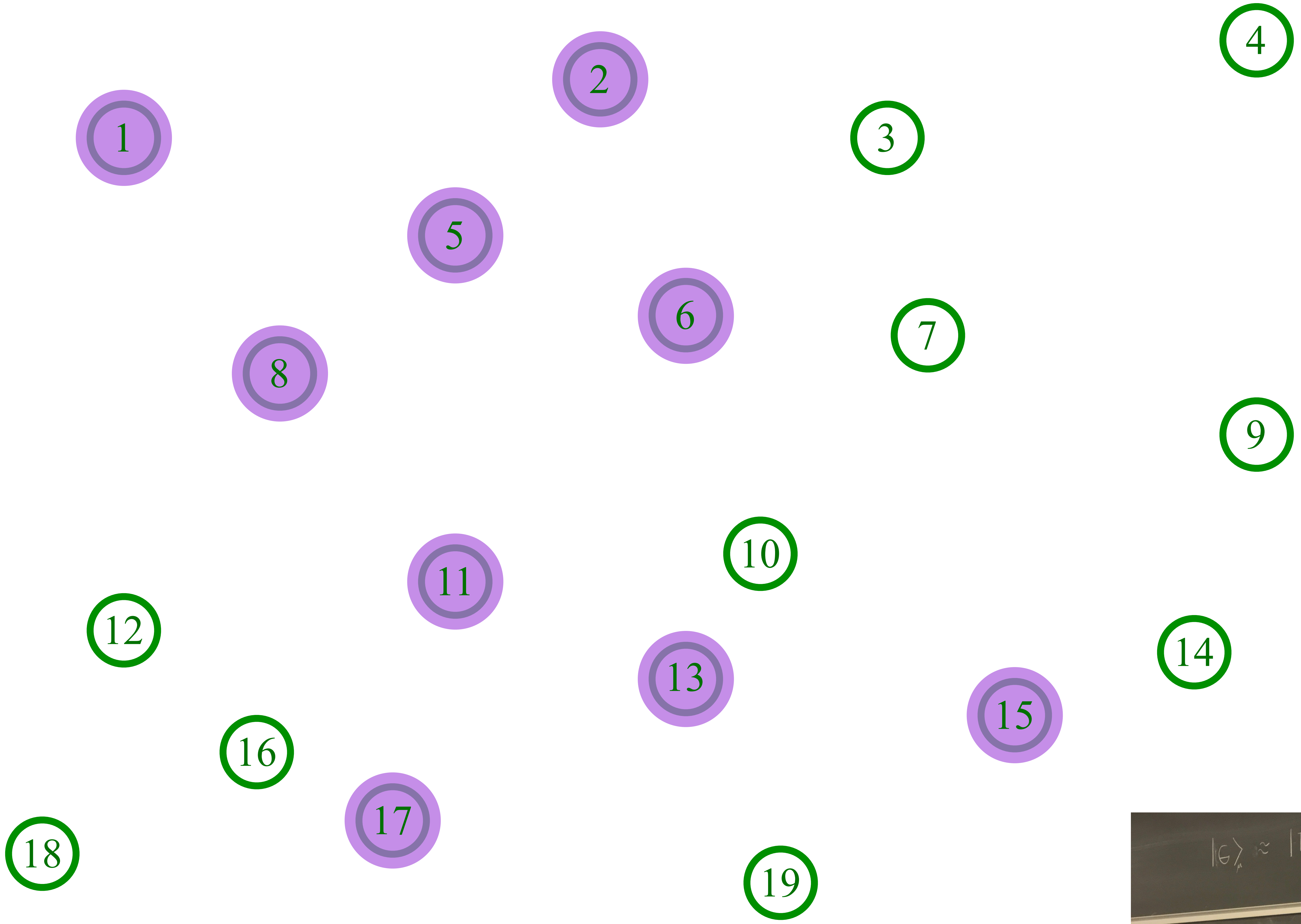
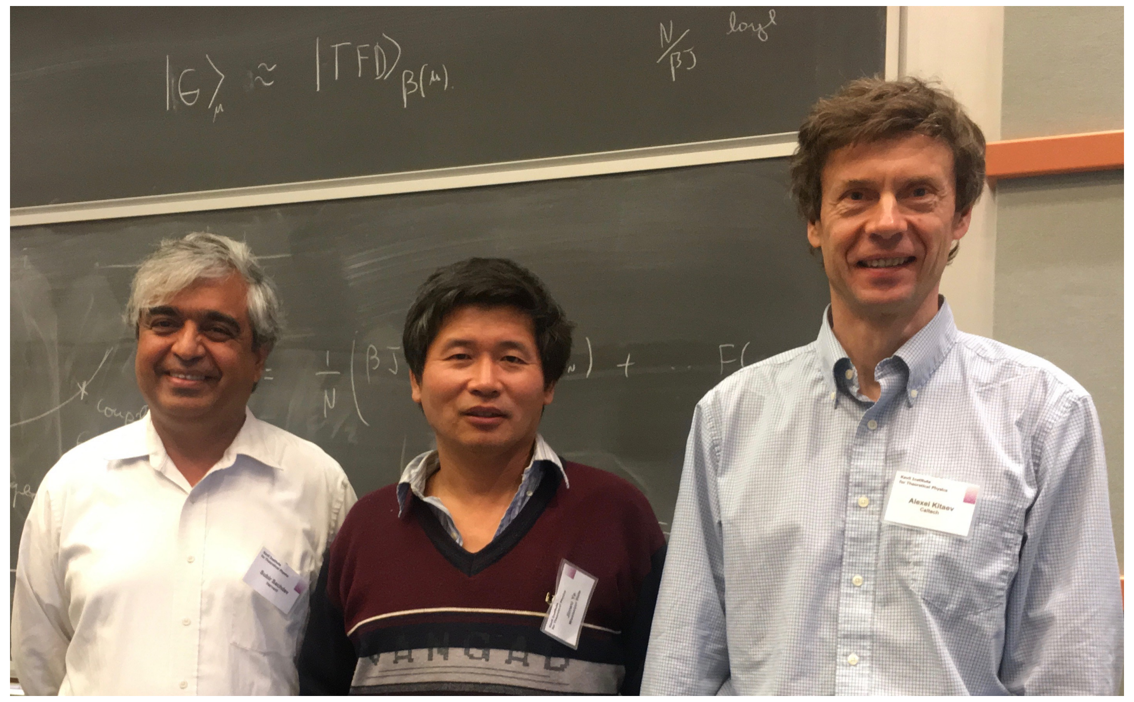


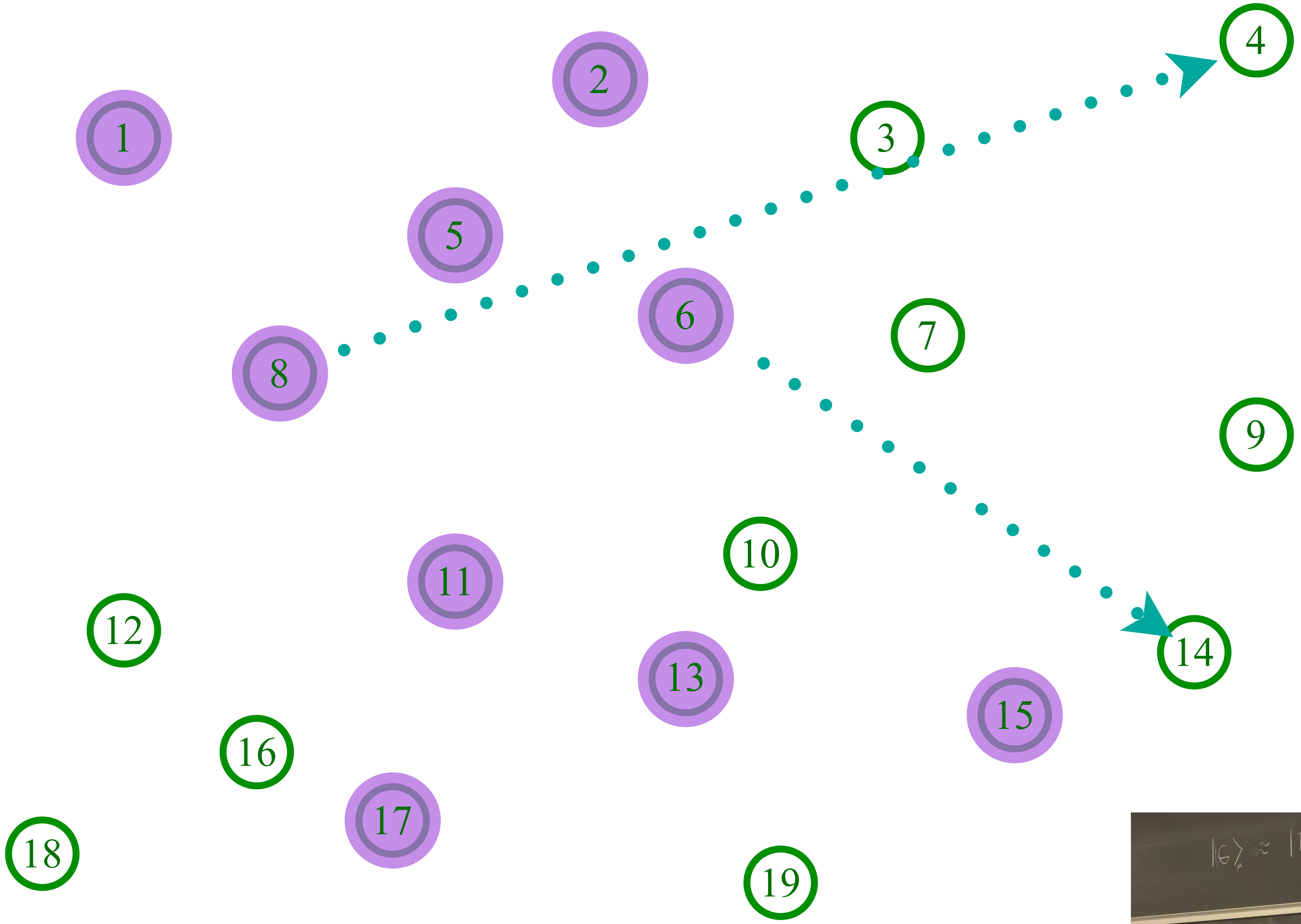
Entangle electrons pairwise randomly



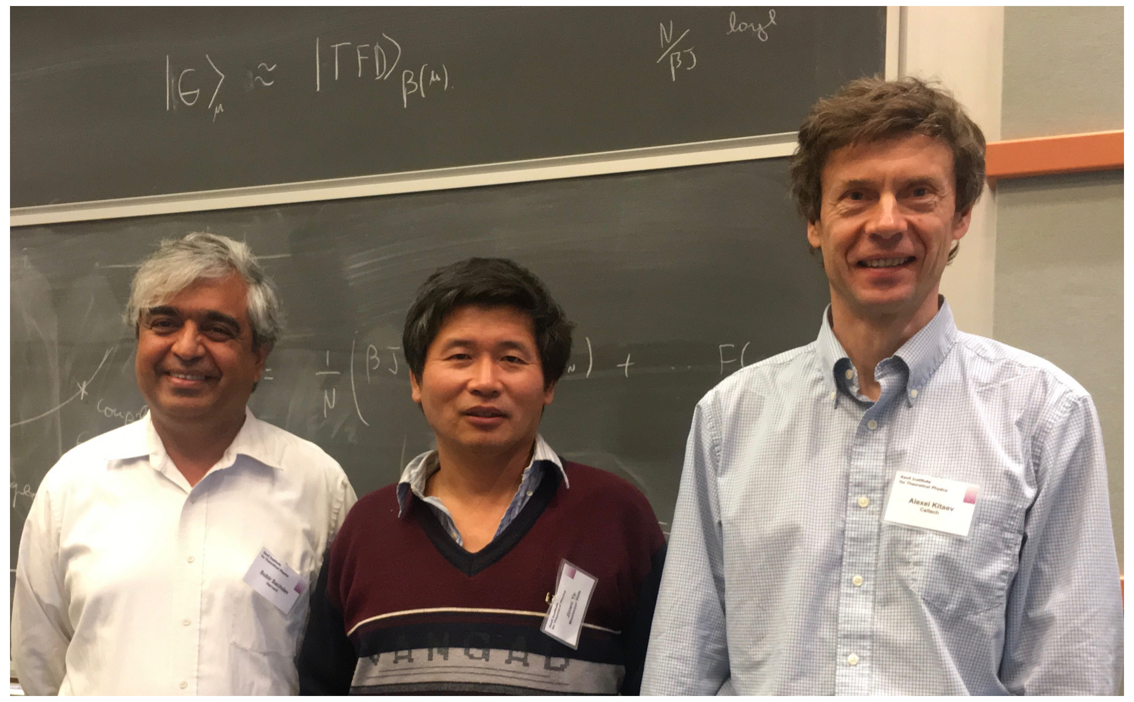
The SYK model

Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015) (unpublished)

$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



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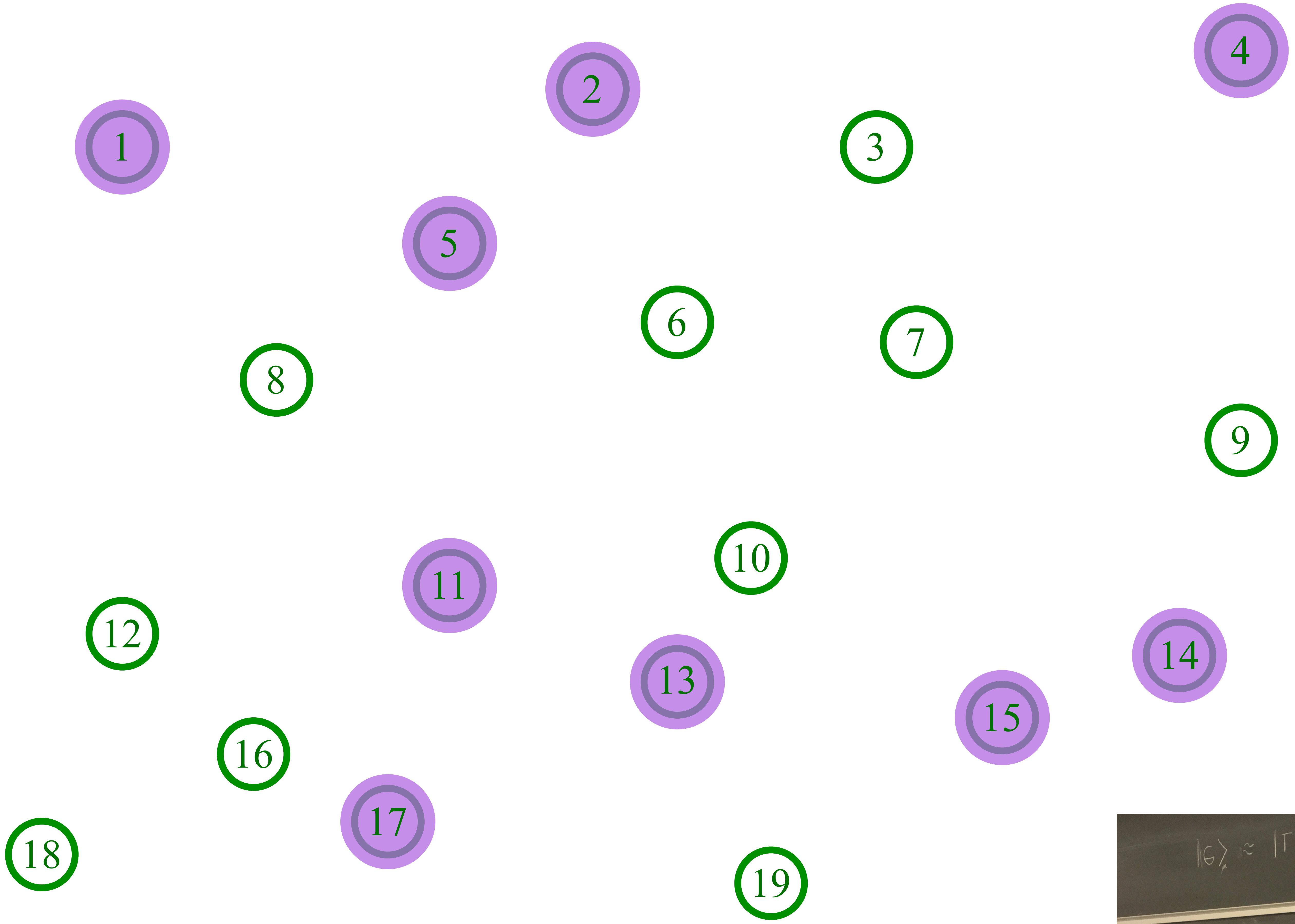
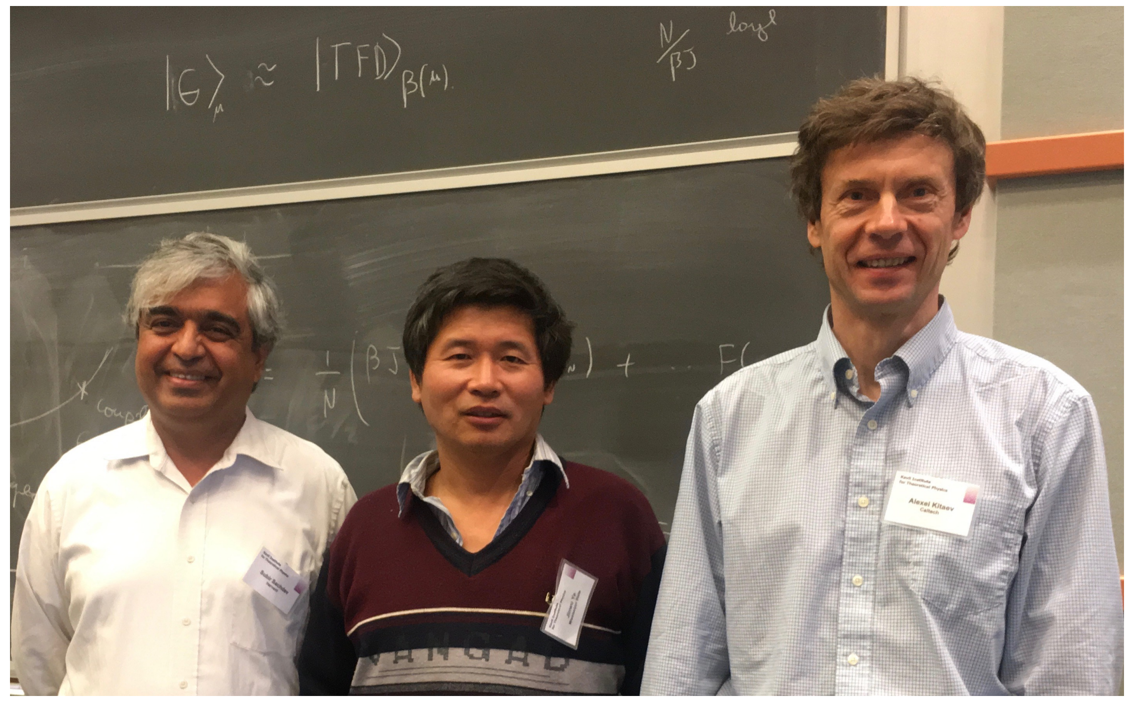
$$U_{6,8;4,14}$$



Entangle electrons pairwise randomly



The SYK model

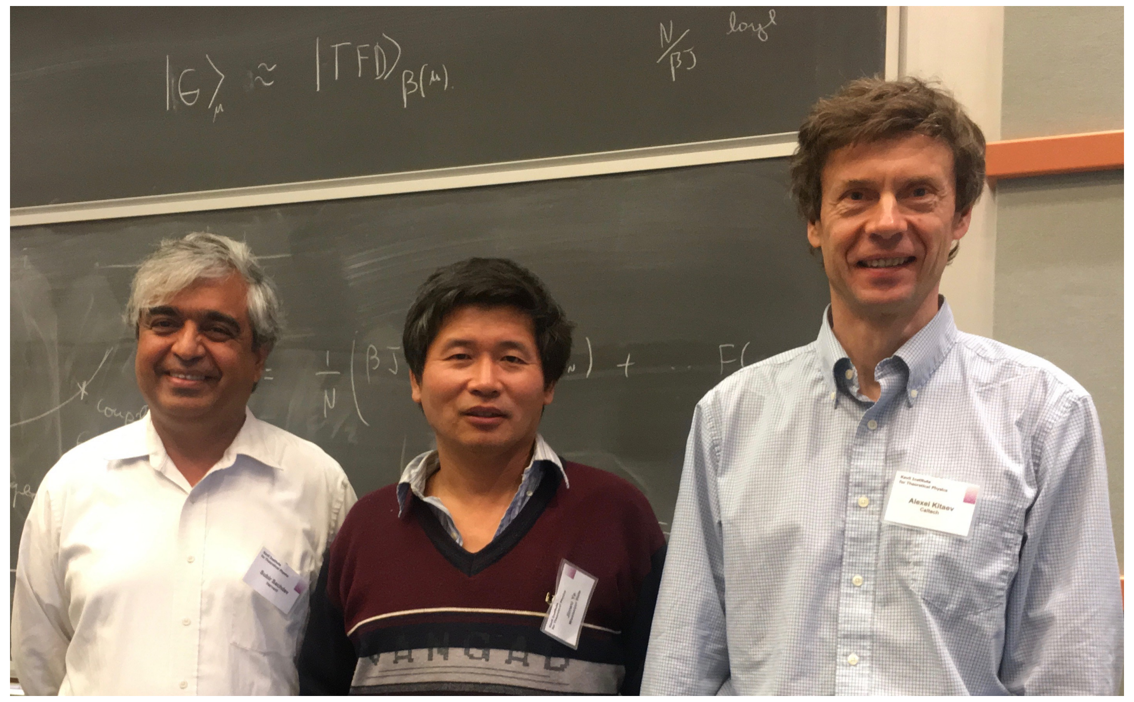
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
Brody et al. *Rev. Mod. Phys.* **53**, 385 (1981))

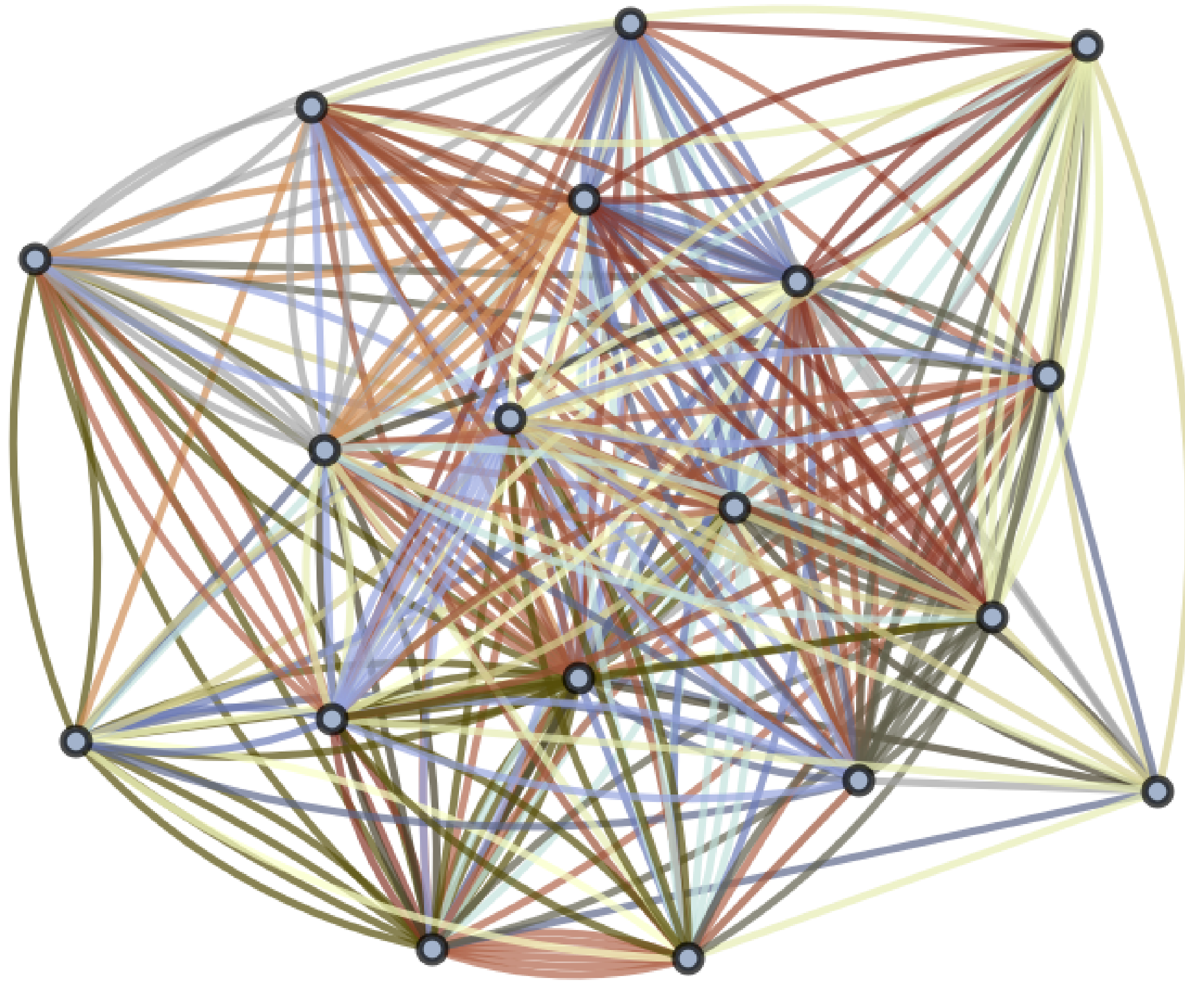
$$\mathcal{H} = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}\, c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta - \mu \sum_\alpha c^\dagger_\alpha c_\alpha$$

$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c^\dagger_\beta + c^\dagger_\beta c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c^\dagger_\alpha c_\alpha \,; \quad [\mathcal{H}, \mathcal{Q}] = 0\,; \quad 0 \leq \mathcal{Q} \leq 1$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



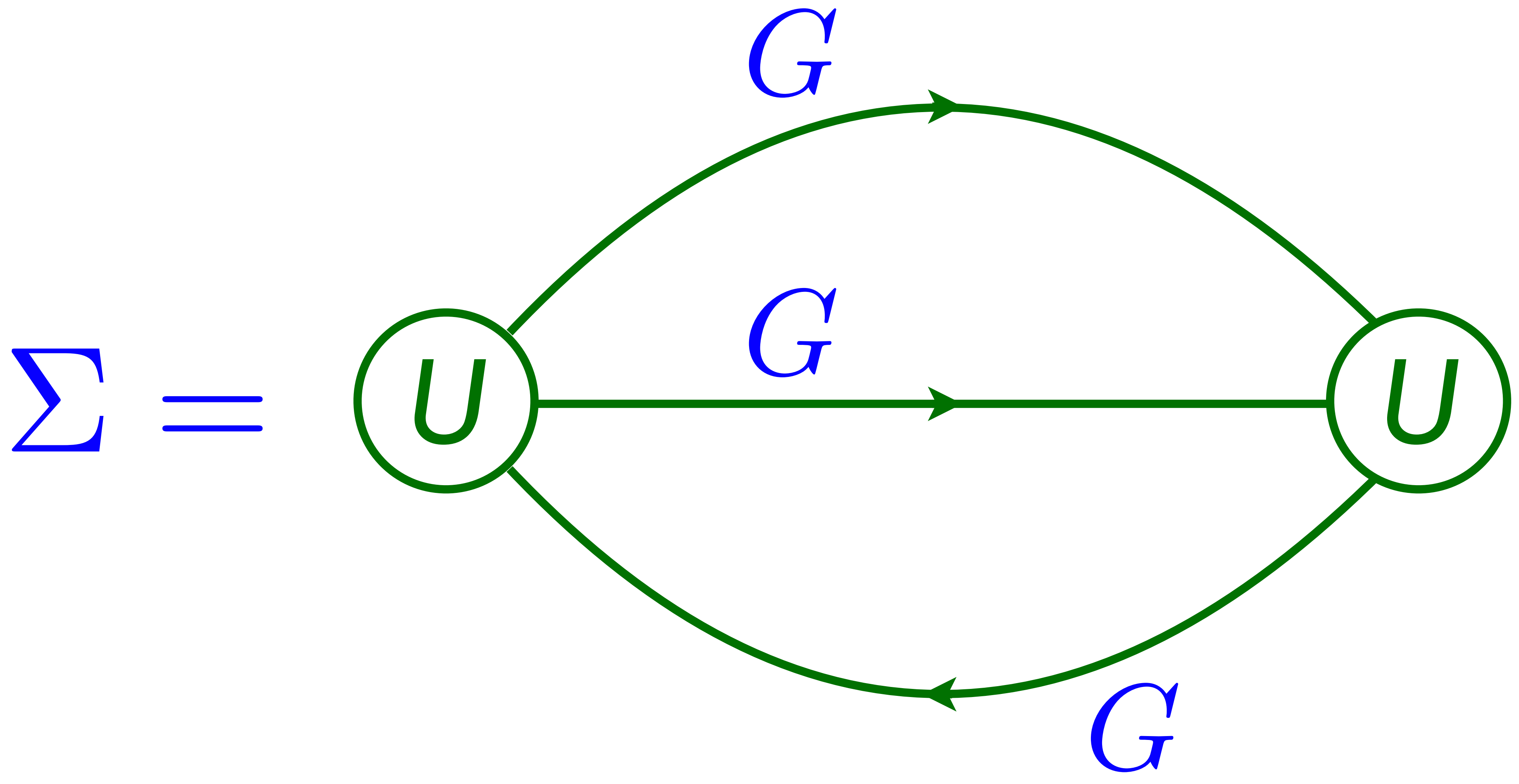


Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Kitaev (2015) (unpublished)
Sachdev *Phys. Rev. X* **5**, 041025 (2015)

The SYK model

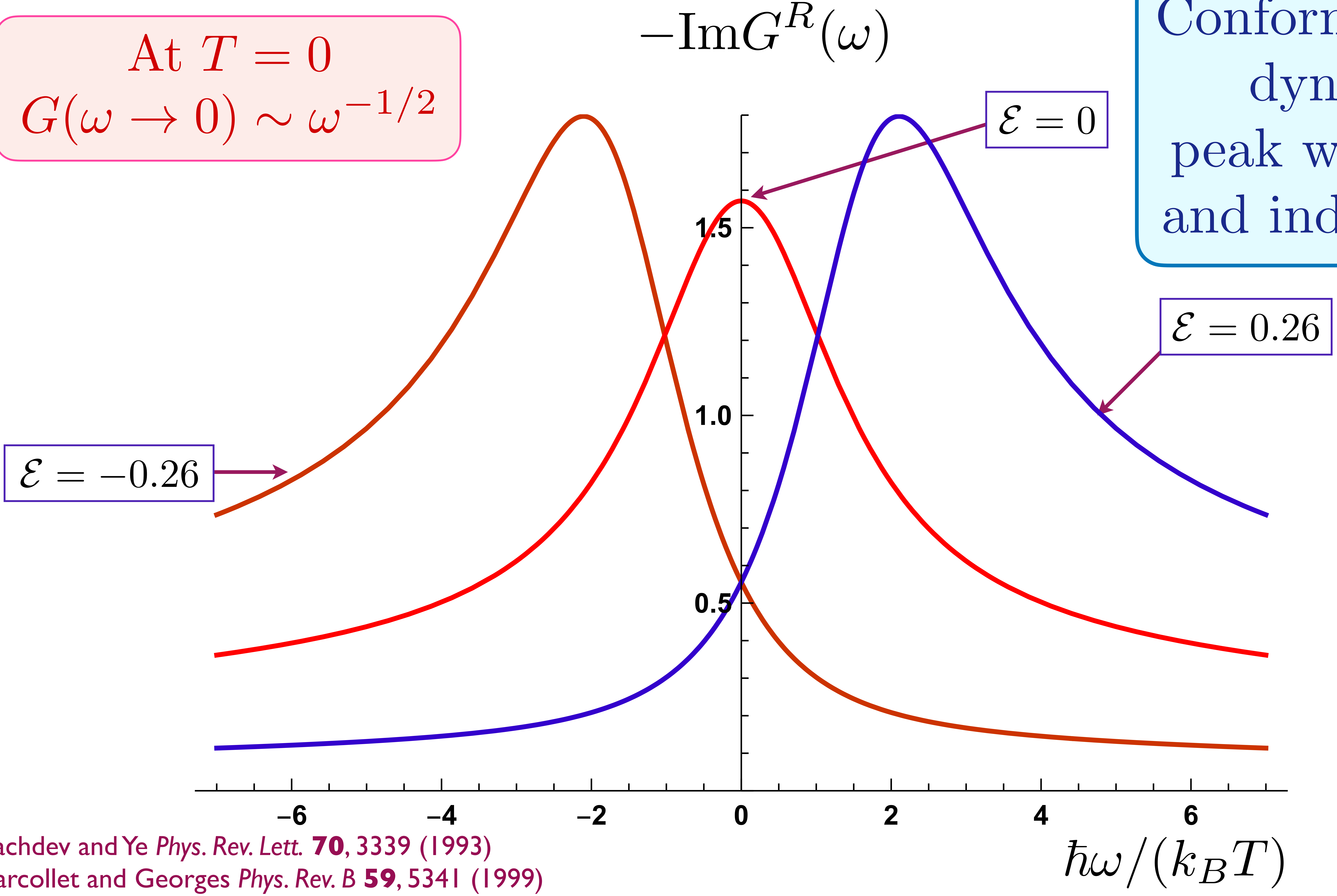
Feynman graph expansion in $U_{\alpha\beta;\gamma\delta}$, and graph-by-graph average, yields exact equations for the fermion Green's function $G(\tau) = -\sum_\alpha \langle c_\alpha(\tau) c_\alpha^\dagger(\tau)\rangle / N$ in the large N limit:

$$\boxed{\begin{gathered} G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau) \\ G(\tau = 0^-) = \mathcal{Q}. \end{gathered}}$$



Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)

The SYK model



At $T = 0$
$G(\omega \to 0) \sim \omega^{-1/2}$

Conformal 'Planckian' dynamics with peak width $\sim k_B T/\hbar$ and independent of U

Sachdev and Ye *Phys. Rev. Lett.* **70**, 3339 (1993)
Parcollet and Georges *Phys. Rev. B* **59**, 5341 (1999)
Sachdev *Phys. Rev. X* **5**, 041025 (2015)

The SYK model

The (averaged) partition function can be written as path integral over the bilocal fermion Green's function $G(\tau_1, \tau_2) \sim \frac{1}{N} \sum_\alpha c_\alpha(\tau_1) c_\alpha^\dagger(\tau_2)$

$$\overline{\mathcal{Z}} = \int \mathcal{D}G(\tau_1, \tau_2) \exp\left(-N S_{\text{eff}}[G]\right)$$

The large N saddle point equation $\delta S_{\text{eff}}/\delta G = 0$ for $G(\tau_1, \tau_2) = G_s(\tau_1 - \tau_2)$ is

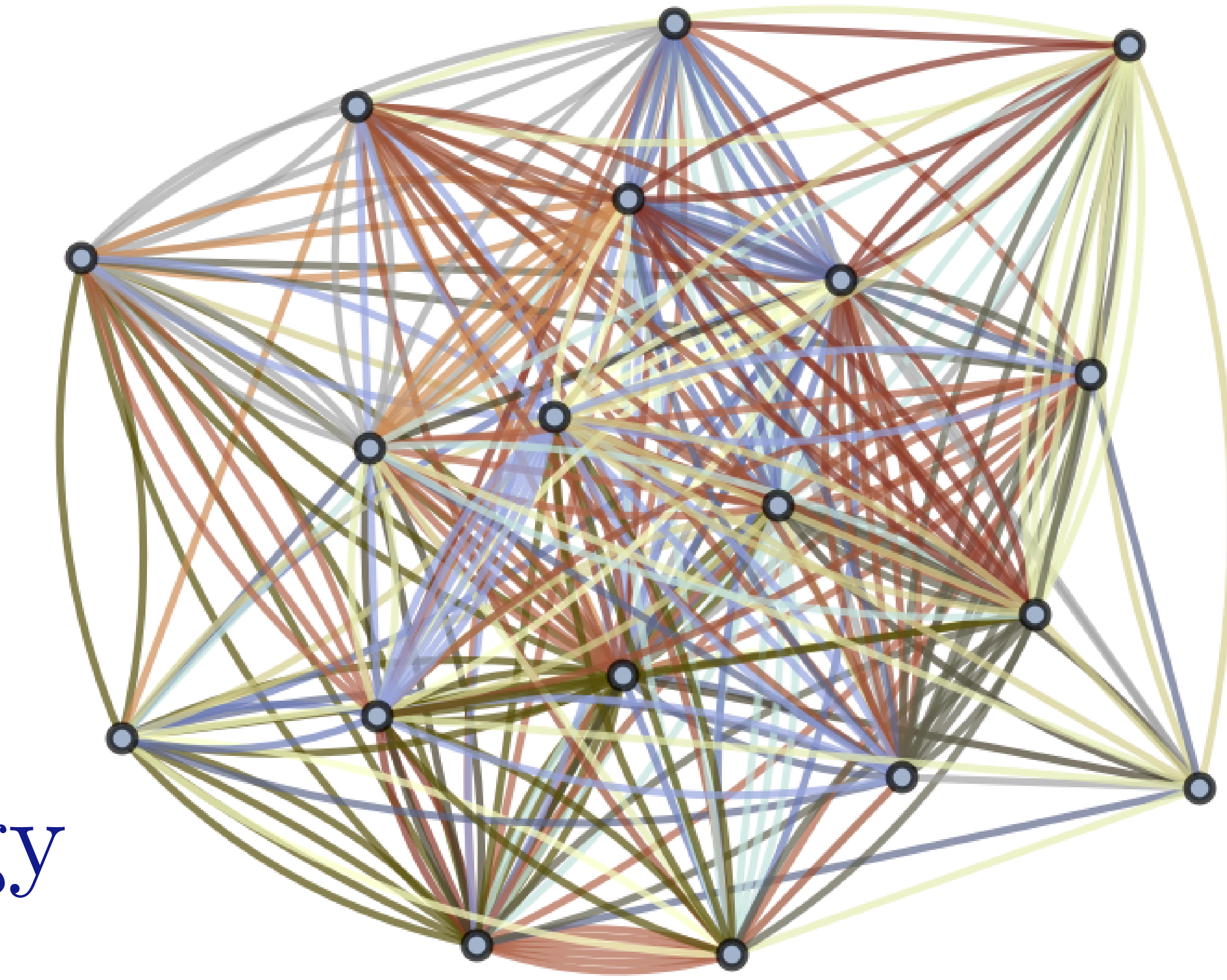
$$G_s(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma_s(\tau) = -U^2 G_s^2(\tau) G_s(-\tau)$$

Parcollet and Georges *Phys. Rev. B* **59**, 5341 (1999)
Kitaev (2015) (unpublished)
Sachdev *Phys. Rev. X* **5**, 041025 (2015)

Time reparameterization symmetry:
At frequencies $\ll U$, the path integral for is invariant under time reparametrization $f(\sigma)$

$$\tau = f(\sigma)$$

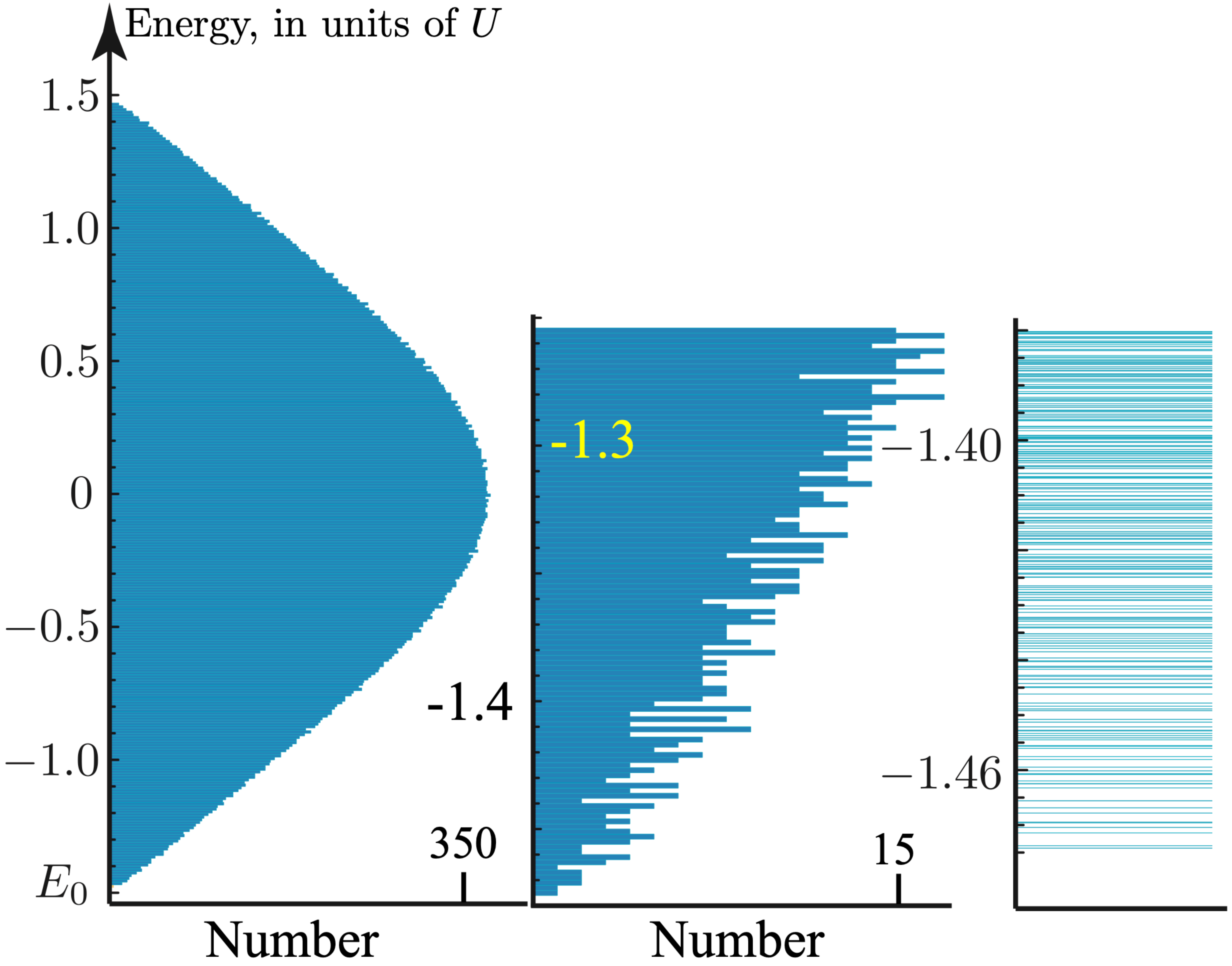
$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \, \widetilde{G}(\sigma_1, \sigma_2)$$



There is also an emergent U(1) gauge symmetry. Hints that the low energy theory is quantum gravity+electromagnetism!

Many-body density of states

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



Complex SYK model

(Numerics: G. Tarnopolsky)

Many-body density of states

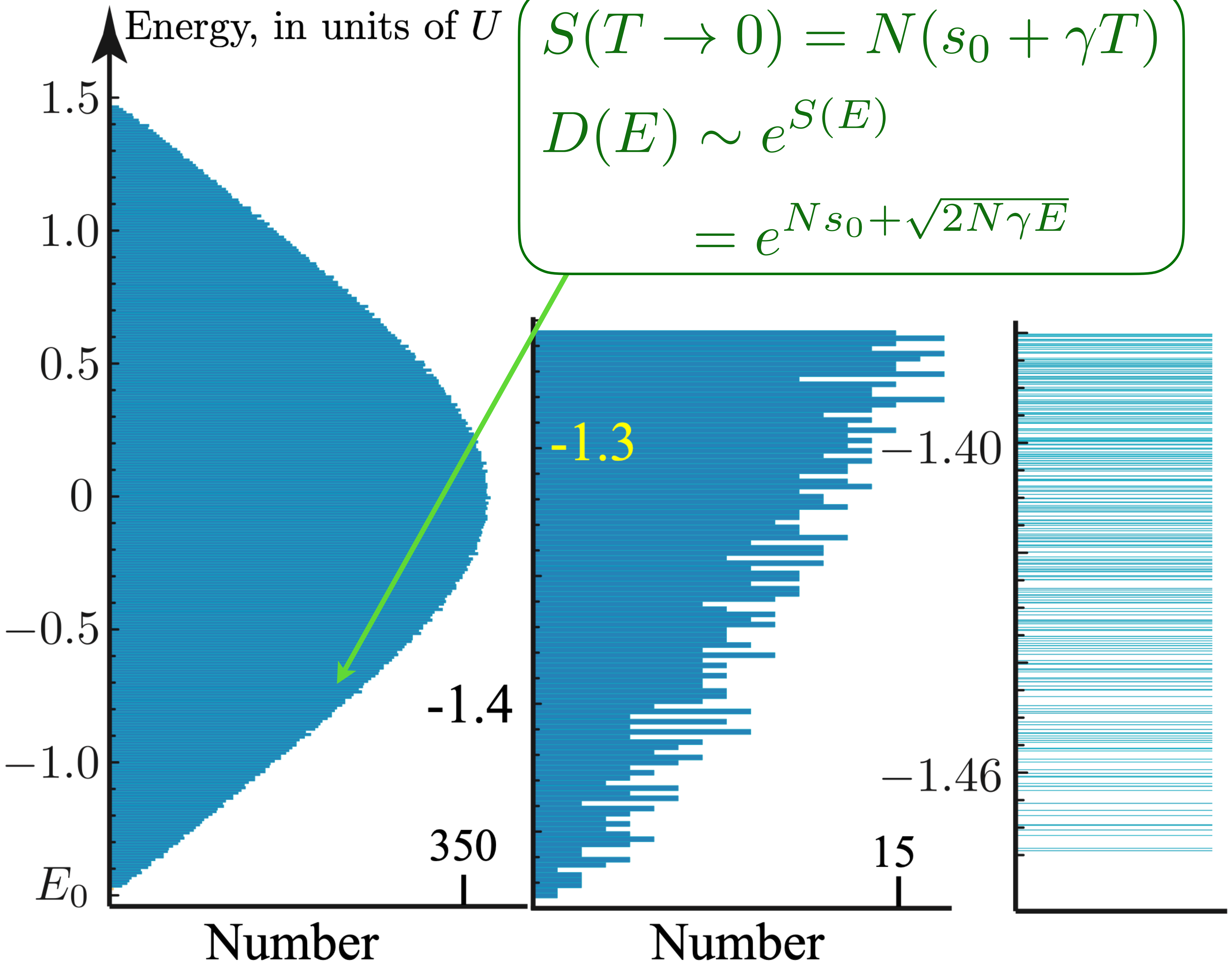
Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$

$$S(T \to 0) = N(s_0 + \gamma T)$$

$$D(E) \sim e^{S(E)}$$

$$= e^{N s_0 + \sqrt{2 N \gamma E}}$$



At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$

$$= 0.46484769917\ldots.$$

Georges, Parcollet and Sachdev
Phys. Rev. B **63**, 134406 (2001)

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(Numerics: G. Tarnopolsky)

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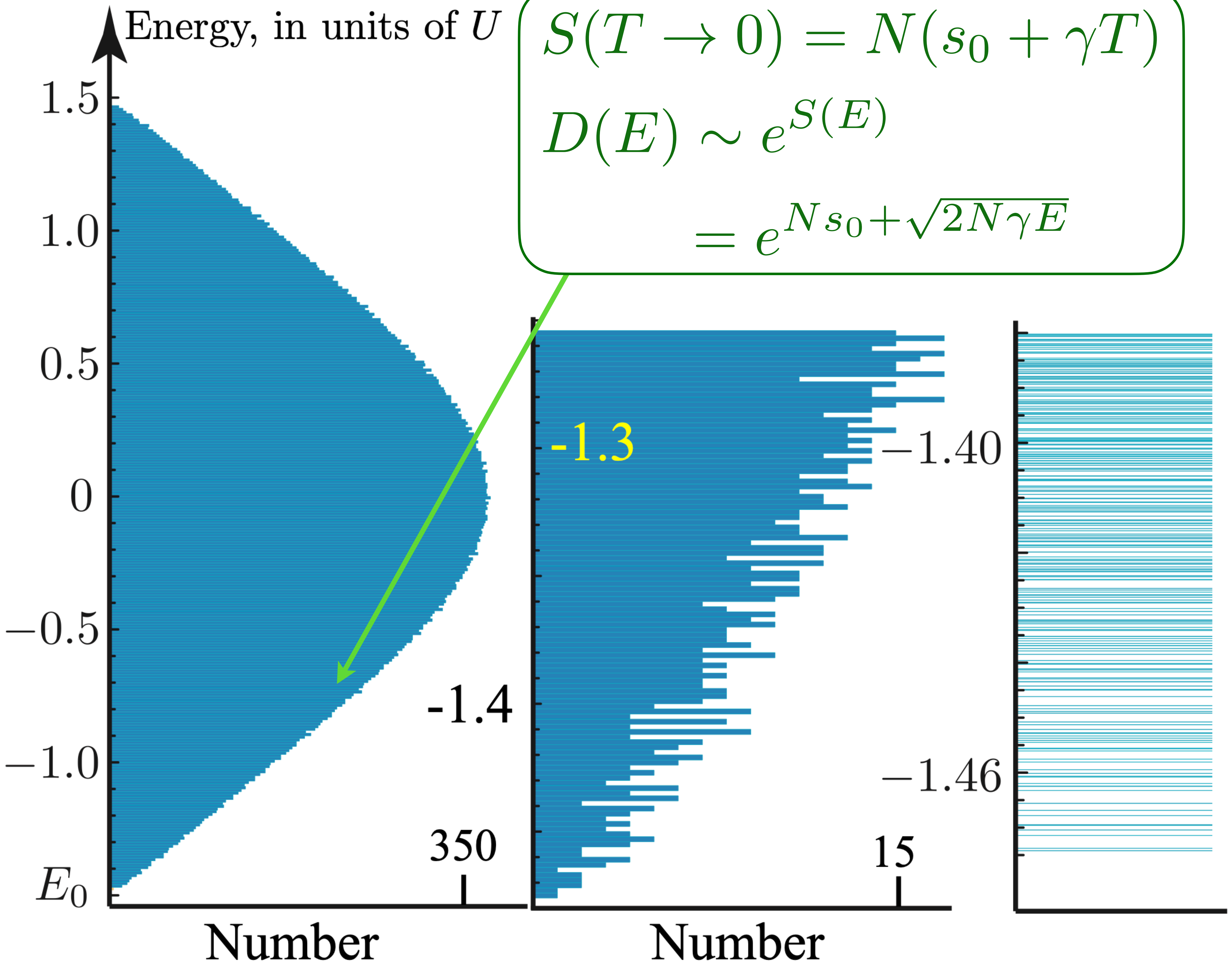


Energy level spacing $\sim e^{-Ns_0}$!

No quasiparticle decomposition: wavefunctions change chaotically from one state to the next.

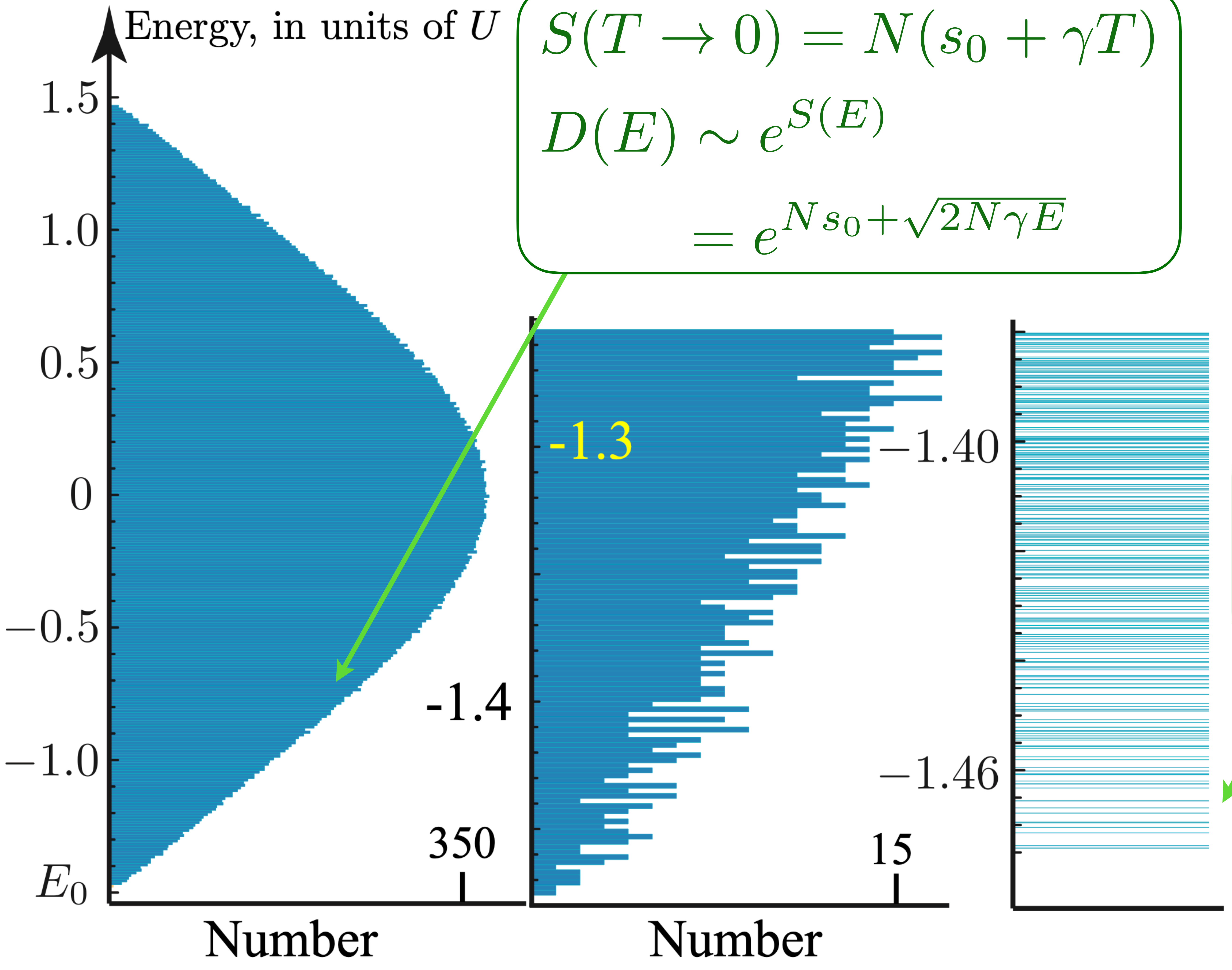
Complex SYK model

(Numerics: G. Tarnopolsky)

Many-body density of states

Beyond Boltzmann

$$D(E) = \sum_i \delta(E - E_i); \quad E_0 + E_i \Rightarrow \text{Many body eigenvalue}$$



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At $\mathcal{Q} = 1/2$

$$s_0 = \frac{\text{Catalan}}{\pi} + \frac{\ln 2}{4}$$

$$= 0.464847699 17\ldots.$$

$$D(E) \sim N^{-1} \exp\left(N s_0\right) \sinh\left(\sqrt{2N\gamma E}\right)$$

Georges, Parcollet and Sachdev Phys. Rev. B **63**, 134406 (2001)

Cotler et al. *J. High Energ. Phys.* **118** (2017)

Gu, Kitaev, Sachdev and Tarnopolsky *J. High Energ. Phys.* **157** (2020)

Complex SYK model

(Numerics: G. Tarnopolsky)

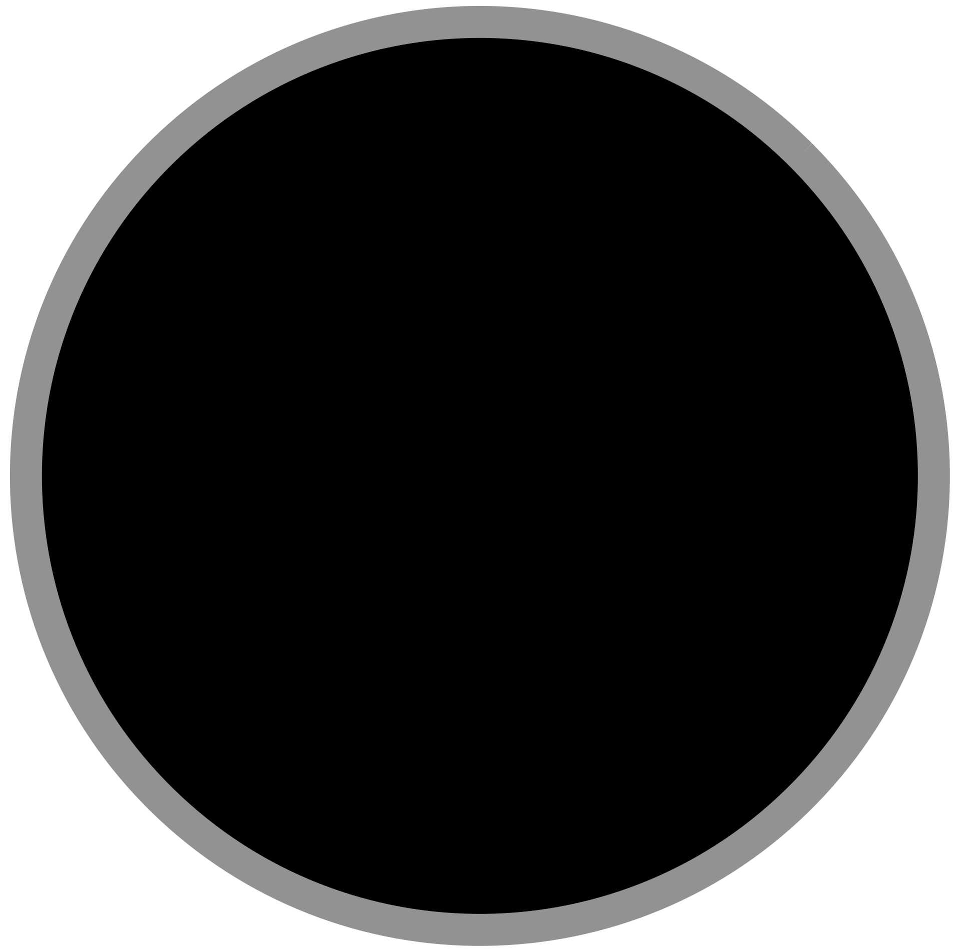
From the SYK model to a quantum theory of charged black holes

Introduction to black holes

Black Holes

Objects so dense that light is gravitationally bound to them.

Horizon radius $R = \dfrac{2GM}{c^2}$

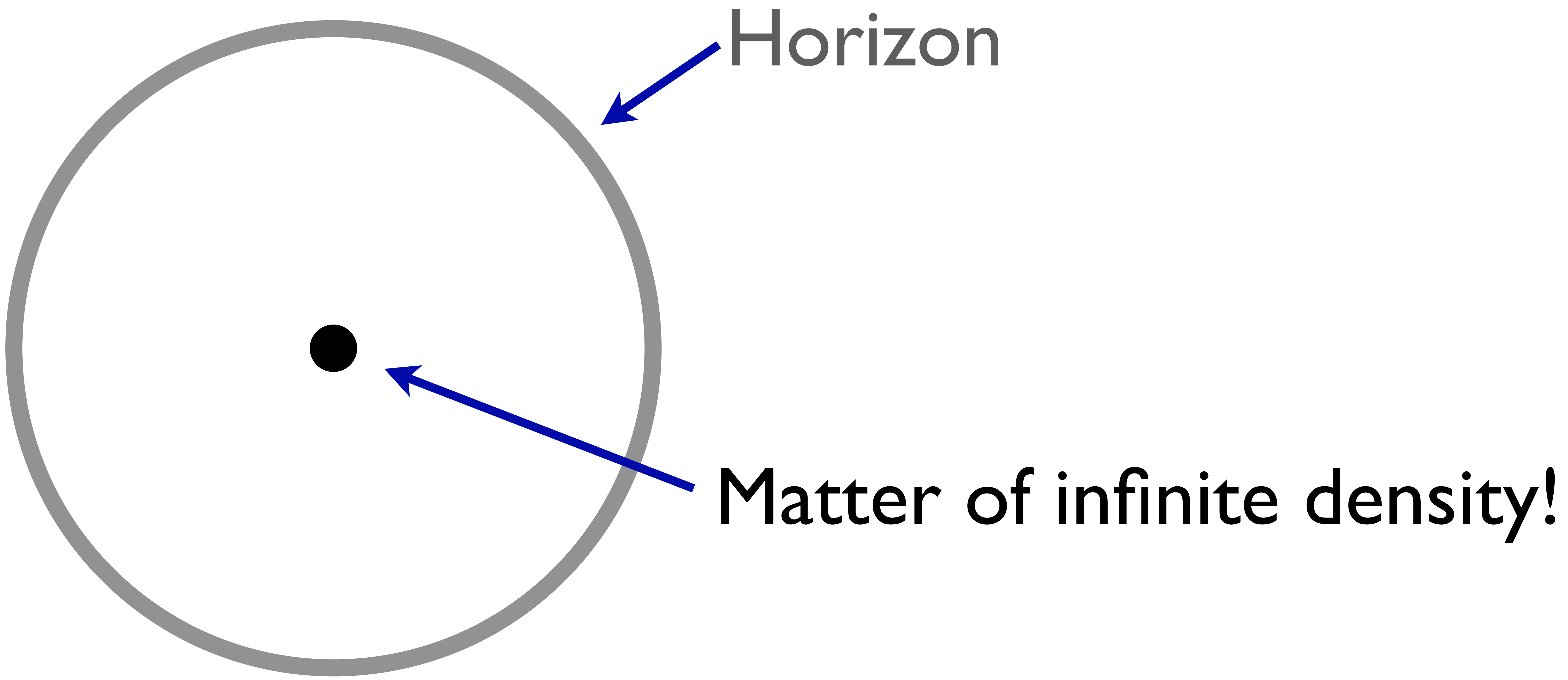


G Newton's constant, c velocity of light, M mass of black hole

For $M =$ earth's mass, $R \approx 9\,mm$!

What is inside a black hole ???

In Einstein's theory, all the matter in a black hole collapses to a singularity at the center of the black hole.



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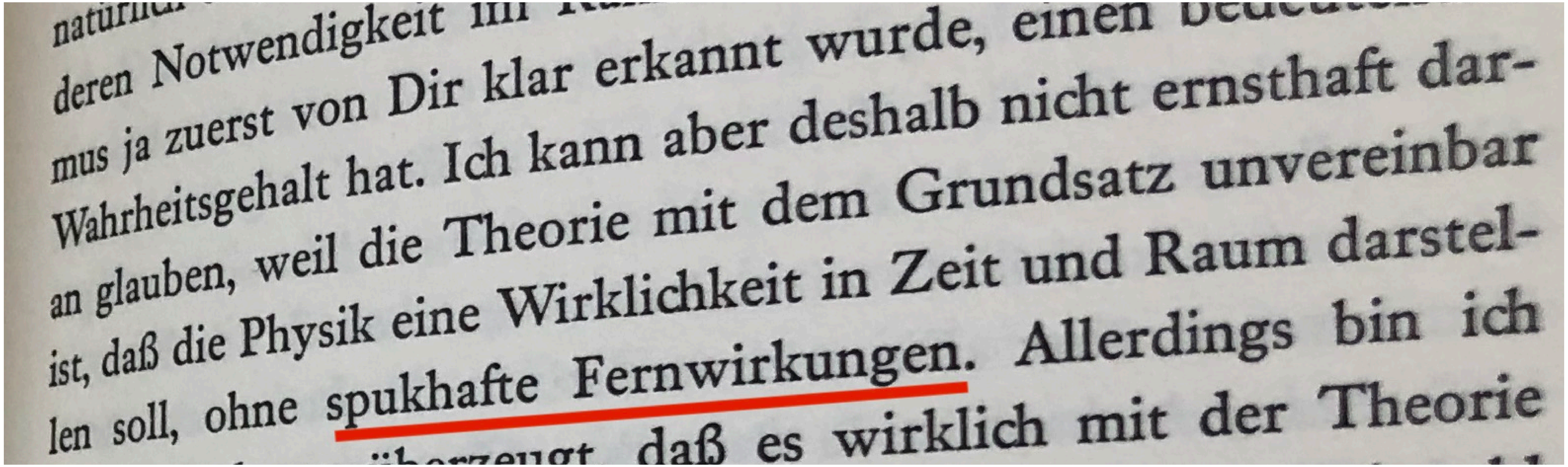
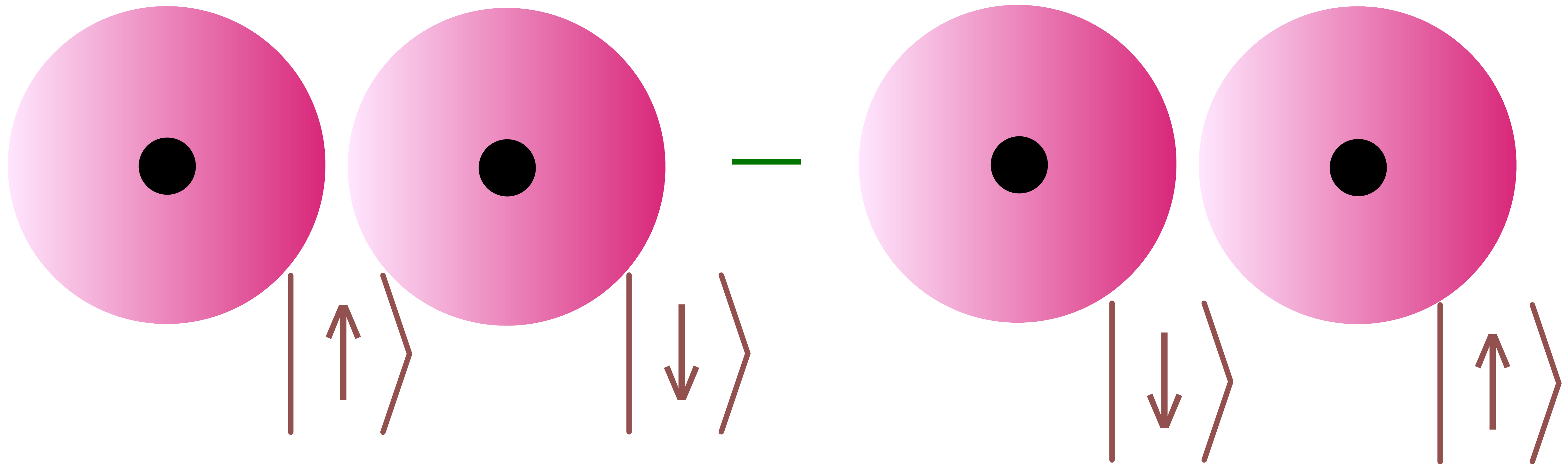

deren Notwendigkeit im
mus ja zuerst von Dir klar erkannt wurde, einen
Wahrheitsgehalt hat. Ich kann aber deshalb nicht ernsthaft dar-
an glauben, weil die Theorie mit dem Grundsatz unvereinbar
ist, daß die Physik eine Wirklichkeit in Zeit und Raum darstel-
len soll, ohne spukhafte Fernwirkungen. Allerdings bin ich
daß es wirklich mit der Theorie

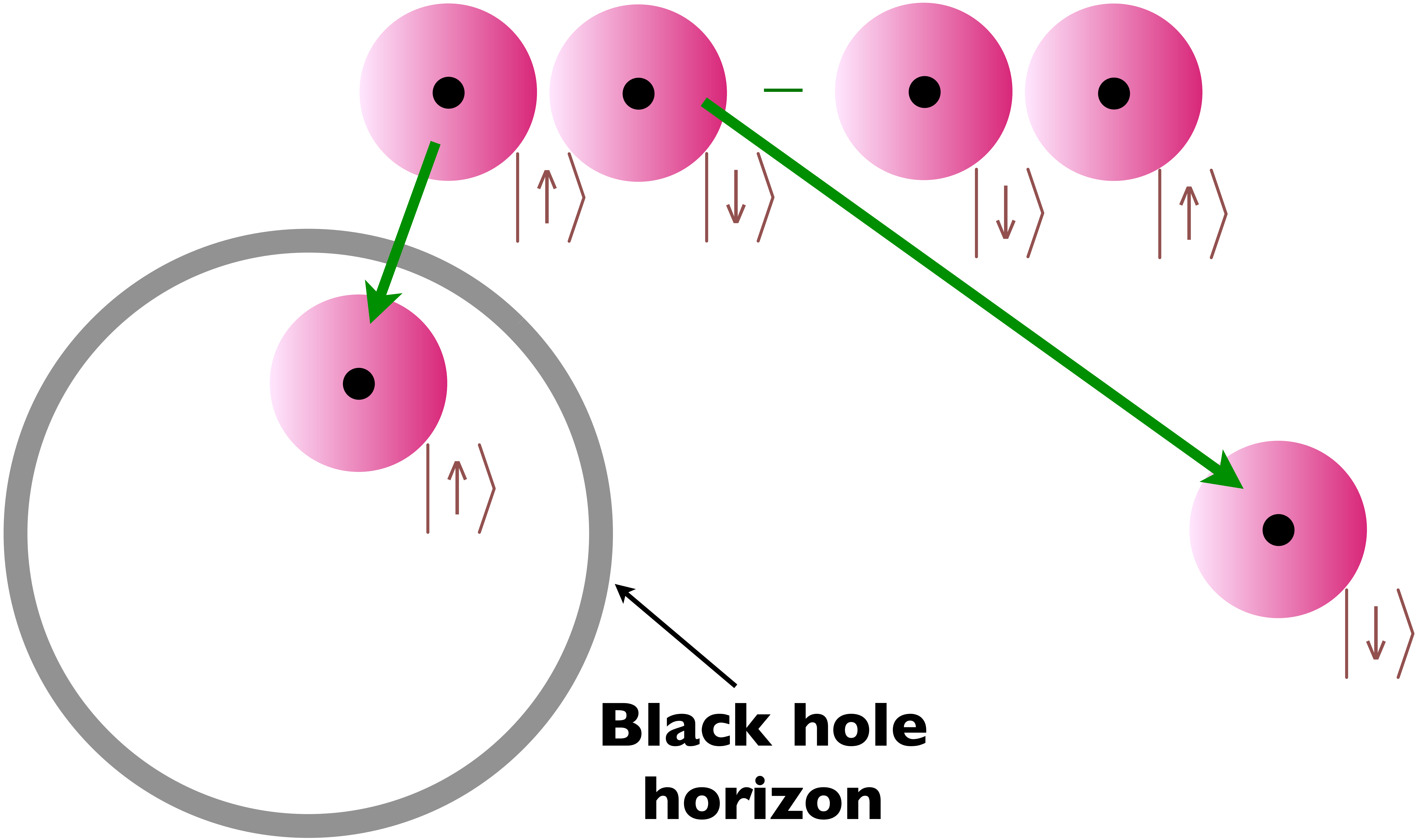
I cannot seriously believe in it because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at distance

Albert Einstein to Max Born, 3 March 1947

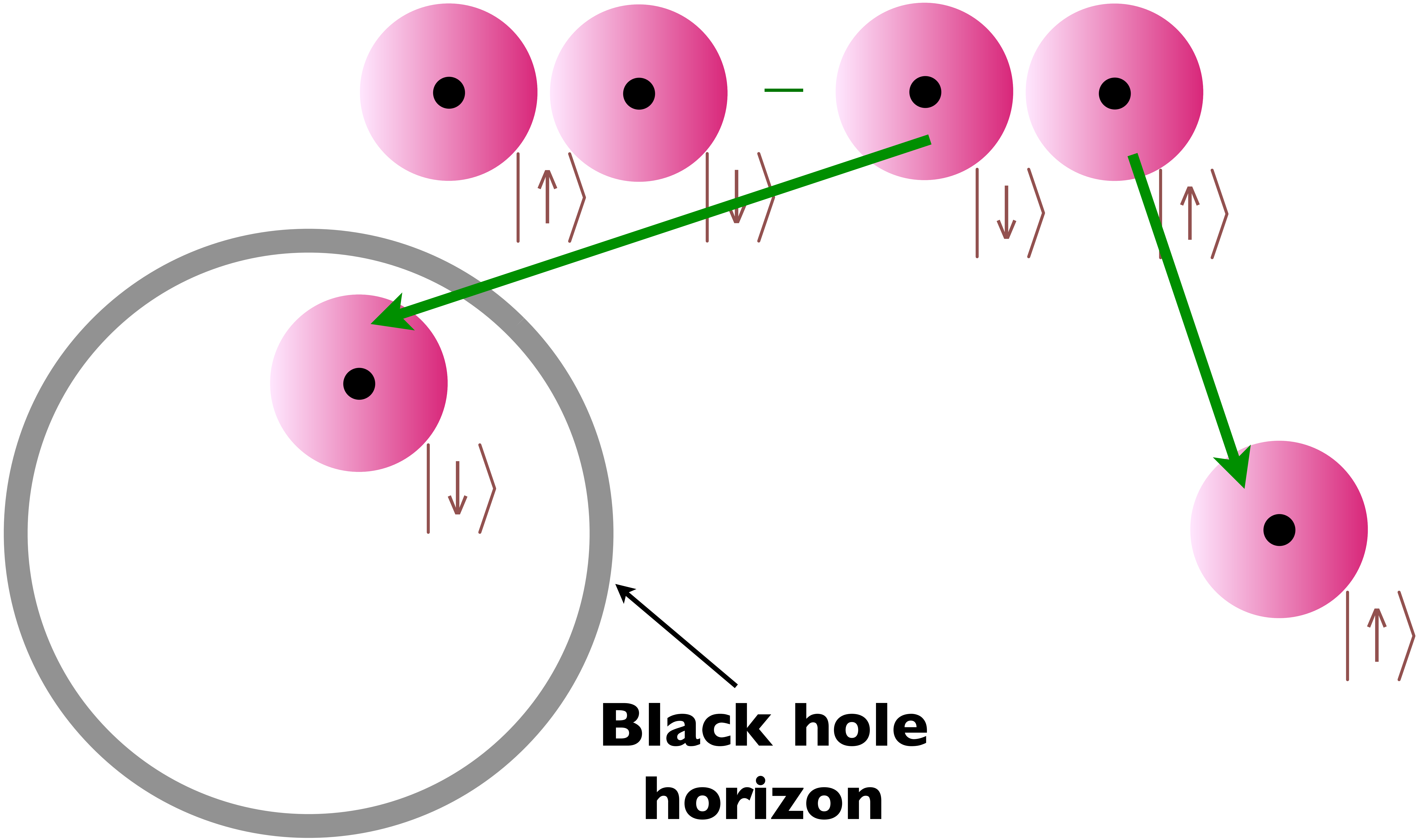
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon



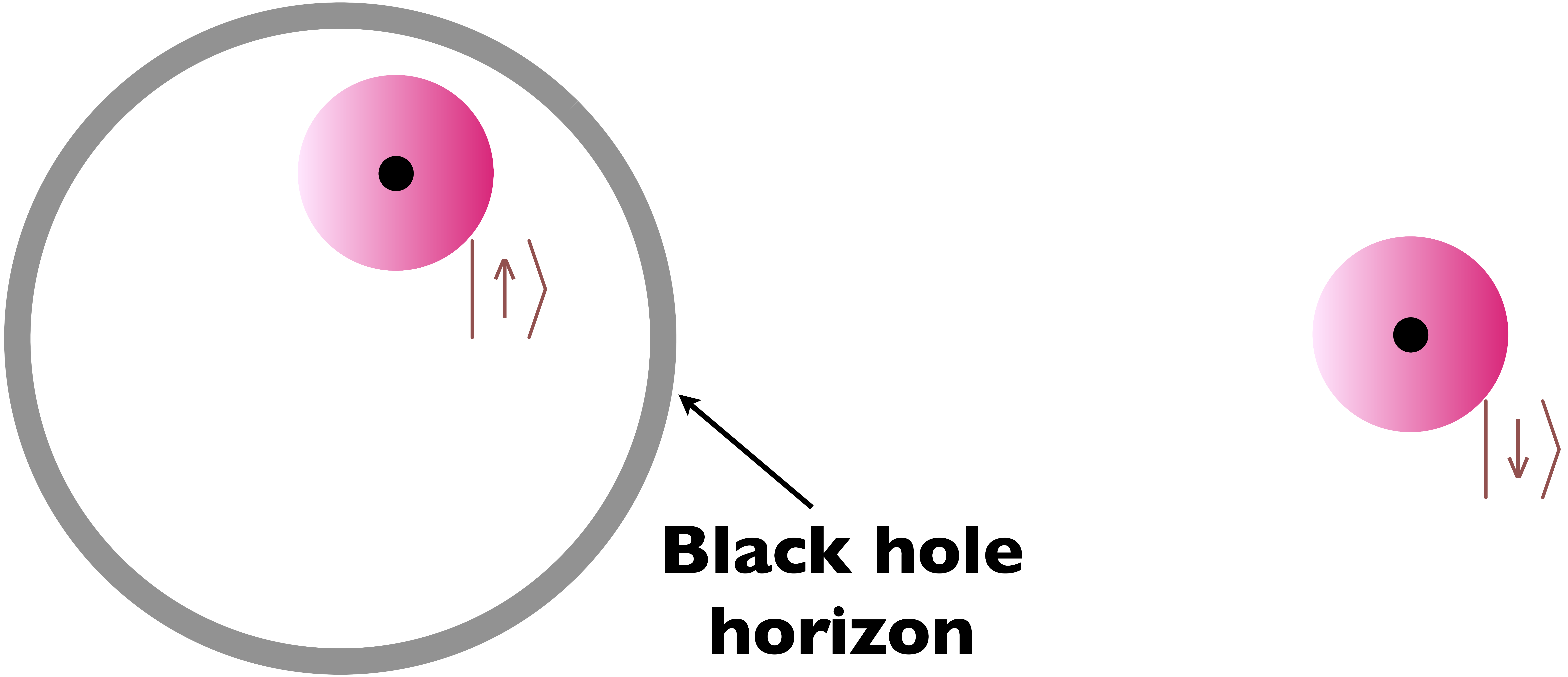
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

Hawking (1975): Black holes have a temperature and an entropy!

To an outside observer, the state of the electron inside the black hole cannot be known, and so the outside electron is in a random state.



Quantum Entanglement across a black hole horizon

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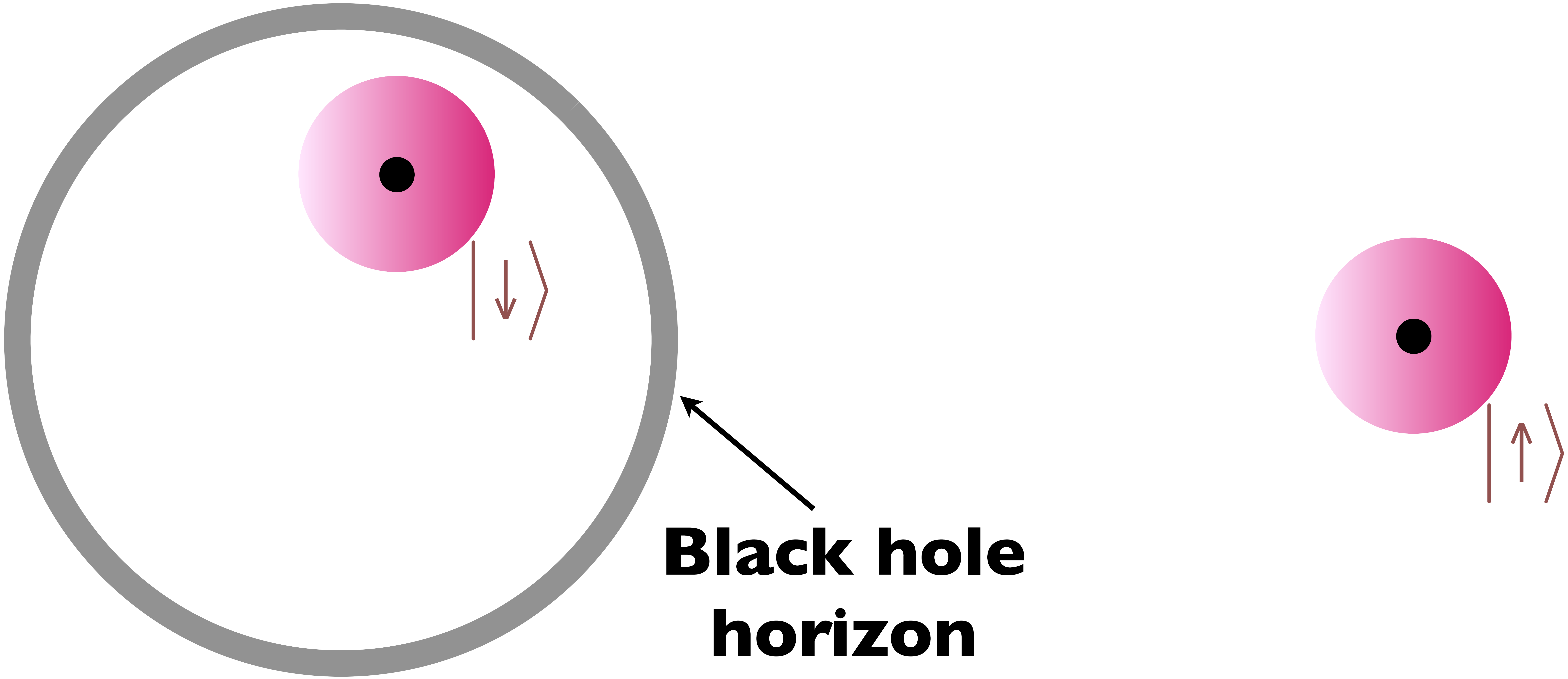
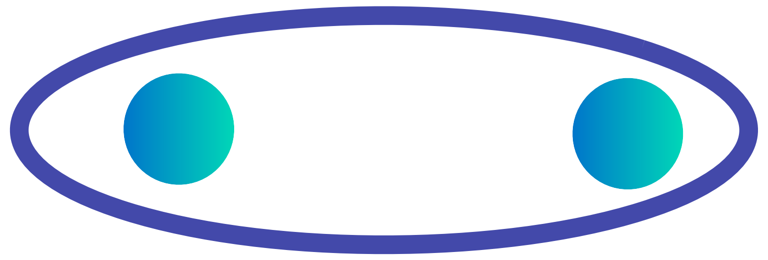
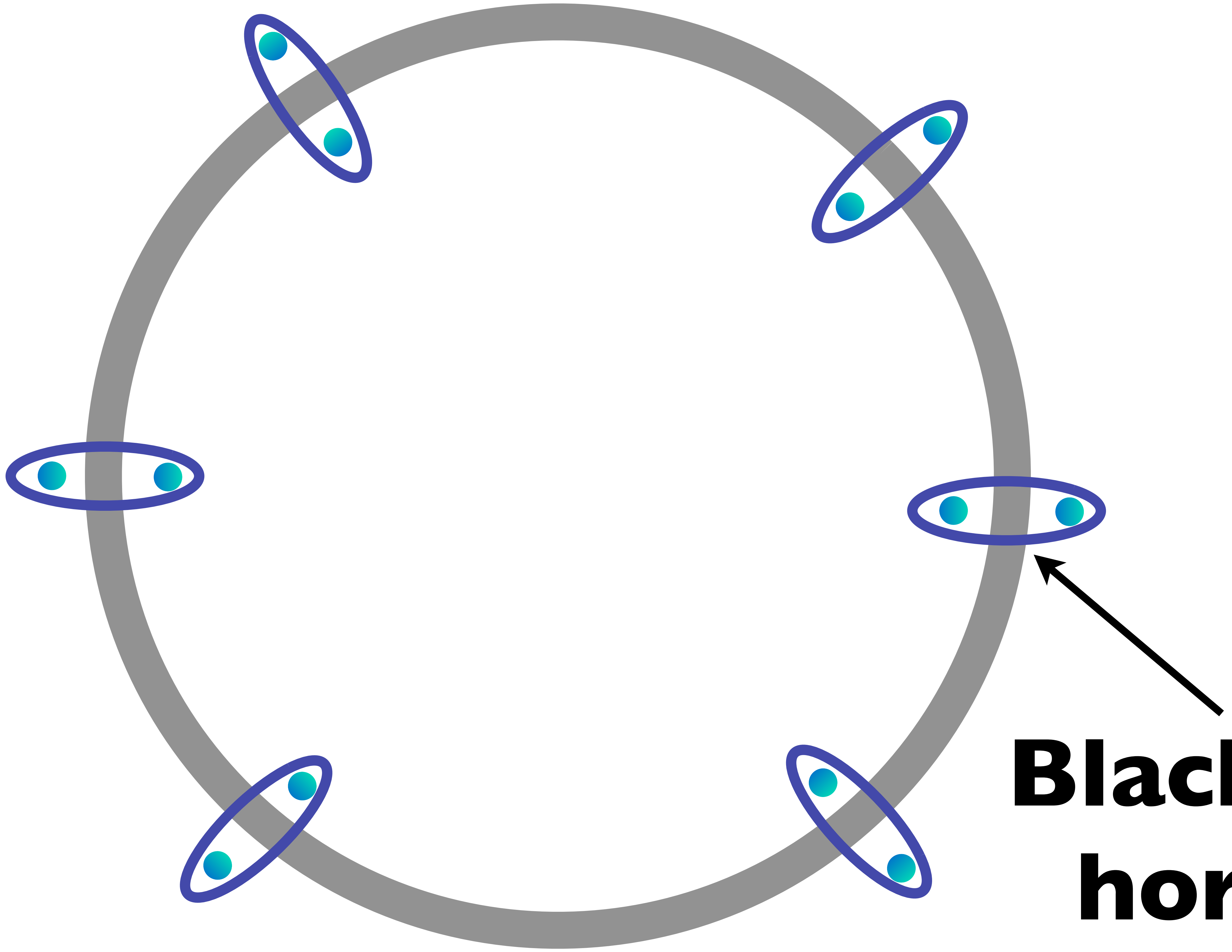
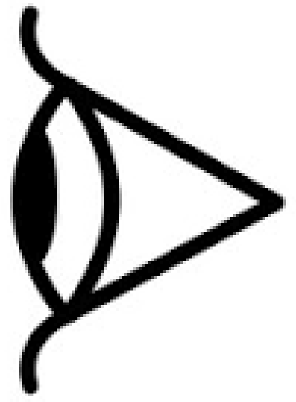


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$





By computations *outside* the black hole, Hawking obtained

$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

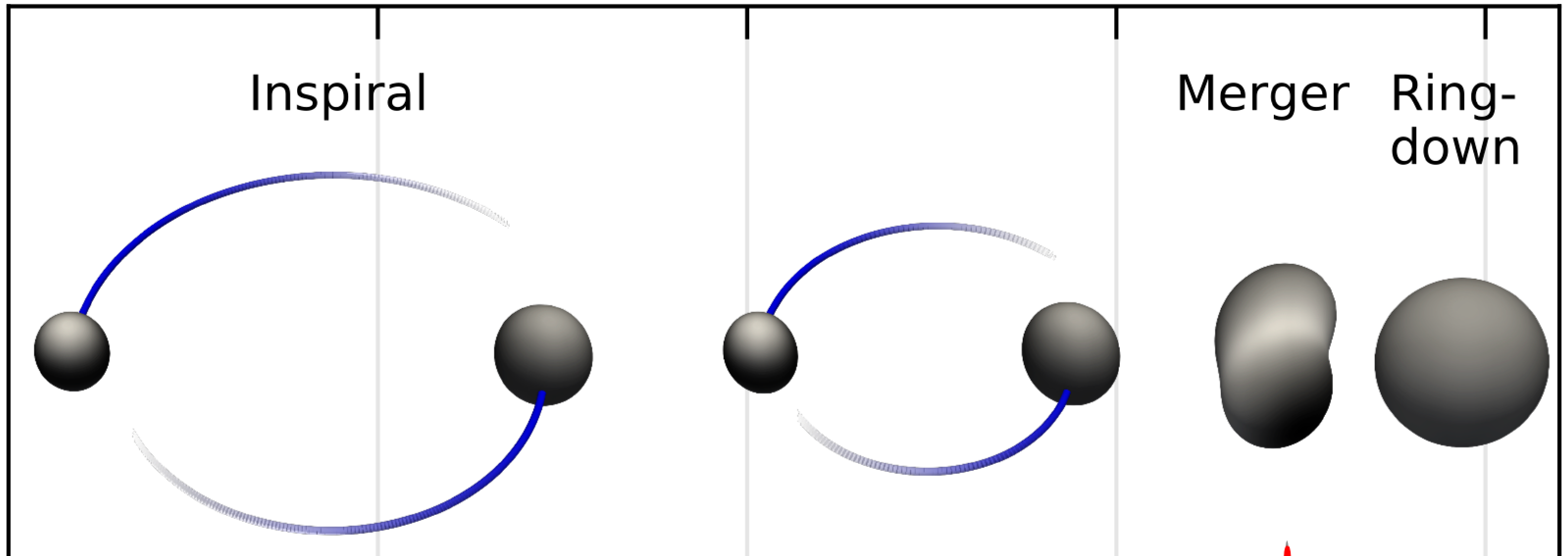
All other systems have entropy proportional to their volume.

Quantum black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.

- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.

- They relax to thermal equilibrium in a time $\sim 8\pi GM/c^3 = \hbar/(k_B T_H)$ which is Planckian!



Bekenstein *Phys. Rev. D* **7**, 2333 (1973)
Hawking *Nature* **248**, 30 (1974)
Vishveshwara *Nature* **227**, 936 (1970)

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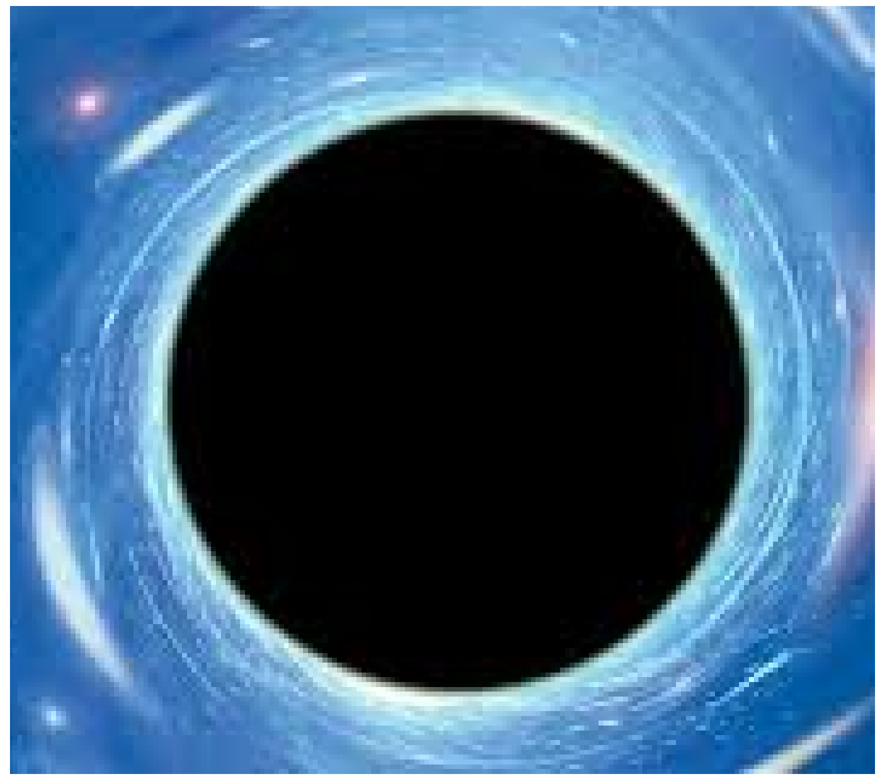
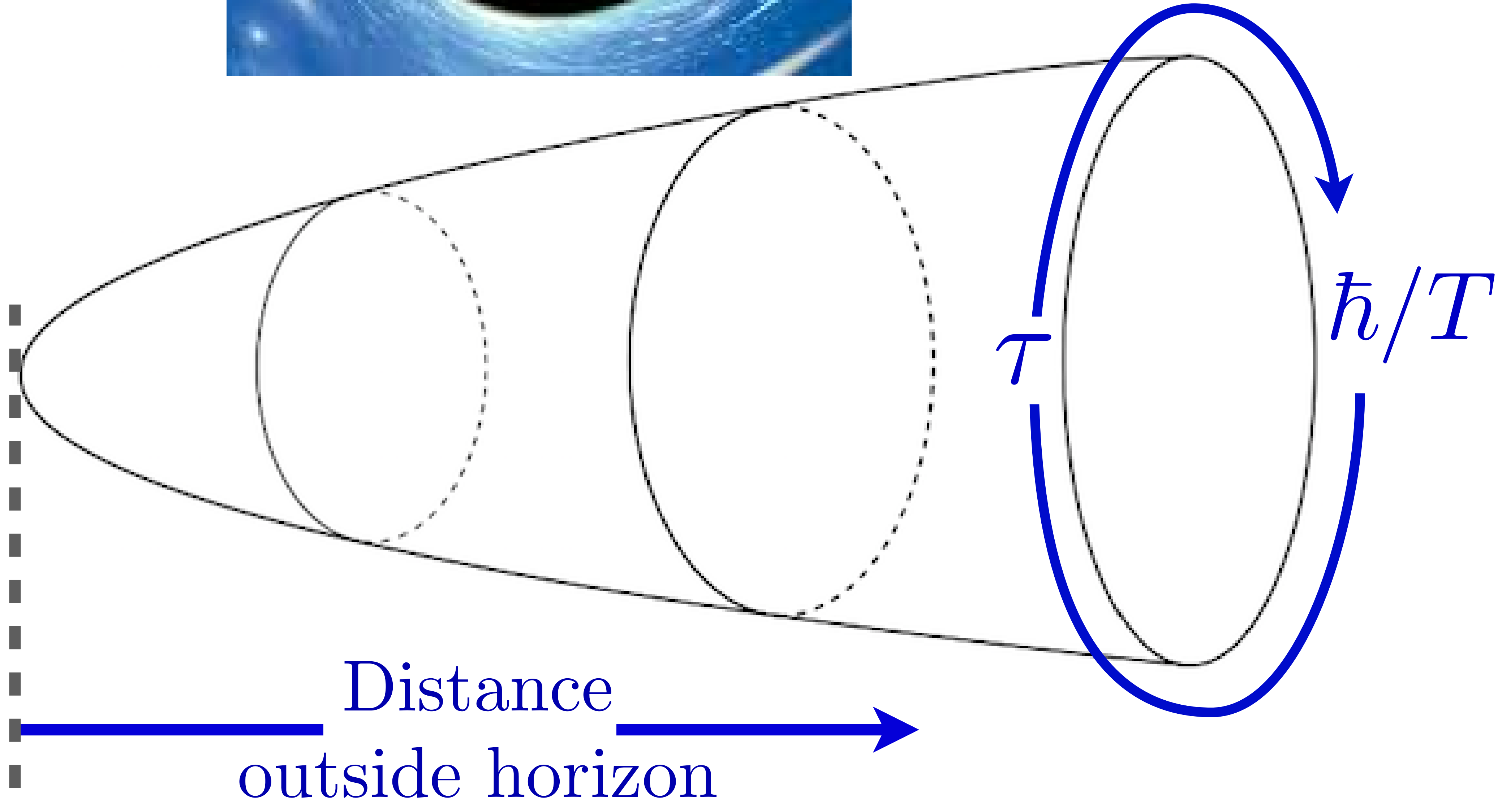
It was clear that quantum theory should be applied to the collapsed matter, but no one knew how to.

The many-body quantum system describing black holes should have no quasiparticle excitations!

Thermodynamics of quantum black holes with charge $\mathcal{Q}$:

$$\mathcal{Z}(\mathcal{Q},T) = \int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(3+1)}_{\text{Einstein gravity+Maxwell EM}}[g_{\mu\nu}, A_\mu]\right)$$





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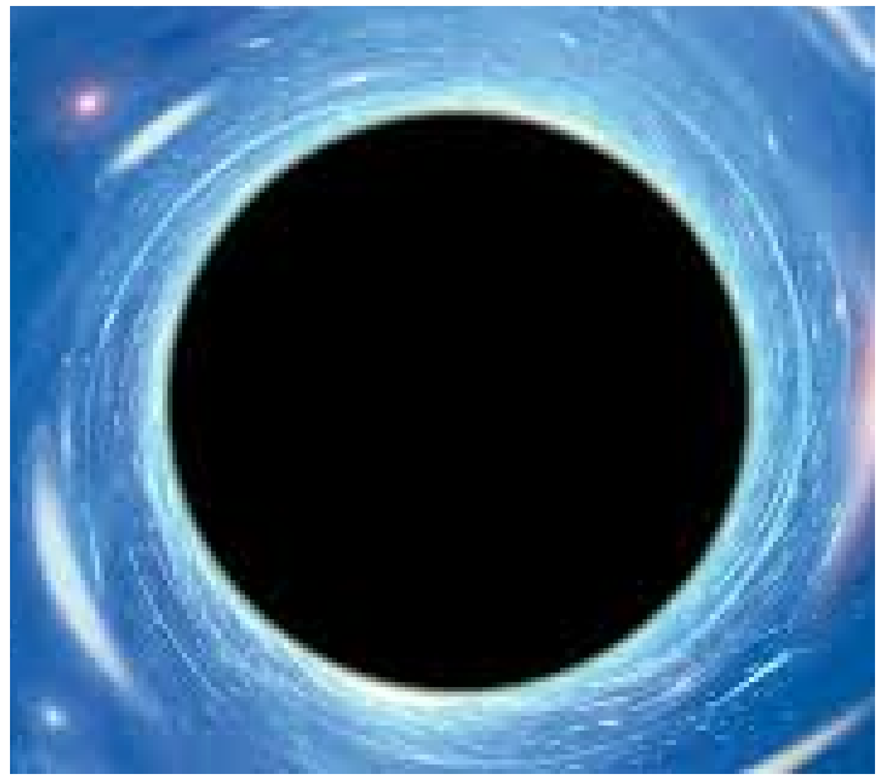
$$= \exp\left(S_{BH}\right) \times \left(\quad ....????... \quad \right)$$

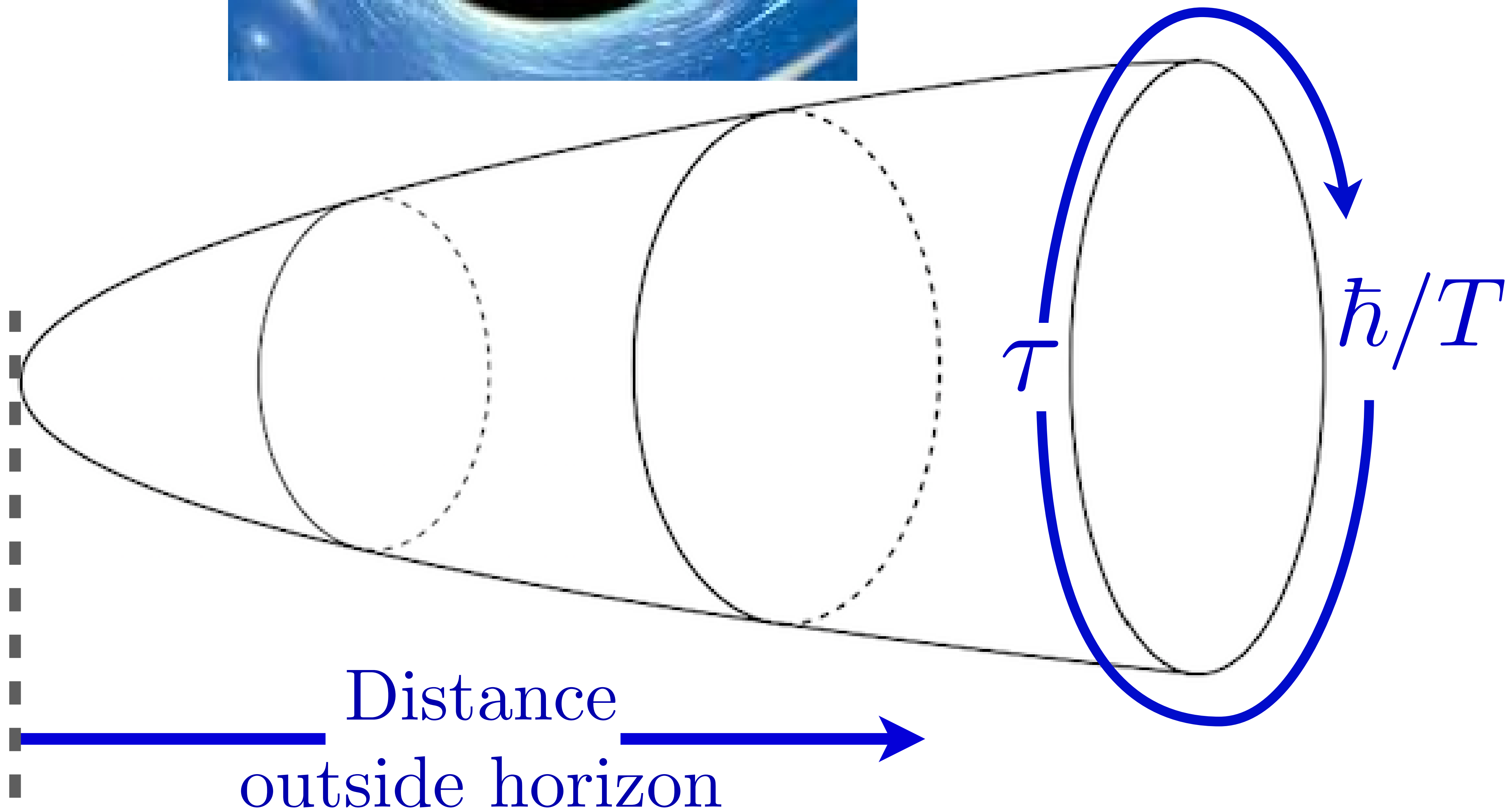
Gibbons and Hawking *Phys. Rev. D* **15**, 2752 (1977)
Chamblin et al. *Phys. Rev. D* **60**, 064018 (1999)



$$S_{BH}(T \to 0, \mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1 + \frac{2(\pi A_0)^{1/2}T}{\hbar c}\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

Obtained from the saddle-point of the gravity path integral in the imaginary time spacetime outside the black hole.



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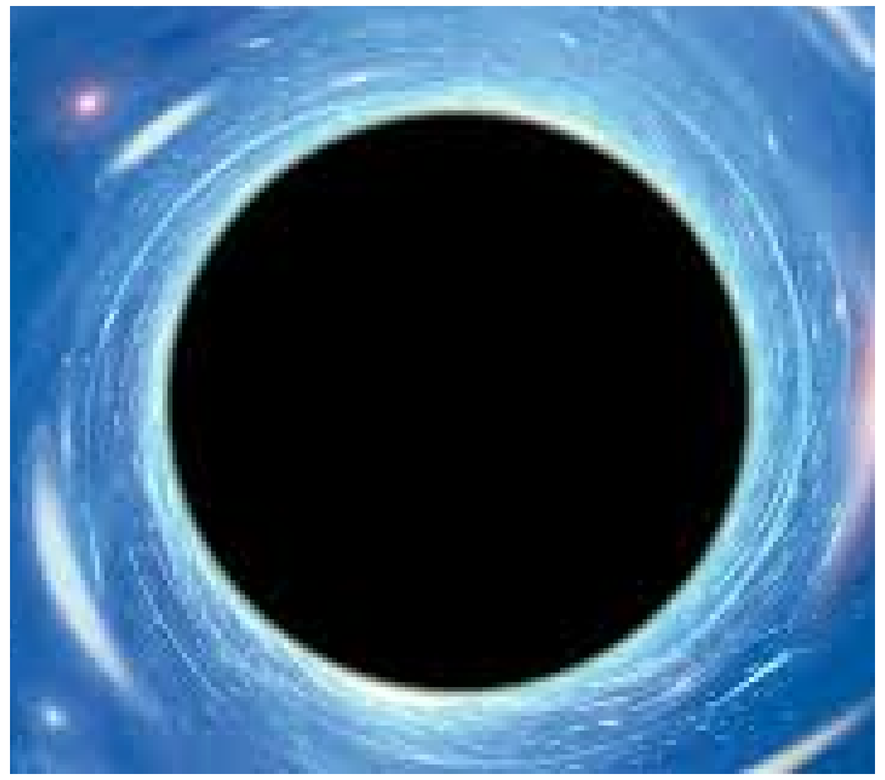

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PHYSICAL REVIEW LETTERS **105**, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

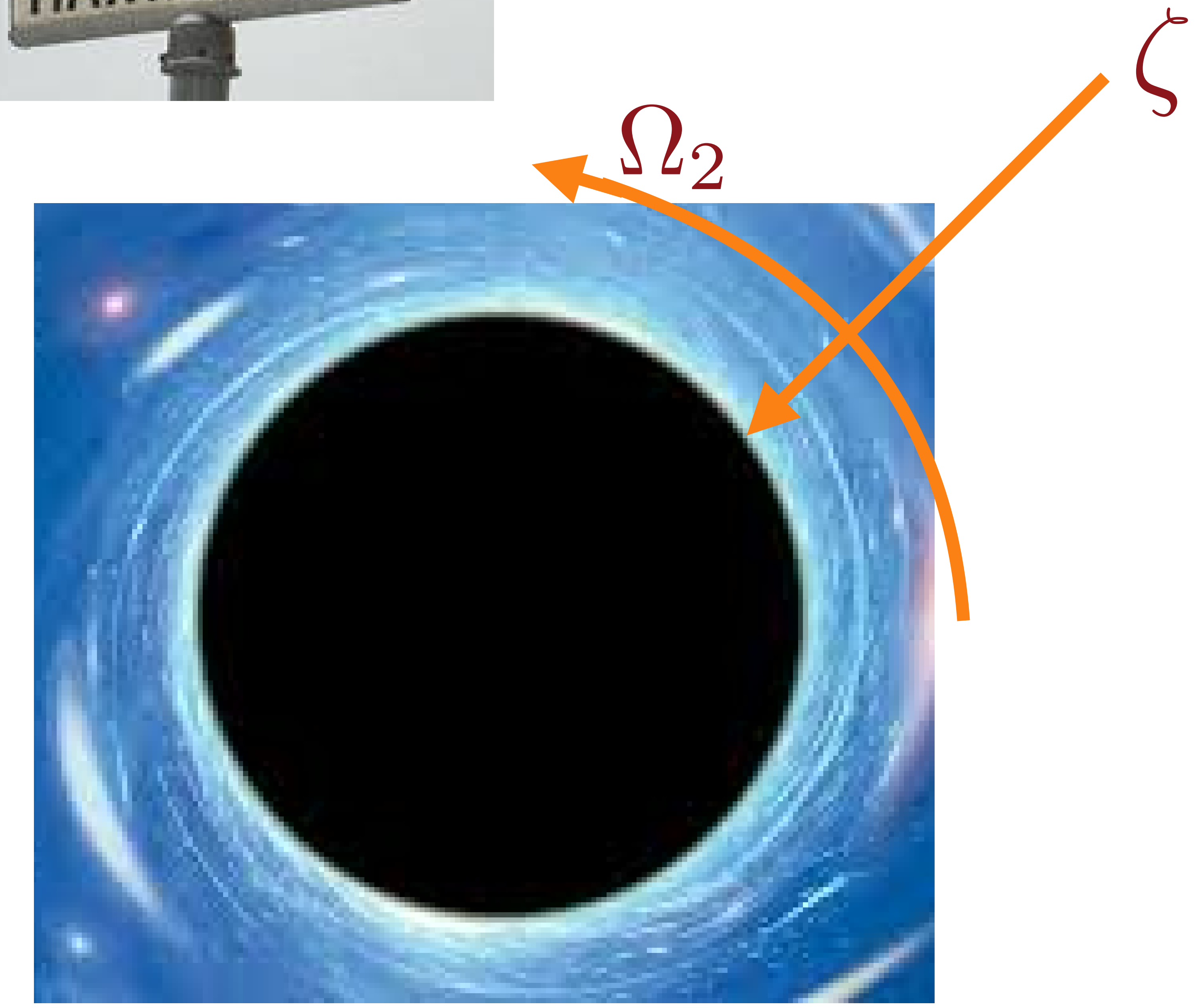
Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

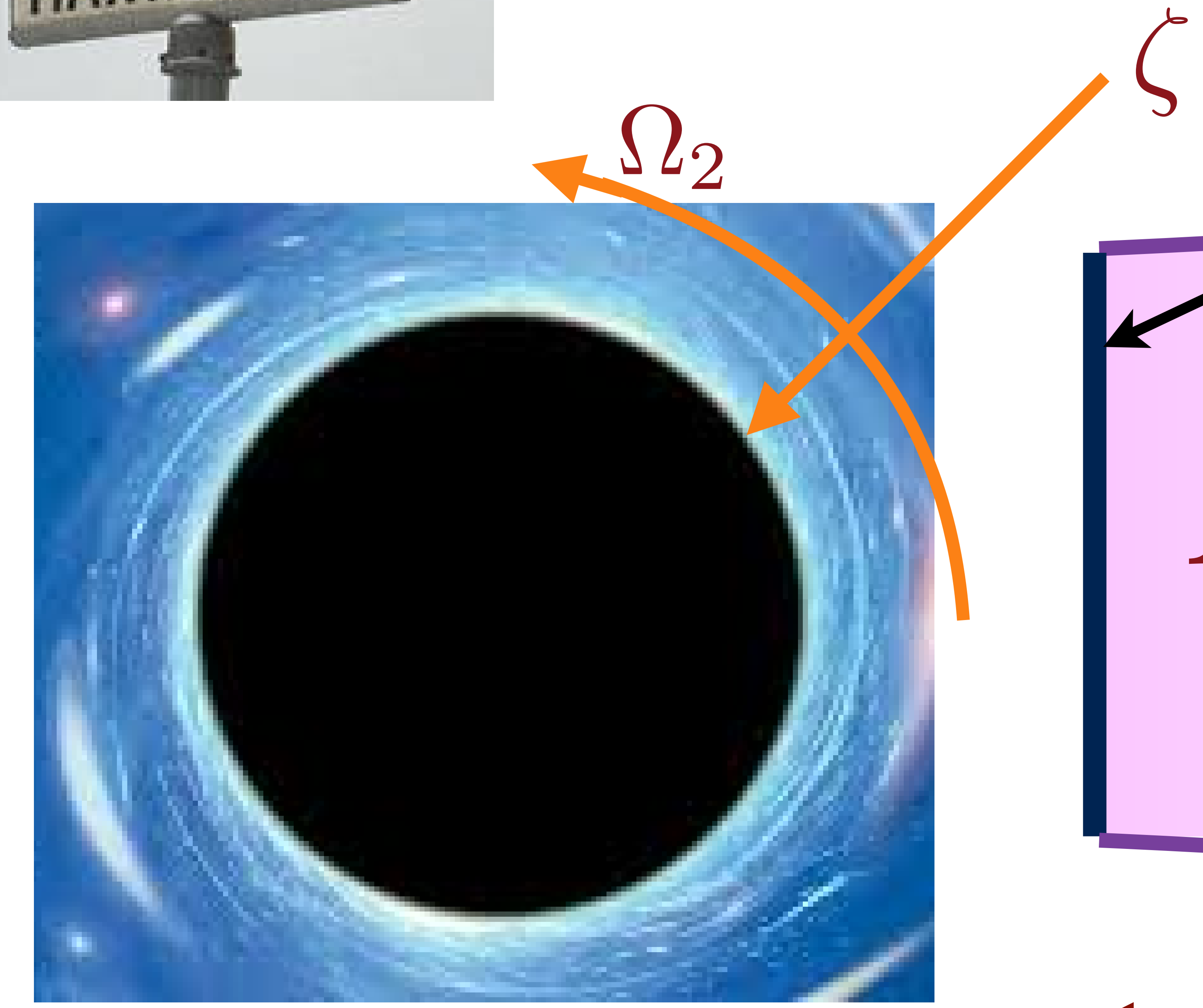
Reissner-Nordstrom black hole of Einstein-Maxwell theory

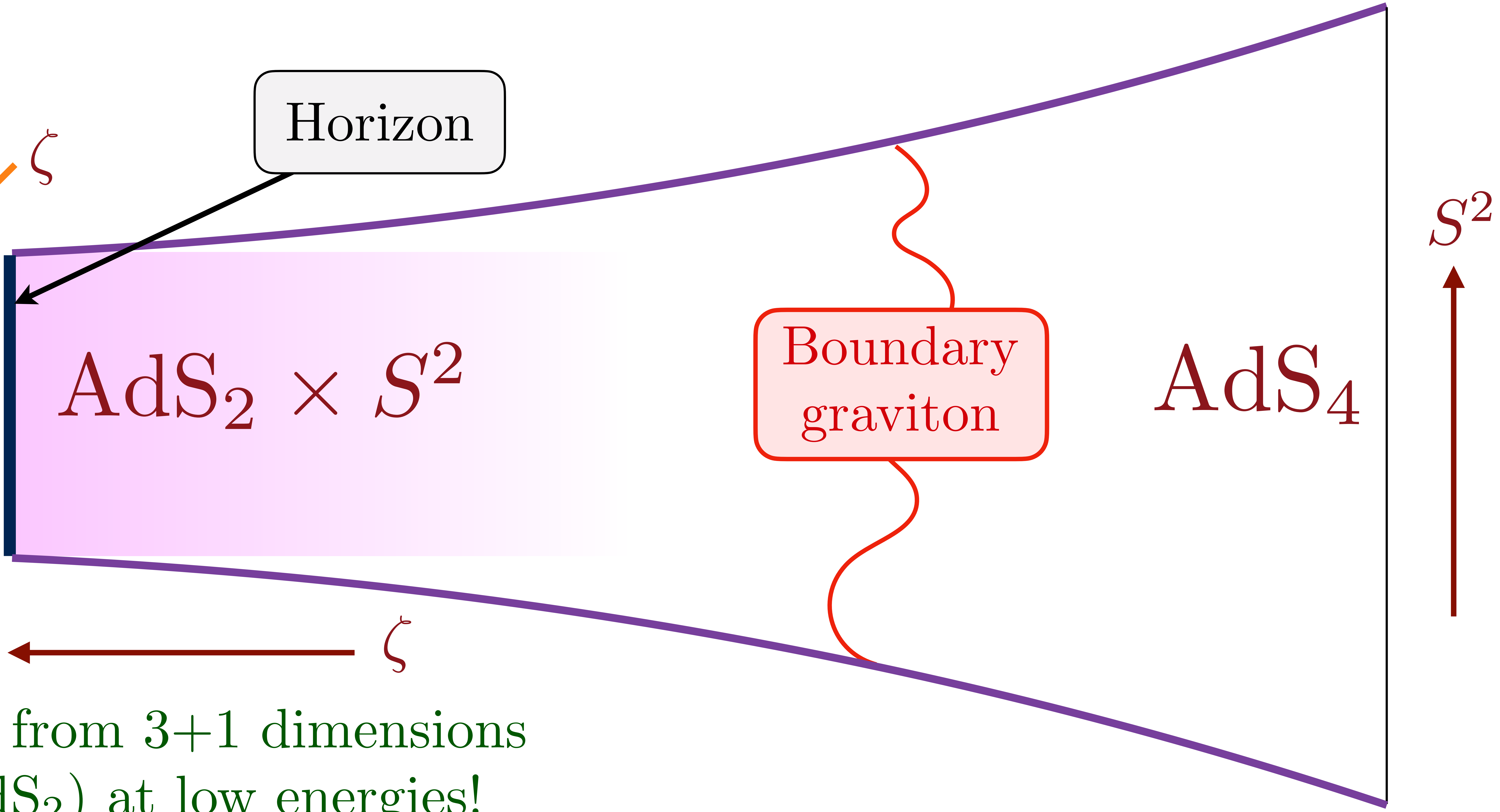


Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!

Faulkner, Liu, McGreevy and Vegh *Phys. Rev. D* **83**, 125002 (2011)
Cubrovic, Zaanen and Schalm *Science* **325**, 439 (2009)
Sachdev *Phys. Rev. Lett.* **105**, 151602 (2010)

Reissner-Nordstrom black hole of Einstein-Maxwell theory





Dimensional reduction from 3+1 dimensions to 1+1 dimensions (AdS_2) at low energies!
The isometry group of AdS_2 is the 0+1 dimensional conformal group SL(2,R).

Faulkner et al. *Phys. Rev. D* **83**, 125002 (2011); Cubrovic et al. *Science* **325**, 439 (2009); Sachdev *Phys. Rev. Lett.* **105**, 151602 (2010)

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Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}+\ldots\right)$$

$A_0 = 2G\mathcal{Q}^2/c^4$ is the area of the charged black hole horizon at $T=0$.

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$$\approx \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\int \mathcal{D}g_{\mu\nu}\mathcal{D}A_\mu \exp\left(-\frac{1}{\hbar}\, I^{(1+1)}_{\text{JT gravity of AdS}_2\text{+boundary graviton}}[g_{\mu\nu},A_\mu]\right)$$

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Maldacena, Stanford and Yang *PTEP* **2016**, 12C104 (2016)

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$$= \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)\exp\left(-\frac{1}{\hbar}I_{\text{SYK}}[\text{time reparameterizations } f(\tau),\text{ phase rotations } \phi(\tau)]\right)$$

Saddle-point:

$$S_{BH}(T\to 0,\mathcal{Q}) = \frac{A(T)c^3}{4G\hbar} = \frac{A_0 c^3}{4G\hbar}\left(1+\frac{2(\pi A_0)^{1/2}T}{\hbar c}+\ldots\right)$$

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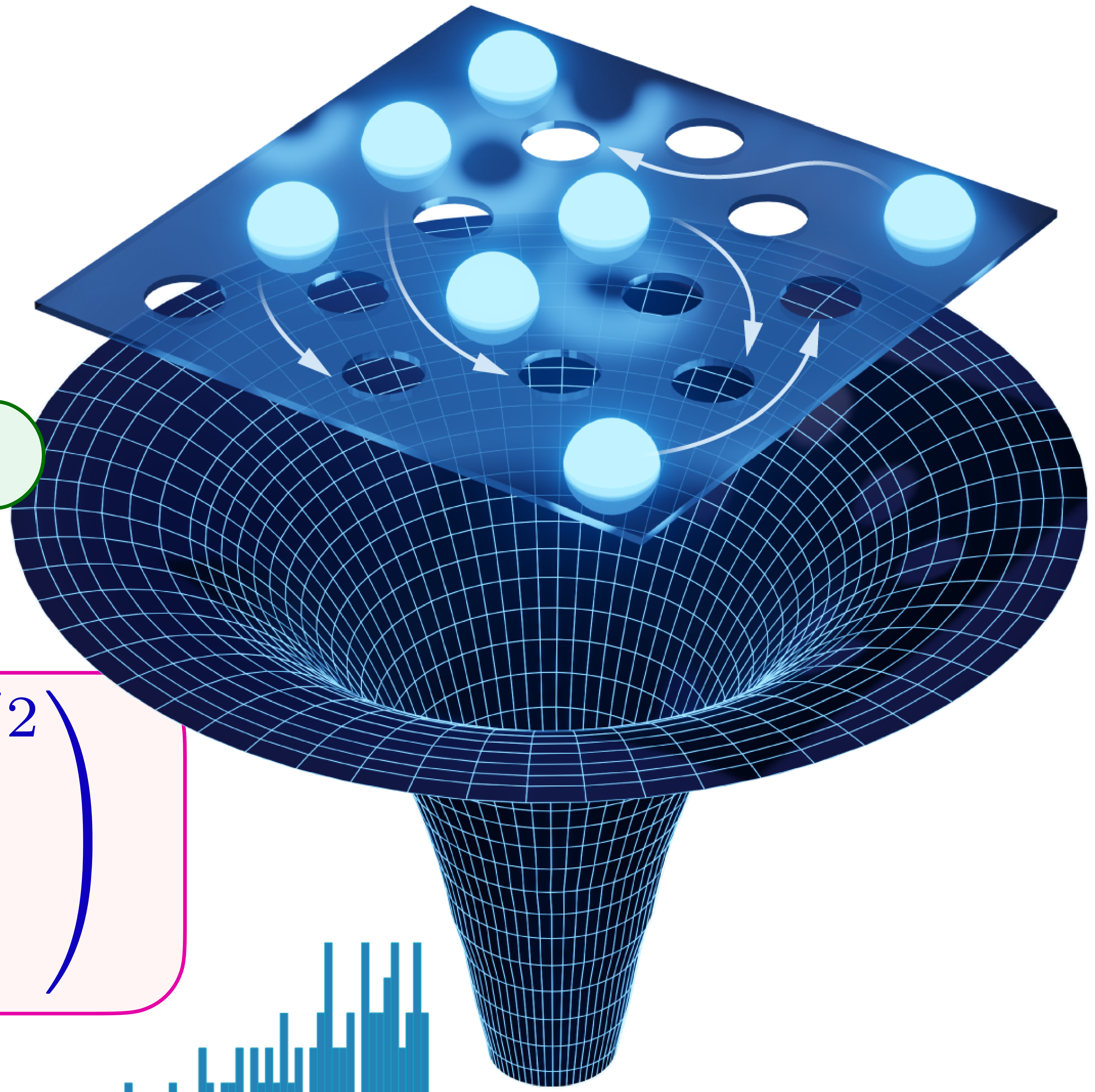
Maldacena, Stanford and Yang *PTEP* **2016**, 12C104 (2016)

Quantum simulation of charged black holes by the SYK model



- For generic charged black holes in 3+1 dimensions, the SYK model yields, in terms of $A_0 = 2GQ^2/c^4$ the horizon area at $T = 0$:

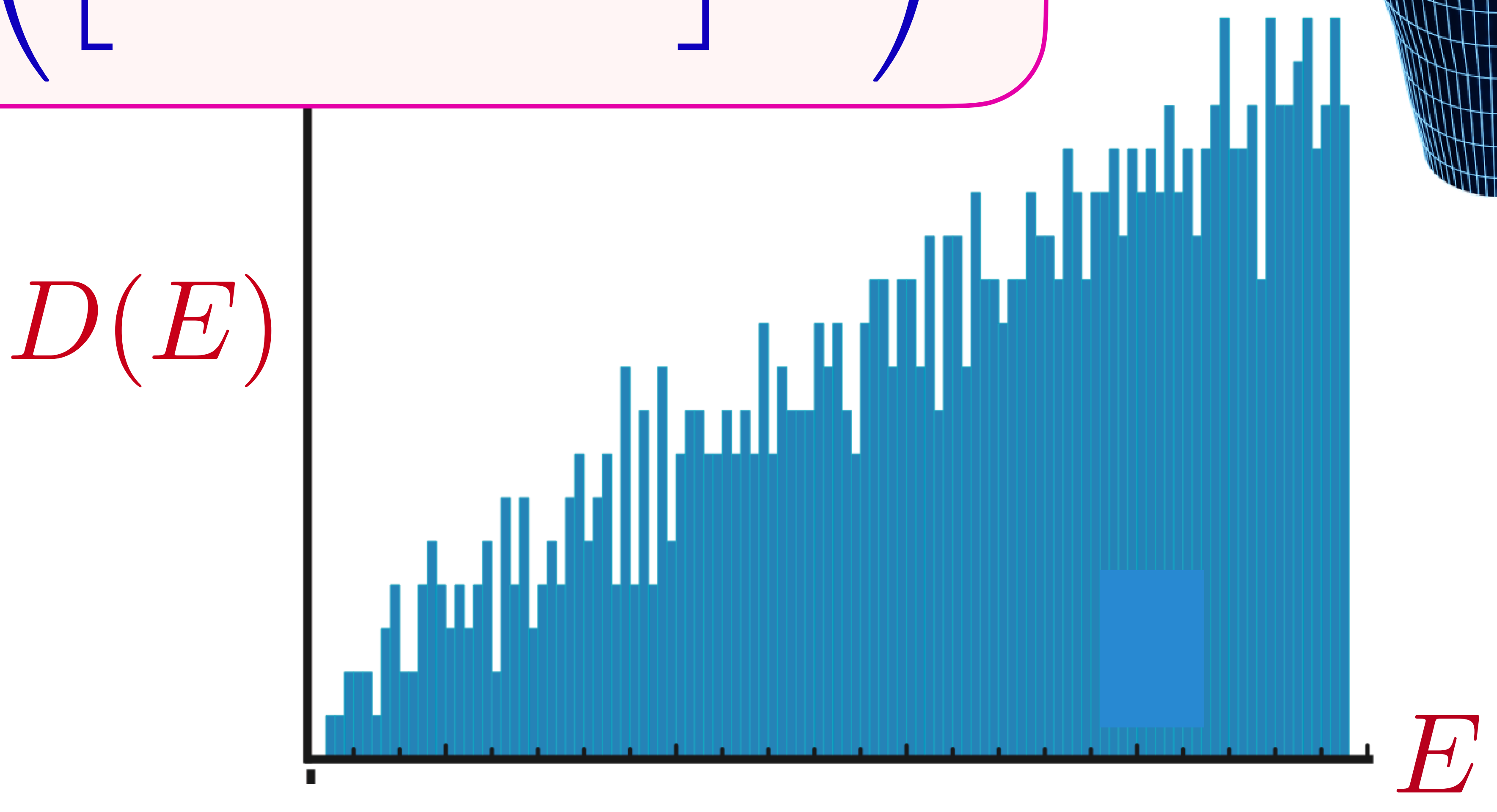
Iliesiu, Murthy, Turiaci arXiv:2209.13608 (2022)

Bekenstein-Hawking

Developments from the SYK model

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} A_0^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

There is no degeneracy, but an exponentially small level spacing down to the ground state.



Chowdhury, Georges, Parcollet and Sachdev *Rev. Mod. Phys.* **94**, 035004 (2022)

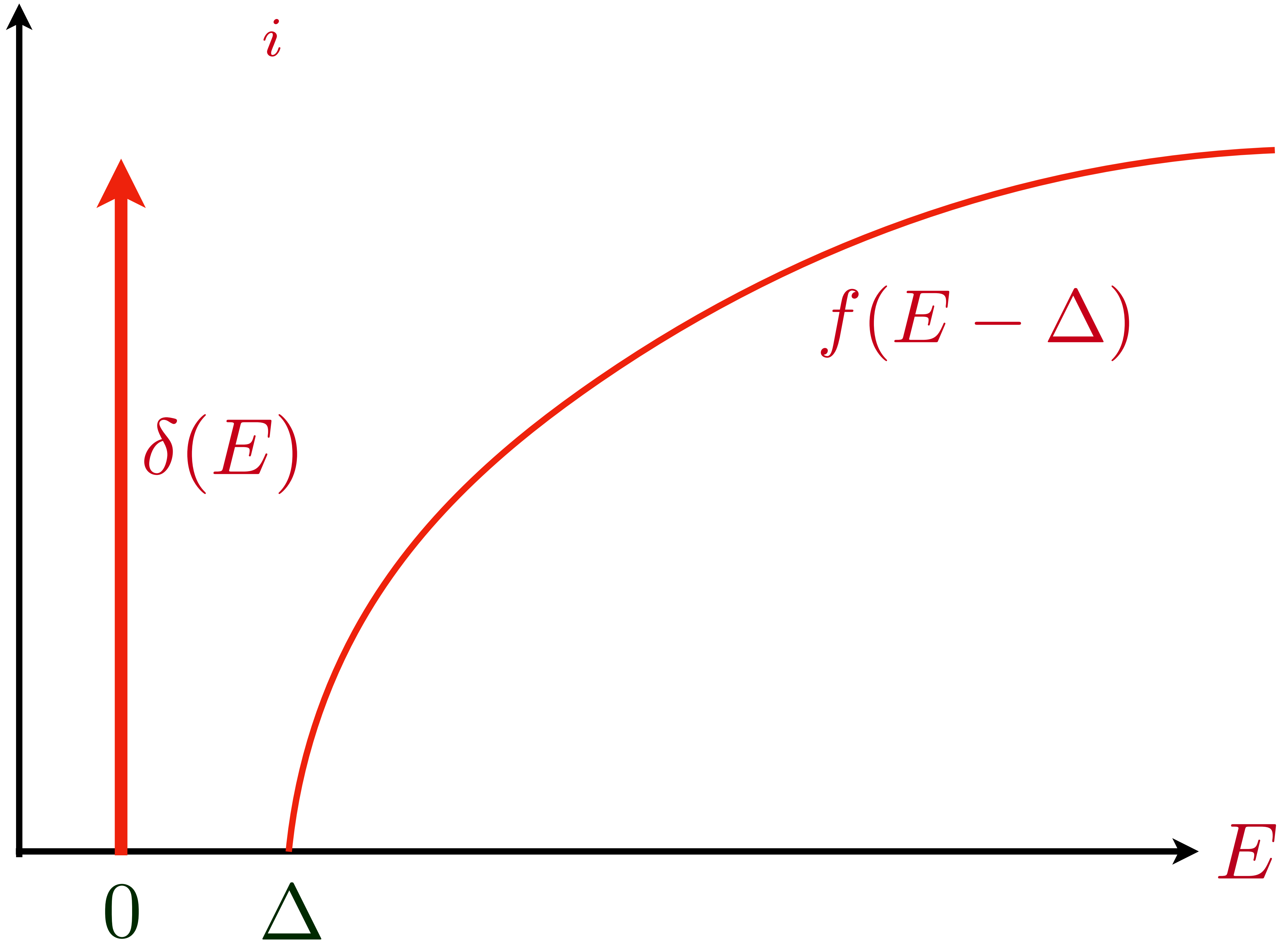
String theory of charged black holes

- With sufficient low energy supersymmetry, string theory yields:

$$D(E) = \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\delta(E) + \theta(E-\Delta) f(E-\Delta) + \ldots$$

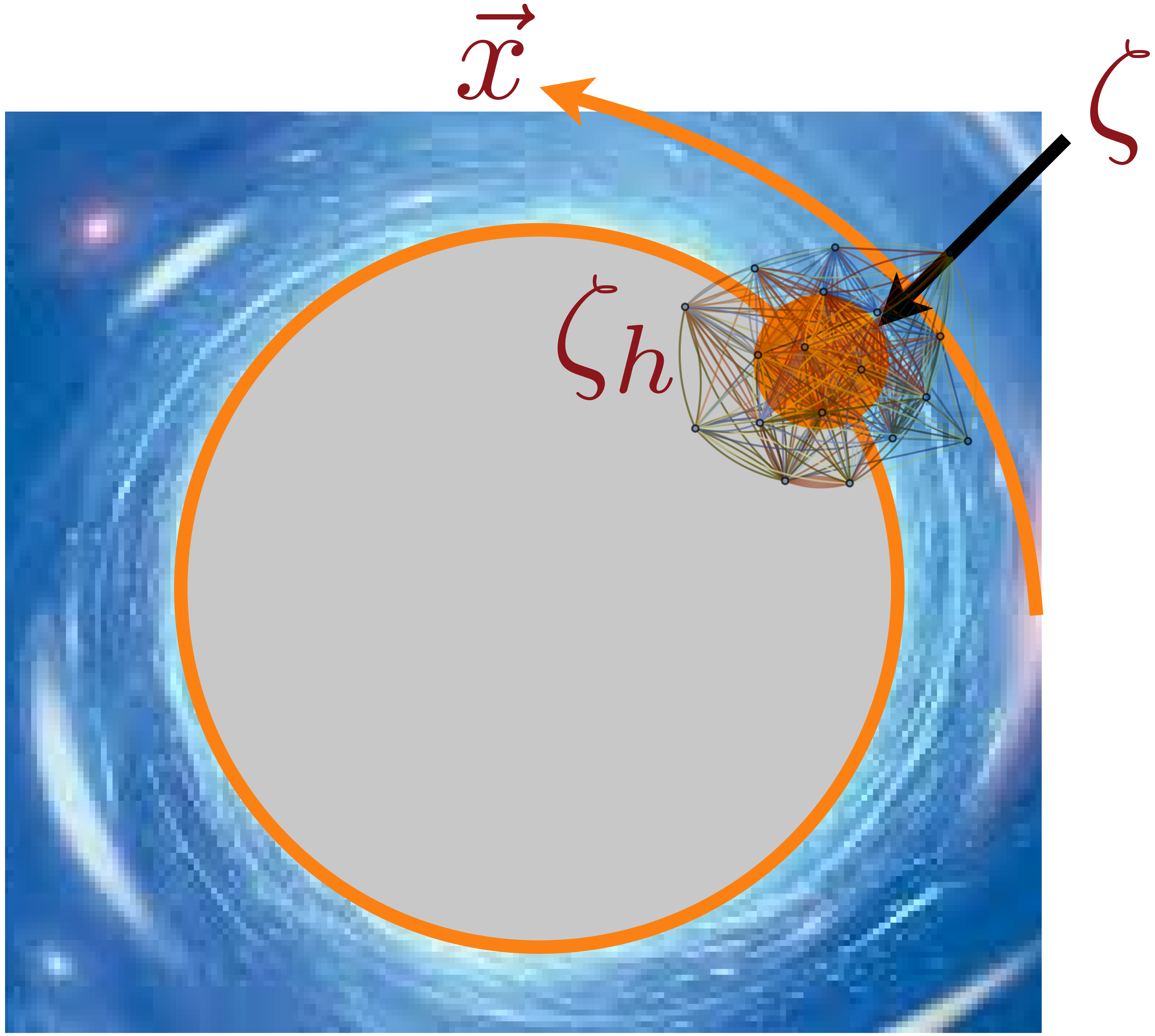
There are exponentially many degenerate BPS ground states, and an energy gap Δ above the ground state.

$$D(E) = \sum_i \delta(E - E_i)$$



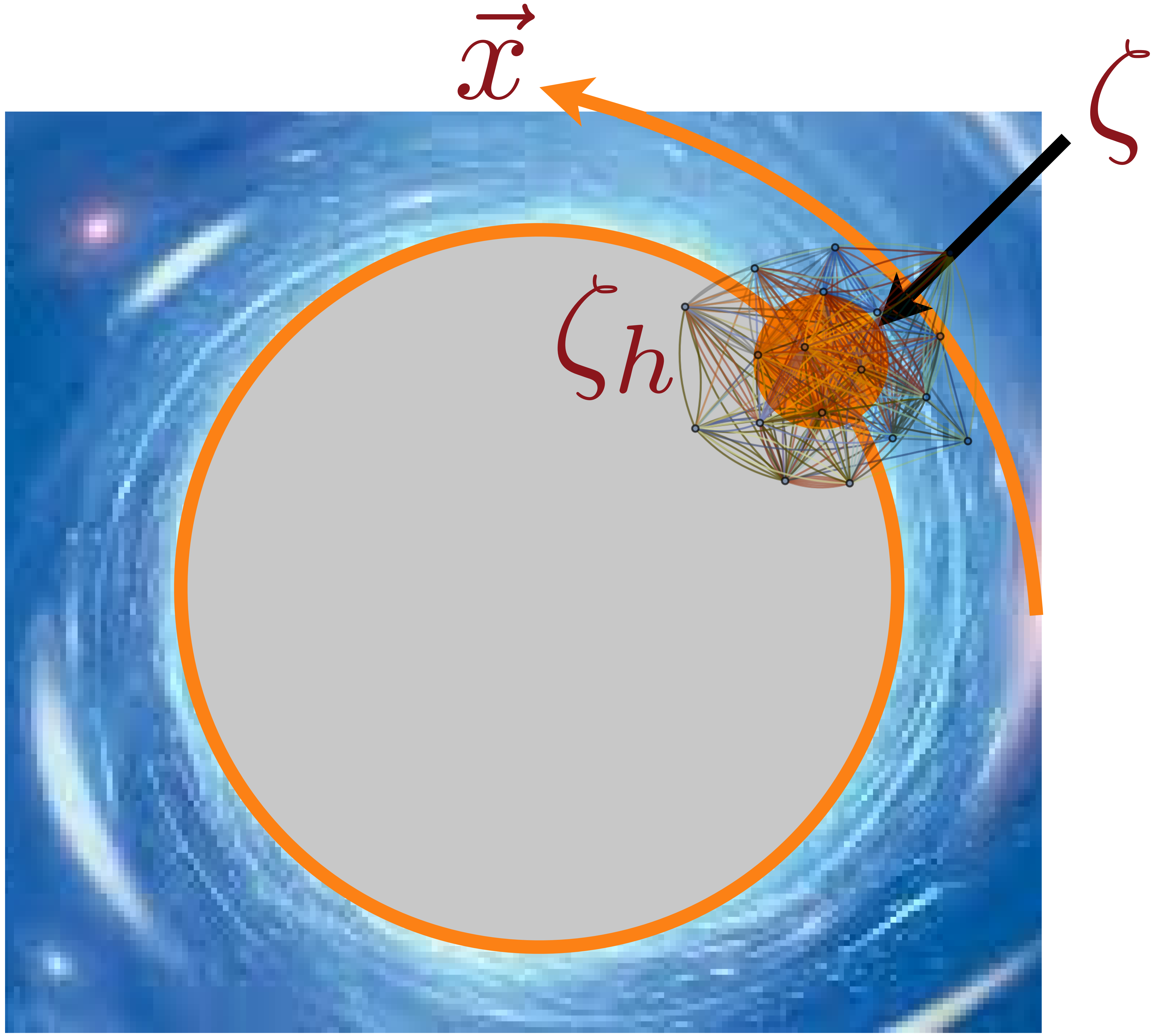
Heydeman, Iliesiu, Turiaci and Zhao *J. Phys. A: Math. Theor.* **55**, 014004 (2021)
Iliesiu, Murthy and Turiaci arXiv:2209.13608 (2022)

Quantum simulation of charged black holes by the SYK model



The SYK model provides a realization of the interior of a charged black hole !

Quantum simulation of charged black holes by the SYK model



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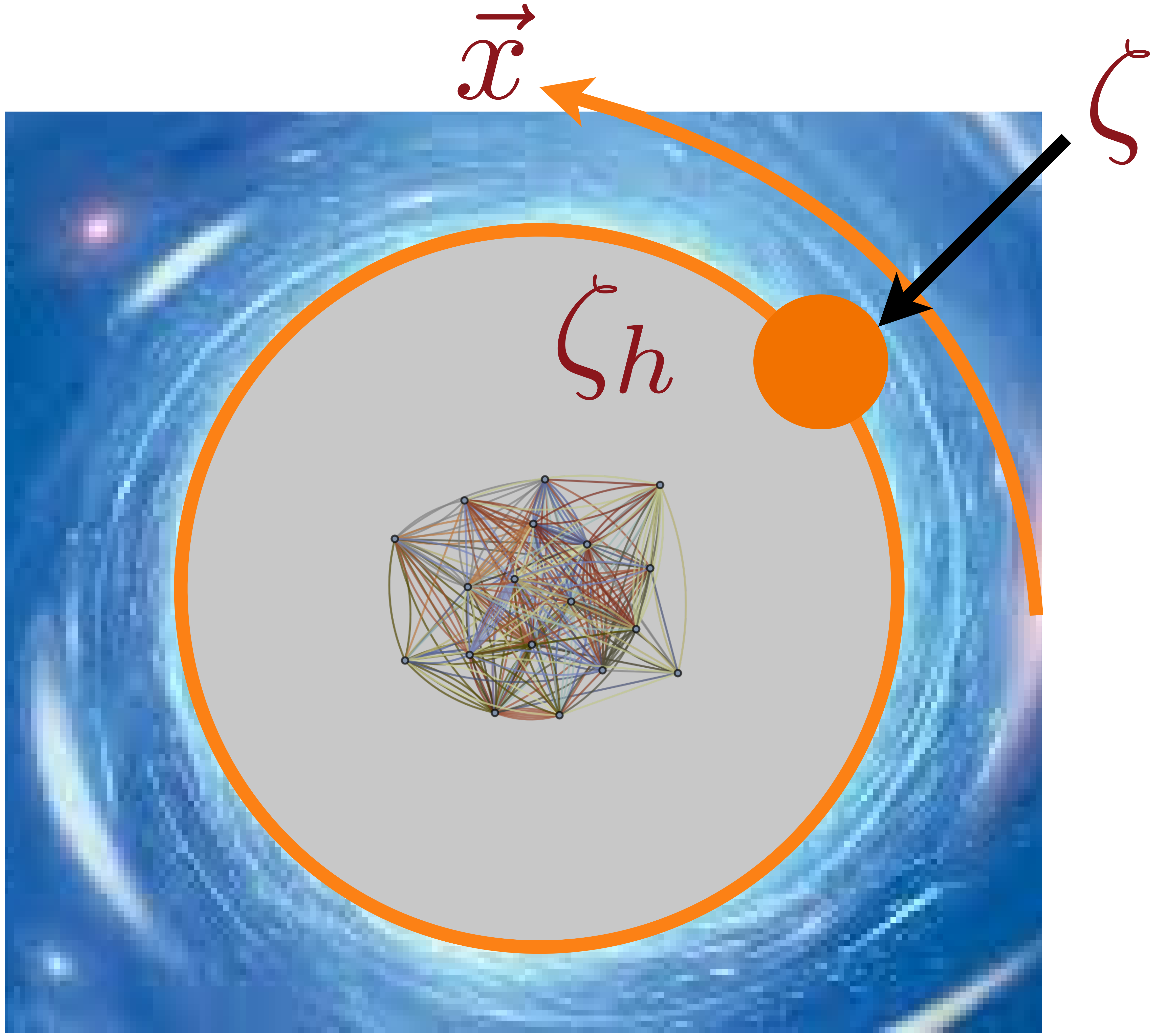


The SYK model provides a realization of the interior of a charged black hole !

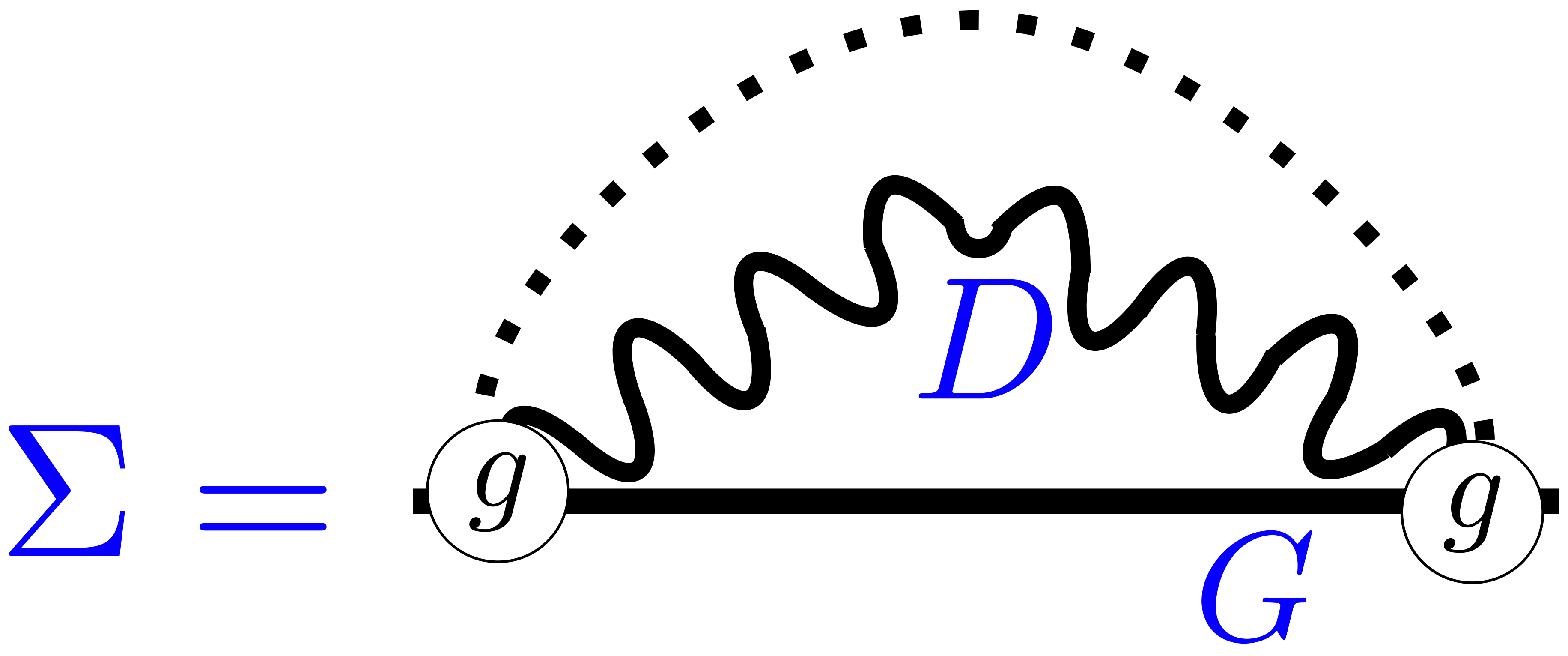
From the SYK model to a universal theory of strange metals

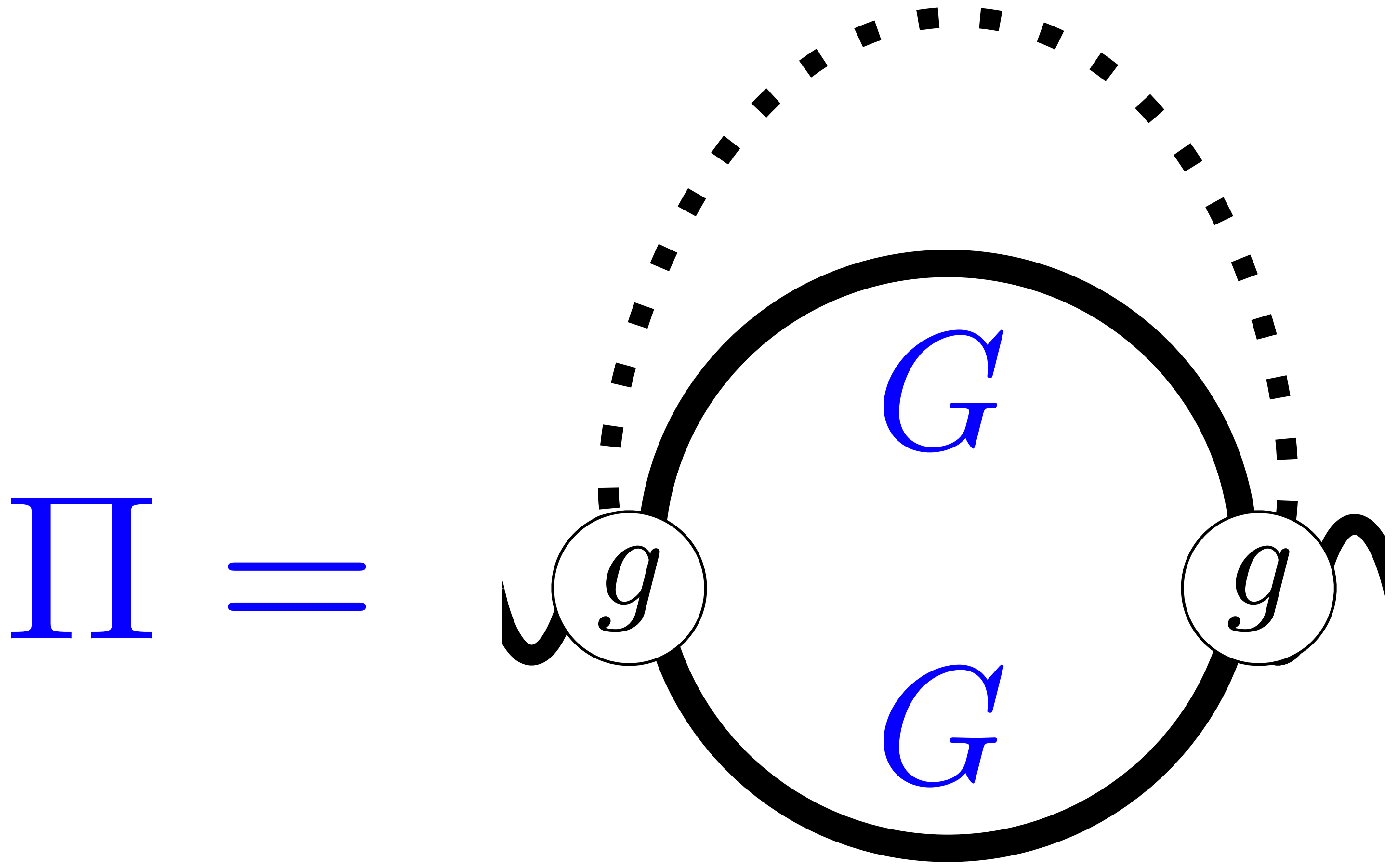
Patel, Guo, Esterlis and Sachdev *Science* **381**, 790 (2023)

Yukawa-SYK model

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2\right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.





Fu, Gaiotto, Maldacena and Sachdev *Phys. Rev. D* **95**, 026009 (2017); Murugan, Stanford and Witten *J. High Energ. Phys.* **2017** 146 (2017); Patel and Sachdev *Phys. Rev. B* **98**, 125134 (2018); Marcus and Vandoren *J. High Energ. Phys.* **2019**, 166 (2019); Wang *Phys. Rev, Lett.* **124**, 017002 (2020); Esterlis and Schmalian *Phys. Rev. B* **100**, 115132 (2019); Wang and Chubukov *Phys. Rev. Research* **2**, 033084 (2020)

Yukawa-SYK model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N}\sum_{ij\ell} g_{ij\ell}\psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$\boxed{\begin{aligned} G(i\omega_n) &= \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)} \\ \Sigma(\tau) &= g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau) \end{aligned}}$$

Make the low frequency ansatz

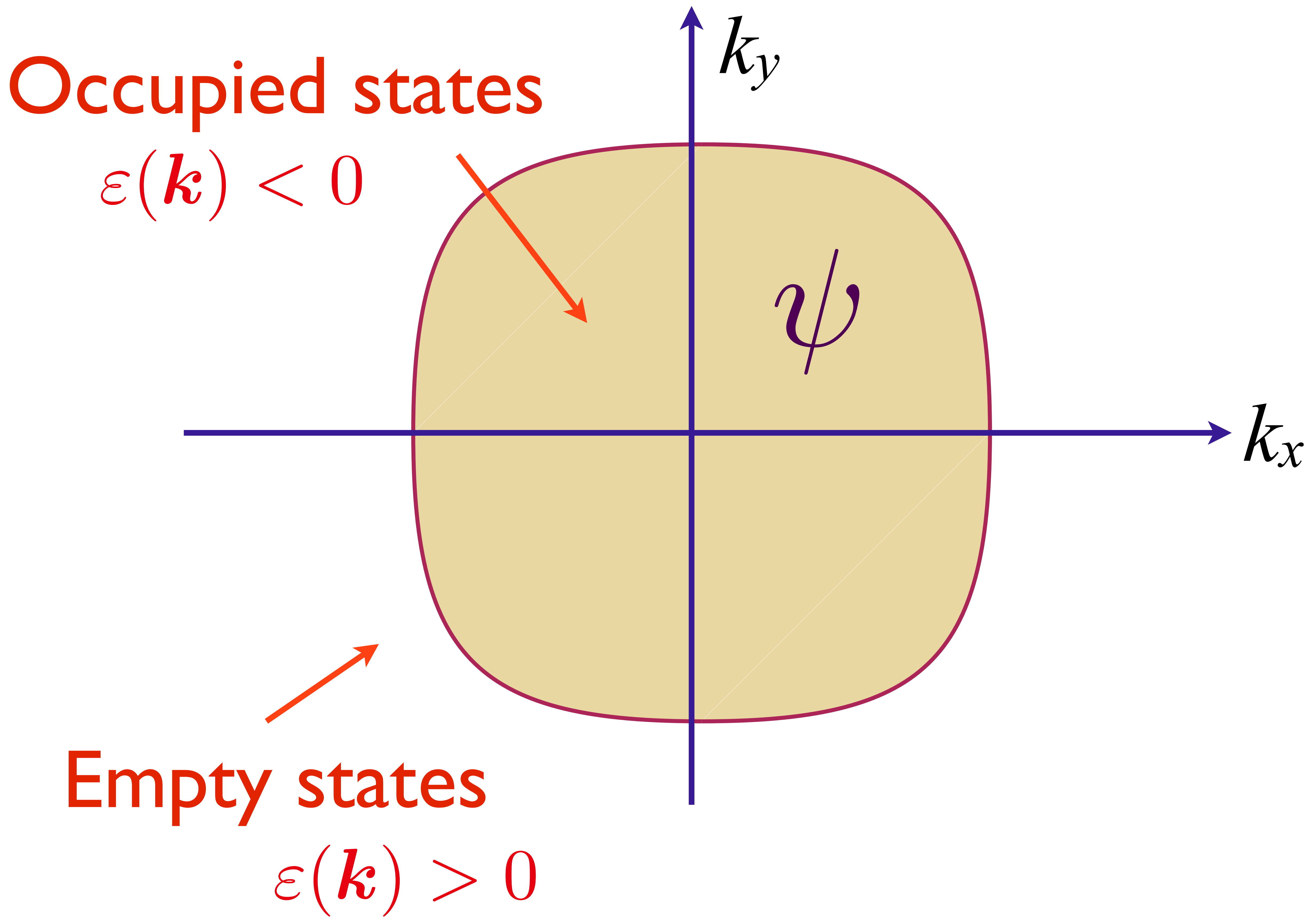
$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

Esterlis and Schmalian
Phys. Rev. B **100**, 115132 (2019)
See also Wang
Phys. Rev, Lett. **124**, 017002 (2020)

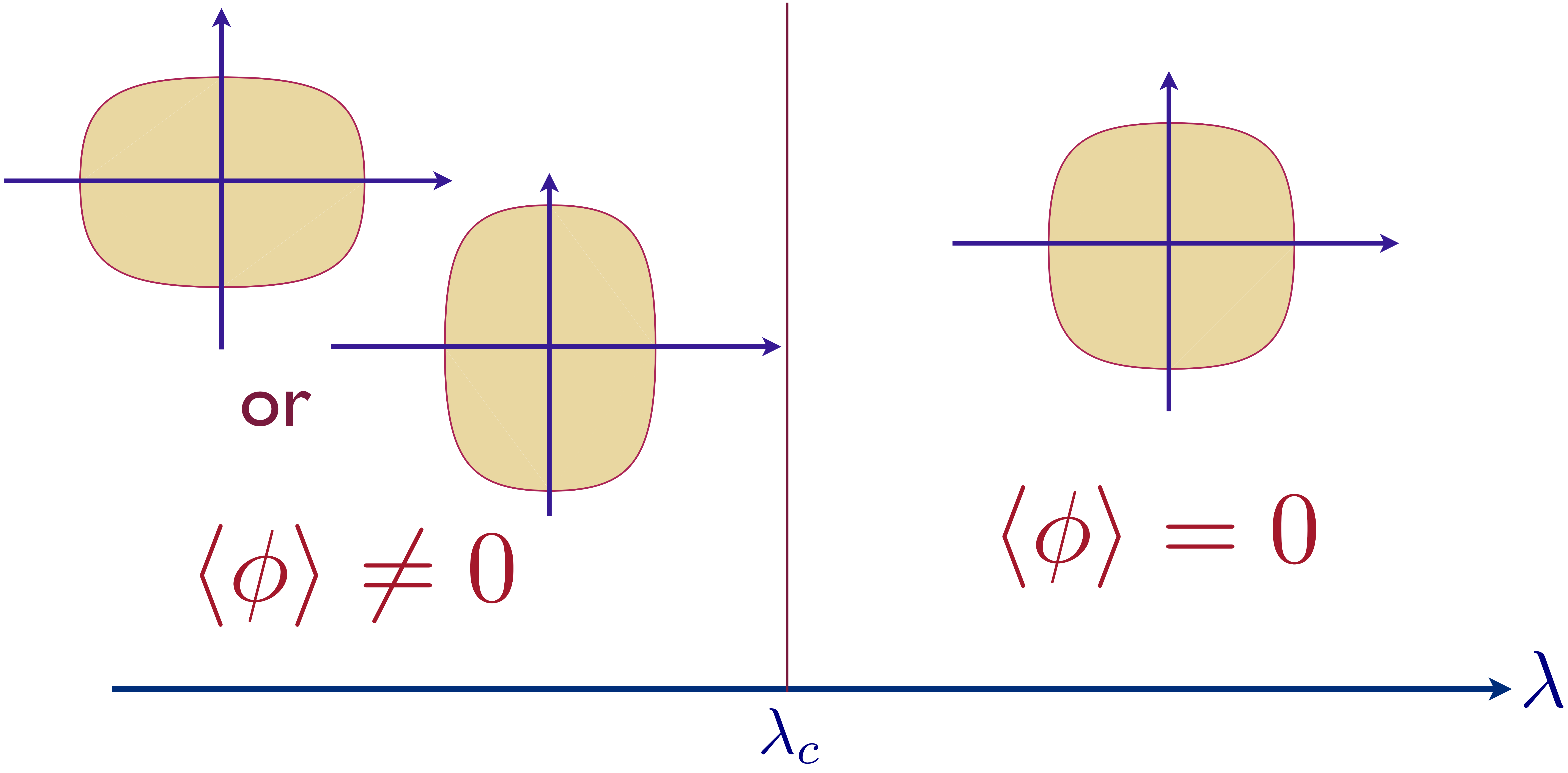
Fermi surface coupled to a critical boson



$+$

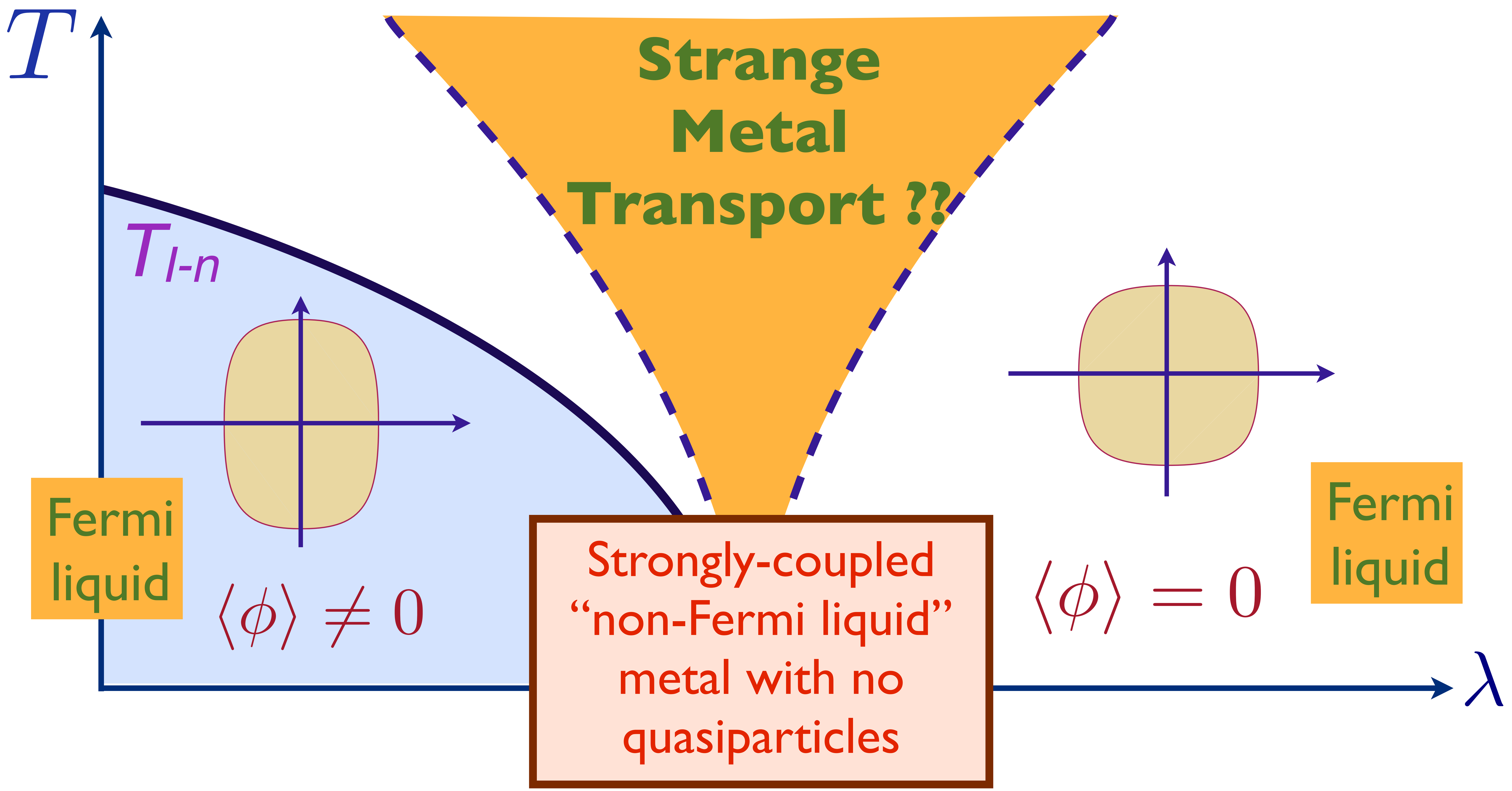
a
critical
boson
ϕ

Quantum criticality of Ising-nematic ordering in a metal

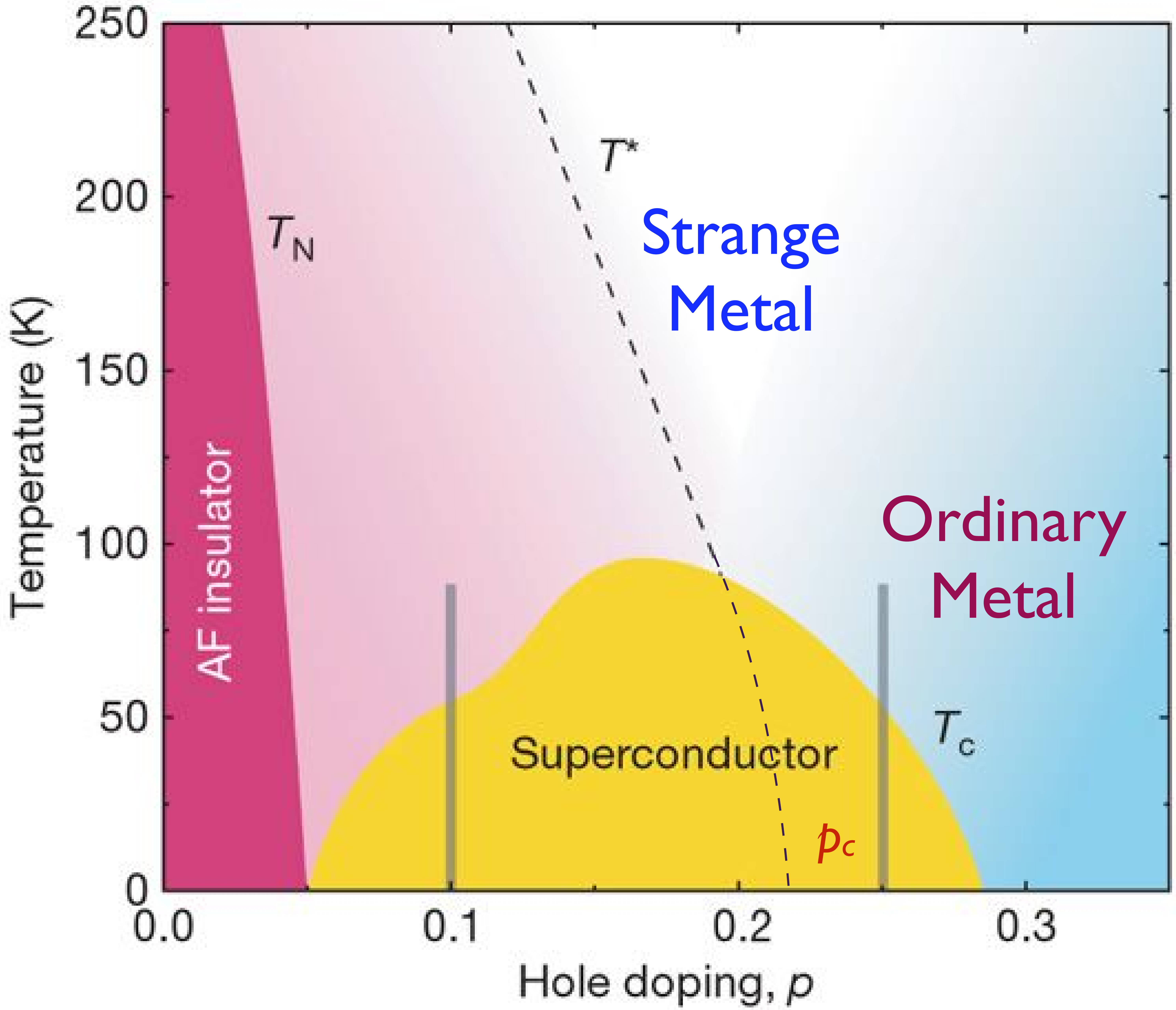


Pomeranchuk instability as a function of coupling λ

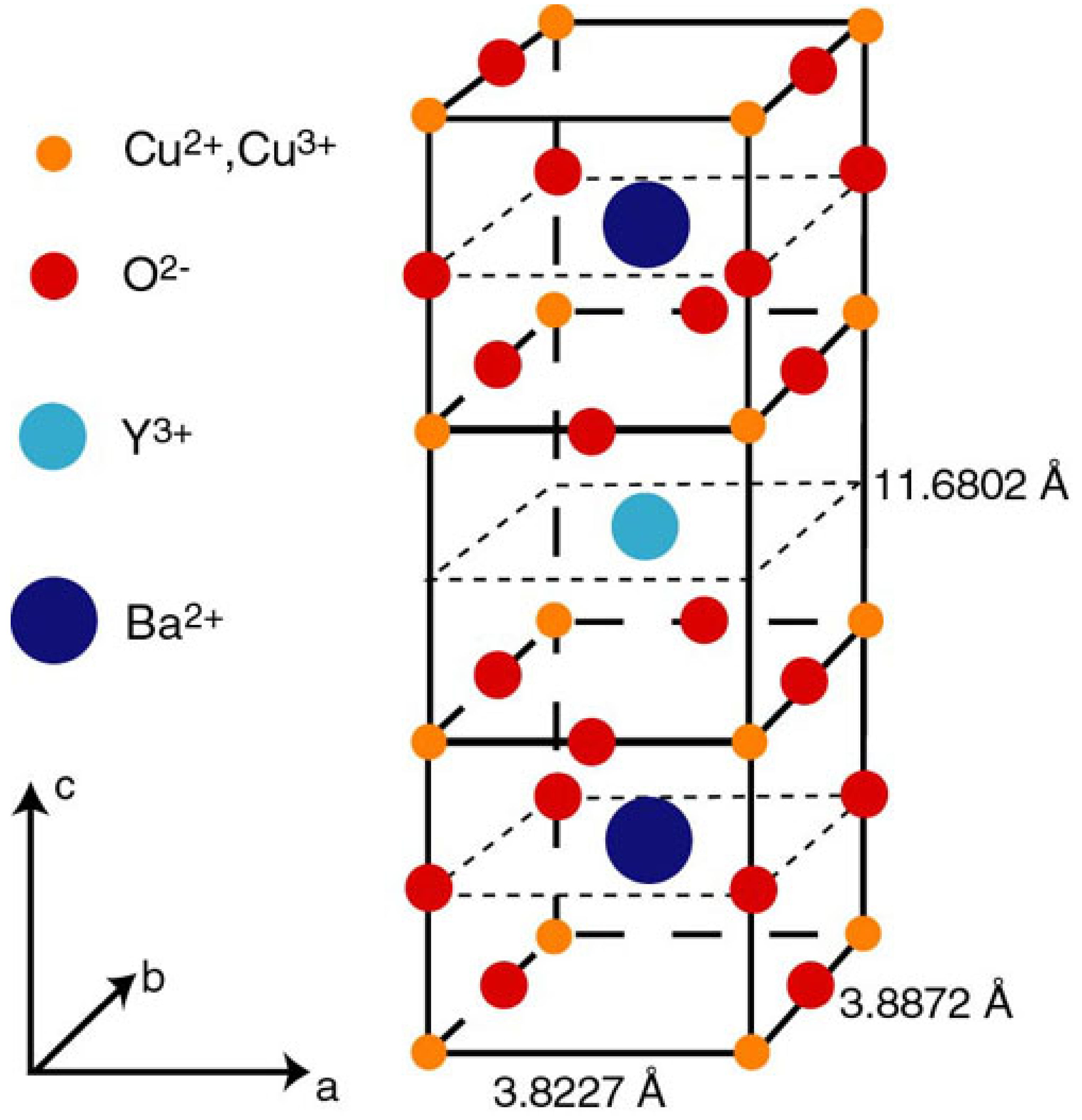
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

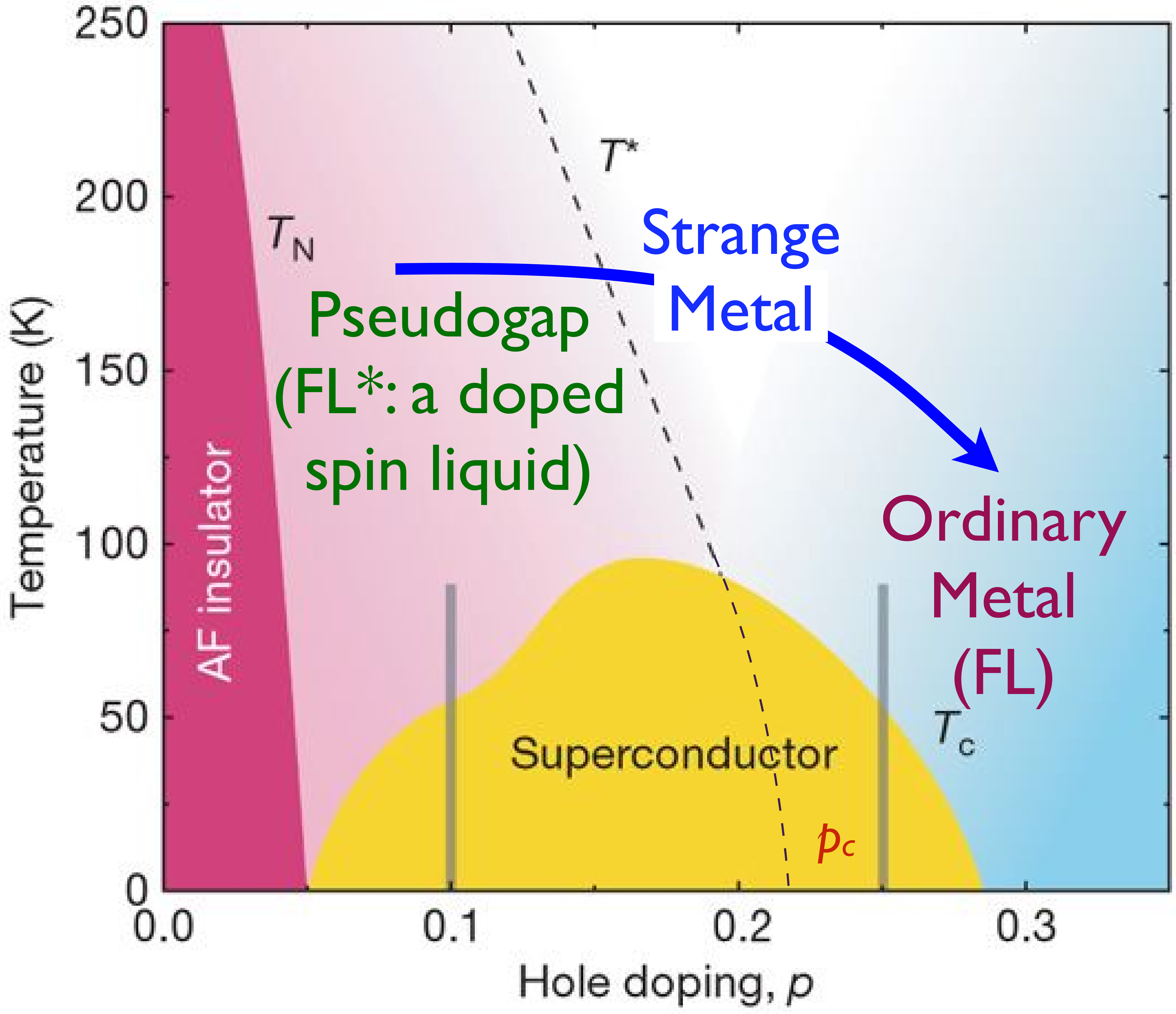


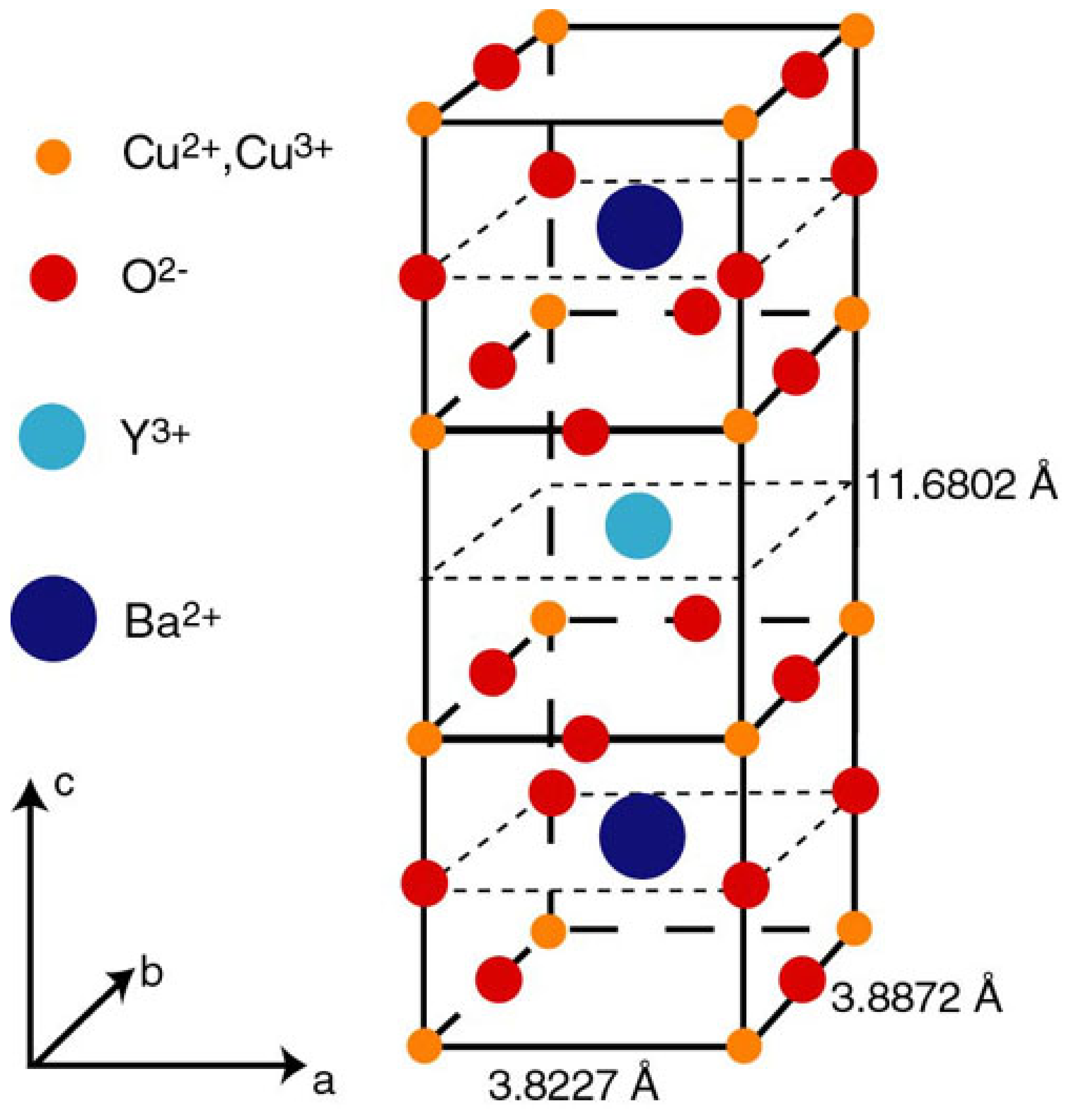
250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T*
Strange Metal
Ordinary Metal
Superconductor
T_c
p_c



Cu2+,Cu3+
O2-
Y3+
Ba2+
c
b
a
11.6802 Å
3.8872 Å
3.8227 Å

$YBa_2Cu_3O_{6+x}$


250
200
150
100
50
0
0.0
0.1
0.2
0.3
Temperature (K)
Hole doping, p
AF insulator
T_N
T^*
Strange
Metal
Pseudogap
(FL*: a doped
spin liquid)
Ordinary
Metal
(FL)
T_c
Superconductor
p_c


Cu^{2+},Cu^{3+}
O^{2-}
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Ba^{2+}
c
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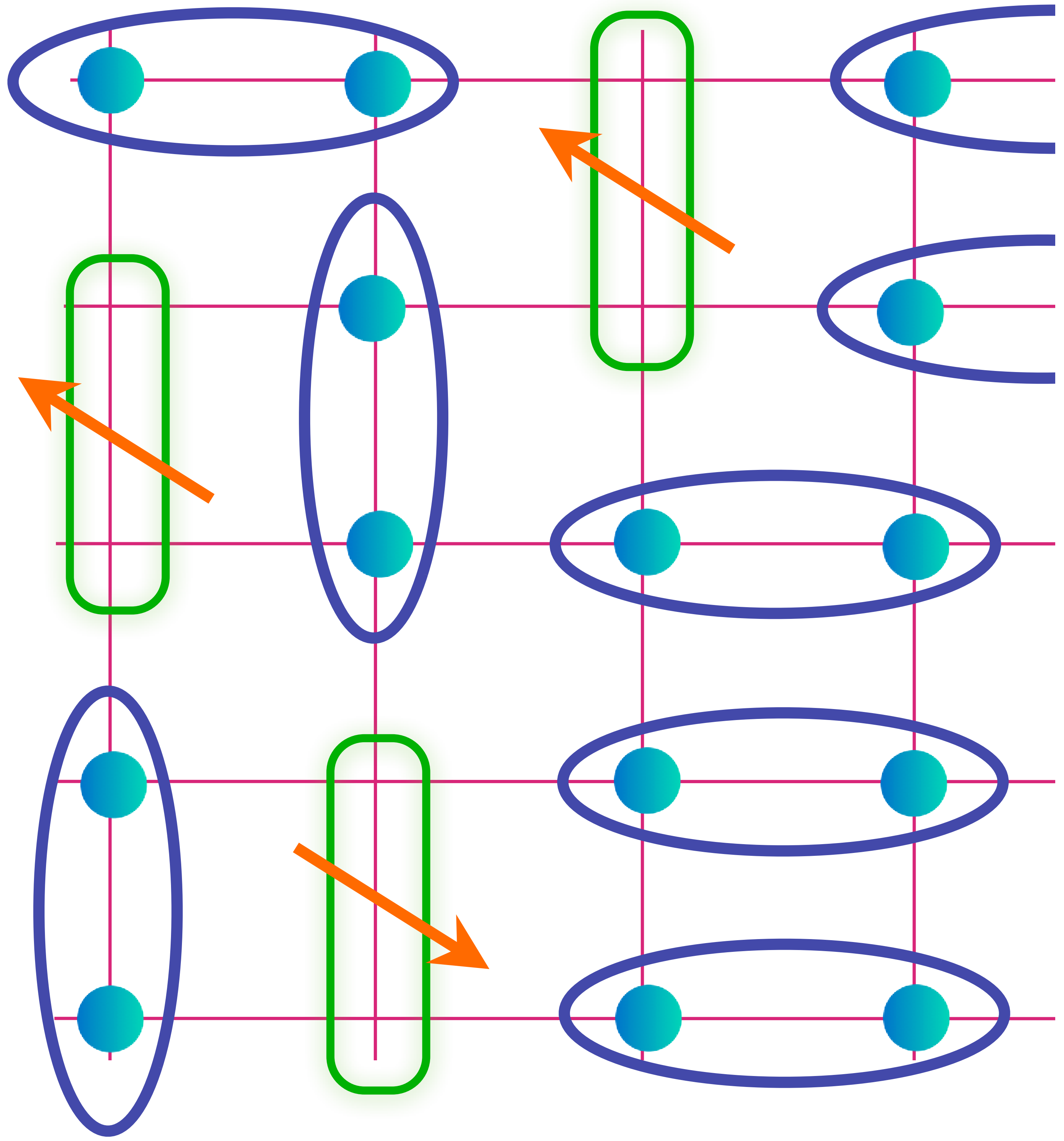
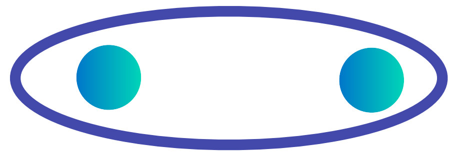
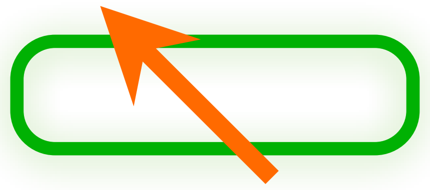


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

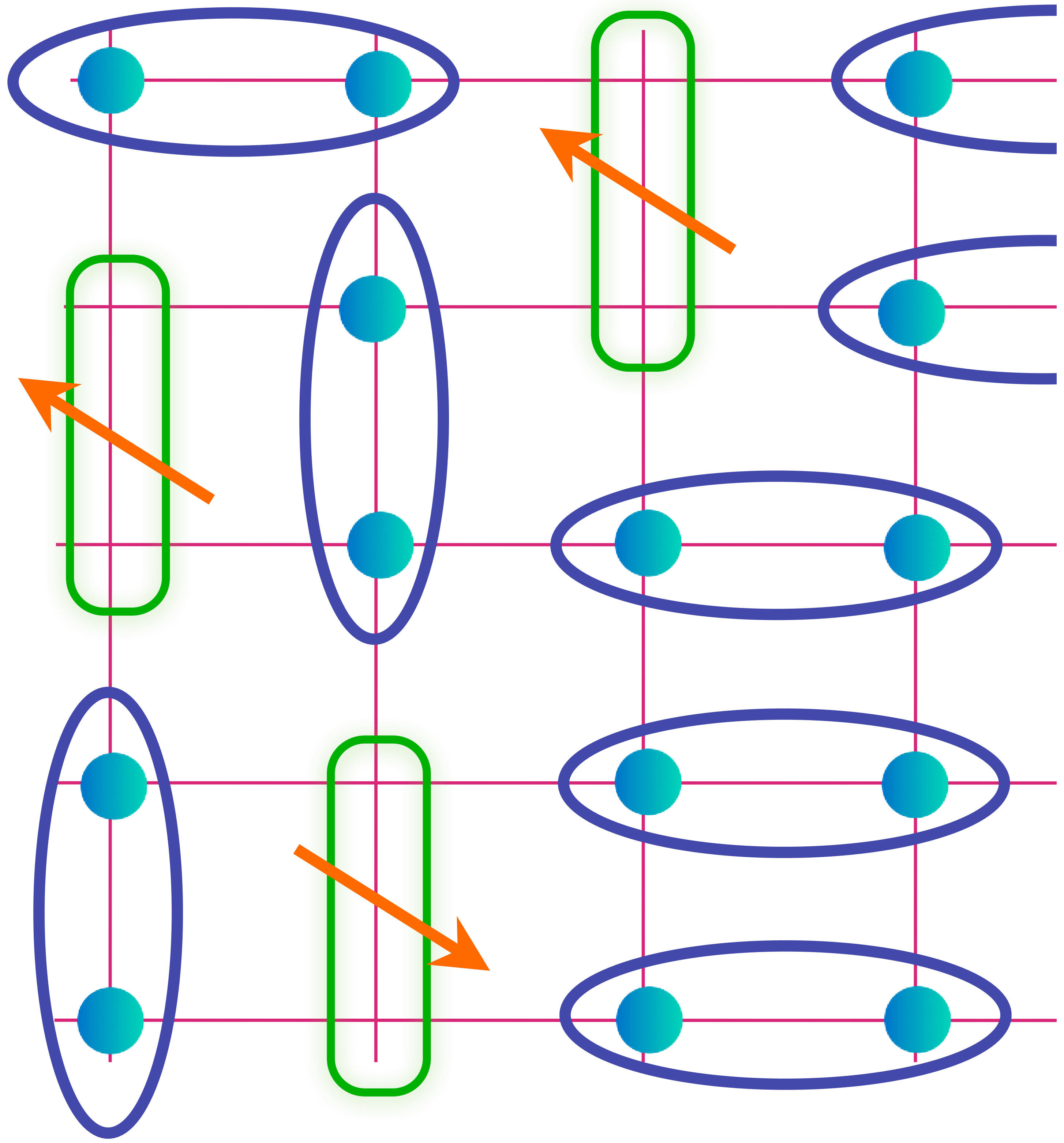
Small Fermi surface
of size p
$+$
spin liquid.

FL*

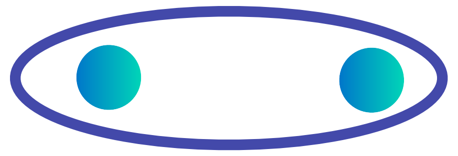
doping p

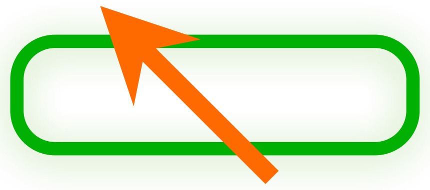
$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$
$= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$
Large Fermi surface of size $1 + p$
Small Fermi surface of size p + spin liquid.
FL*
FL
doping p

Zhang and Sachdev *Phys. Rev. Research* **2**, 023172 (2020); *Phys. Rev. B* **102**, 155124 (2020)



Φ Higgs boson with the fundamental gauge charge of an emergent SU(2) gauge field.

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

Small Fermi surface of size p + spin liquid.

$\boxed{\text{FL*}}$

$\langle \Phi \rangle \neq 0$

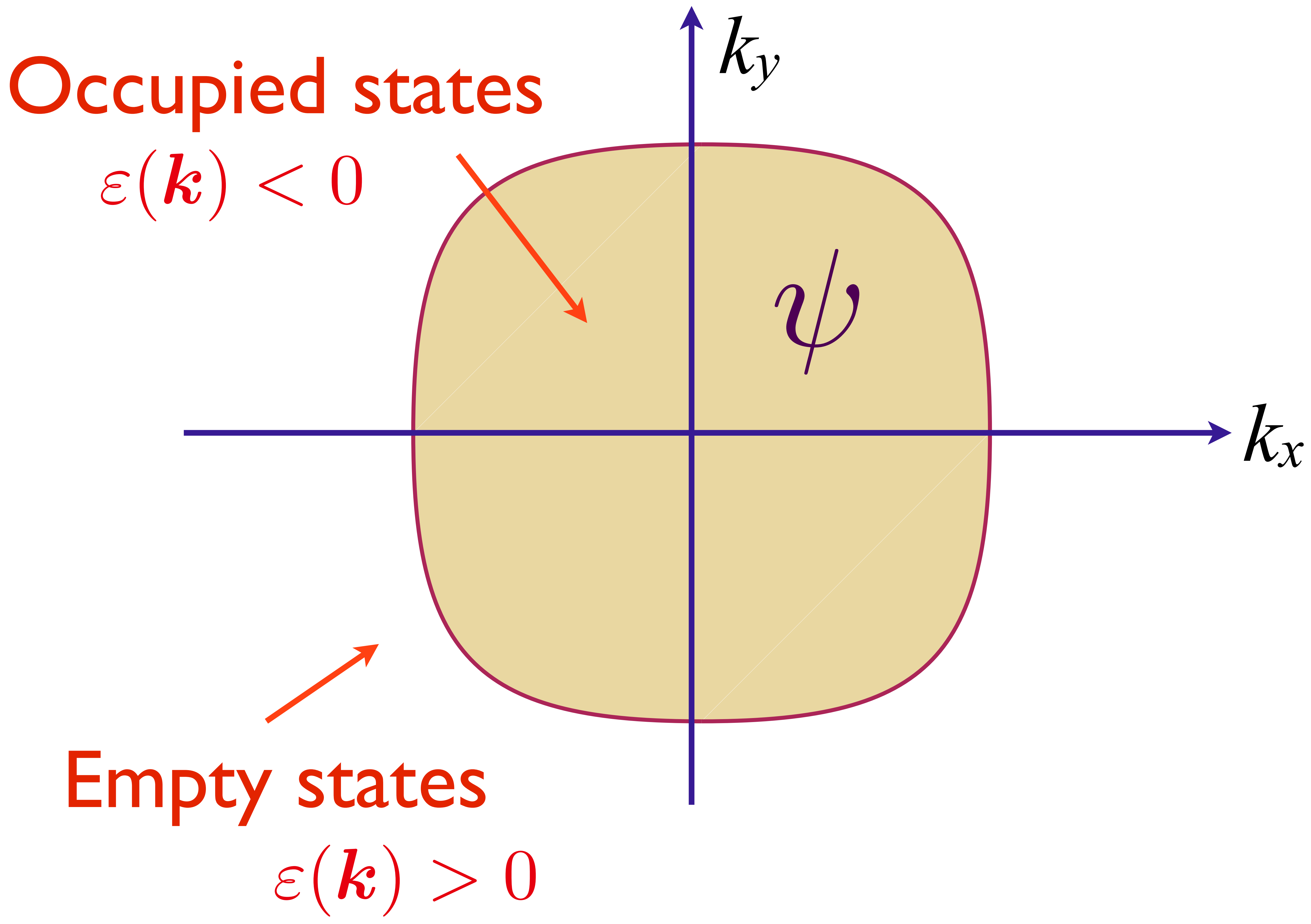
Large Fermi surface of size $1+p$

$\boxed{\text{FL}}$

$\langle \Phi \rangle = 0$

doping p

Fermi surface coupled to a critical boson



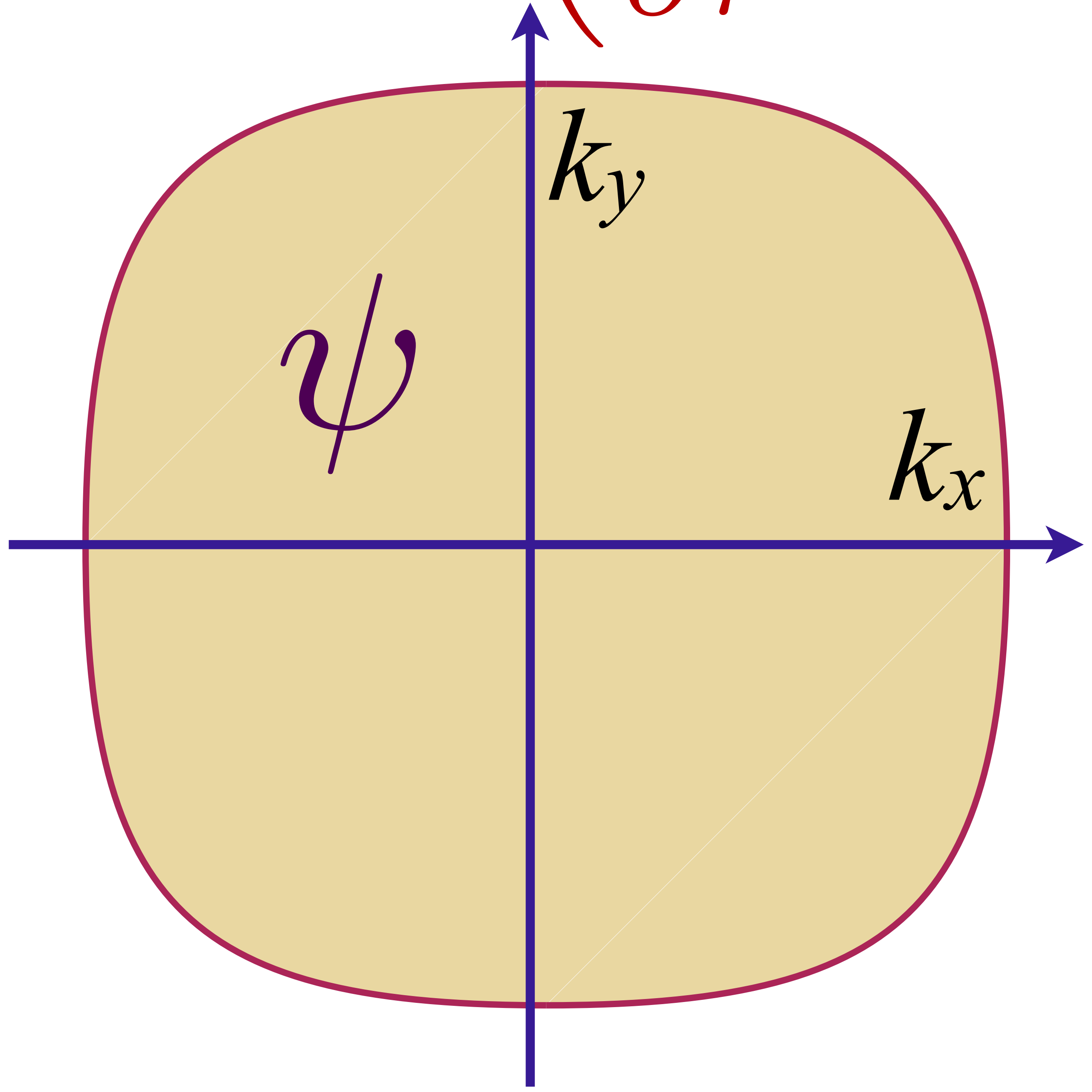
$+$

a
critical
boson
ϕ (or Φ)

Fermi surface + critical boson

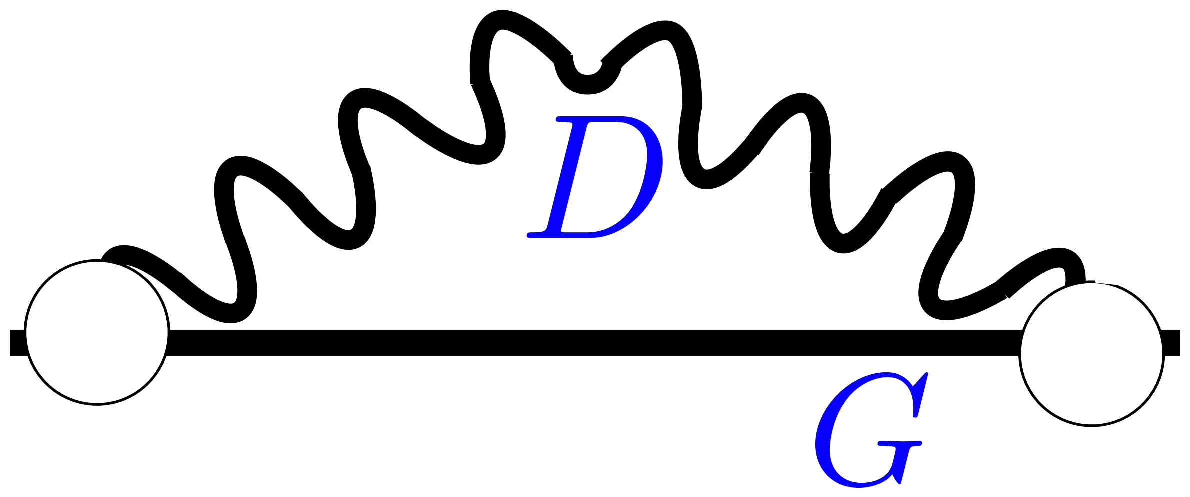
$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Critical boson
ϕ (or Φ)

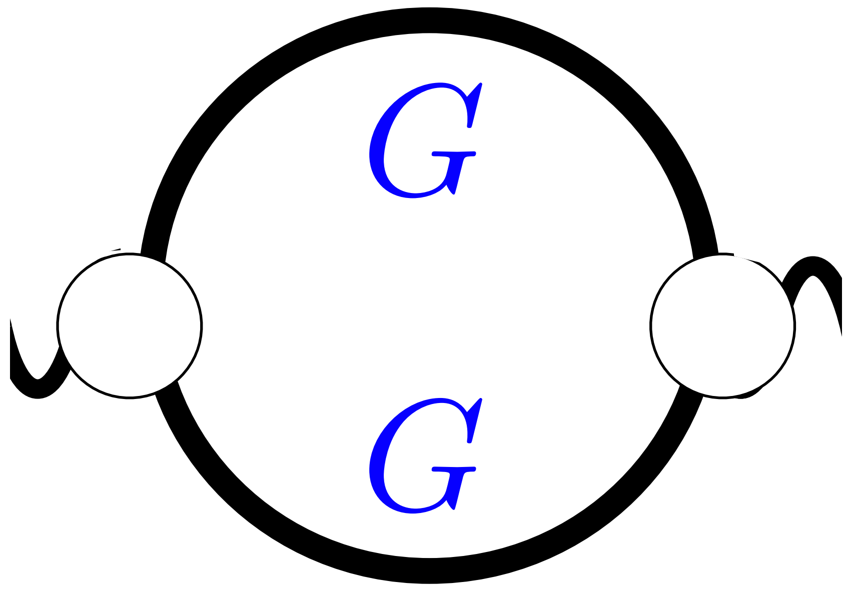


$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

$\Sigma =$



$\Pi =$



Solution of Migdal-Eliashberg equations for electron (G) and boson (D) Green's functions at small ω:

Lee *Phys. Rev. Lett.* **63**, 680 (1989)

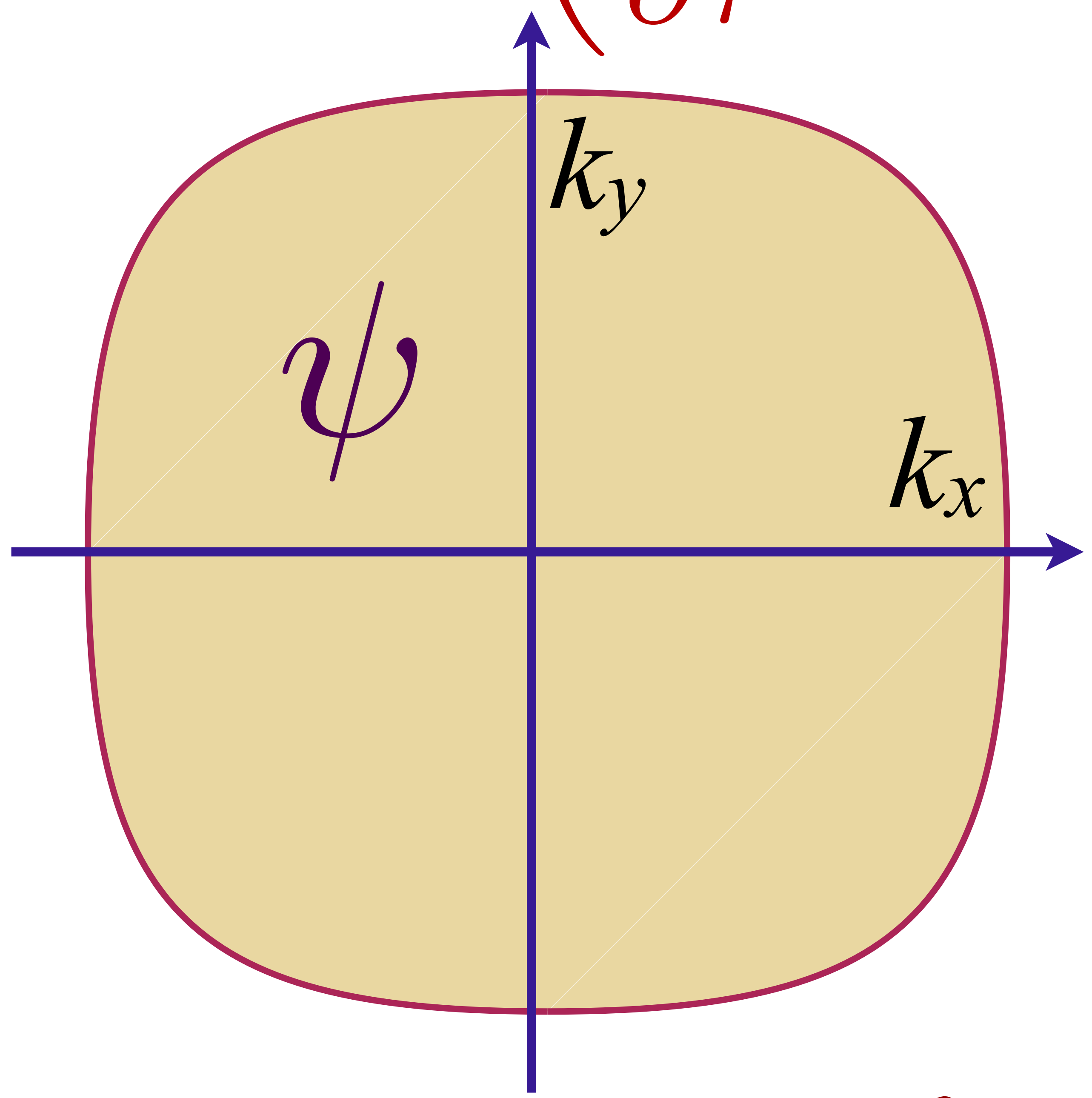
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}\,, \quad G(\boldsymbol{k}, i\omega) = \frac{1}{i\omega - \varepsilon(\boldsymbol{k}) - \Sigma(\hat{\boldsymbol{k}}, i\omega)}\,, \quad D(\boldsymbol{q}, i\Omega) = \frac{1}{\Omega^2 + q^2 + \gamma|\Omega|/q}$$

Fermi surface + critical boson

$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$

Critical boson
ϕ (or Φ)

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$



Transport—a perfect metal!

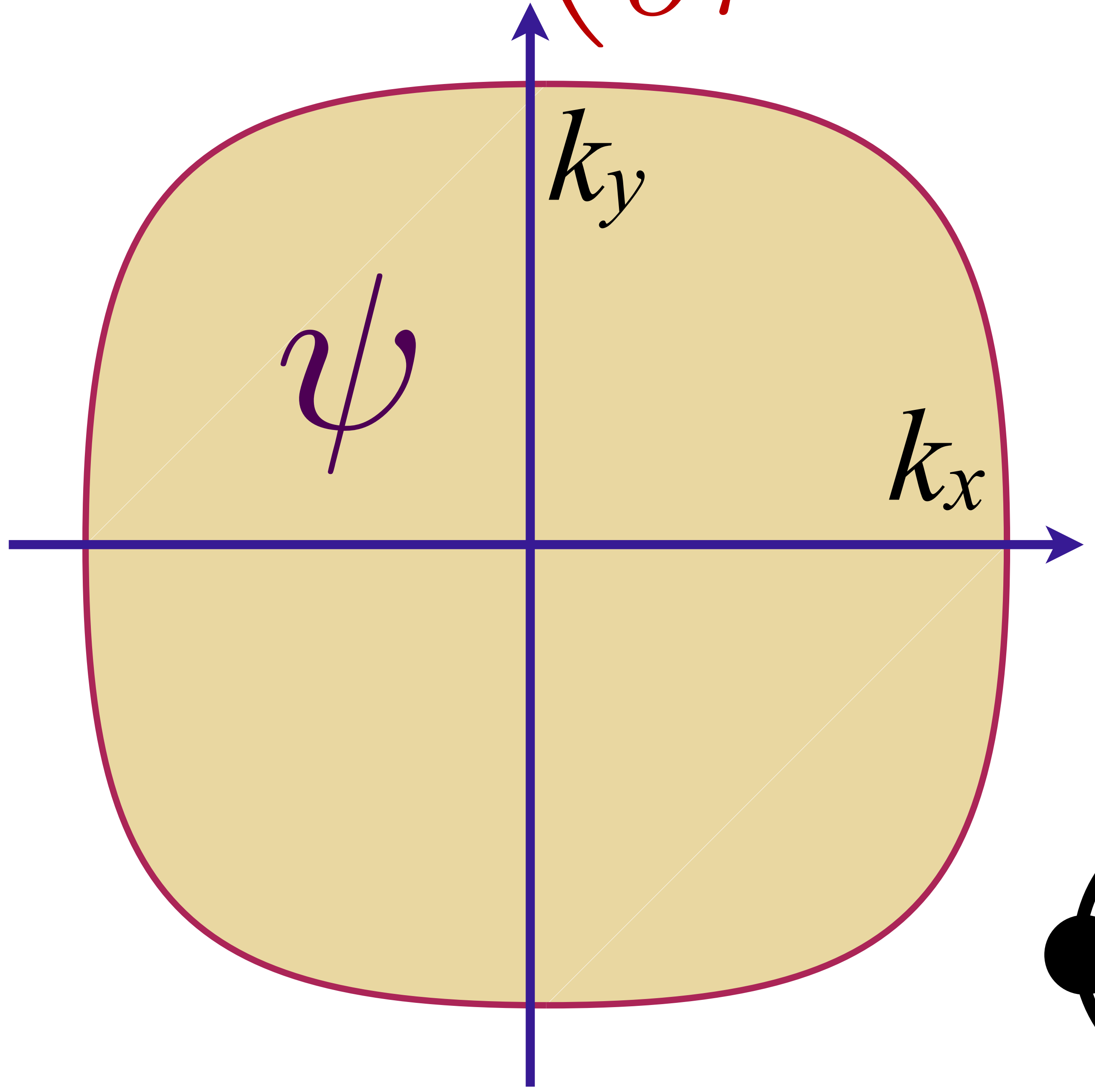
Conservation of momentum and fermion-boson drag imply:

$$\mathrm{Re}\left[\sigma(\omega)\right] = D\delta(\omega) + \ldots$$

Hartnoll, Kovtun, Muller and Sachdev *Phys. Rev. B* **76**, 144502 (2007)
Maslov, Yudson and Chubukov *Phys. Rev. Lett.* **106**, 106403 (2011)
Hartnoll, Mahajan, Punk and Sachdev *Phys. Rev. B* **89**, 155130 (2014)
Eberlein, Mandal and Sachdev *Phys. Rev. B* **94**, 045133 (2016)

Fermi surface + critical boson

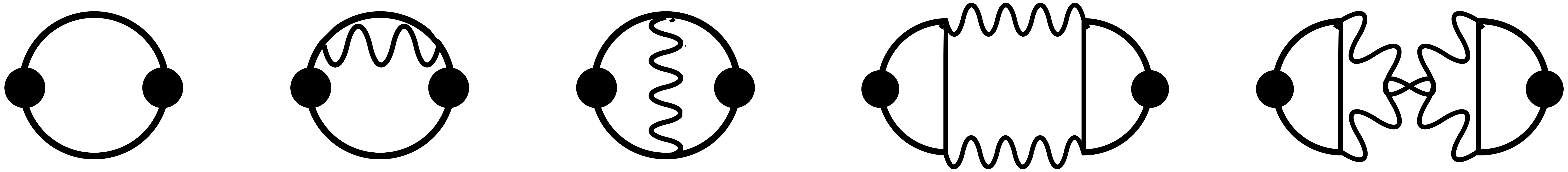
$$\mathcal{L}_\psi = \psi^\dagger_{\boldsymbol{k}} \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



Critical boson
ϕ (or Φ)

$$\frac{[\phi(\boldsymbol{r})]^2}{J} + \psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\phi(\boldsymbol{r})$$

Optical conductivity—Diagrams



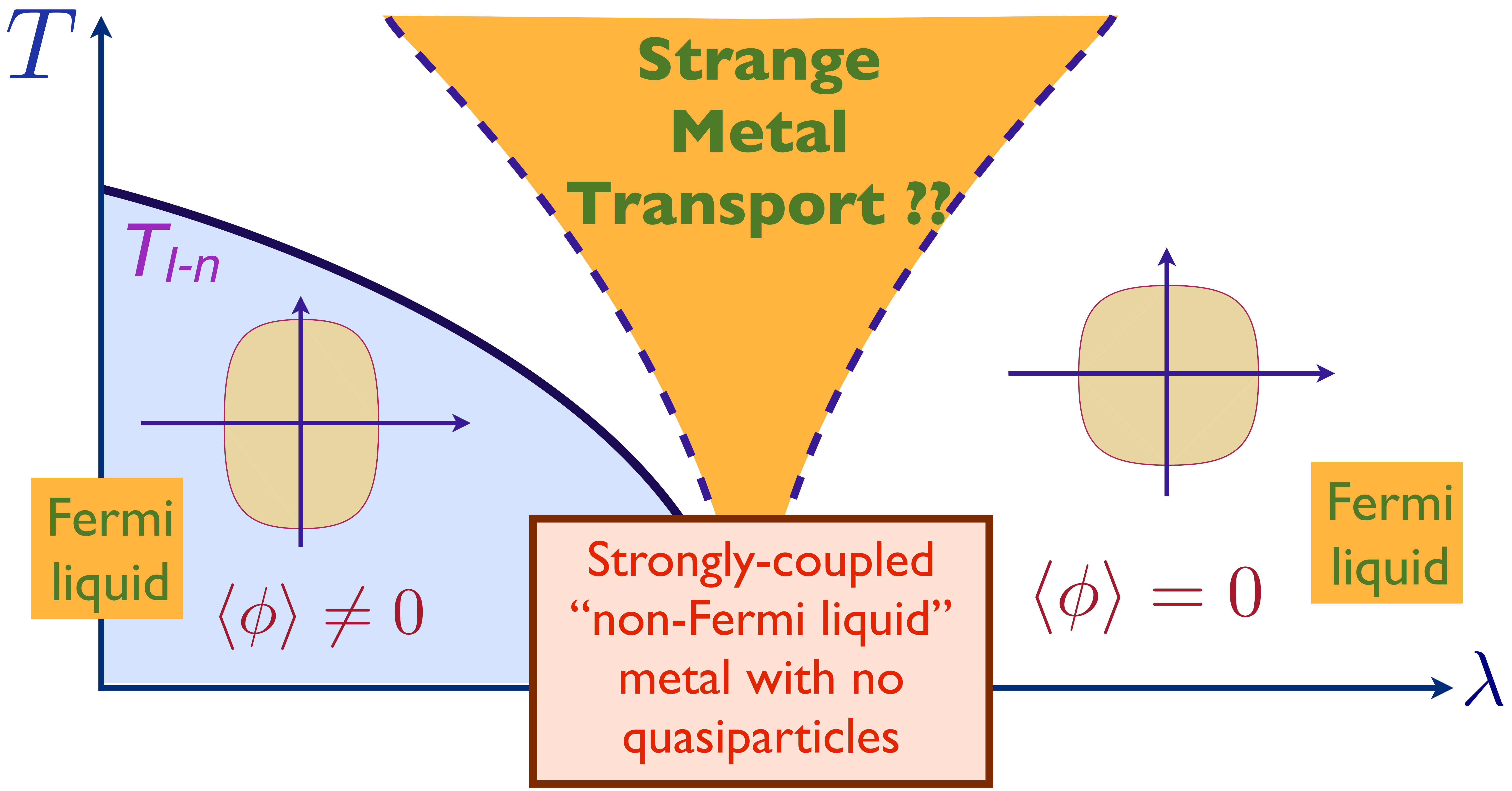
$$\mathrm{Re}\left[\sigma(\omega)\right] = C\,|\omega|^{-2/3}$$

Kim, Furusaki, Wen and Lee
Phys. Rev. B **50**, 17917 (1994)

$$C = 0\,; \quad \sigma(\omega) = iD/(\omega - \omega_c) + \omega^0 + \ldots$$

Guo, Patel, Esterlis and Sachdev *Phys. Rev. B* **106**, 115151 (2022);
Shi, Else, Goldman, Senthil, *SciPost Phys*. 14, 113 (2023); Guo, Sachdev and Patel arXiv:2308.01956 (2023)

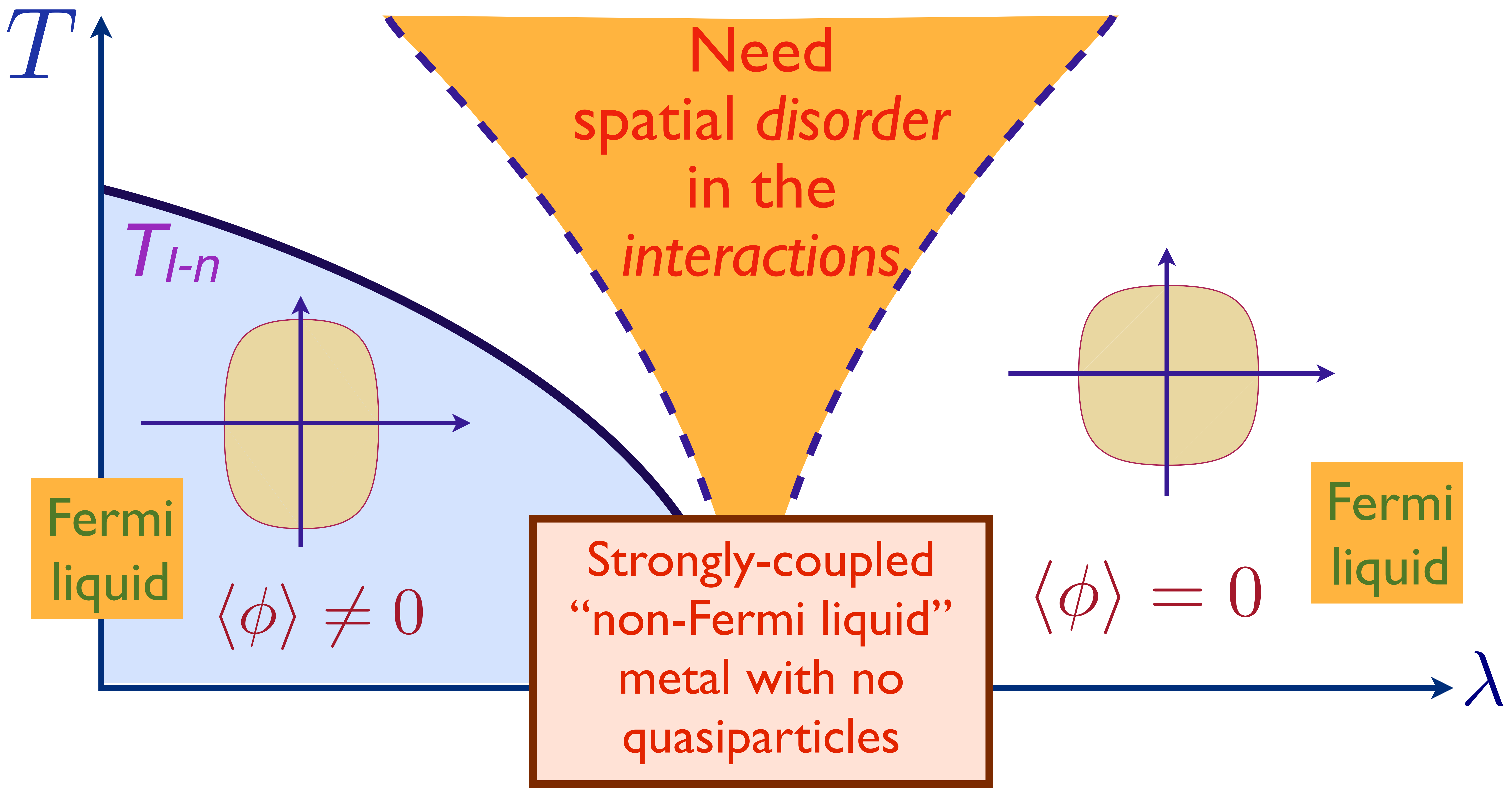
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal

No.



Phase diagram as a function of T and λ

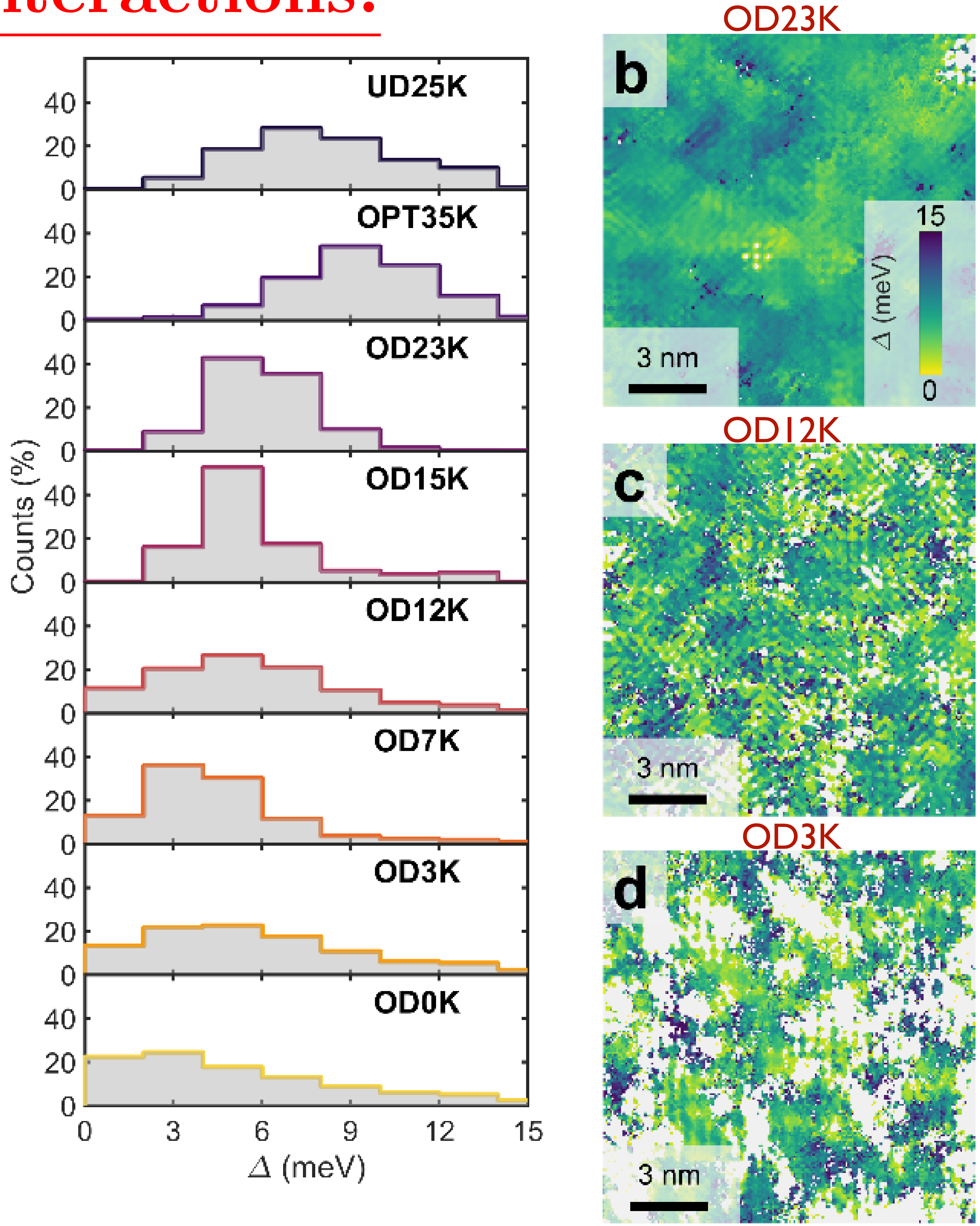
Spatially random interactions!

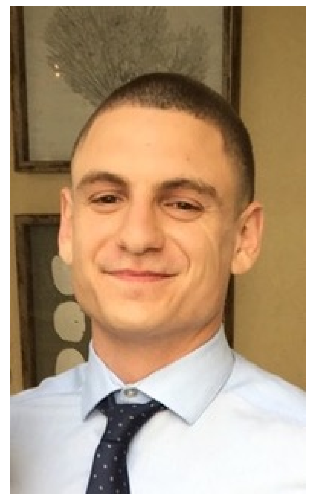
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge,
Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos,
Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden,
Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman,
Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Nature Materials **22**, 703 (2023)

In the Hubbard model, disorder in the hopping t_{ij} generates disorder in the exchange interaction $J_{ij} = \dfrac{t_{ij}^2}{U}$.



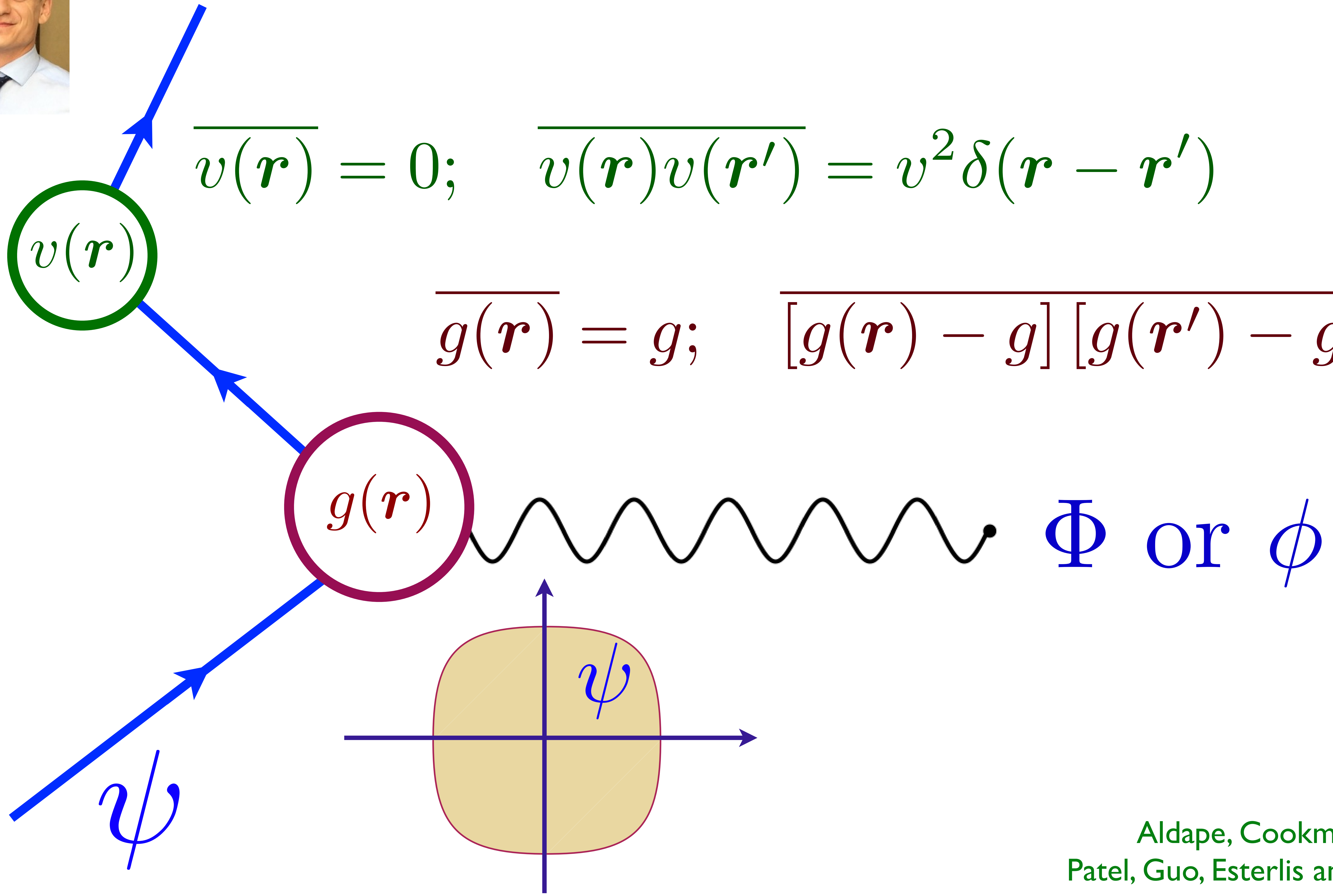


Universal theory of strange metals

2D inhomogeneous Yukawa-SYK model

$$\overline{v(\boldsymbol{r})} = 0; \quad \overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$$

$$\overline{g(\boldsymbol{r})} = g; \quad \overline{[g(\boldsymbol{r}) - g]\,[g(\boldsymbol{r}') - g]} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$$



Aldape, Cookmeyer, Patel and Altman *Phys. Rev. B* **105**, 235111 (2022)
Patel, Guo, Esterlis and Sachdev arXiv: 2203.04990, *Science* **381**, 790 (2023)

2D inhomogeneous Yukawa-SYK model

Electron Green's function: $G(\omega) \sim \dfrac{1}{\omega \dfrac{m^*(\omega)}{m} - \varepsilon(\boldsymbol{k}) + i\left(\dfrac{1}{\tau_e} + \dfrac{1}{\tau_{\rm in}(\omega)}\right)\operatorname{sgn}(\omega)}$

$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2\right)|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi}\left(\frac{g^2}{v^2} + g'^2\right)\ln(\Lambda/\omega)$$

Marginal Fermi liquid self energy and $T\ln(1/T)$ specific heat.

2D inhomogeneous Yukawa-SYK model

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$$\frac{1}{\tau_e} \sim v^2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim \left(\frac{g^2}{v^2} + g'^2 \right) |\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2}{\pi} \left(\frac{g^2}{v^2} + g'^2 \right) \ln(\Lambda/\omega)$$

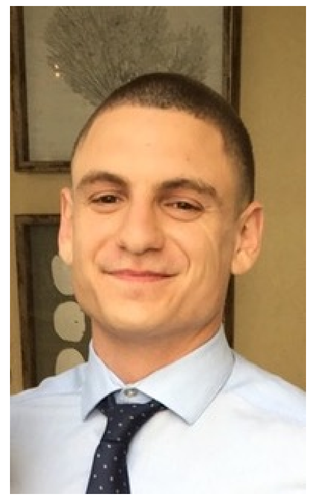
Marginal Fermi liquid self energy and $T \ln(1/T)$ specific heat.

Conductivity: $\sigma(\omega) \sim \dfrac{1}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}}$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2 |\omega| \quad ; \quad \frac{m^*_{\rm trans}(\omega)}{m} \sim \frac{2g'^2}{\pi} \ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2





Universal theory of strange metals

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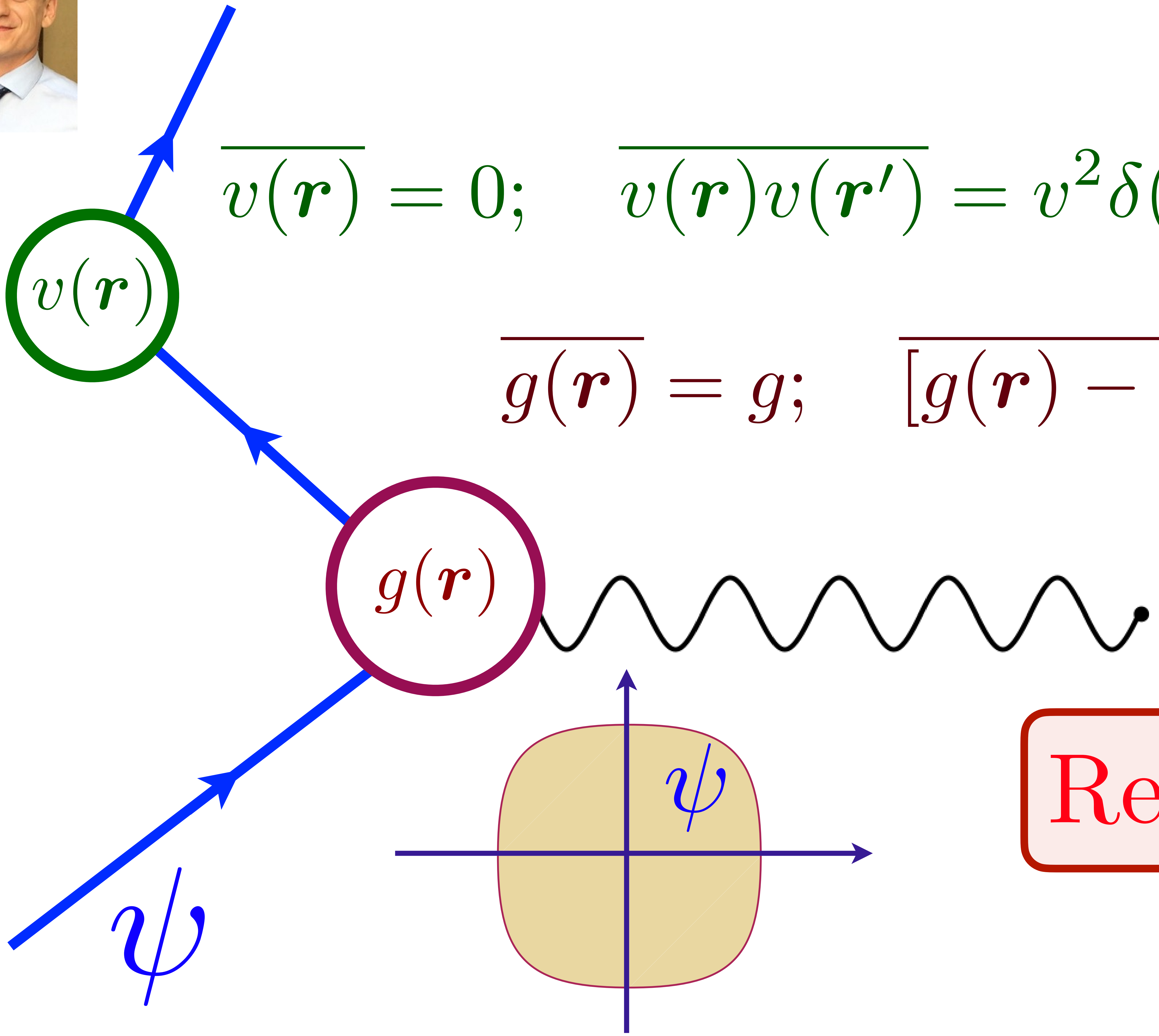

Φ or ϕ

Resistivity $\rho(T) \sim v^2 + g'^2 T$

Aldape, Cookmeyer, Patel and Altman, *Phys. Rev. B* **105**, 235111 (2022)
Patel, Guo, Esterlis and Sachdev arXiv: 2203.04990, *Science* **381**, 790 (2023)

Properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

Hartnoll and MacKenzie *Rev. Mod. Phys.* **94**, 041002 (2002)

3. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

van Heumen et al. *Phys. Rev. B* **106**, 054515 (2022)
Michon et al. *Nat. Commun.* **14**, 3033 (2023)

4. Photoemission: nearly "marginal Fermi liquid" electron spectral density:

$$\mathrm{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

Reber et al. *Nat. Commun.* **10**, 5737 (2019)

The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and strange metals

The many faces of multi-particle entanglement

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The many faces of multi-particle entanglement

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- Fractionalization and new emergent particles, as in spin liquids and FL*.

The many faces of multi-particle entanglement

- Absence of quasiparticles, as in the SYK model and strange metals

- A quantum theory of the interior of a black hole.

- Fractionalization and new emergent particles, as in spin liquids and FL*.

- Higher temperature superconductivity (?)