Quantum spin liquids and the phases of the cuprates

Coshare Science **02**, v3, 1-91 (2024)
Video link: https://www.cosharescience.com/videodetail/127

Subir Sachdev

Christos, Luo, Shackleton, Zhang, Scheurer, and Sachdev,
Proc. Nat. Acad. Sci. **120**, e2302701120 (2023)
Nikolaenko, Milczewski, Joshi, and Sachdev, *Phys. Rev. B* **108**, 045123 (2023)
Christos and Sachdev, arXiv:2308.03835 (2023)

PHYSICS
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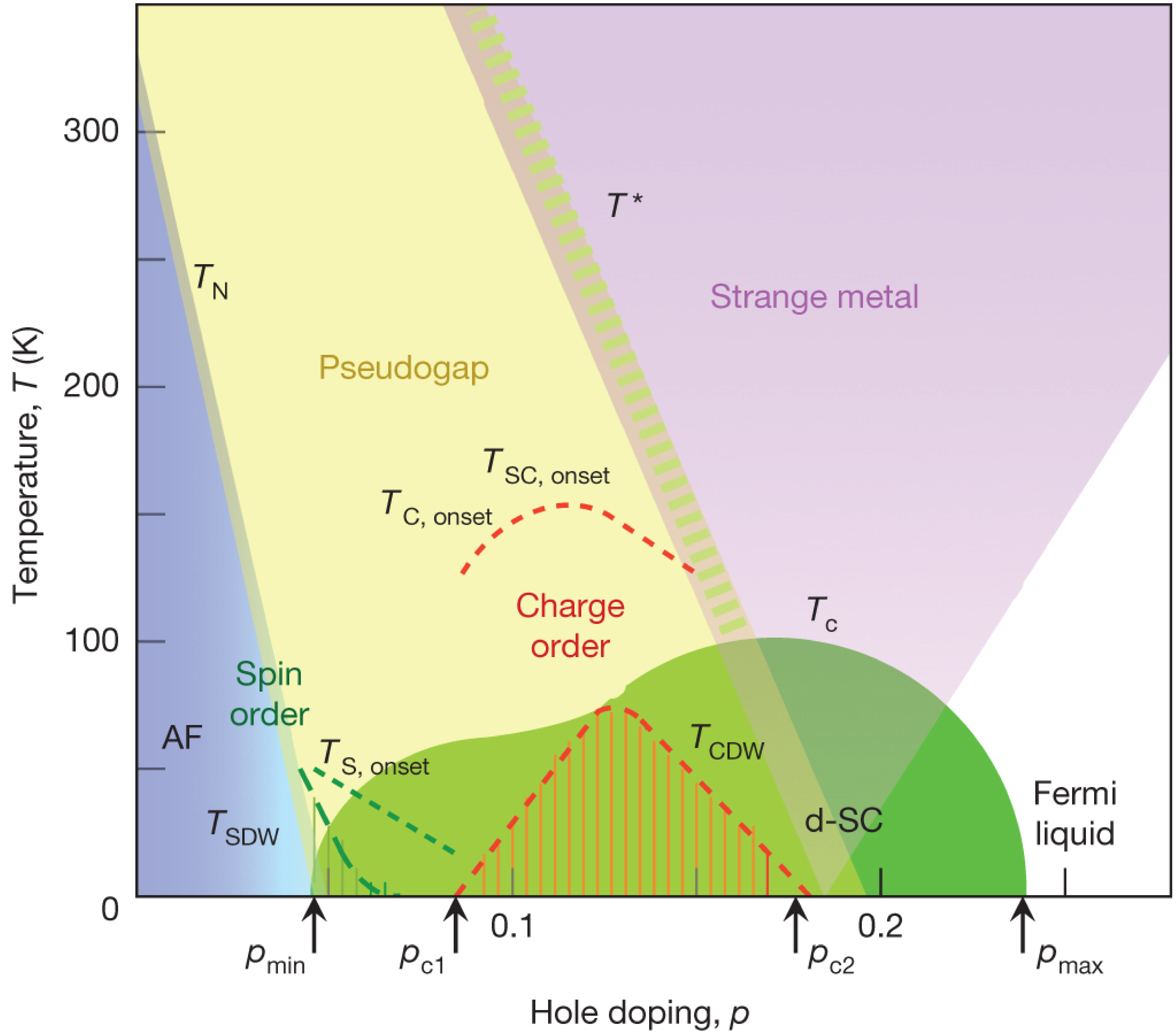
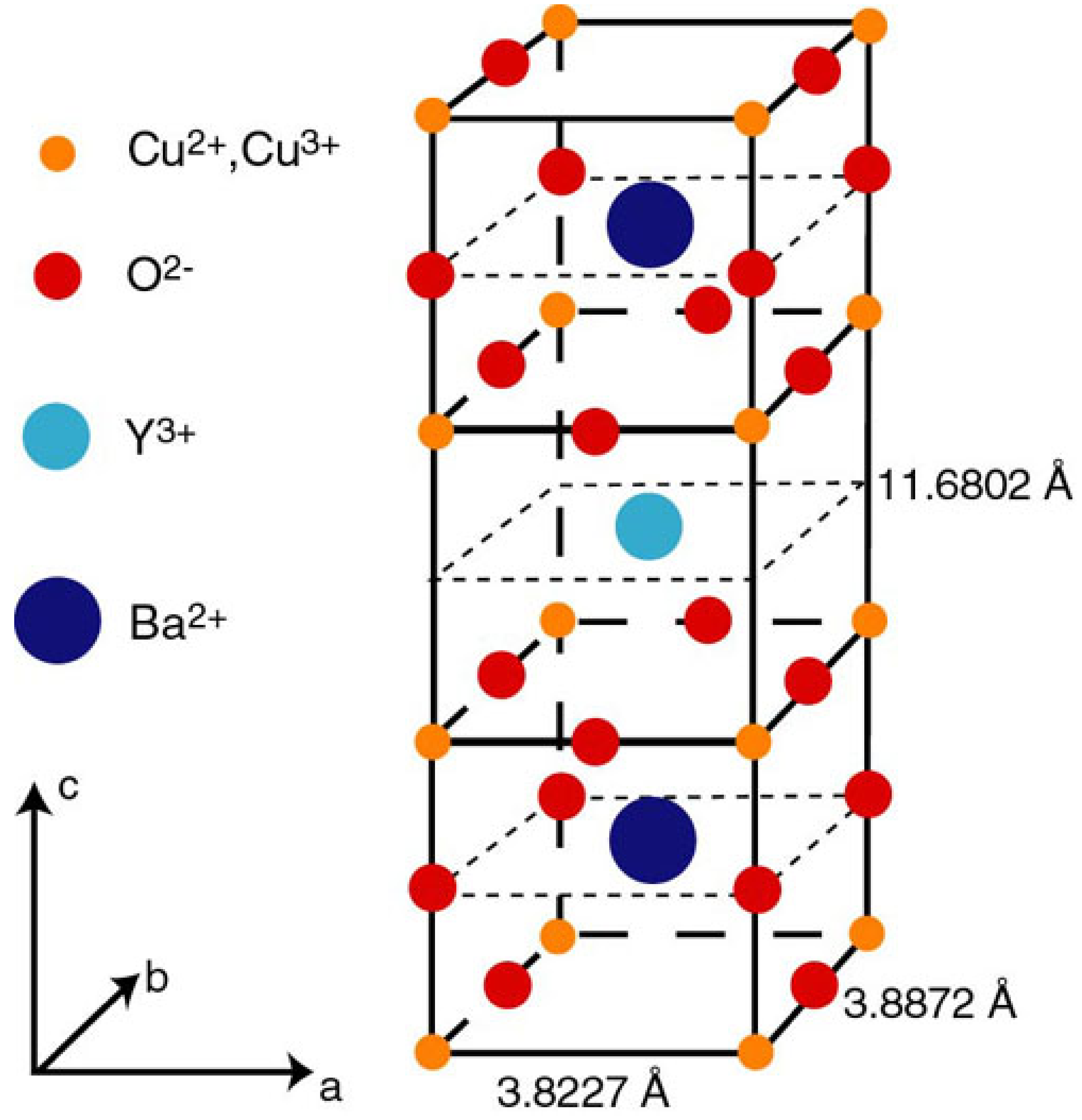
HARVARD

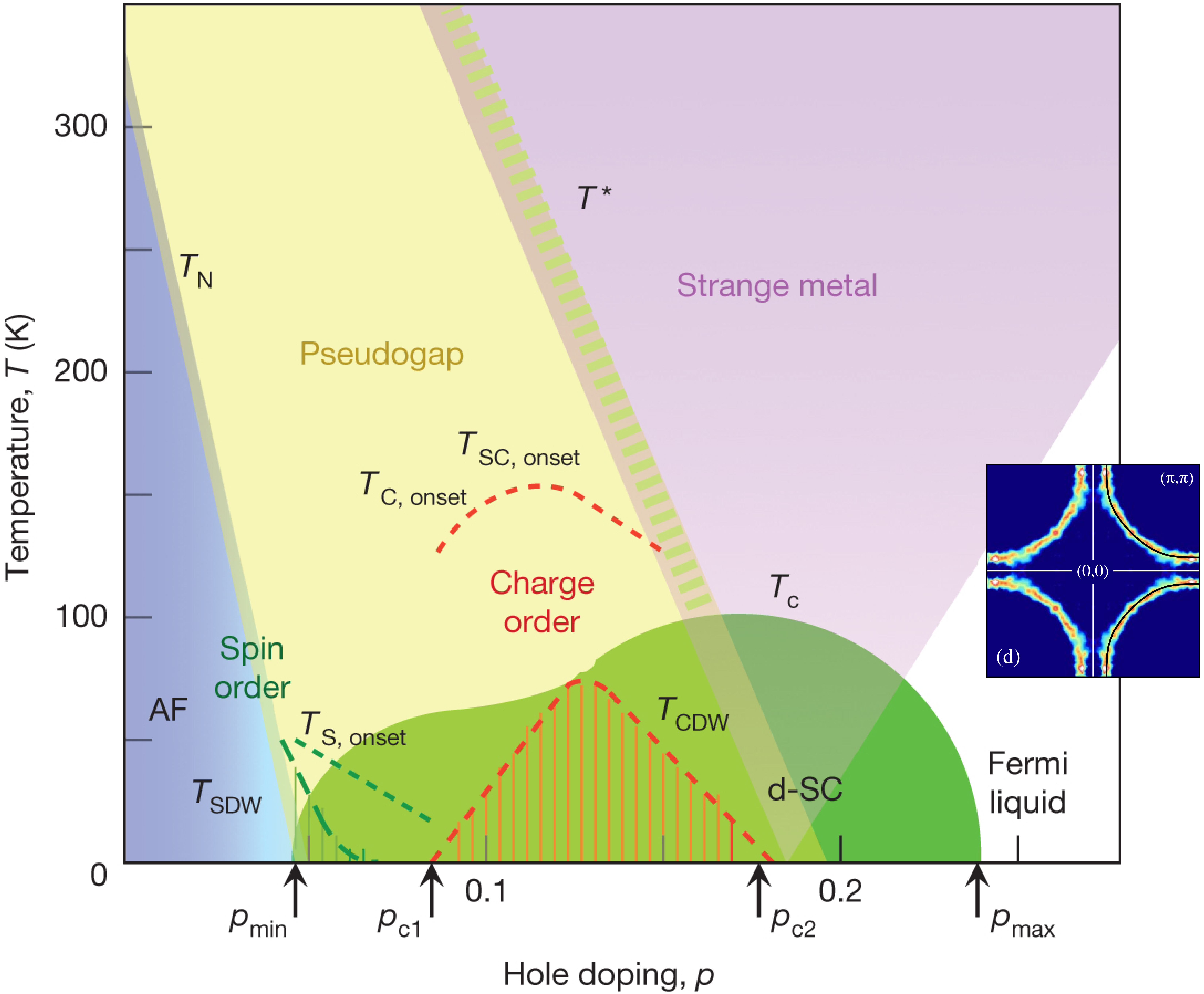
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





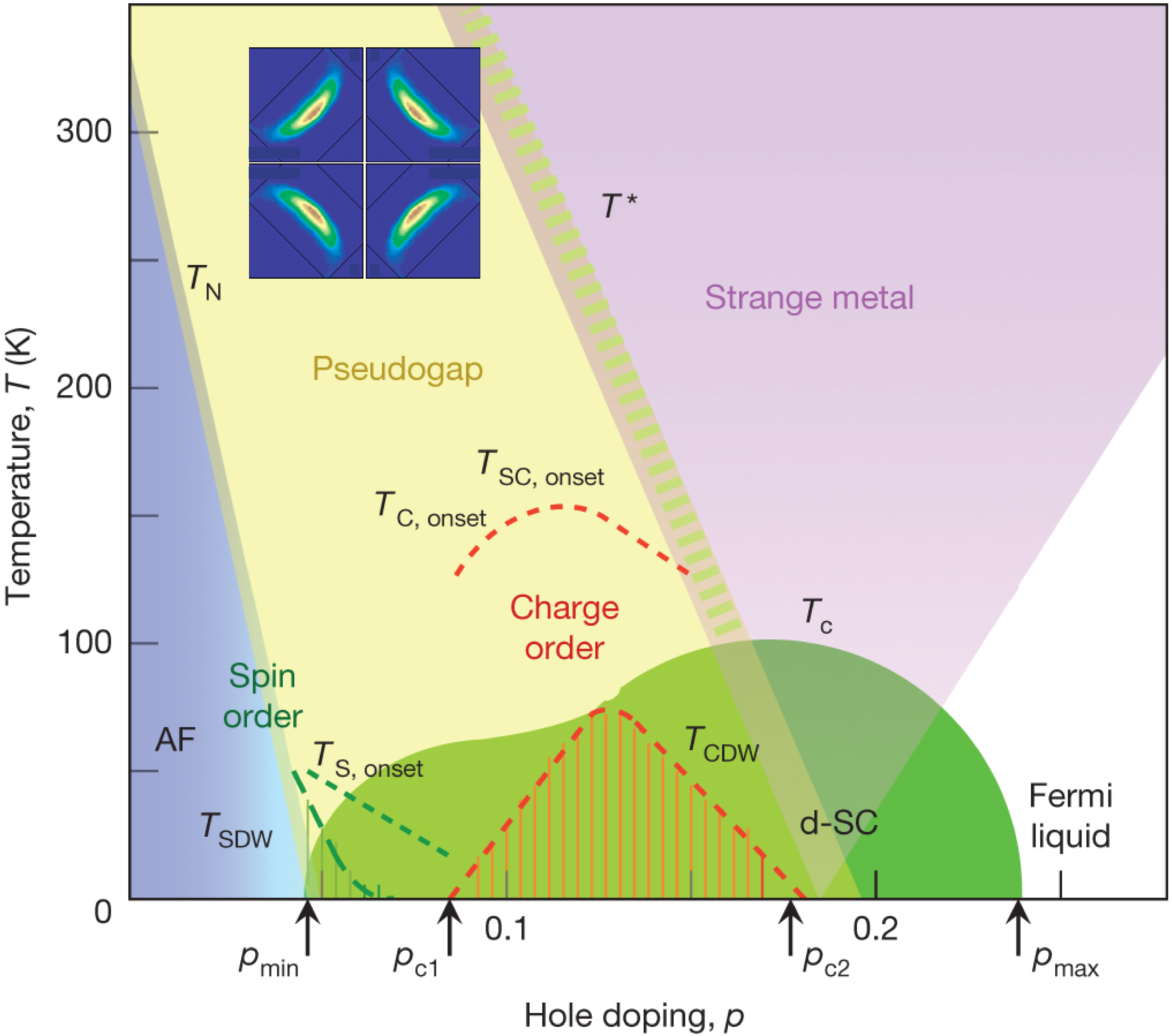
$YBa_2Cu_3O_{6+x}$

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



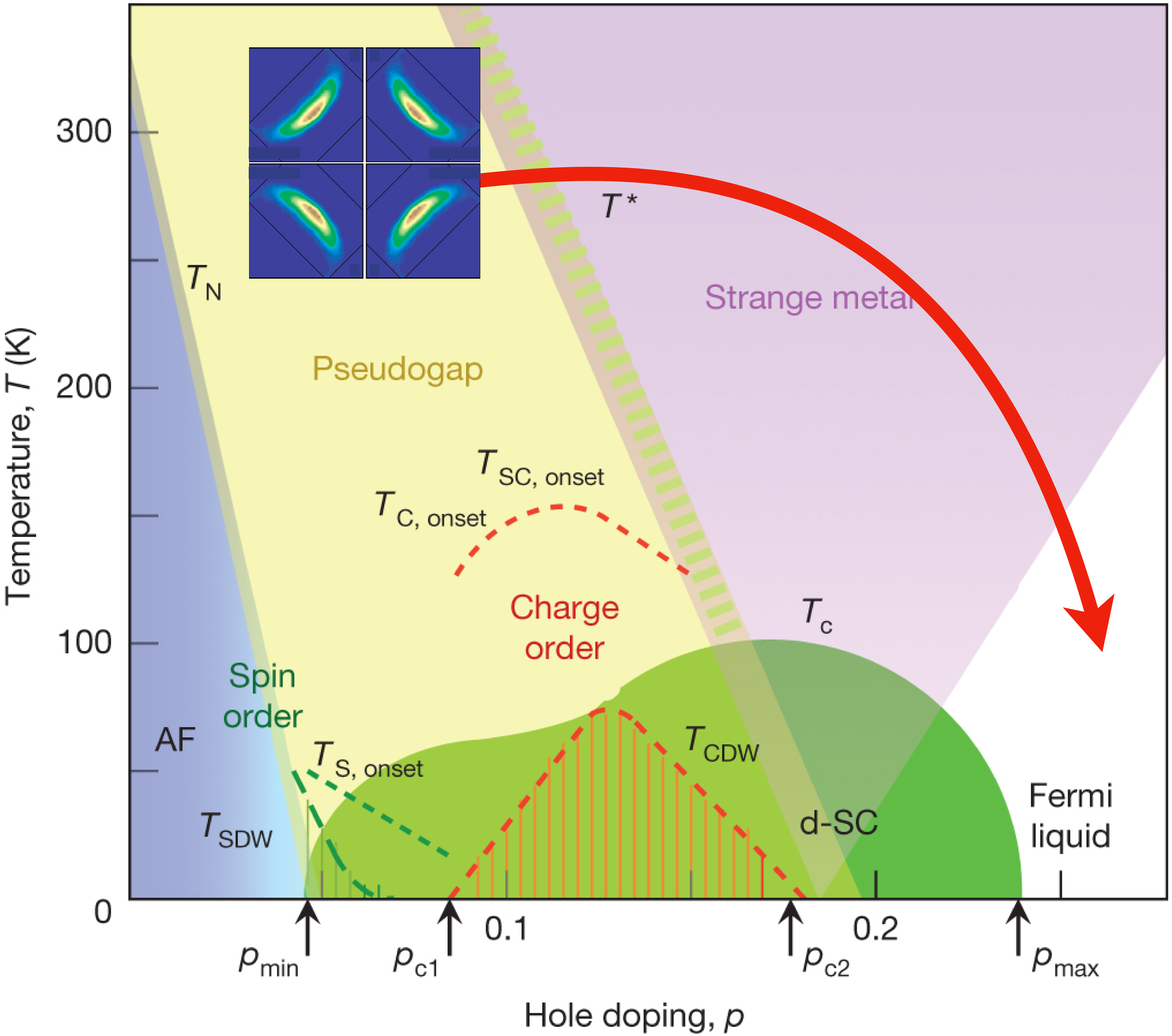
Fermi liquid in the overdoped metal

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



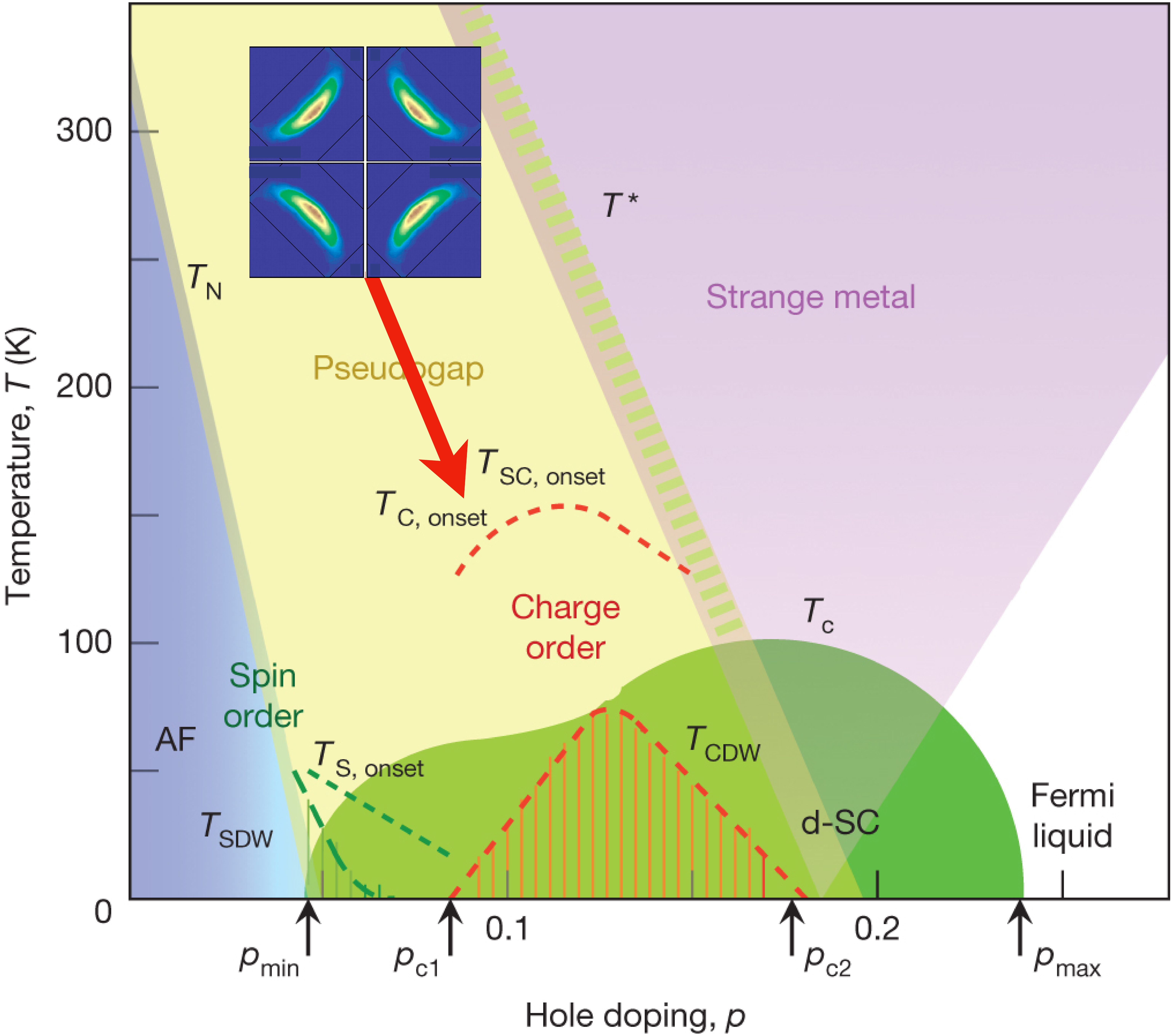
Theory for "pseudogap metal" with "Fermi arcs"?

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Strange metal from quantum criticality between pseudogap and Fermi liquid?

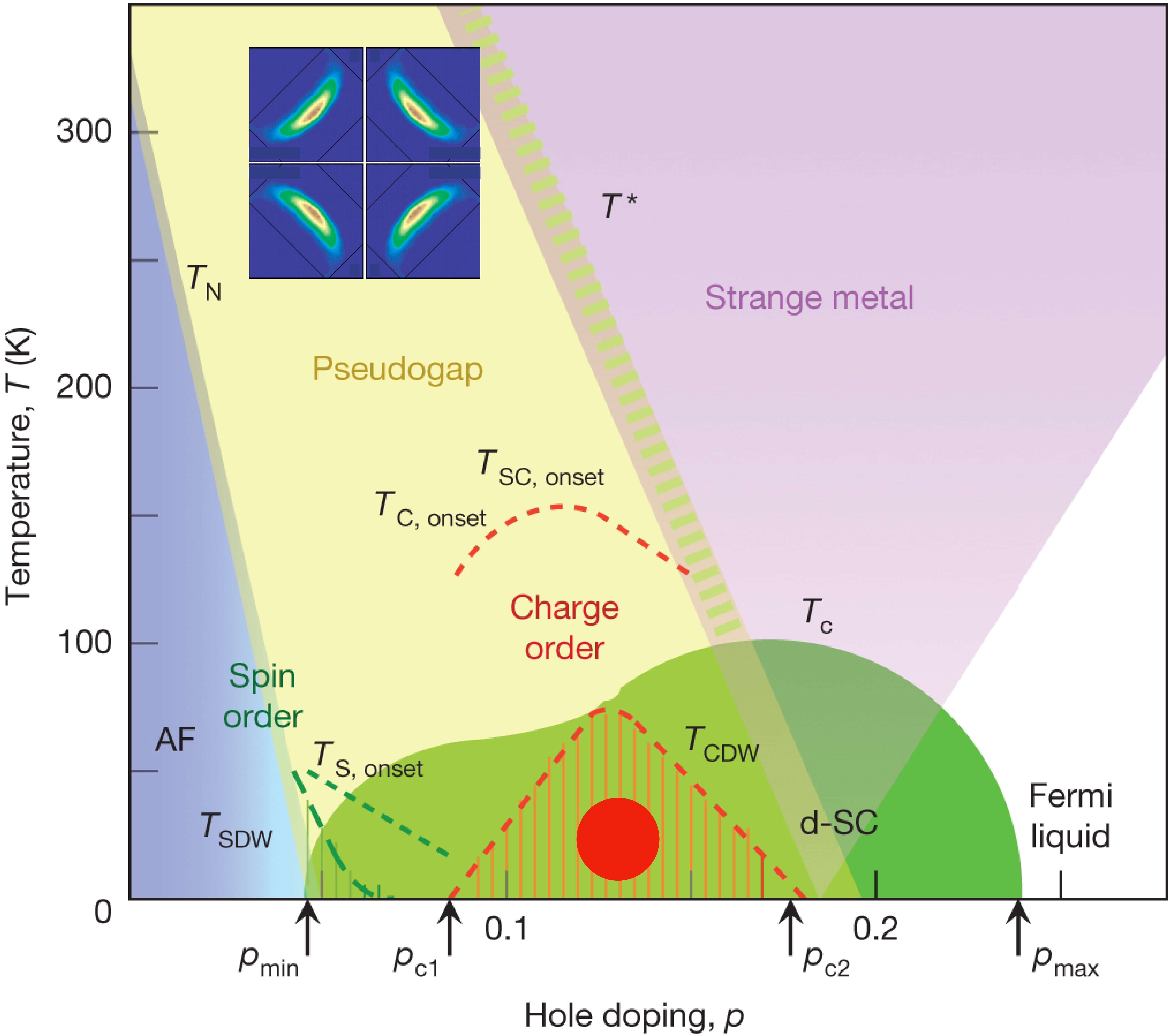
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



A theory for the onset of charge order and d-wave superconductivity from the pseudogap metal.

Why are T_c and $T_{\rm CDW}$ about the same?

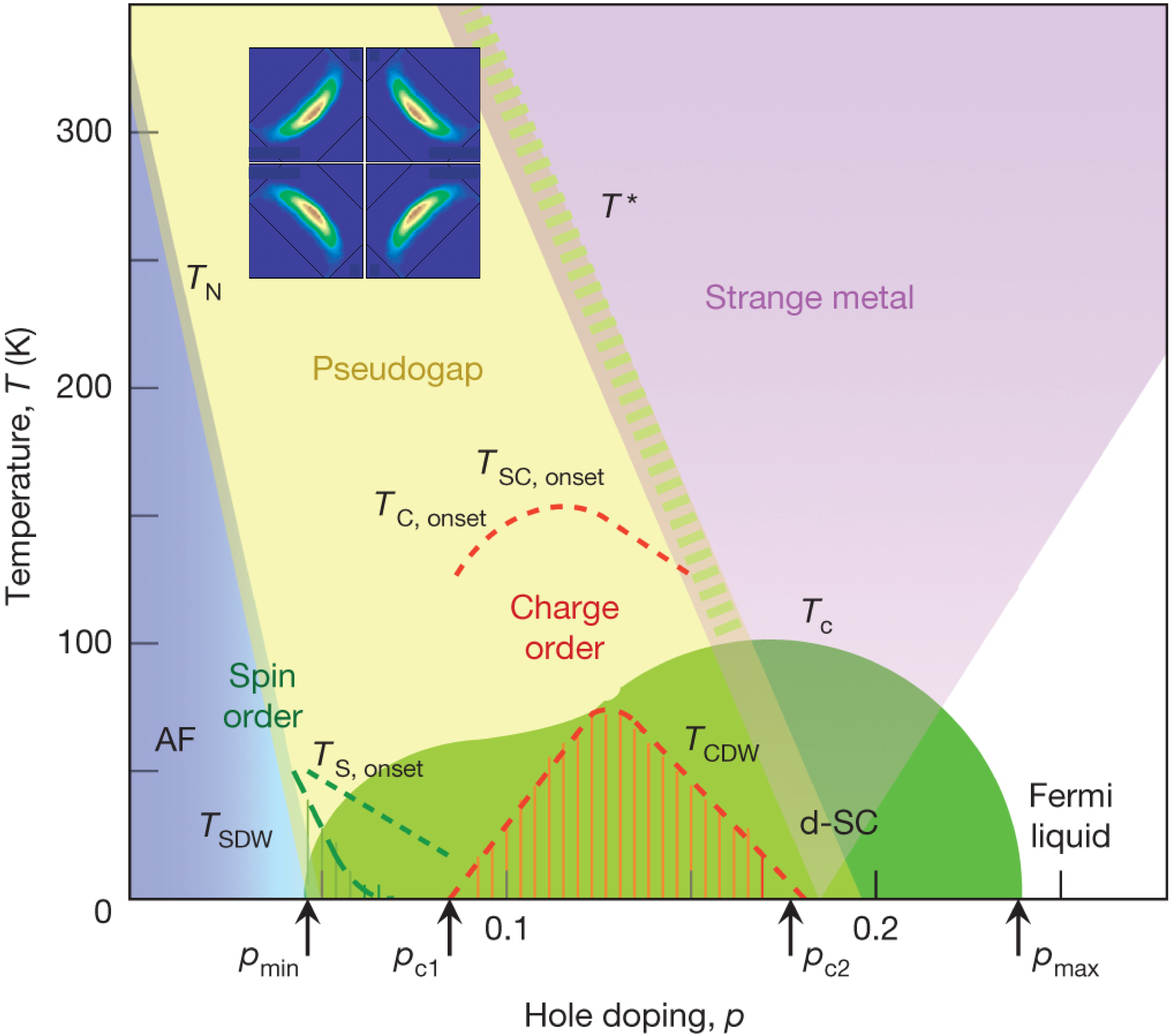
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Quantum oscillations in the CDW phase at low T show only a single electron pocket.

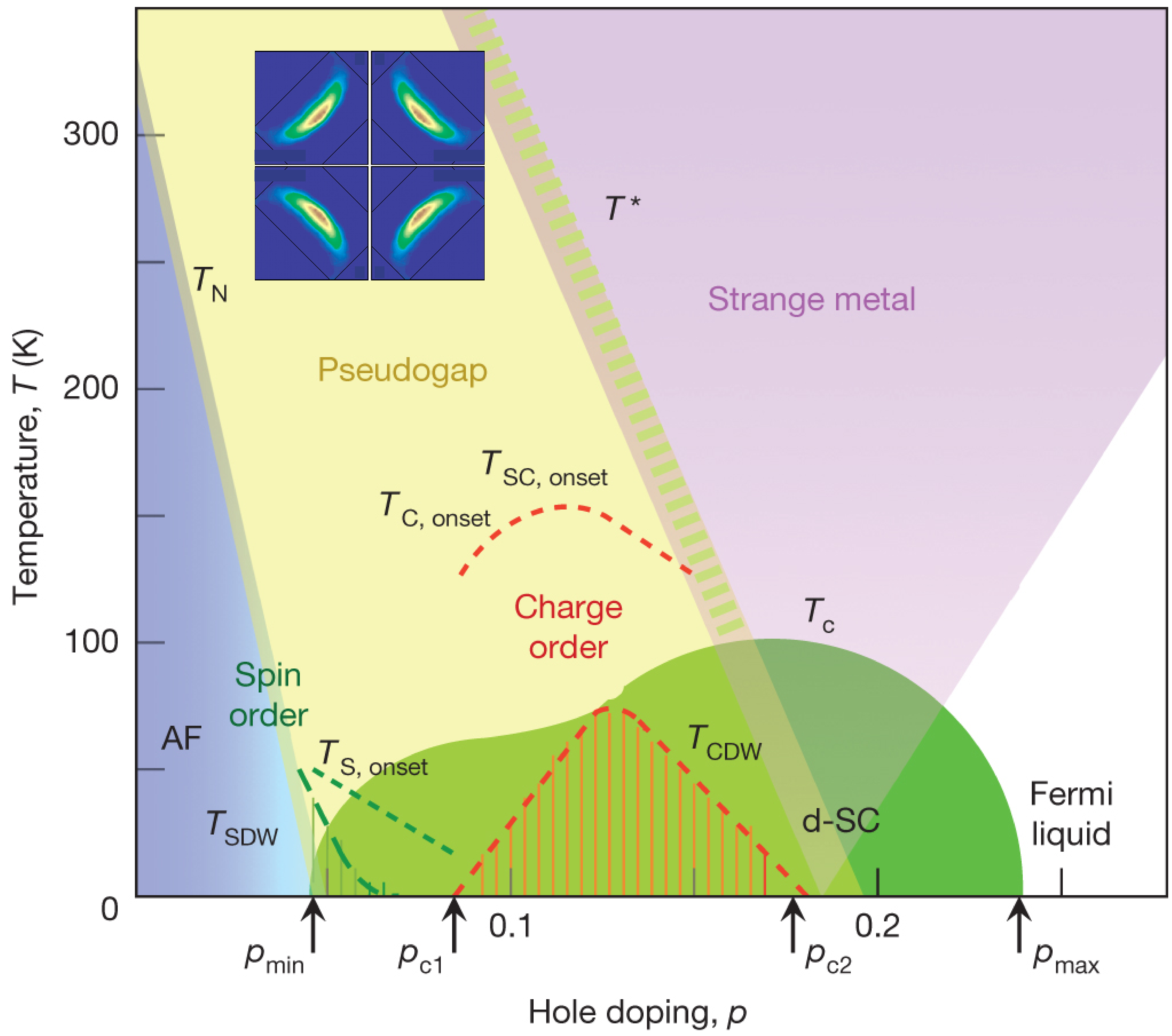
This cannot be obtained as the CDW instability of a Fermi liquid

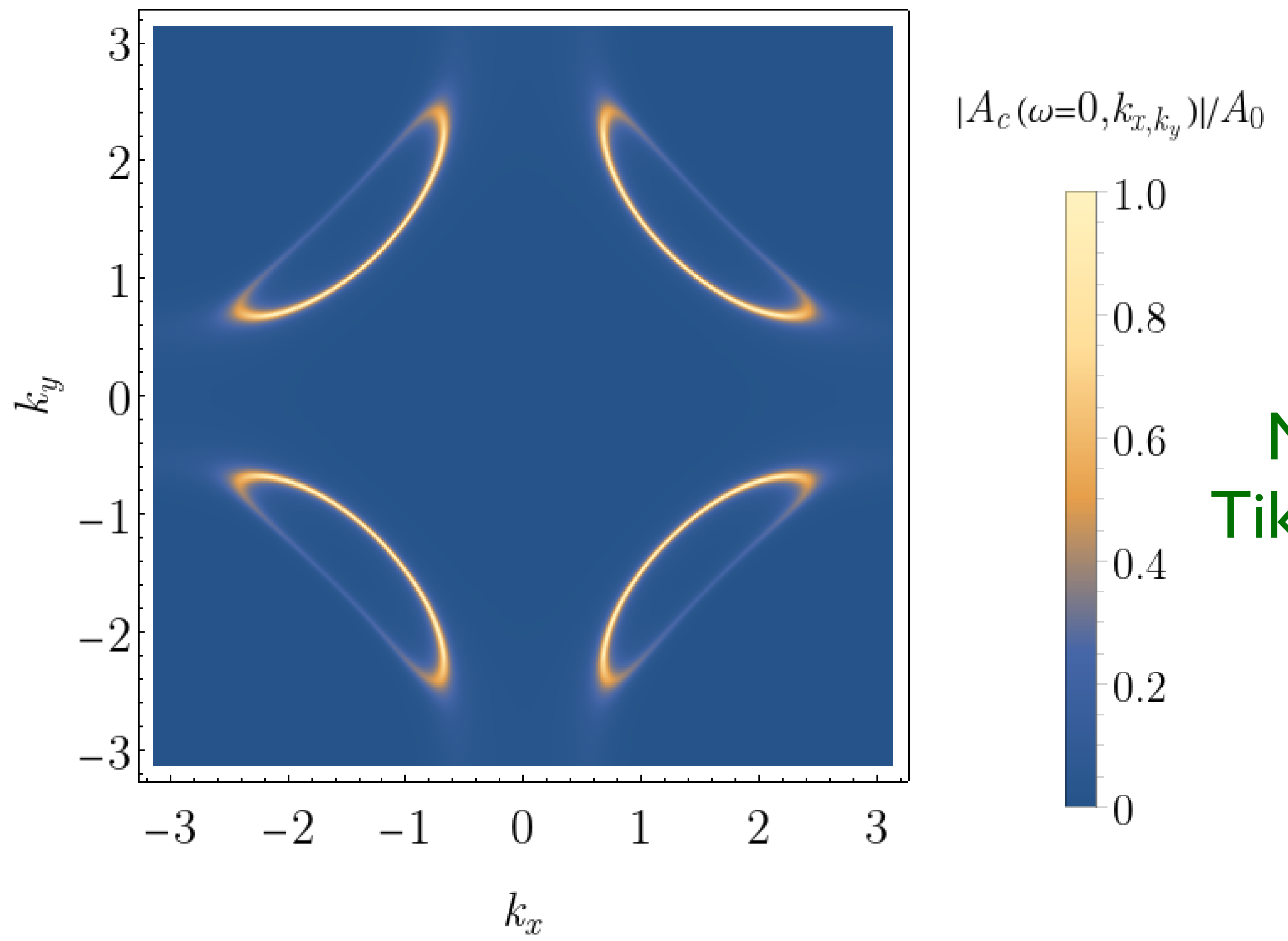
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



Build a theory for the phase diagram from a theory of the pseudogap metal

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





Qi and Sachdev, *Phys. Rev. B* **81**, 115129 (2010)

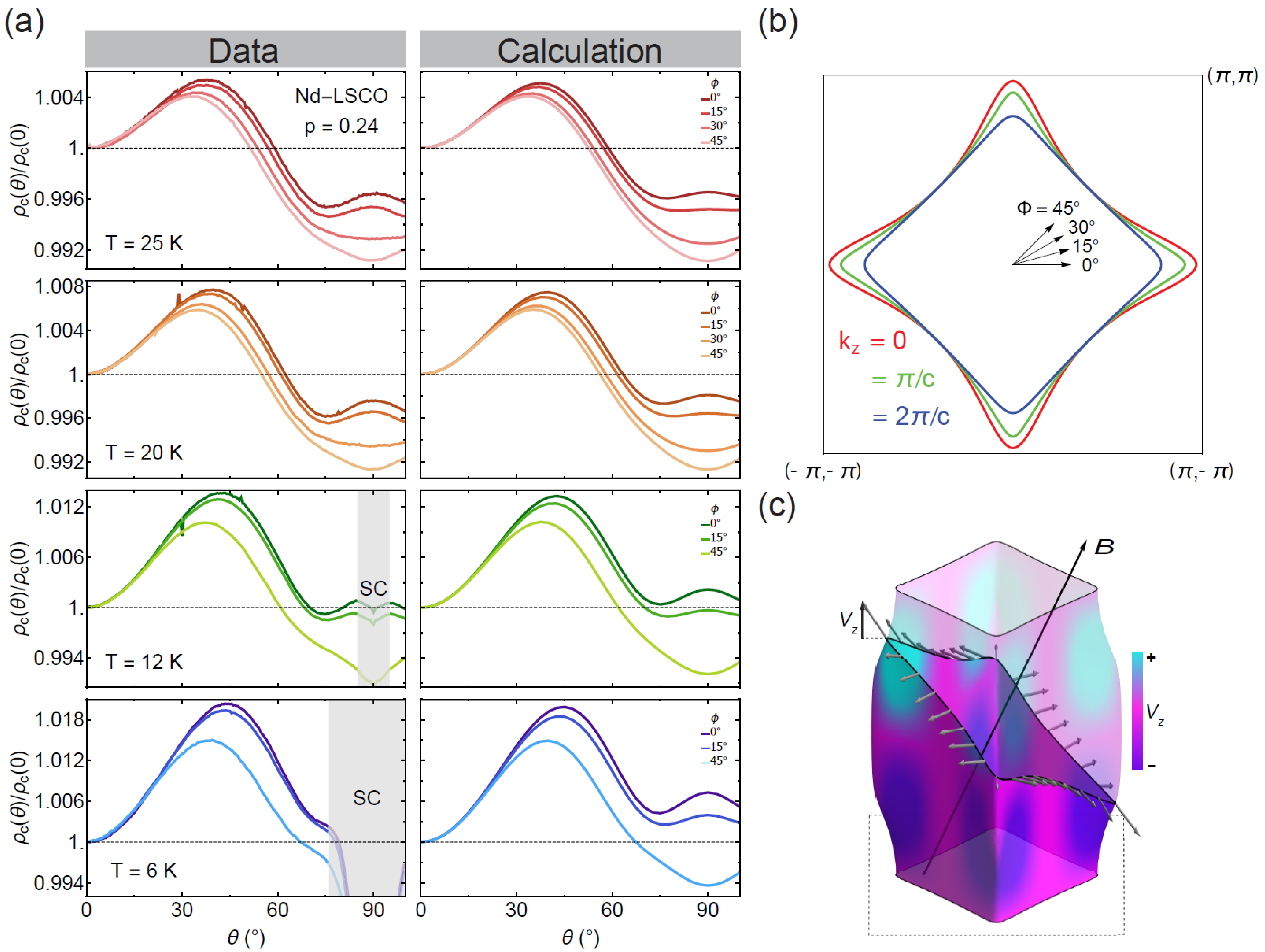
Mascot, Nikolaenko, Tikhanovskaya, Zhang, Morr, and Sachdev, *Phys. Rev. B* **105**, 075146 (2022)

Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

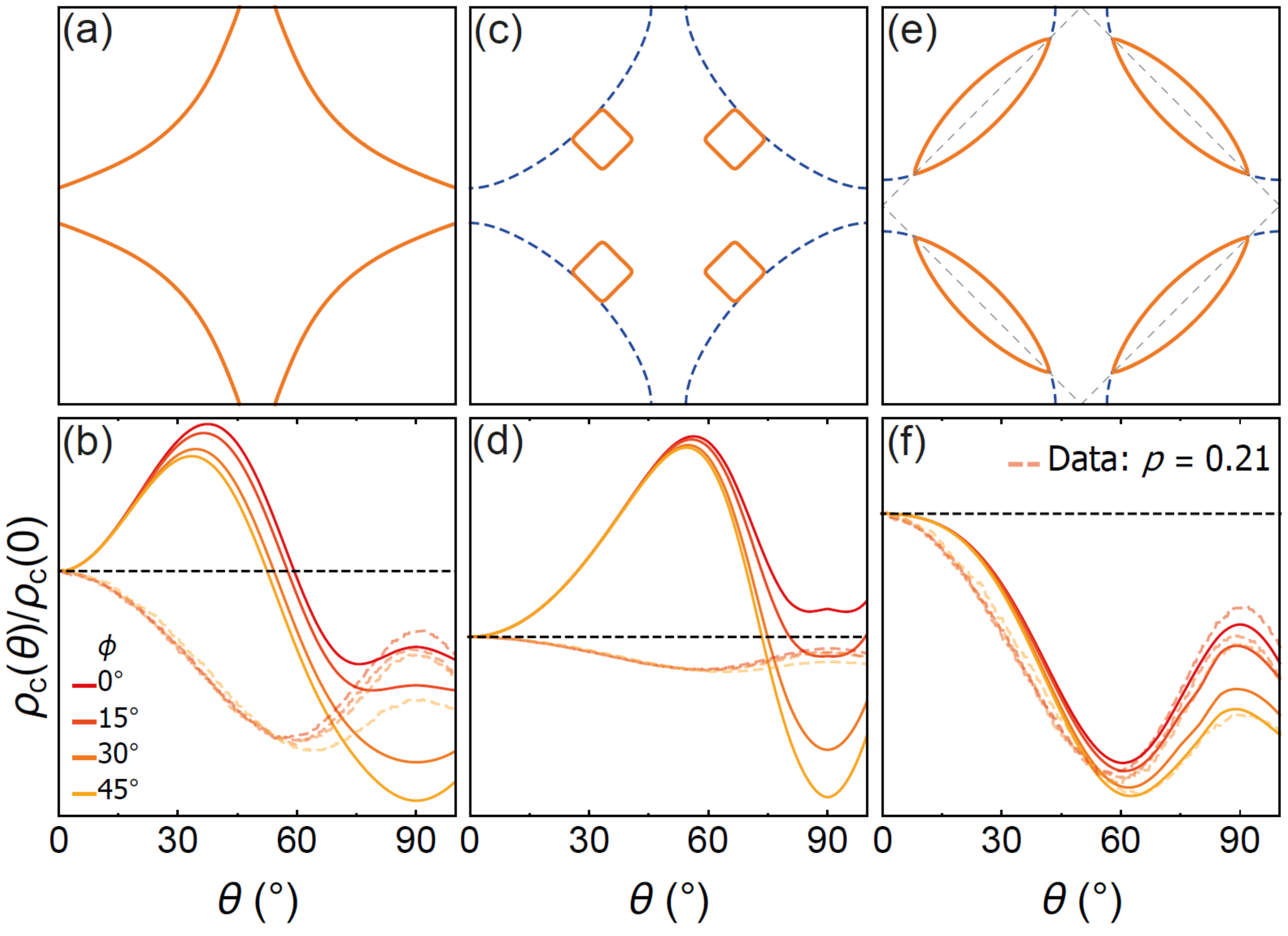
Yang, Rice, and Zhang, *Phys. Rev. B* **73**, 174501 (2006)
Stanescu and Kotliar, *Phys. Rev. B* **74**, 125110 (2006)
Berthod, Giamarchi, Biermann, and Georges, *Phys. Rev. Lett.* **97**, 136401 (2006)
Sakai, Motome, and Imada, *Phys. Rev. Lett.* **102**, 056404 (2009)
Skolimowski and M. Fabrizio, *Phys. Rev. B* **106**, 045109 (2022)
Zhao, Mai, Bradley, and Phillips, arXiv:2305.02341 (2023)

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Fang, Grissonnanche, Legros, Verret, Laliberté, Collignon, Ataei, Dion, Zhou, Graf, Lawler, Goddard, Taillefer, and Ramshaw, *Nat. Phys.* **18**, 558 (2022)

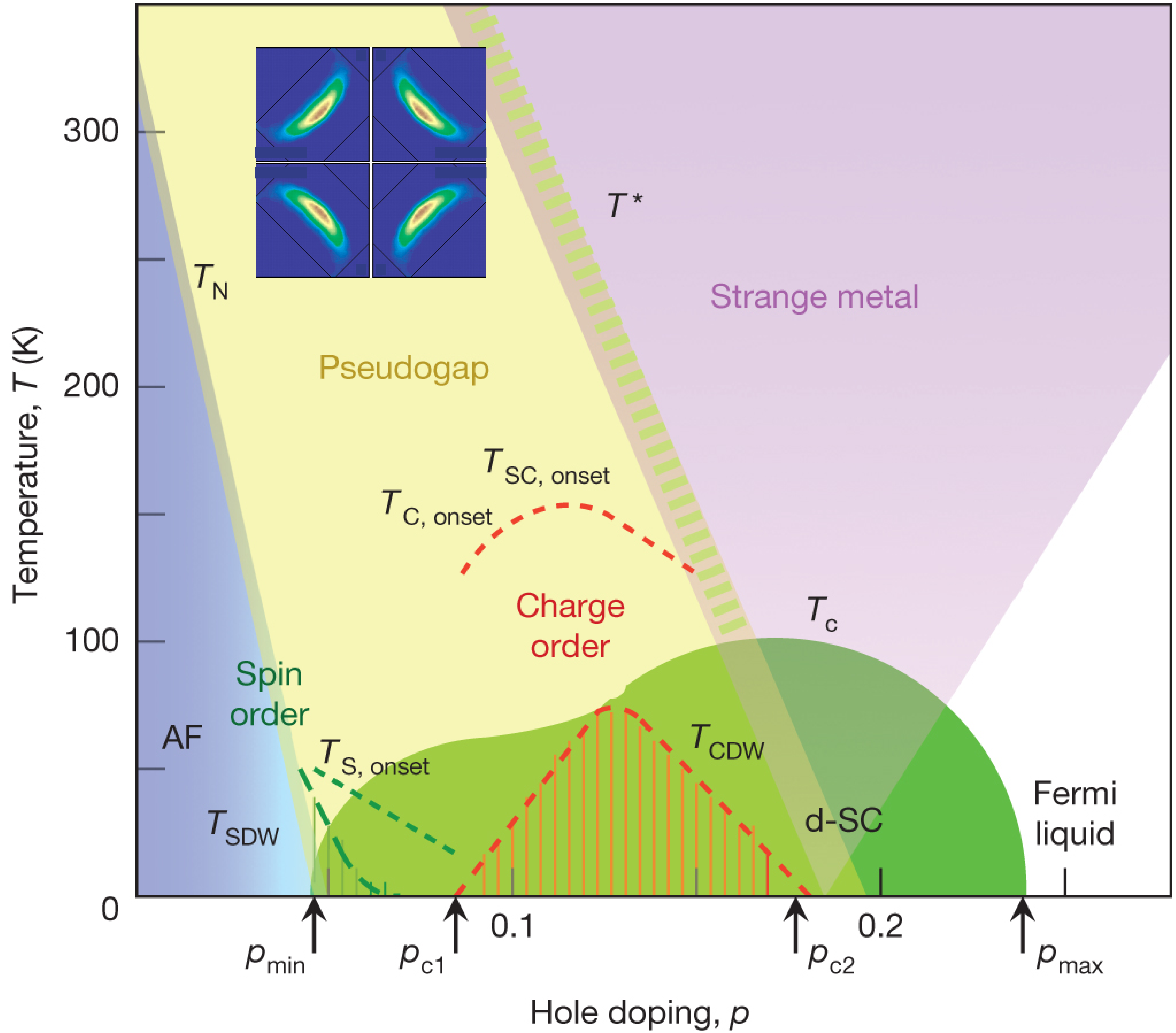


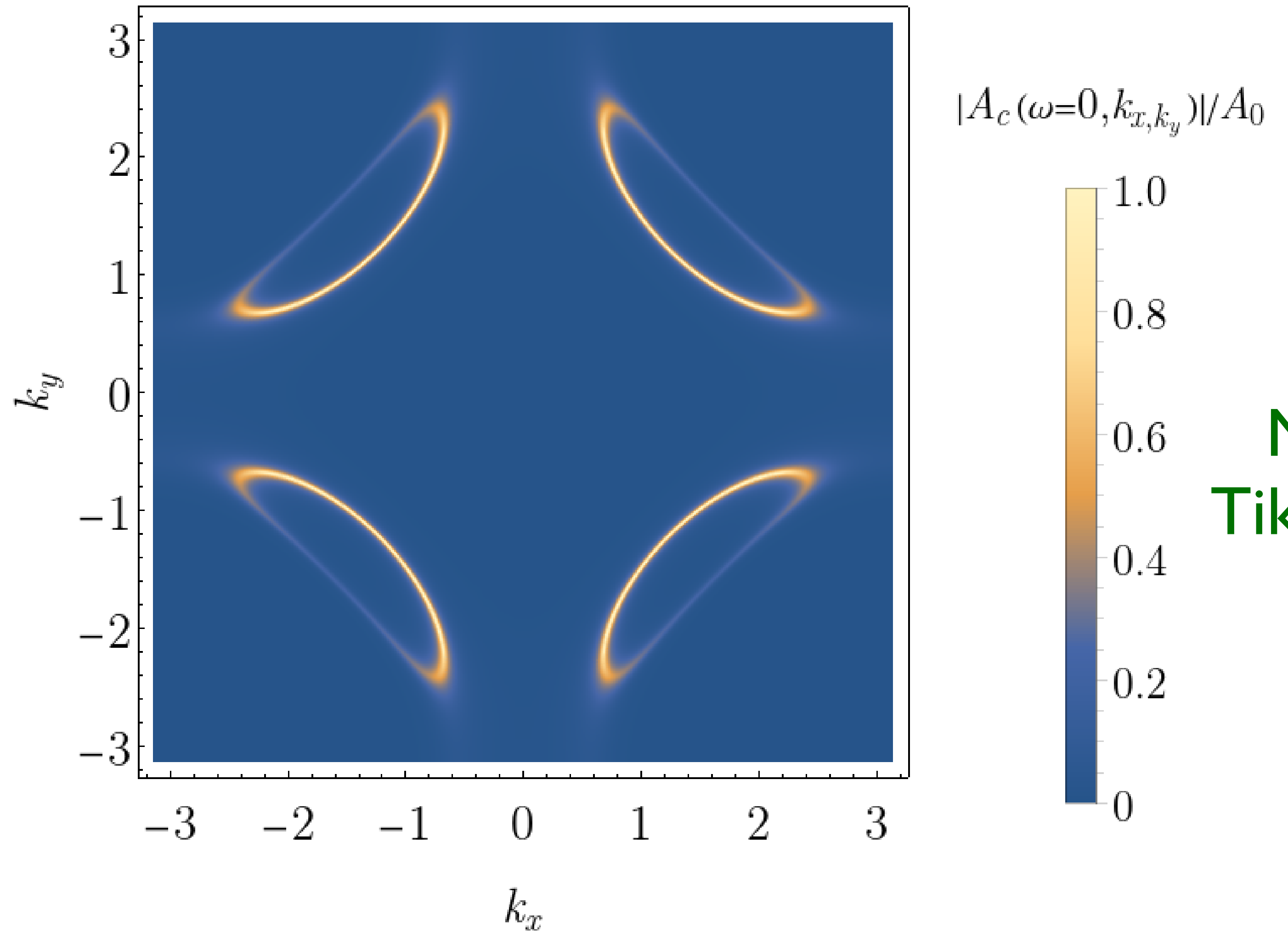
$p > p_c$ Large Fermi surface



$p < p_c$ Reconstructed Fermi surface

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





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Sakai, Motome, and Imada, *Phys. Rev. Lett.* **102**, 056404 (2009)
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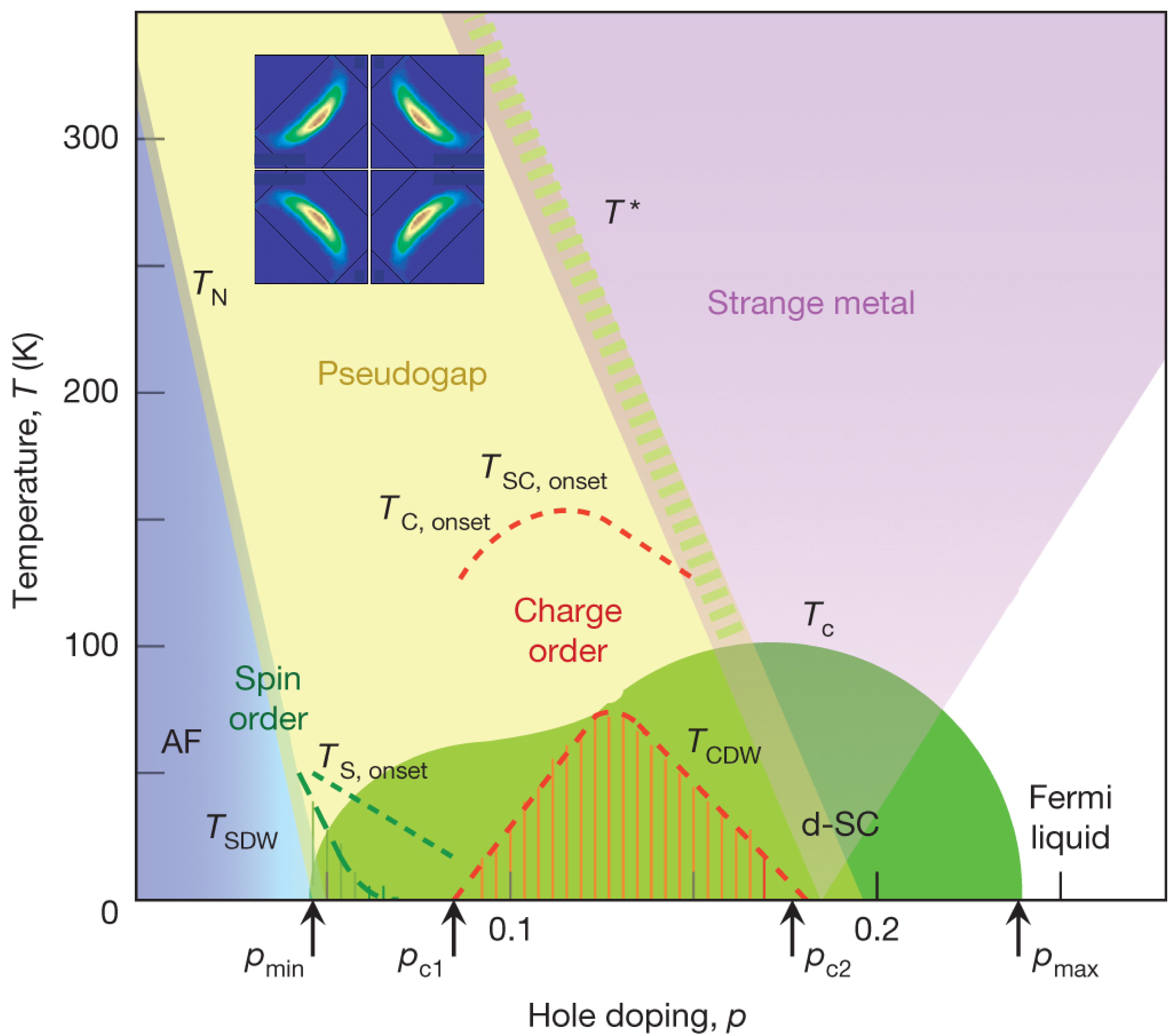

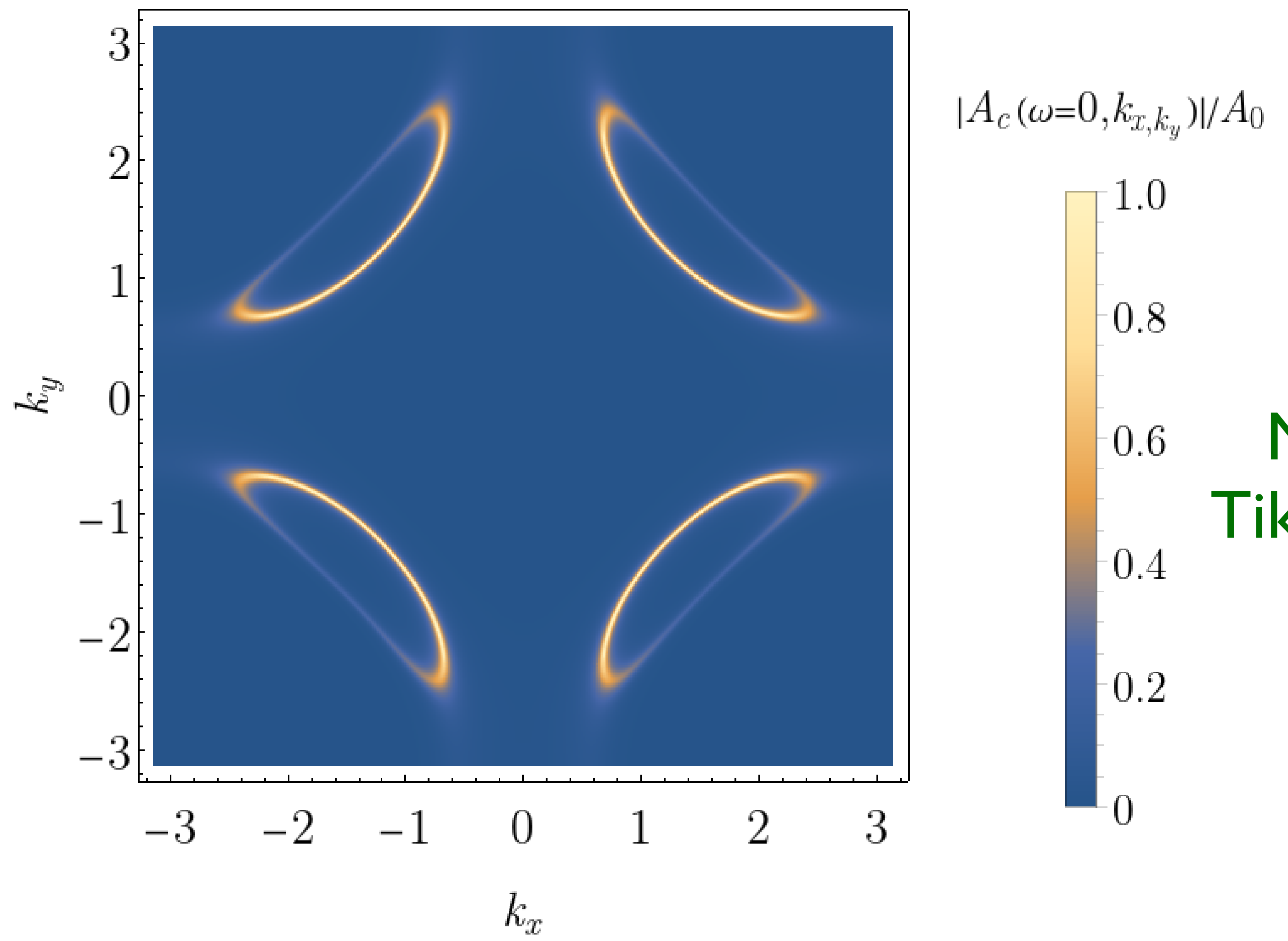

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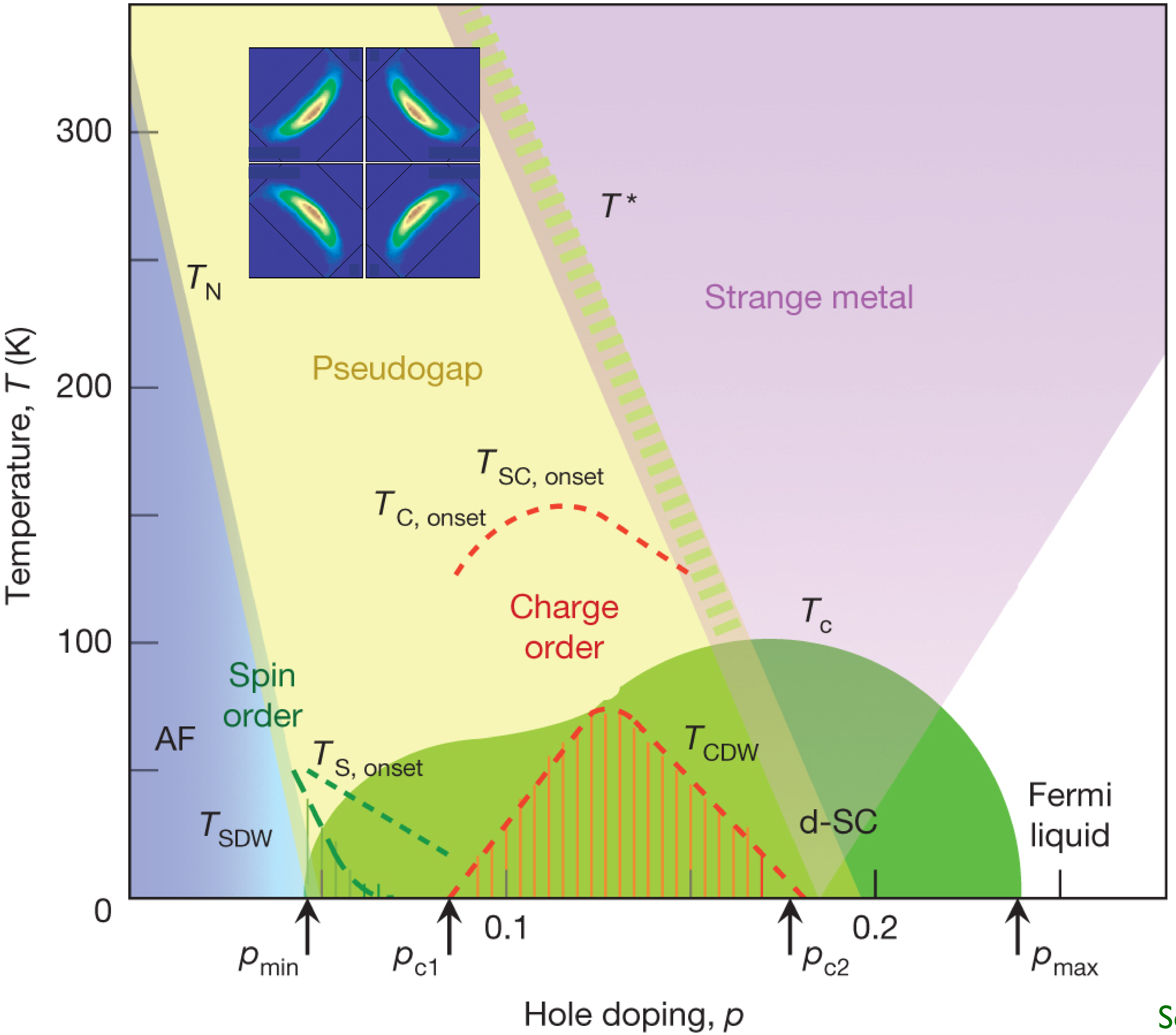
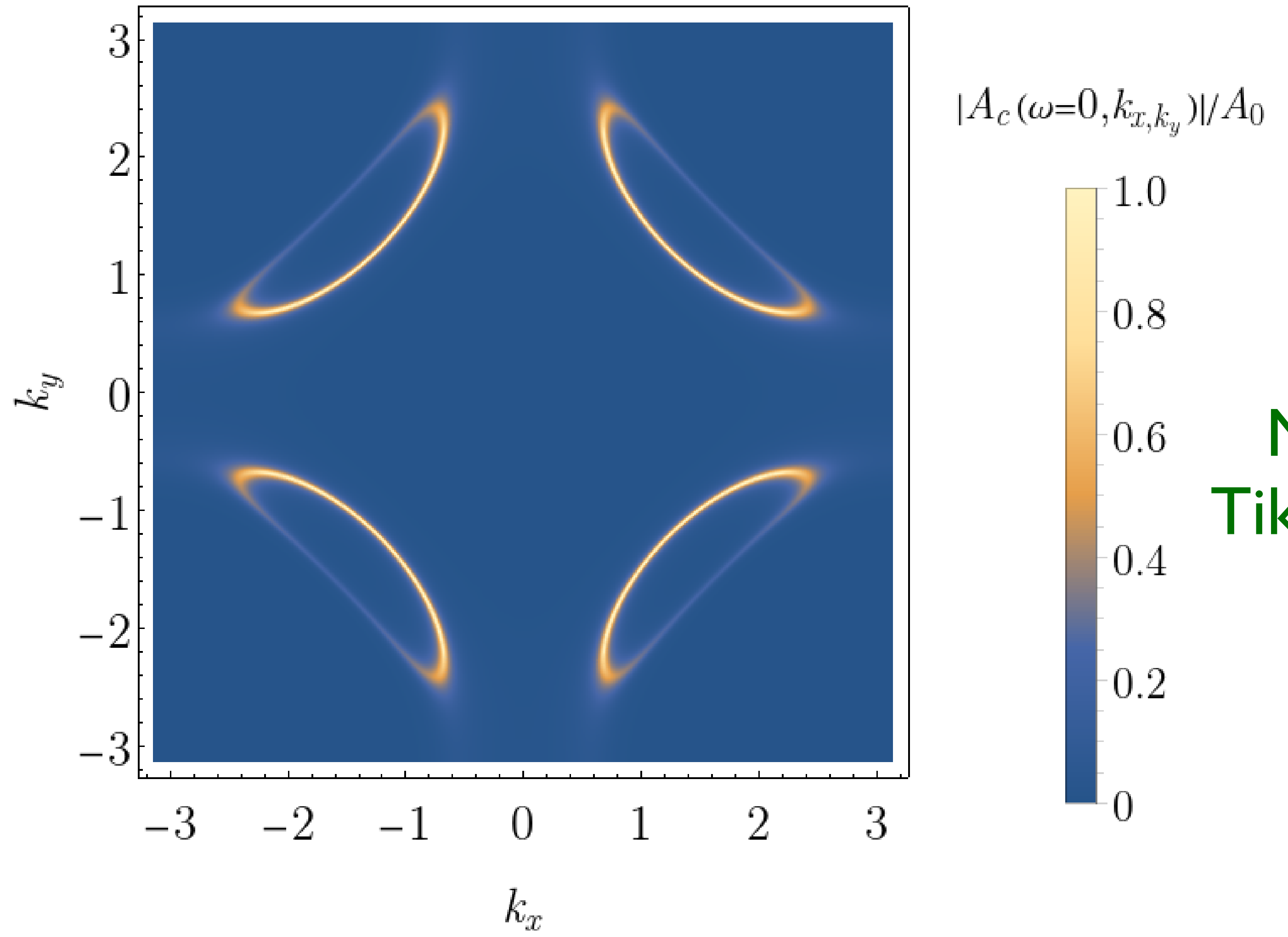
Stanescu and Kotliar, *Phys. Rev. B* **74**, 125110 (2006)
Berthod, Giamarchi, Biermann, and Georges, *Phys. Rev. Lett.* **97**, 136401 (2006)
Sakai, Motome, and Imada, *Phys. Rev. Lett.* **102**, 056404 (2009)
Skolimowski and M. Fabrizio, *Phys. Rev. B* **106**, 045109 (2022)
Zhao, Mai, Bradley, and Phillips, arXiv:2305.02341 (2023)

Green function zeros….

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)





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Hole pocket Fermi surfaces of size p with charge e, spin-1/2 quasiparticles

+

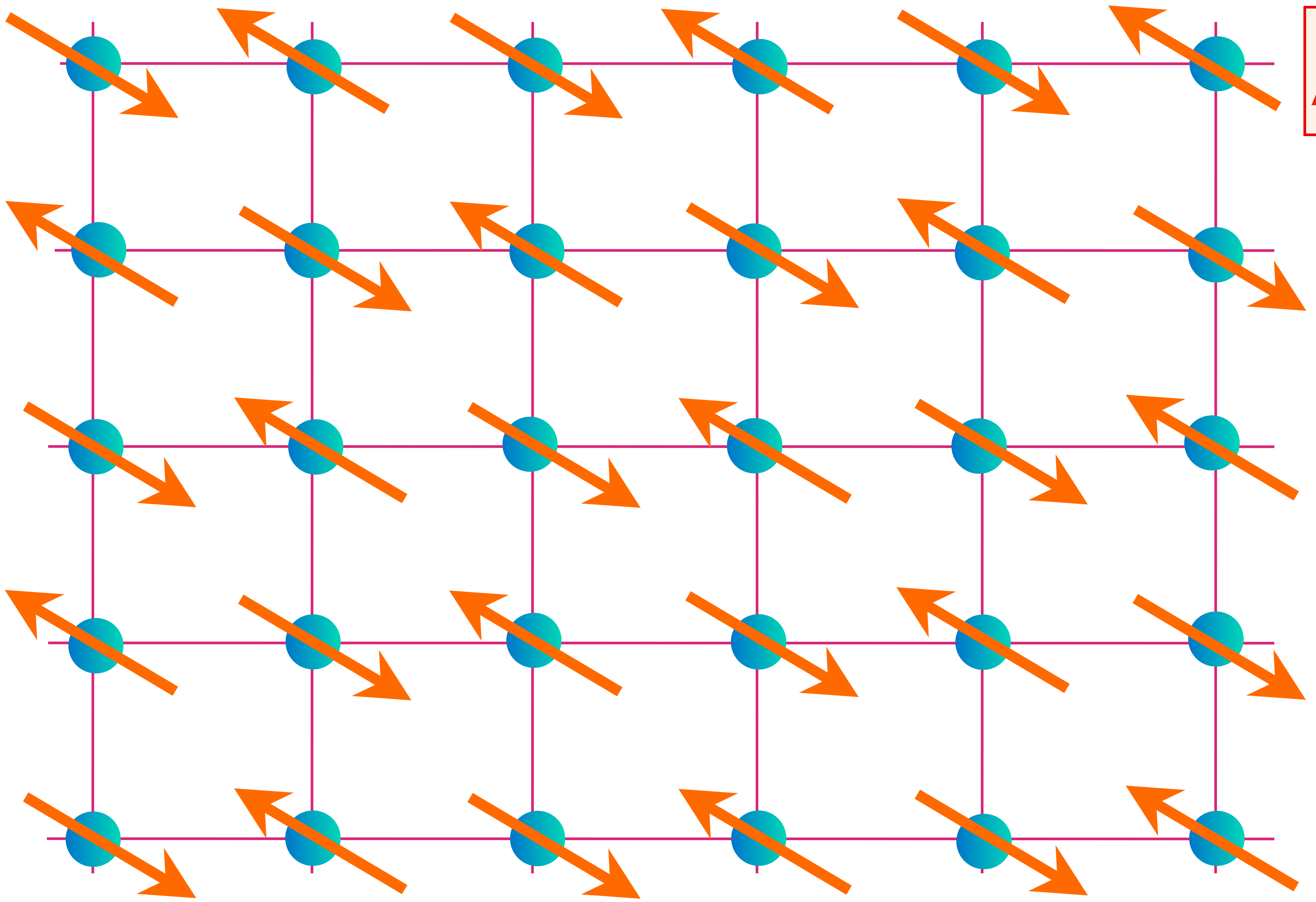
'spectator' square lattice spin liquid *at half-filling.*

FL*: Spin liquid is *required* because the Fermi surface does not enclose the Luttinger volume $(1+p)$.

Senthil, Vojta, and Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Intro to spin liquids and doped spin liquids (FL*)

The dance of electrons on Cu atoms in YBCO

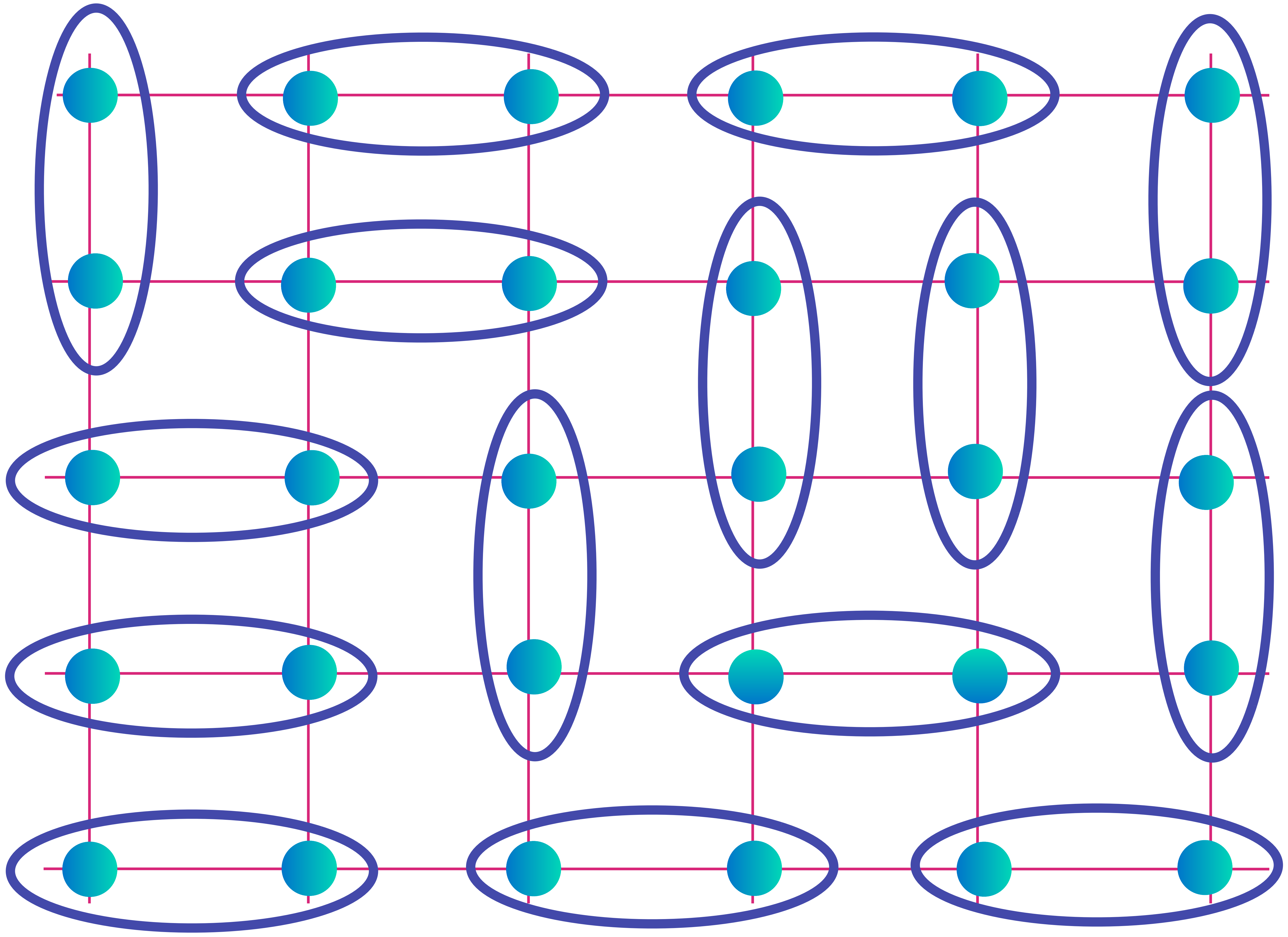


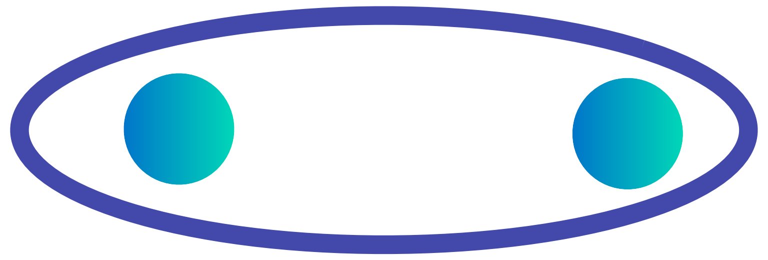
Antiferromagnetism

All nearest-neighbor pairs of electrons have opposite spins

The dance of electrons on Cu atoms in YBCO

Anderson, *Mater. Res. Bull.* **8**, 153 (1973)



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

Spin liquid

Electrons form entangled pairs, and the pairs entangle across the entire sample

Baskaran, Zou, and Anderson, *Solid State Comm.* **63**, 973 (1987)
Kivelson, Rokhsar, and Sethna, *Phys. Rev. B* **35**, 8865 (1987)
Rokhsar and Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)
Fradkin and Kivelson, *Mod. Phys. Lett. B* **4**, 225 (1990)

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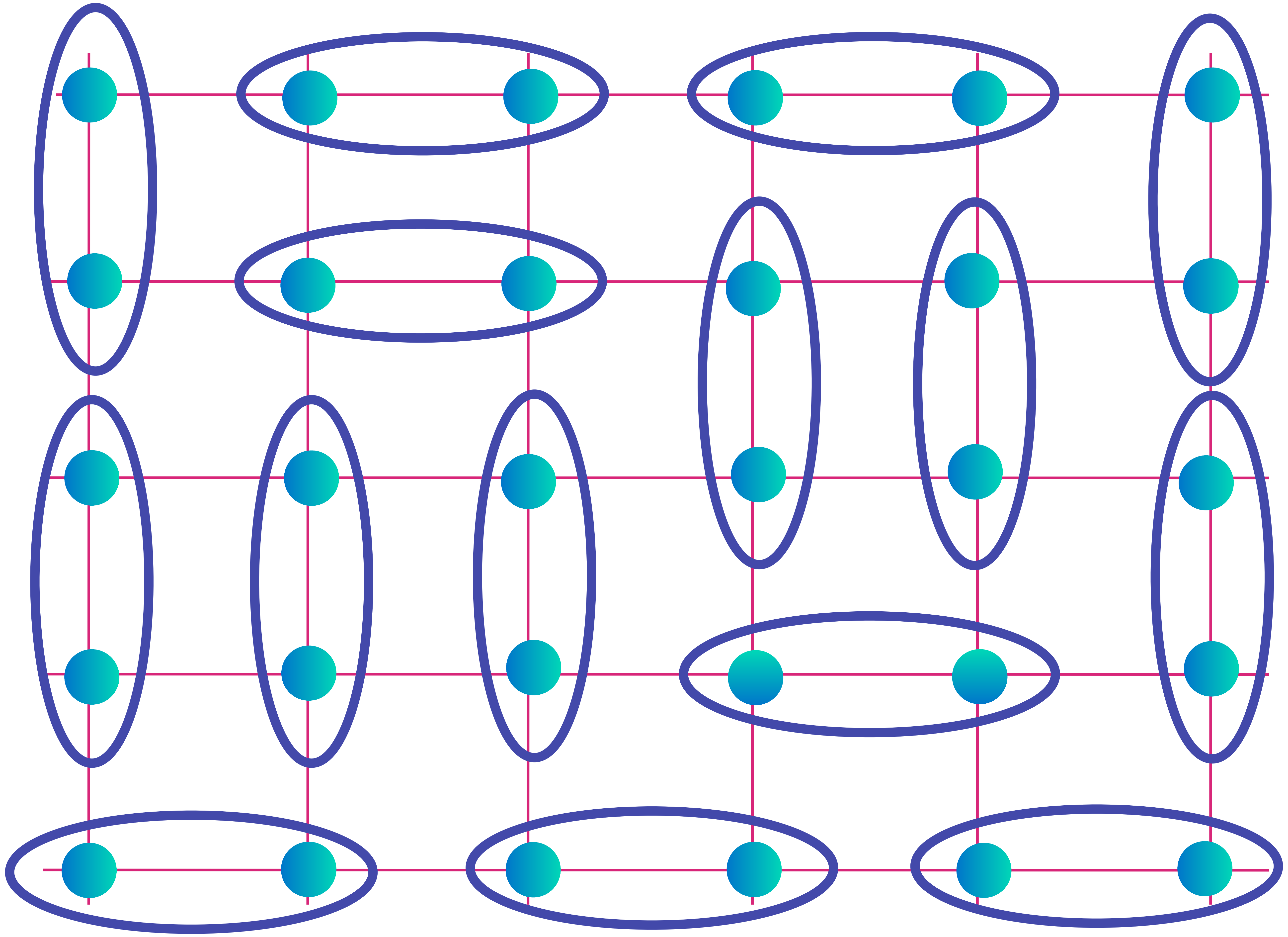

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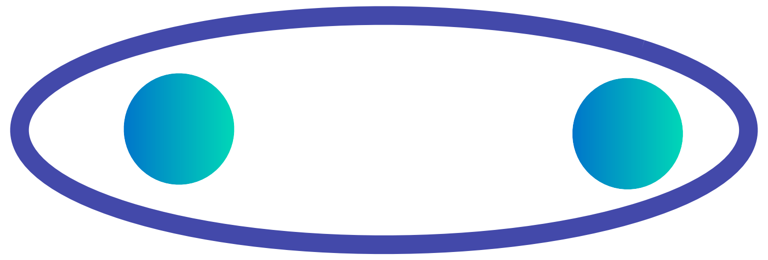

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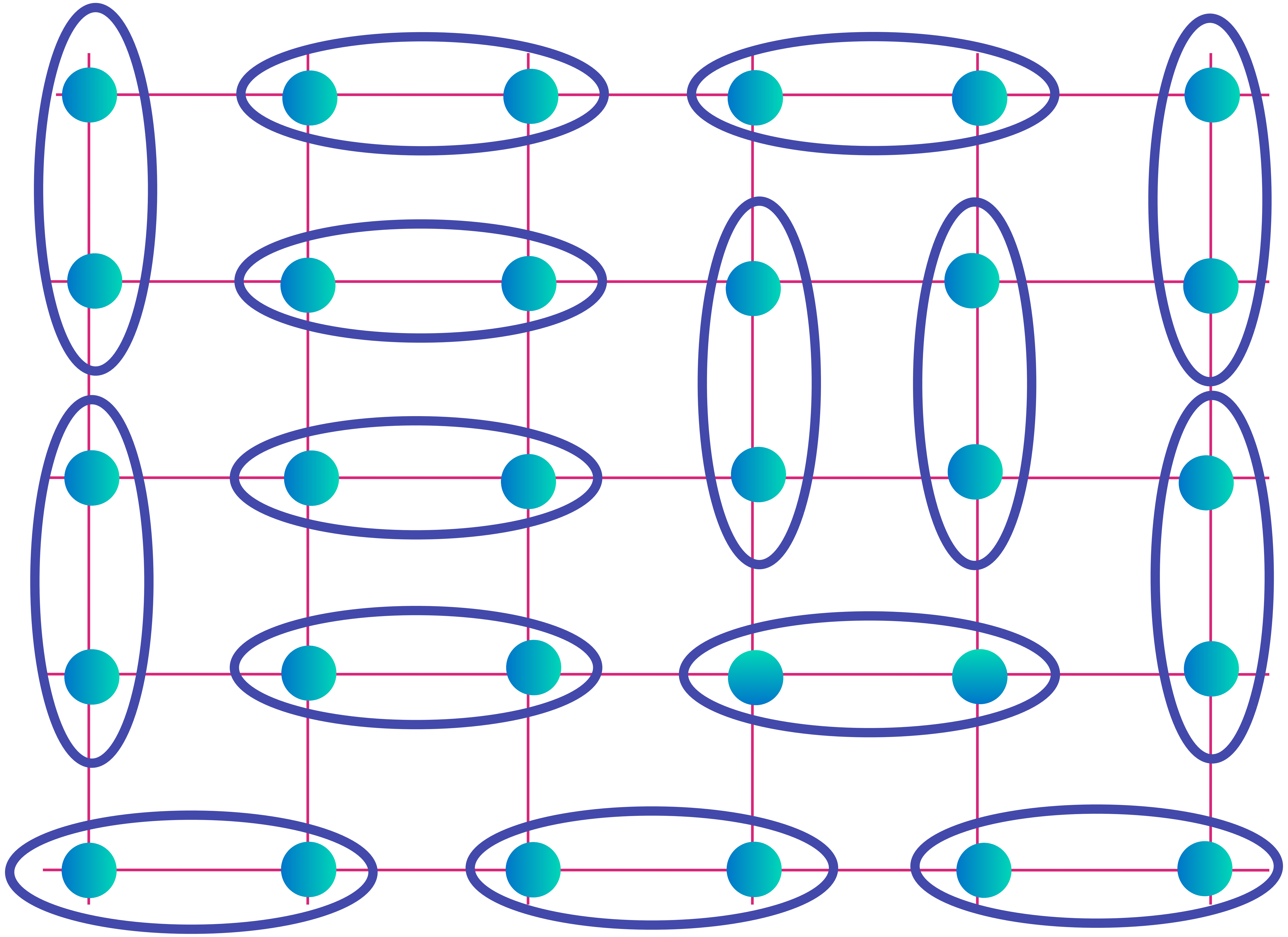

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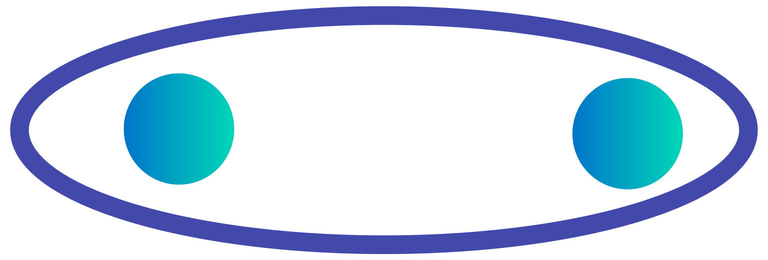 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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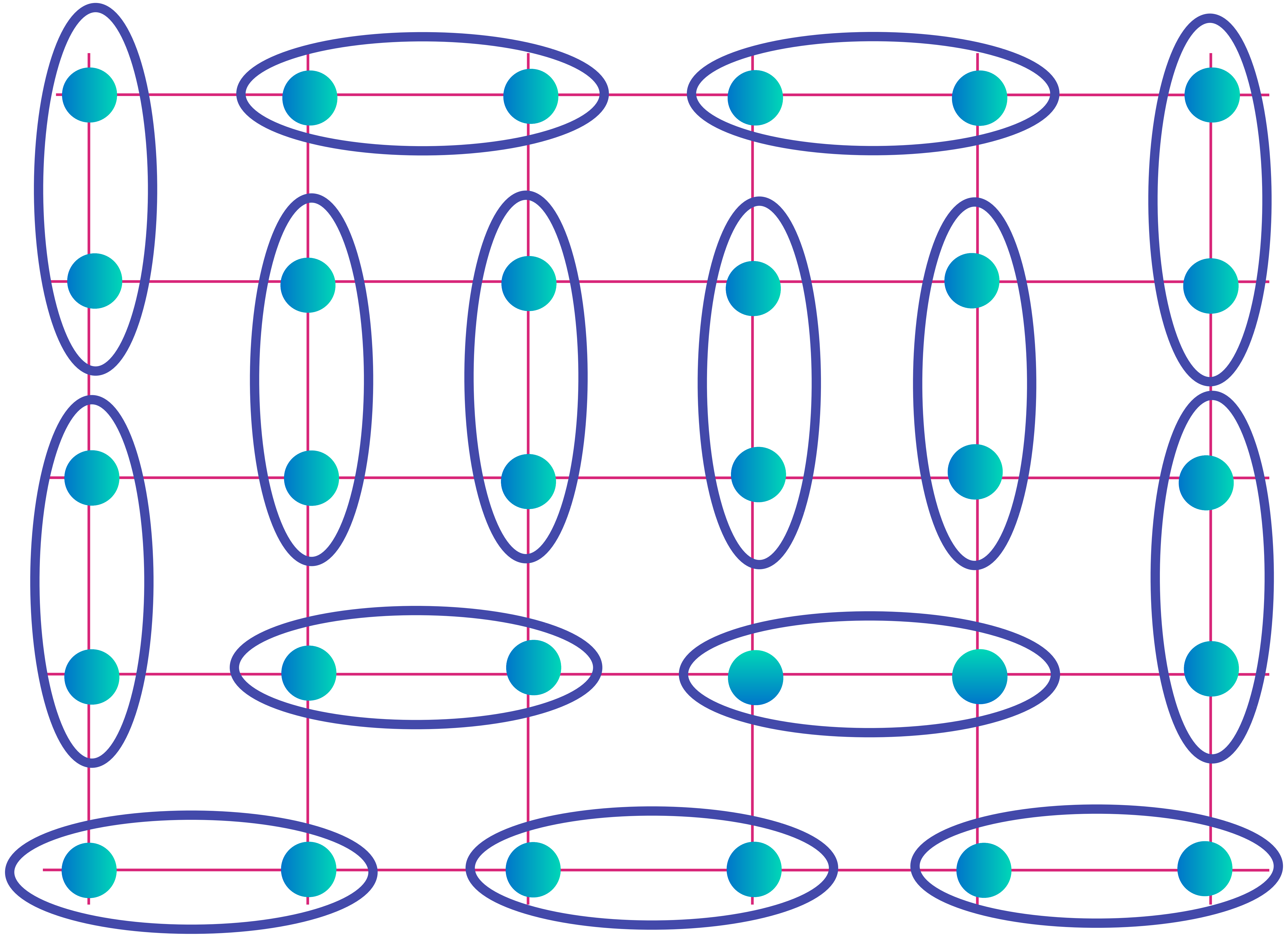

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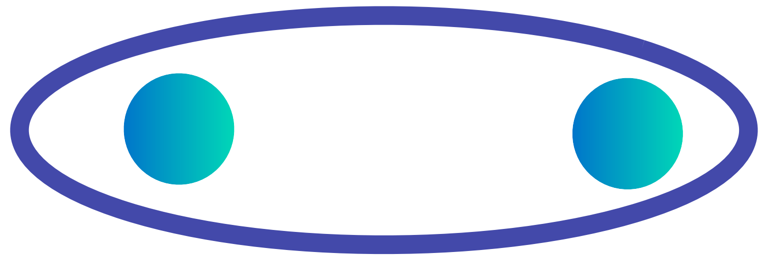 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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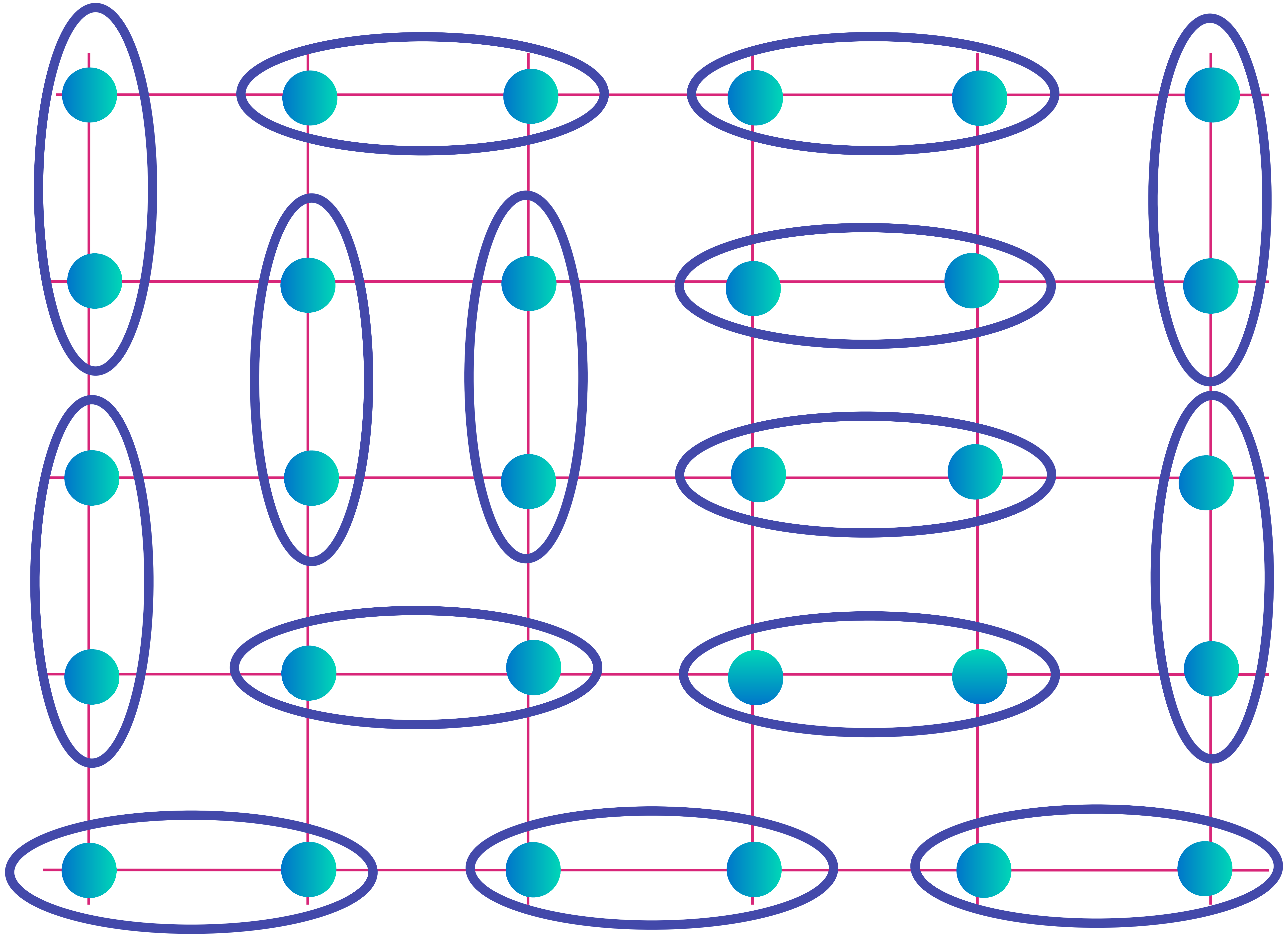

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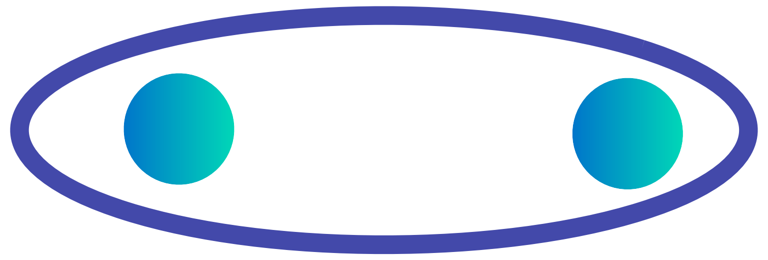

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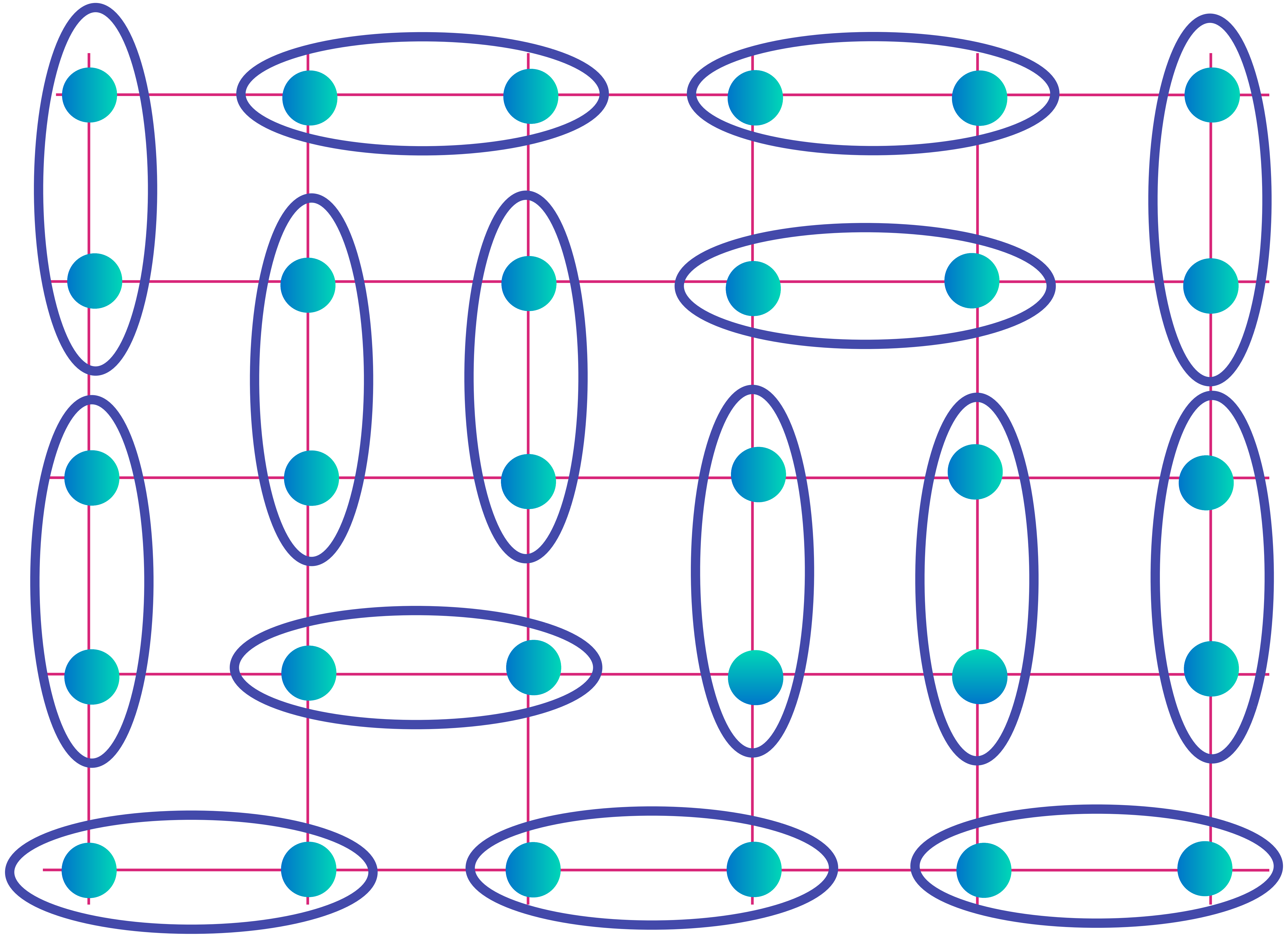

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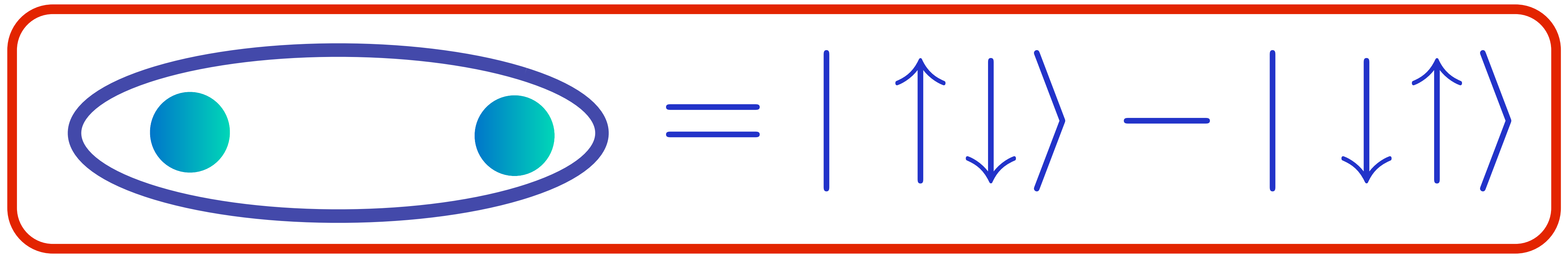

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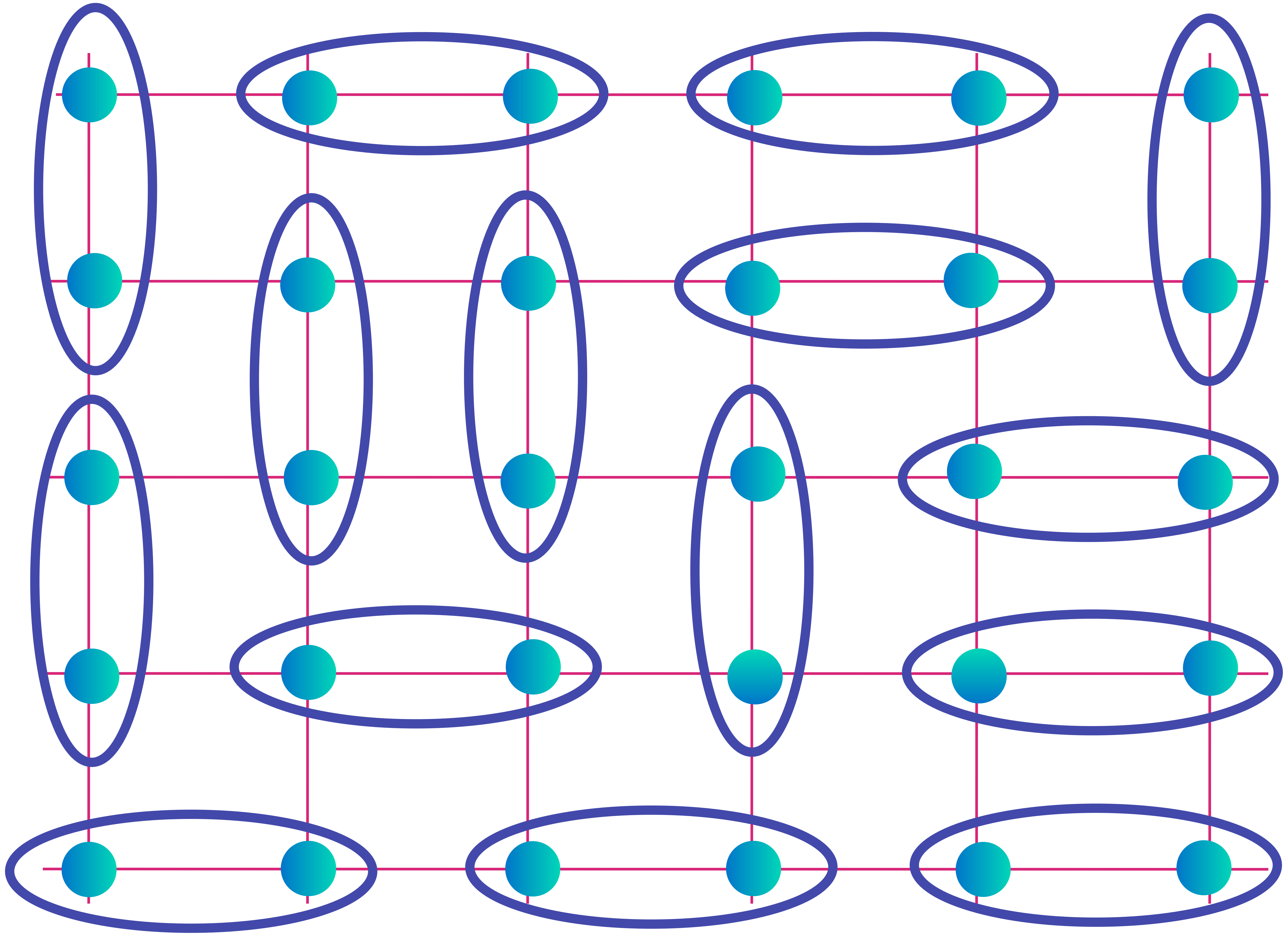

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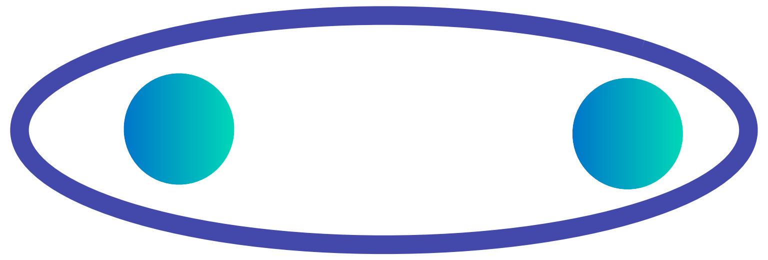

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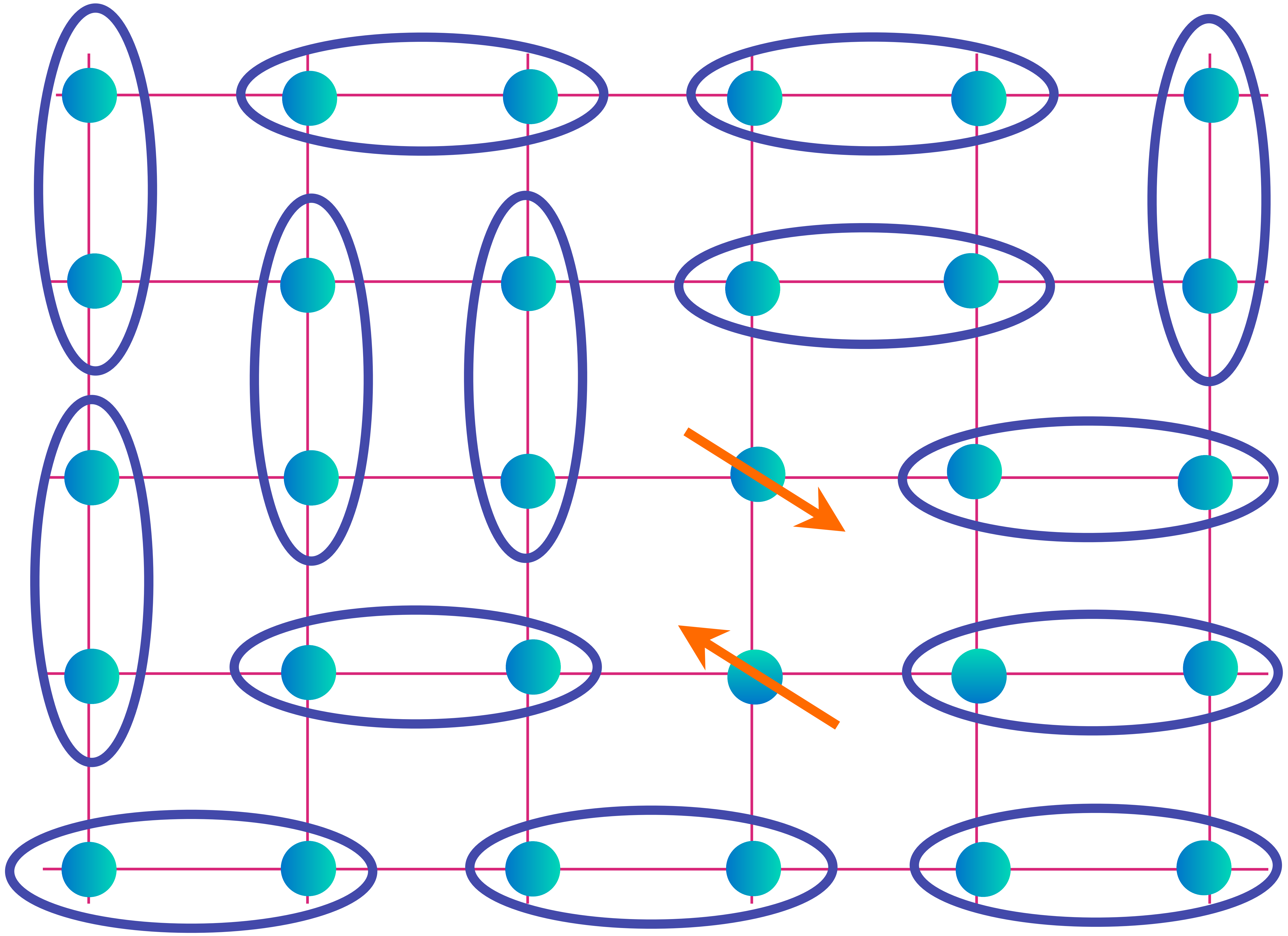

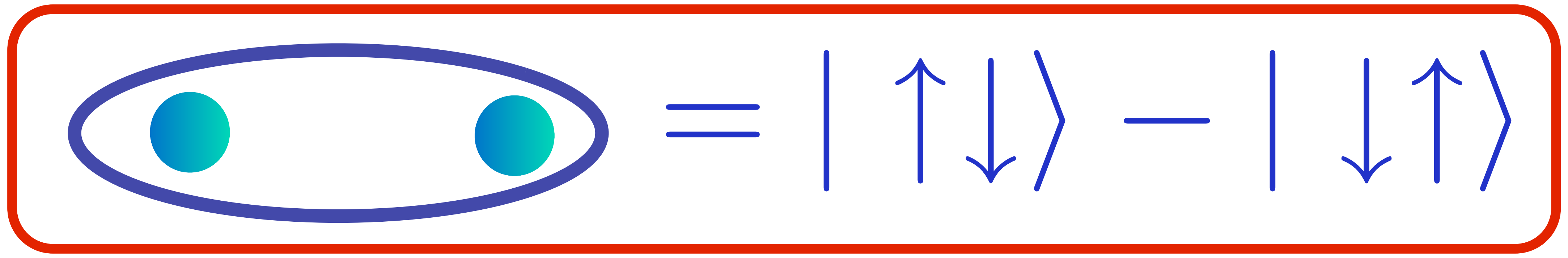

Spin liquid

Fractionalized spinon excitations with spin S=1/2 and charge 0.

Baskaran, Zou, and Anderson, *Solid State Comm.* **63**, 973 (1987)
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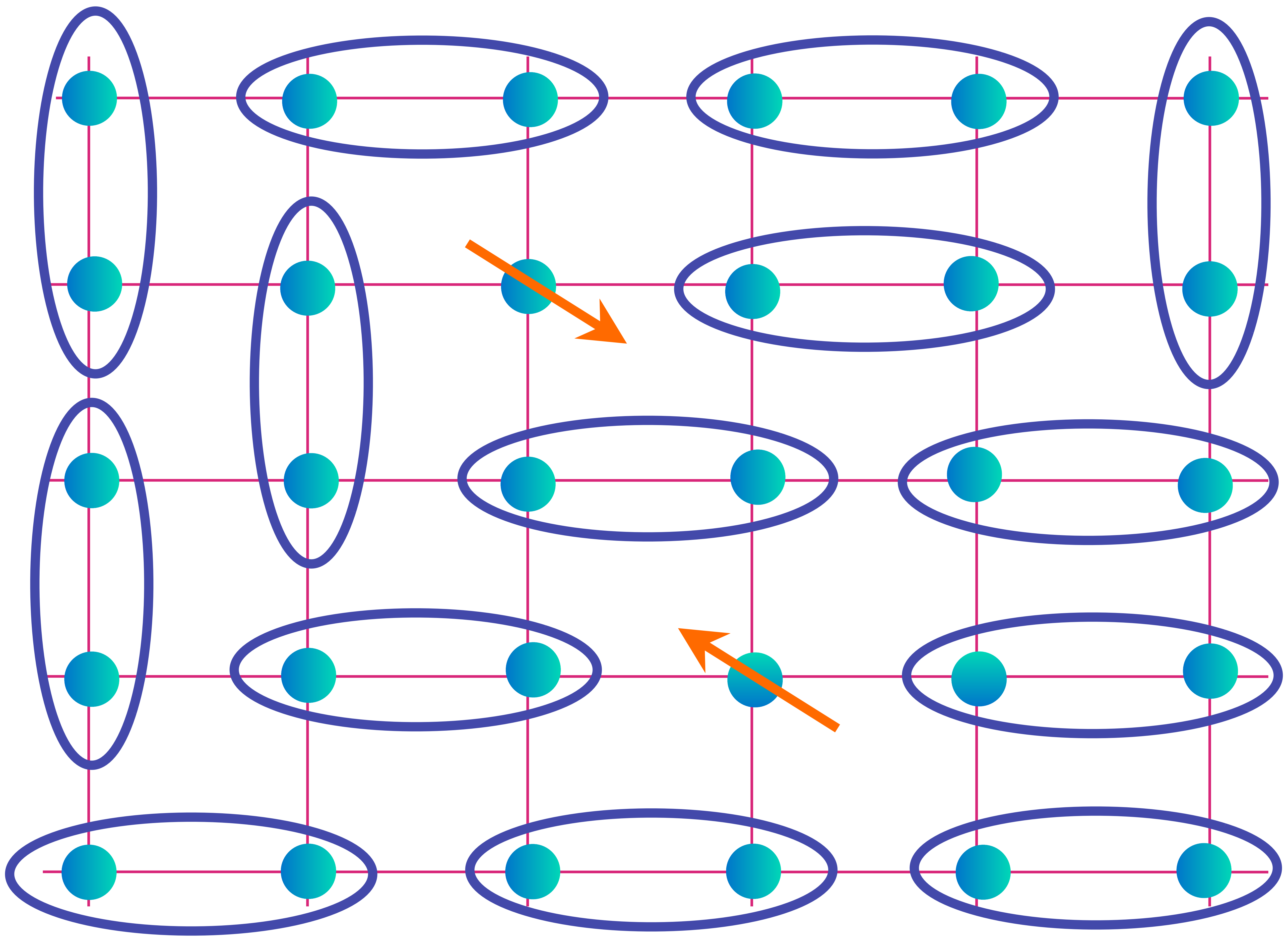

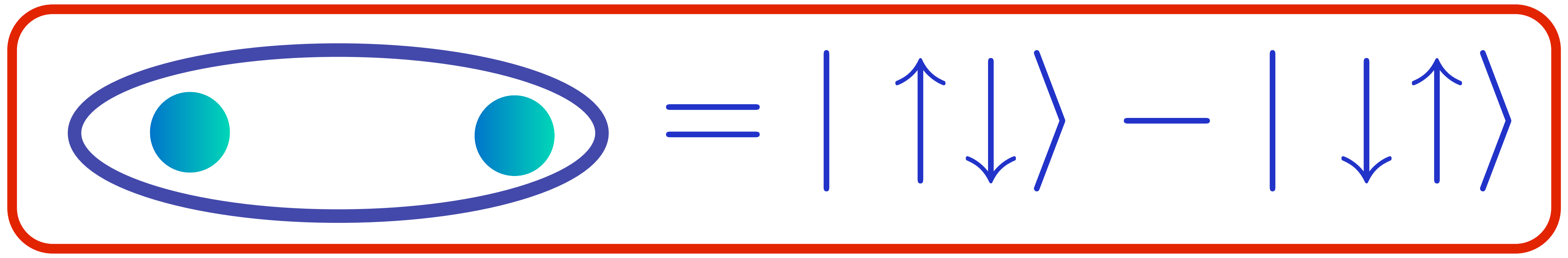

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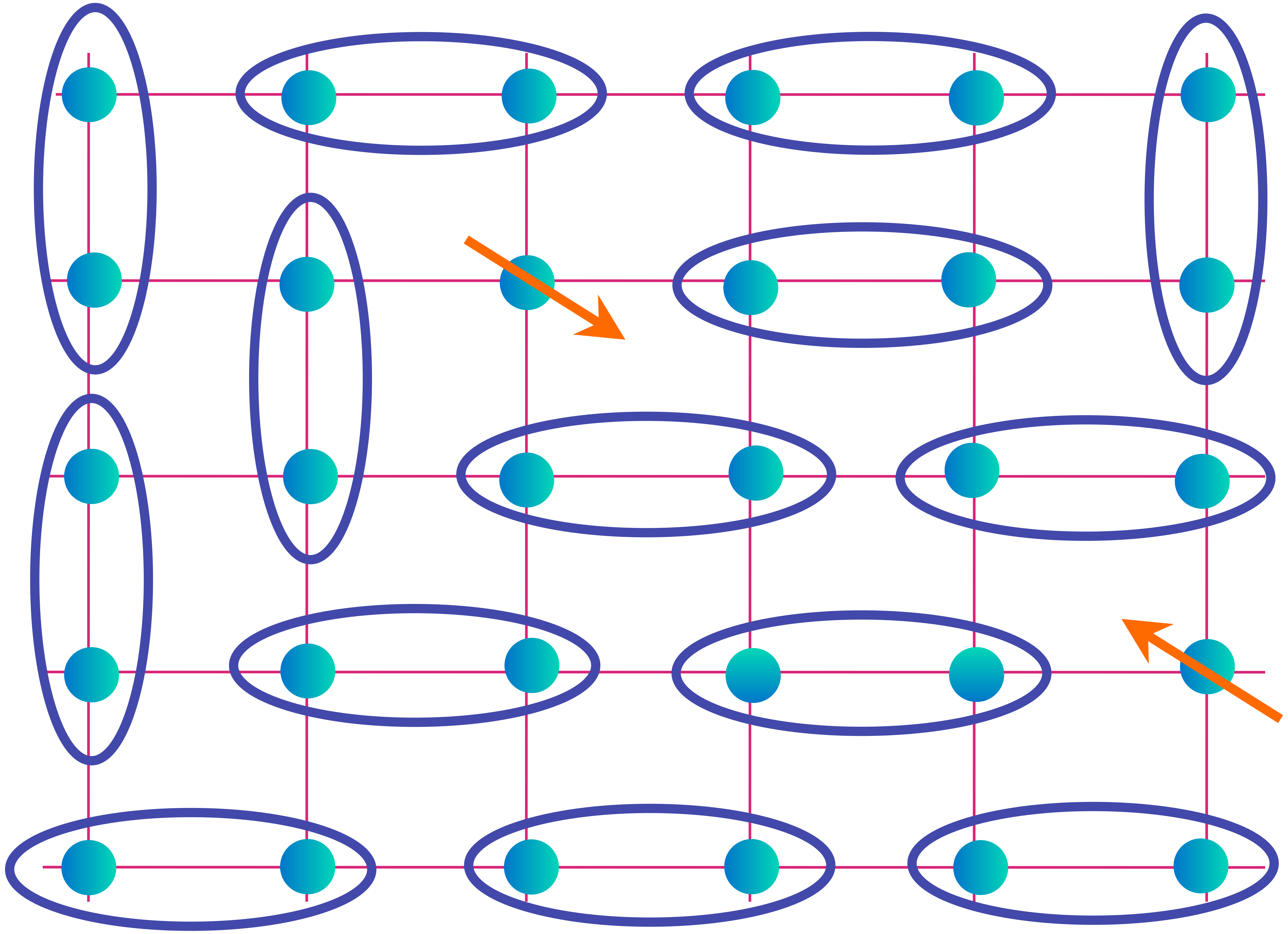

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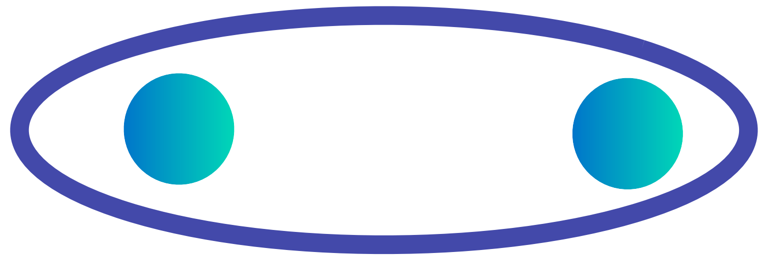

$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

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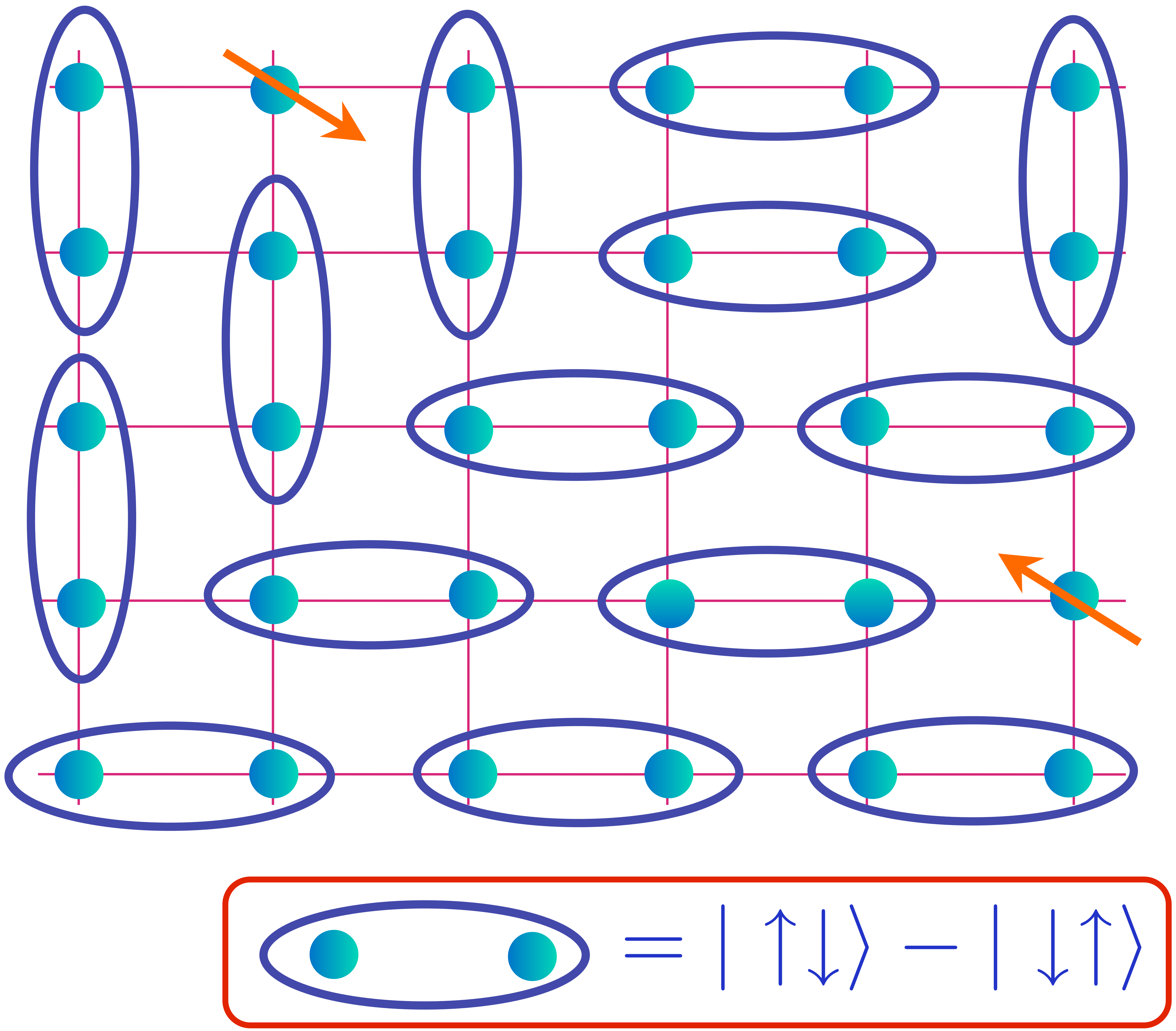

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Spin liquid

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Holons

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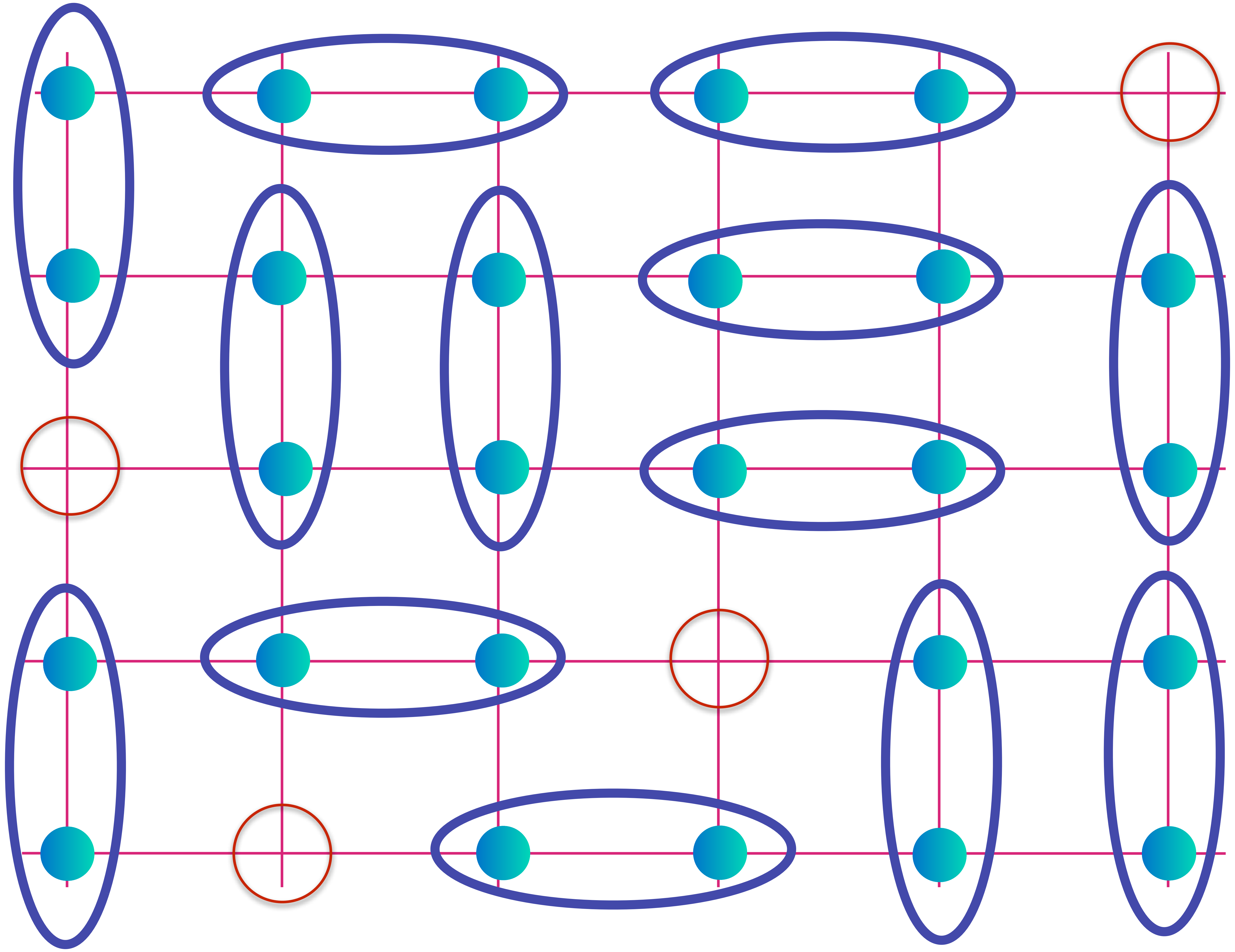
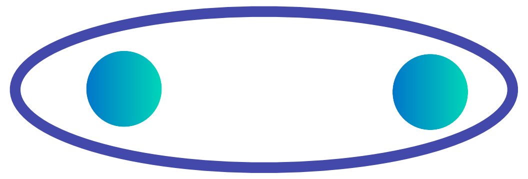


Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Holons

Baskaran, Zou, and Anderson, *Solid State Comm.* **63**, 973 (1987)
Kivelson, Rokhsar, and Sethna, *Phys. Rev. B* **35**, 8865 (1987)
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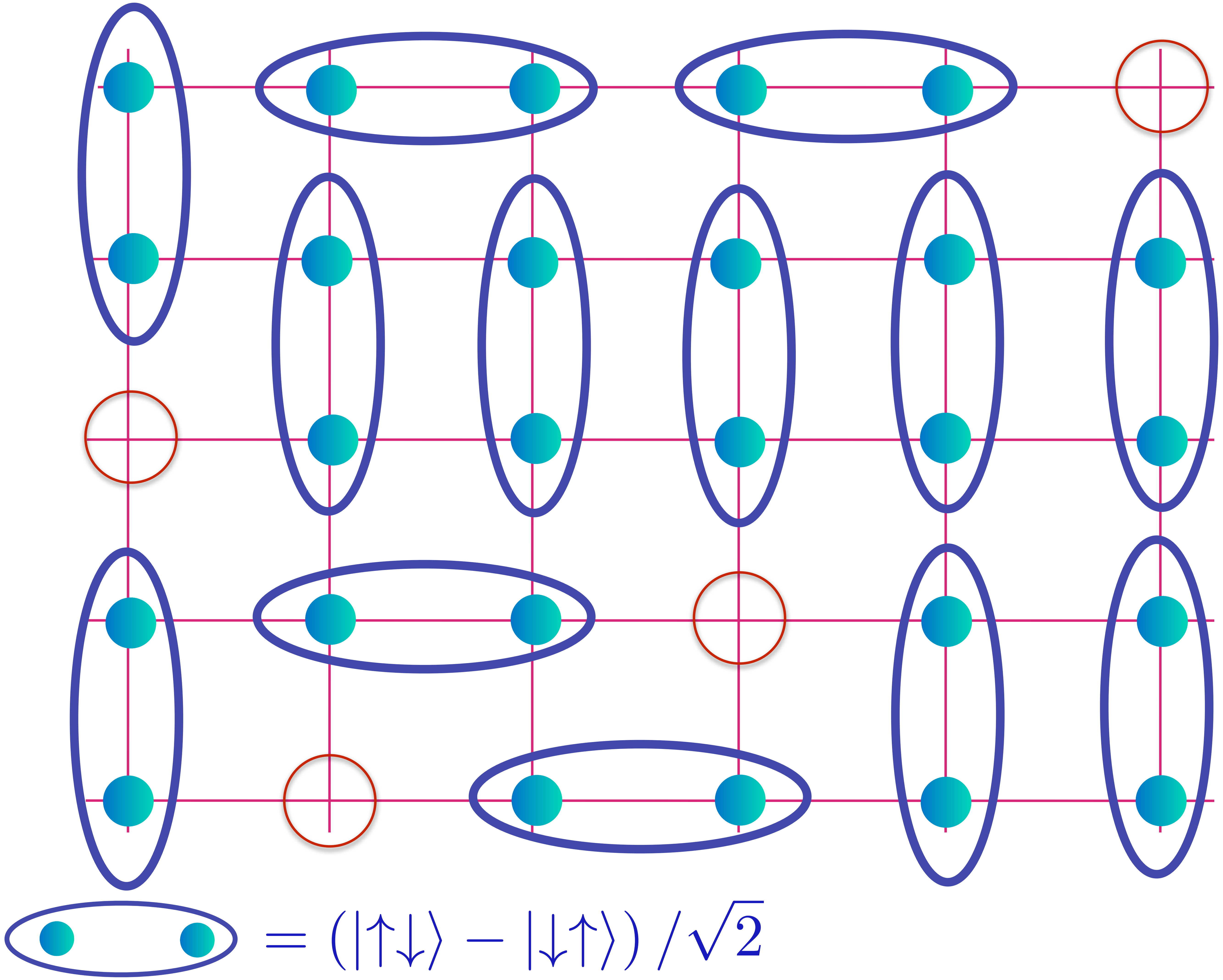

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid

Fractionalized holon excitations with spin S=0 and charge +e.

Holons

Baskaran, Zou, and Anderson, *Solid State Comm.* **63**, 973 (1987)
Kivelson, Rokhsar, and Sethna, *Phys. Rev. B* **35**, 8865 (1987)
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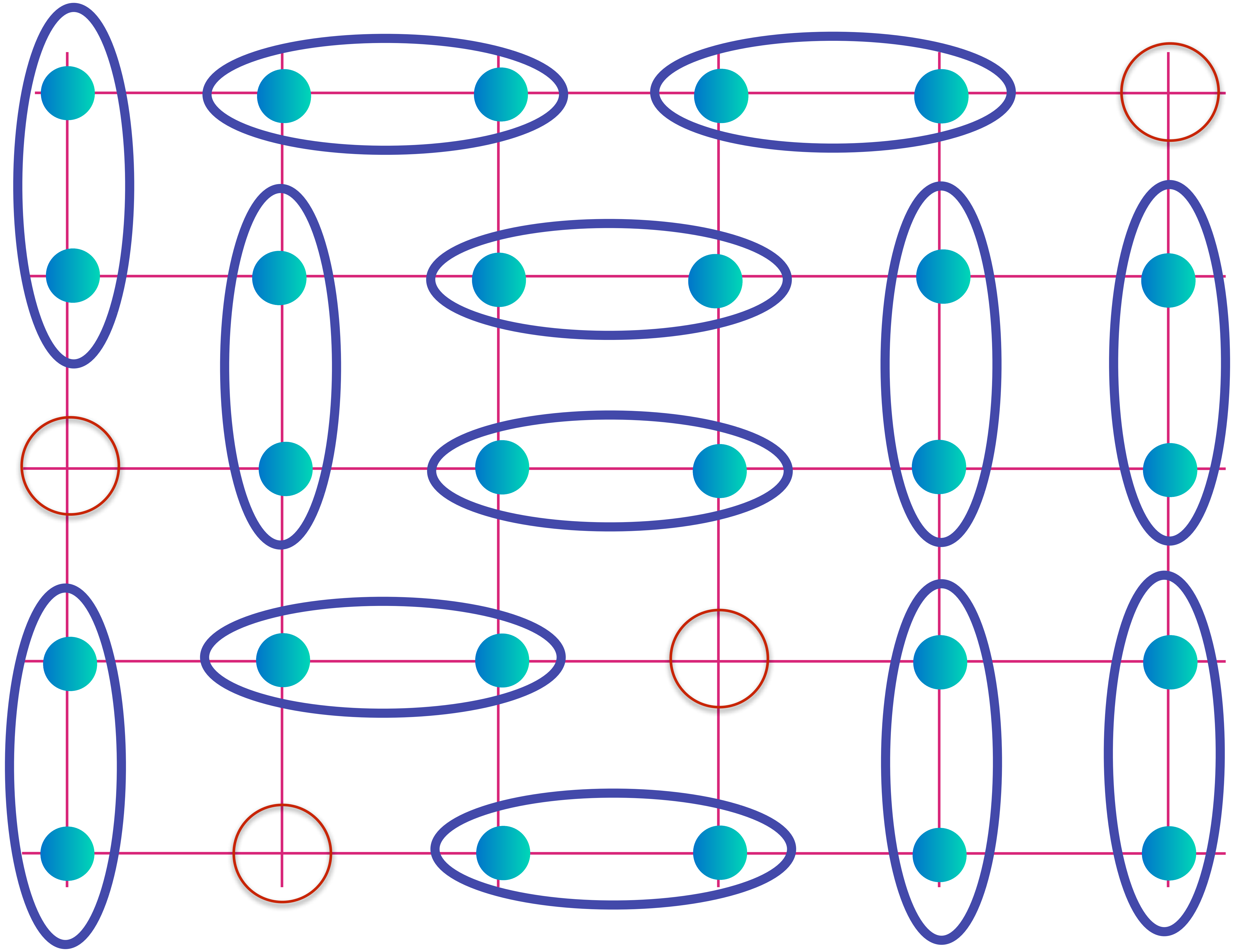

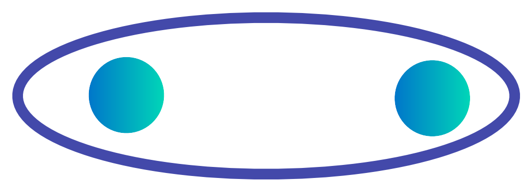 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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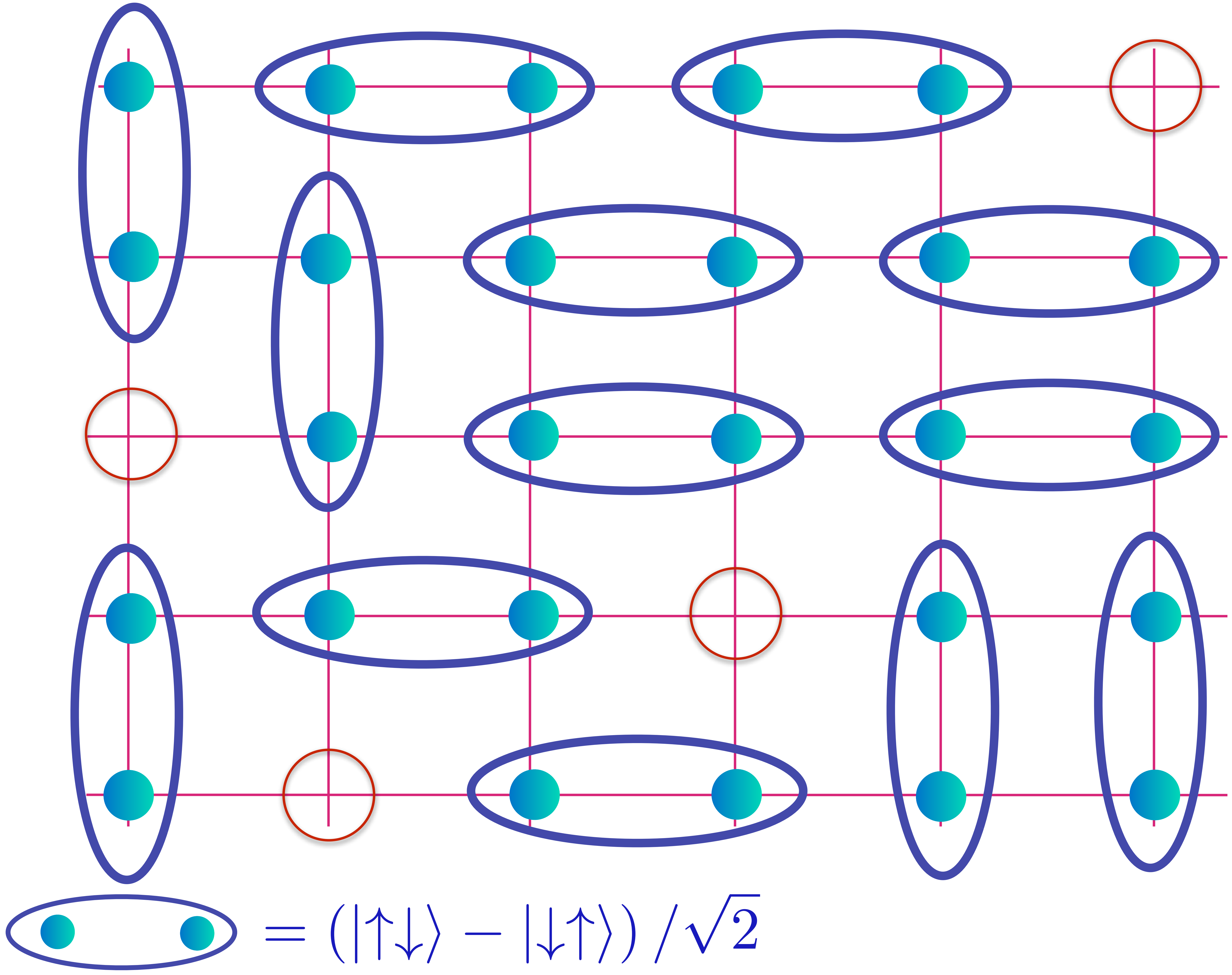

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Holon metal

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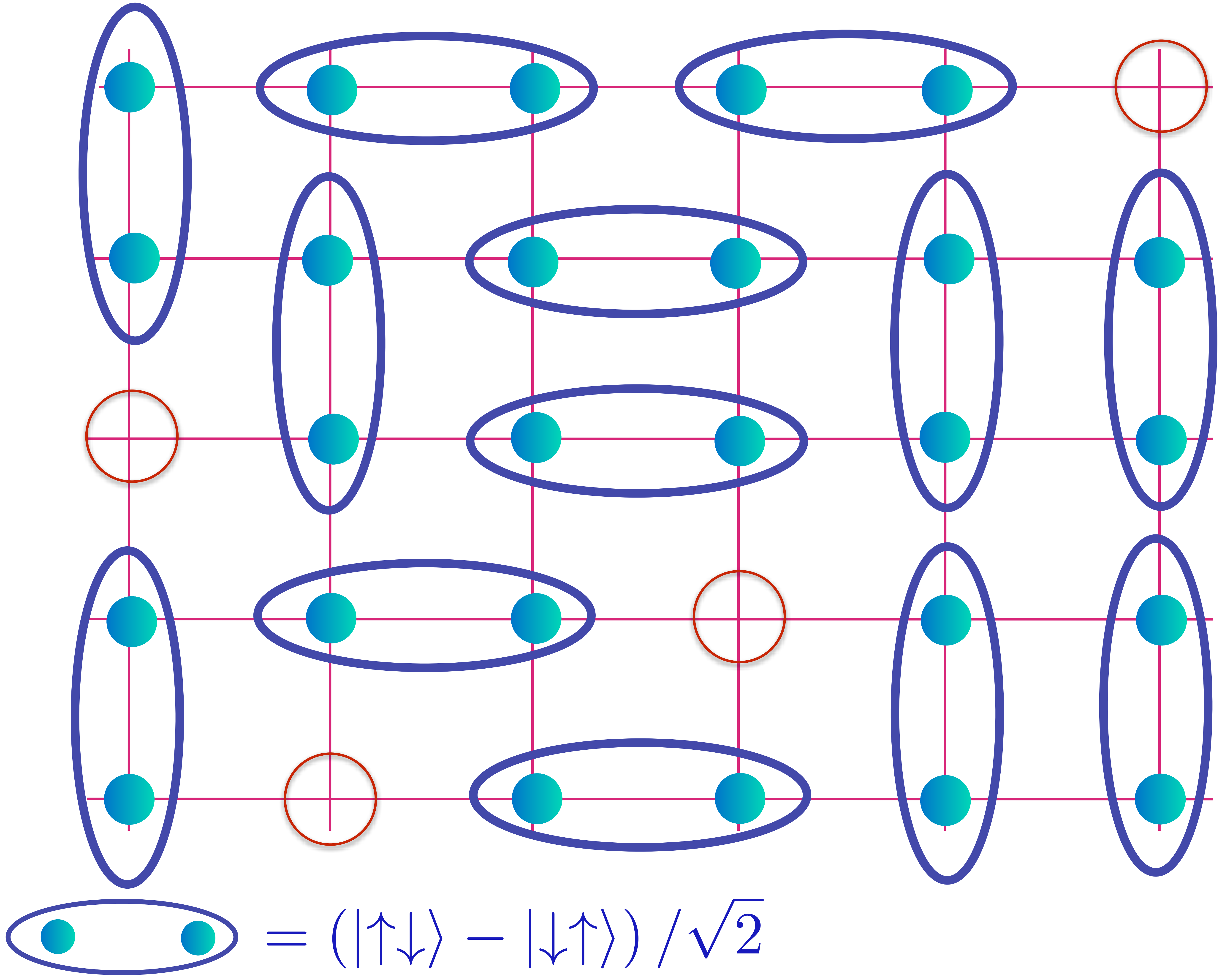

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

If each holon is a fermion, we obtain a Fermi surface of holons of size p

Holon metal

Baskaran, Zou, and Anderson, *Solid State Comm.* **63**, 973 (1987)
Kivelson, Rokhsar, and Sethna, *Phys. Rev. B* **35**, 8865 (1987)
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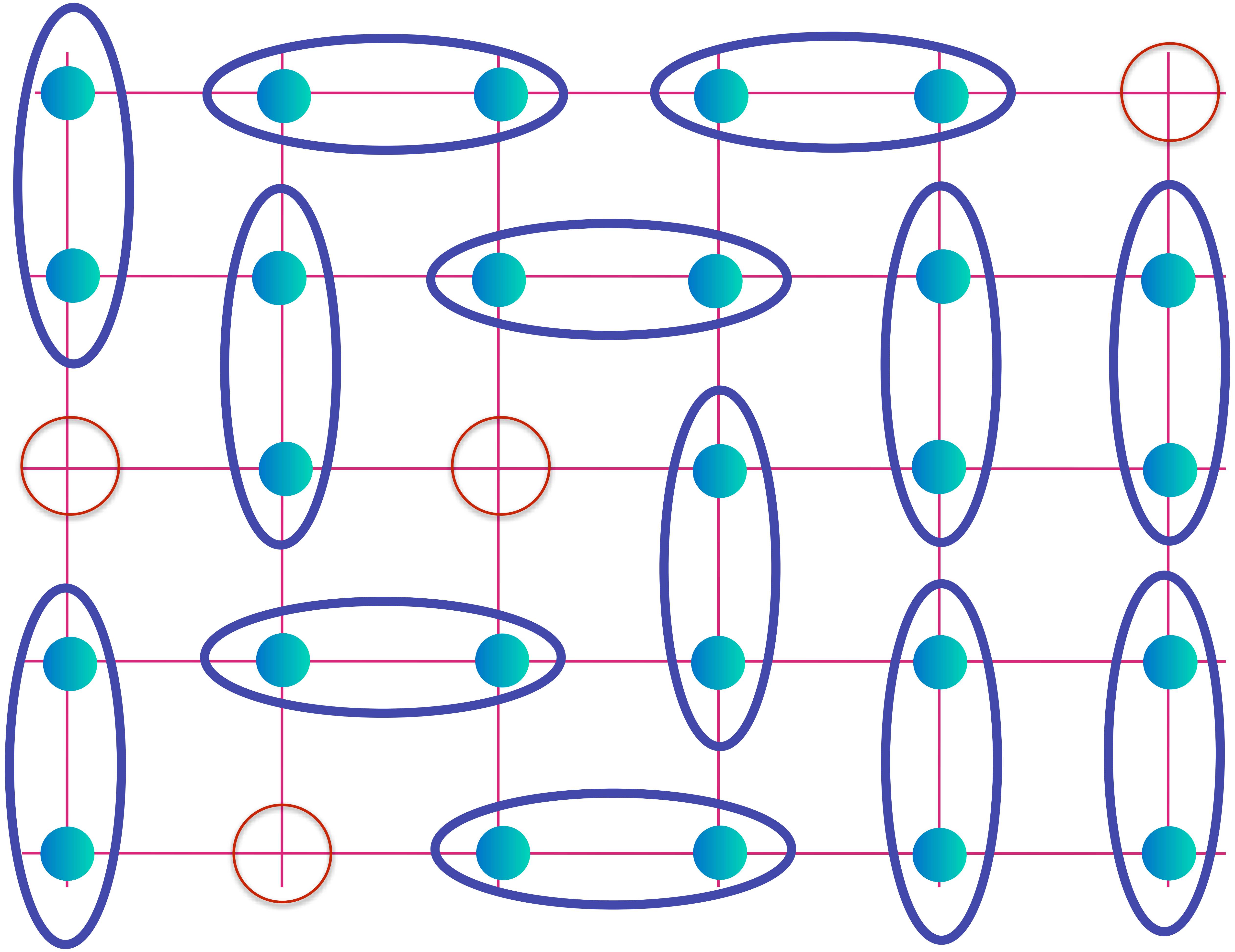

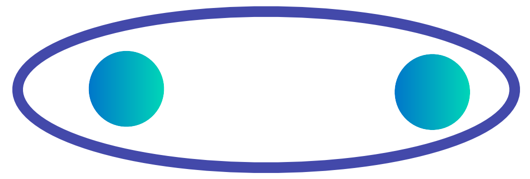
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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~~Holon metal~~

Baskaran, Zou, and Anderson, *Solid State Comm.* **63**, 973 (1987)
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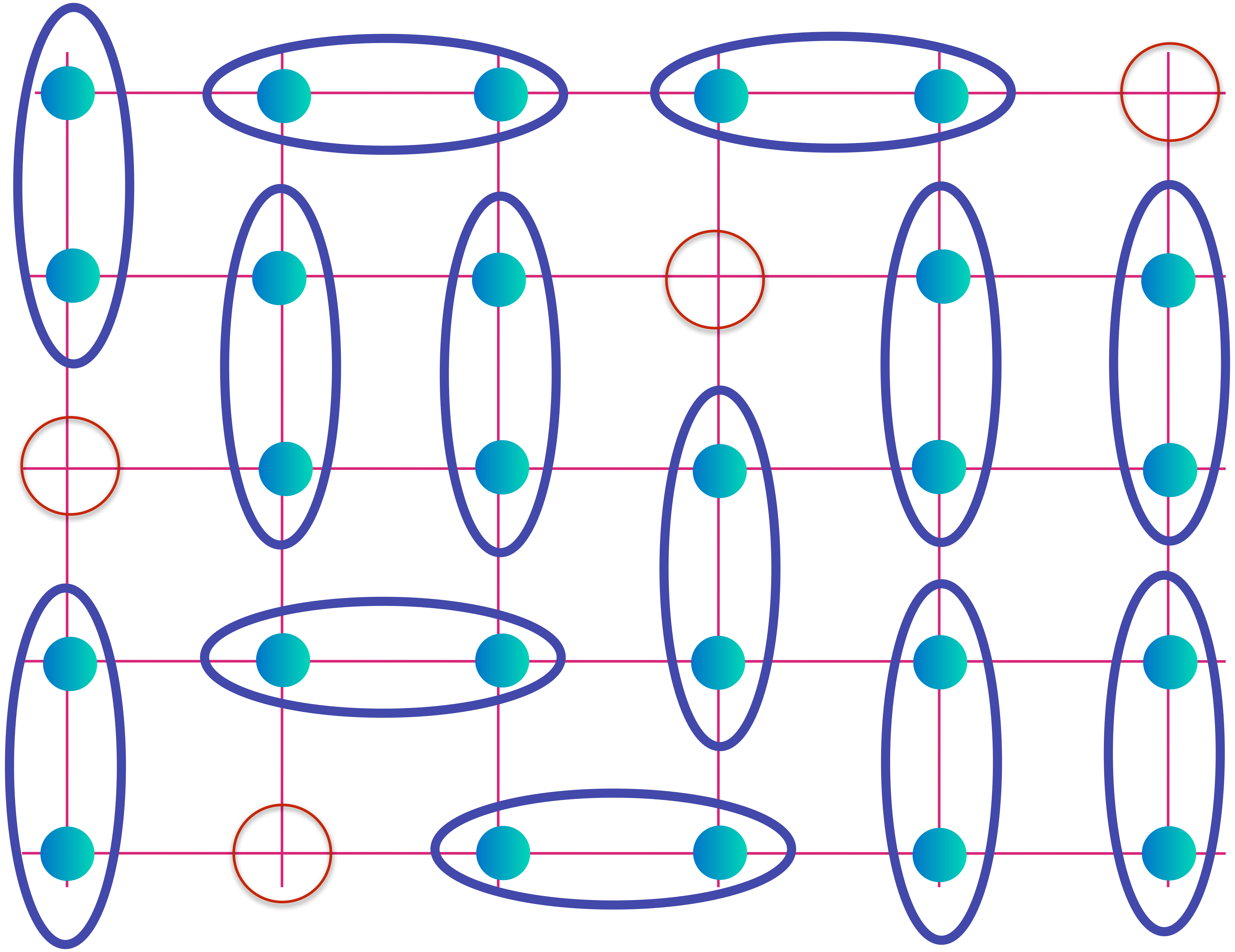

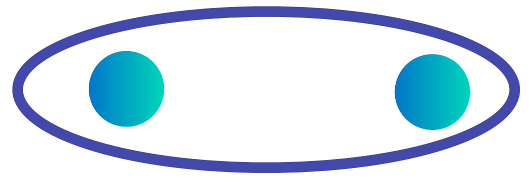 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) / \sqrt{2}$

If each holon is a fermion, we obtain a Fermi surface of holons of size p

Cannot explain pseudogap metal

~~Holon metal~~

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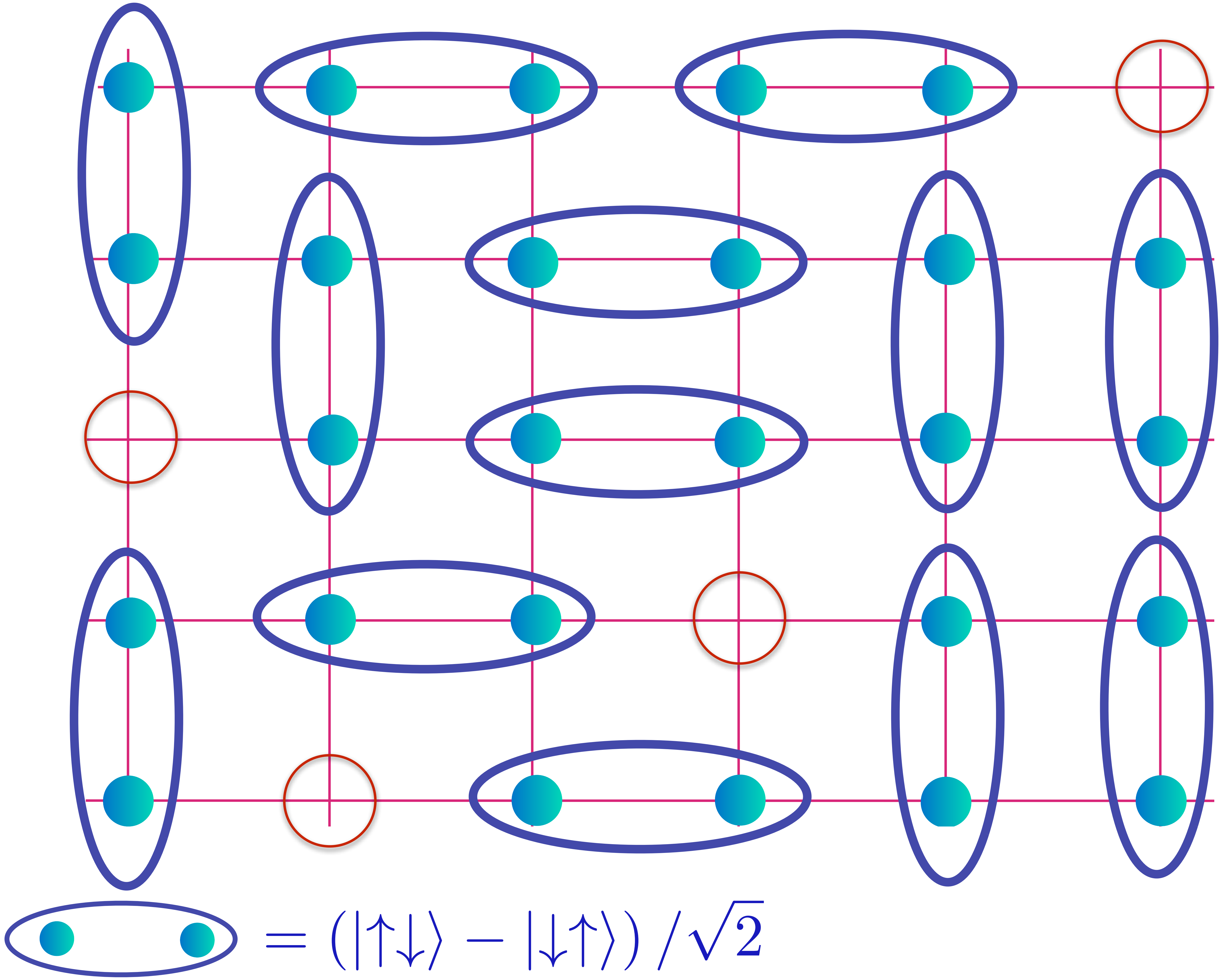

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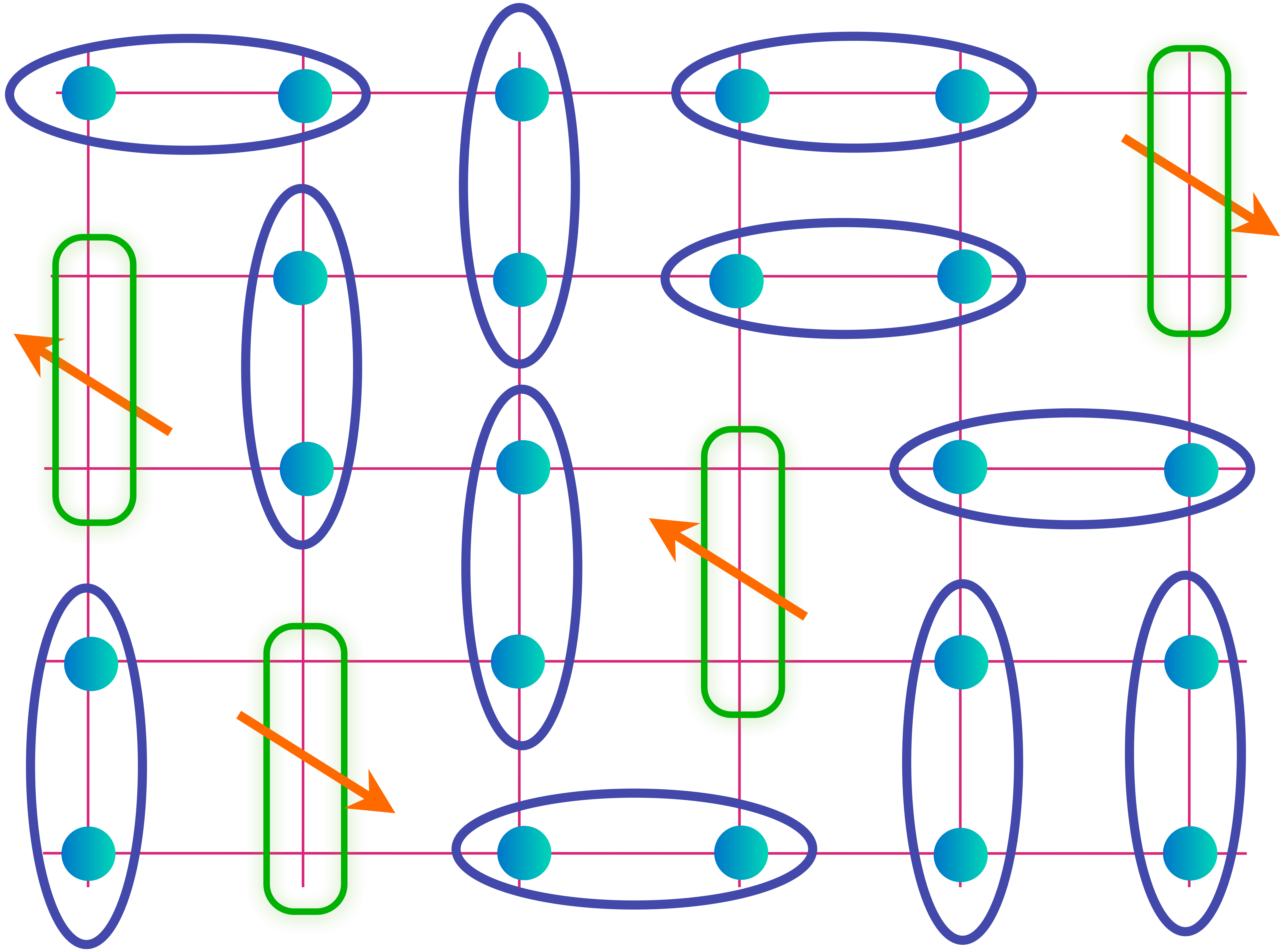
If each holon is a fermion, we obtain a Fermi surface of holons of size p

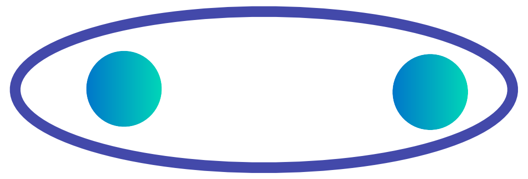
Cannot explain pseudogap metal

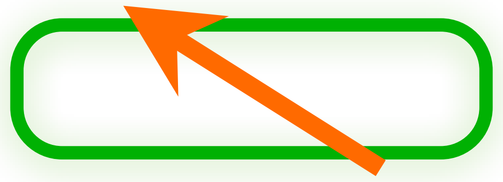
FL* in a ***one-band*** model

Sachdev, *Phys. Rev. B* **49**, 6770 (1994); Wen and Lee, *Phys. Rev. Lett.* **76**, 503 (1996)
Kaul, Kolezhuk, Levin, Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

Moon and Sachdev, *Phys. Rev. B* **83**, 224508 (2011); Punk, Allais, and Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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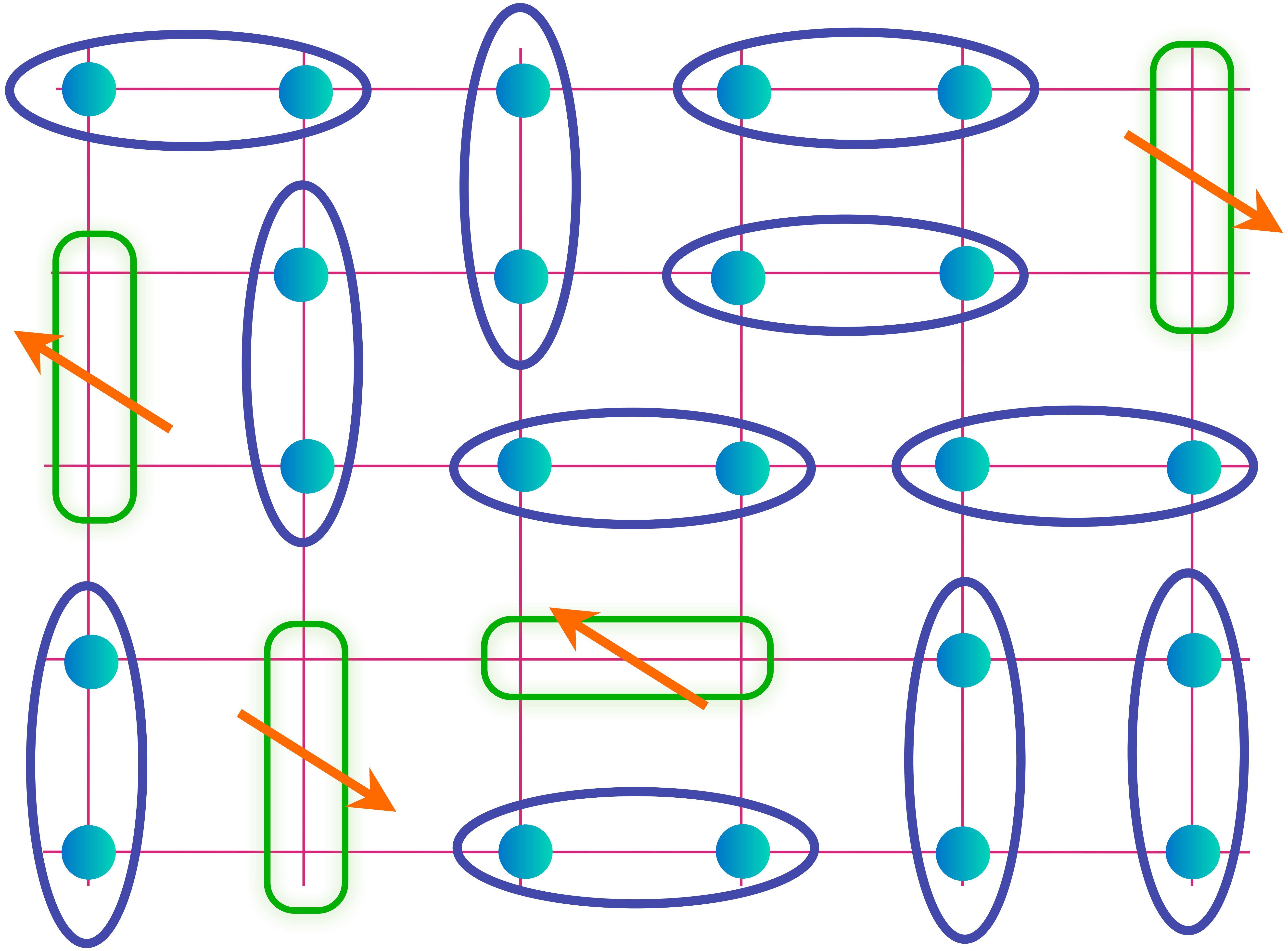
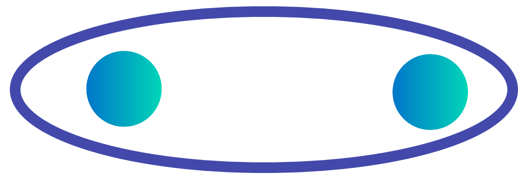
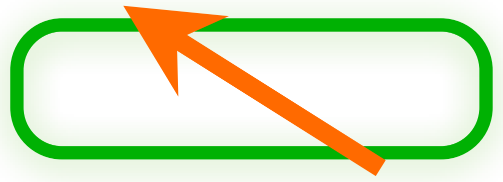


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

Moon and Sachdev, *Phys. Rev. B* **83**, 224508 (2011); Punk, Allais, and Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

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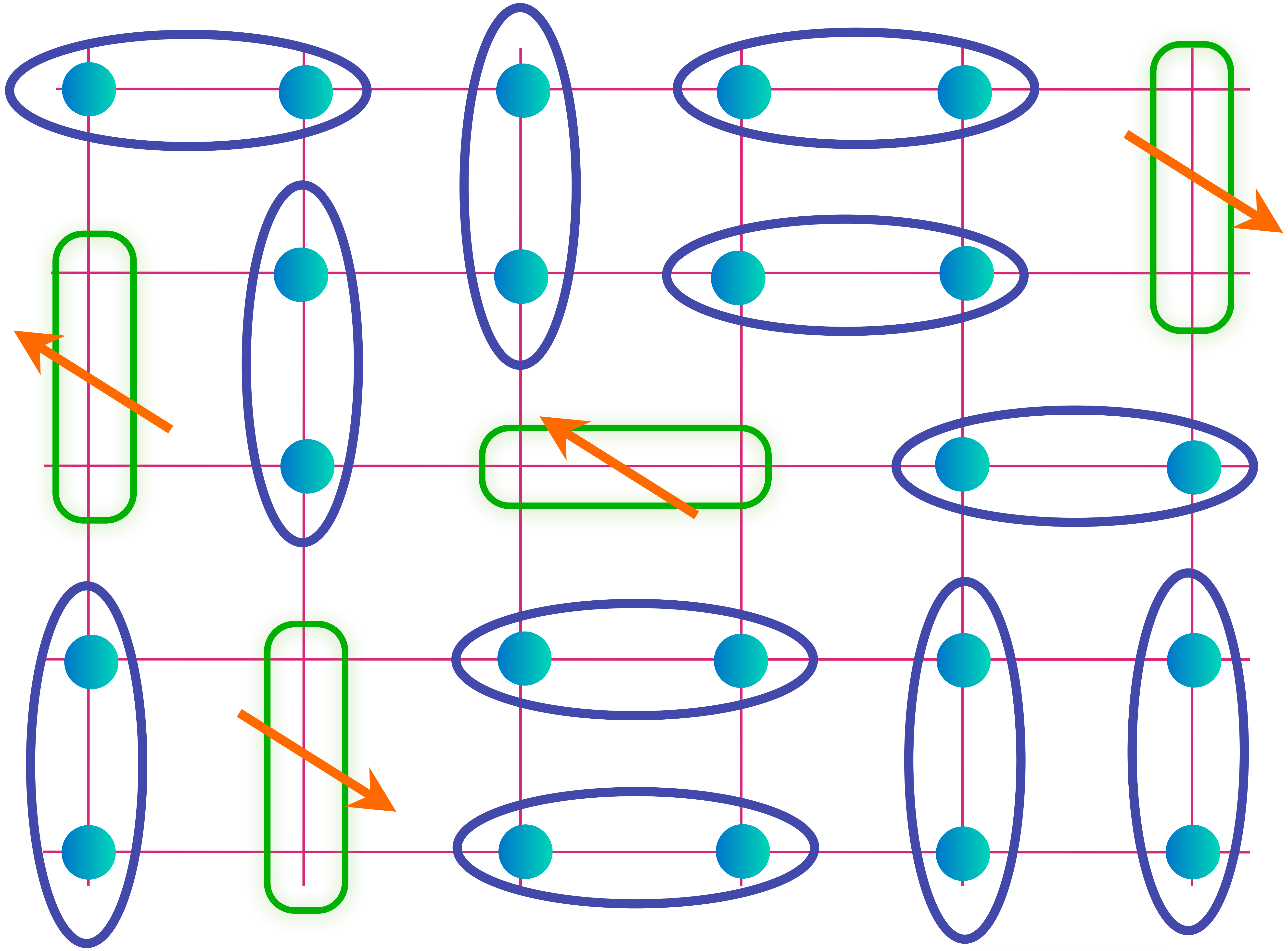

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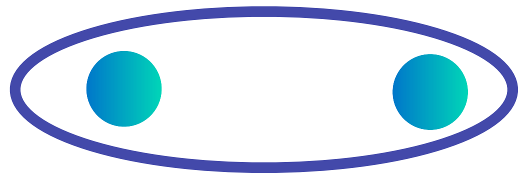
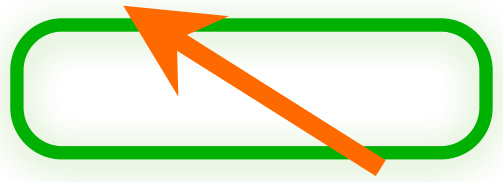
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

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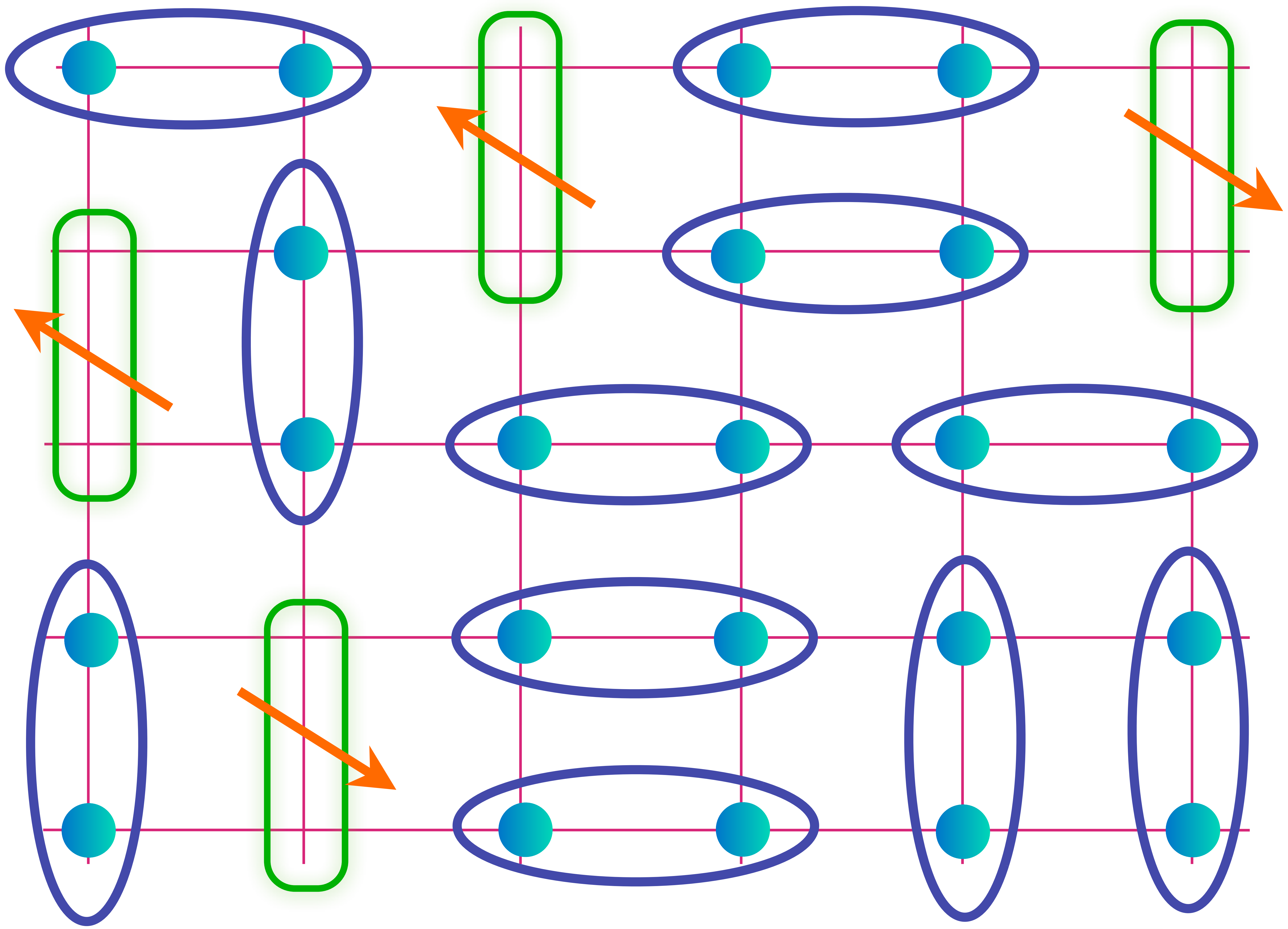

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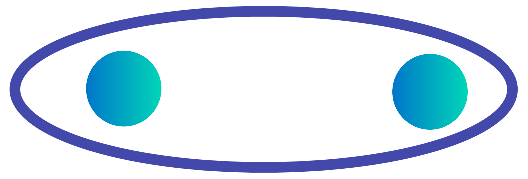
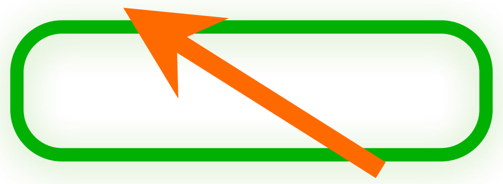


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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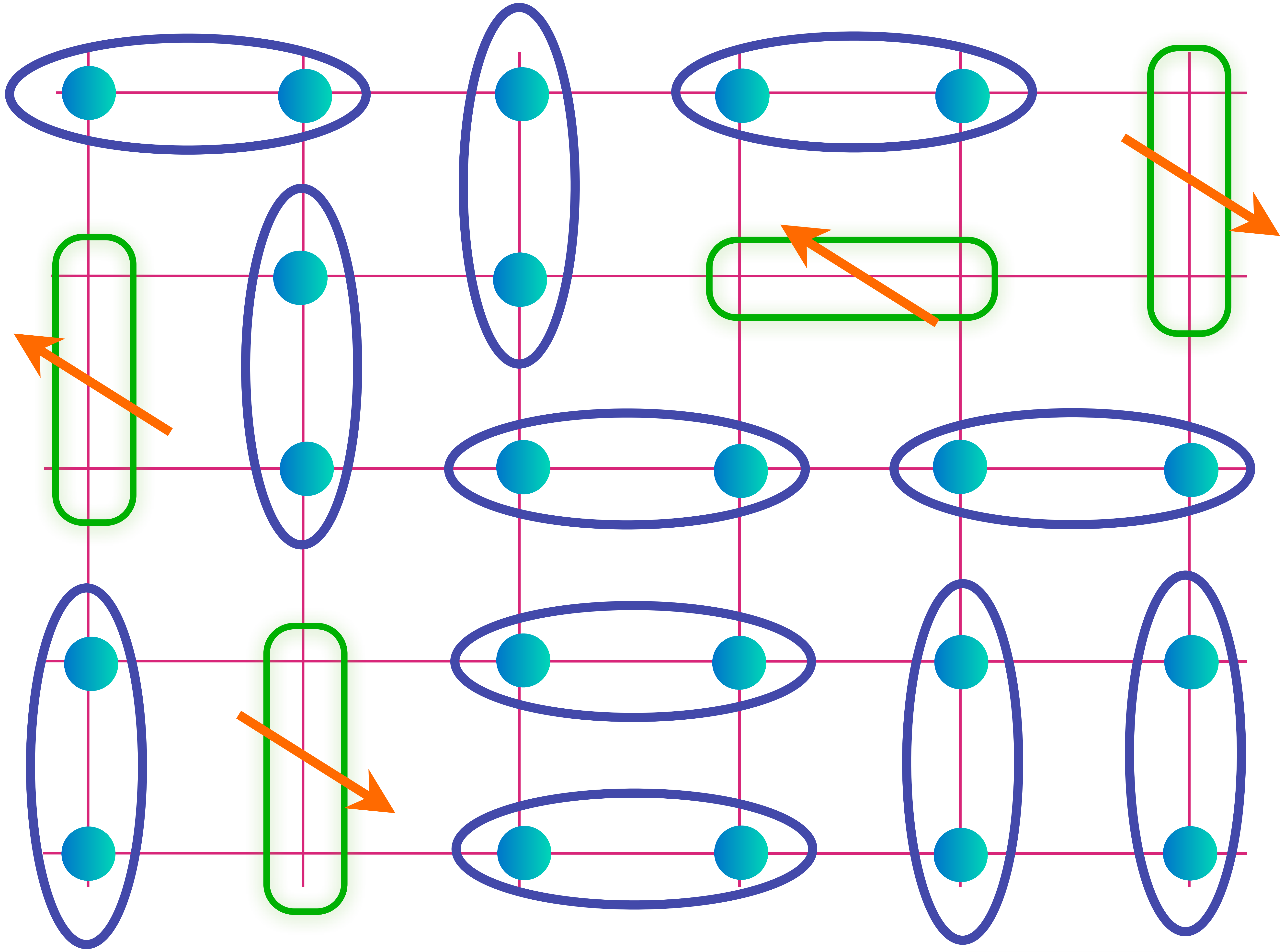

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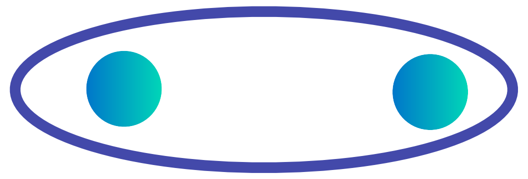
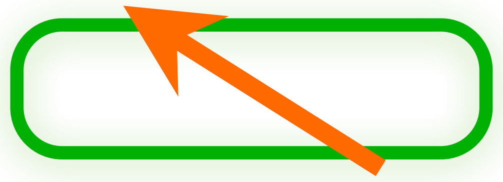


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) / \sqrt{2}$



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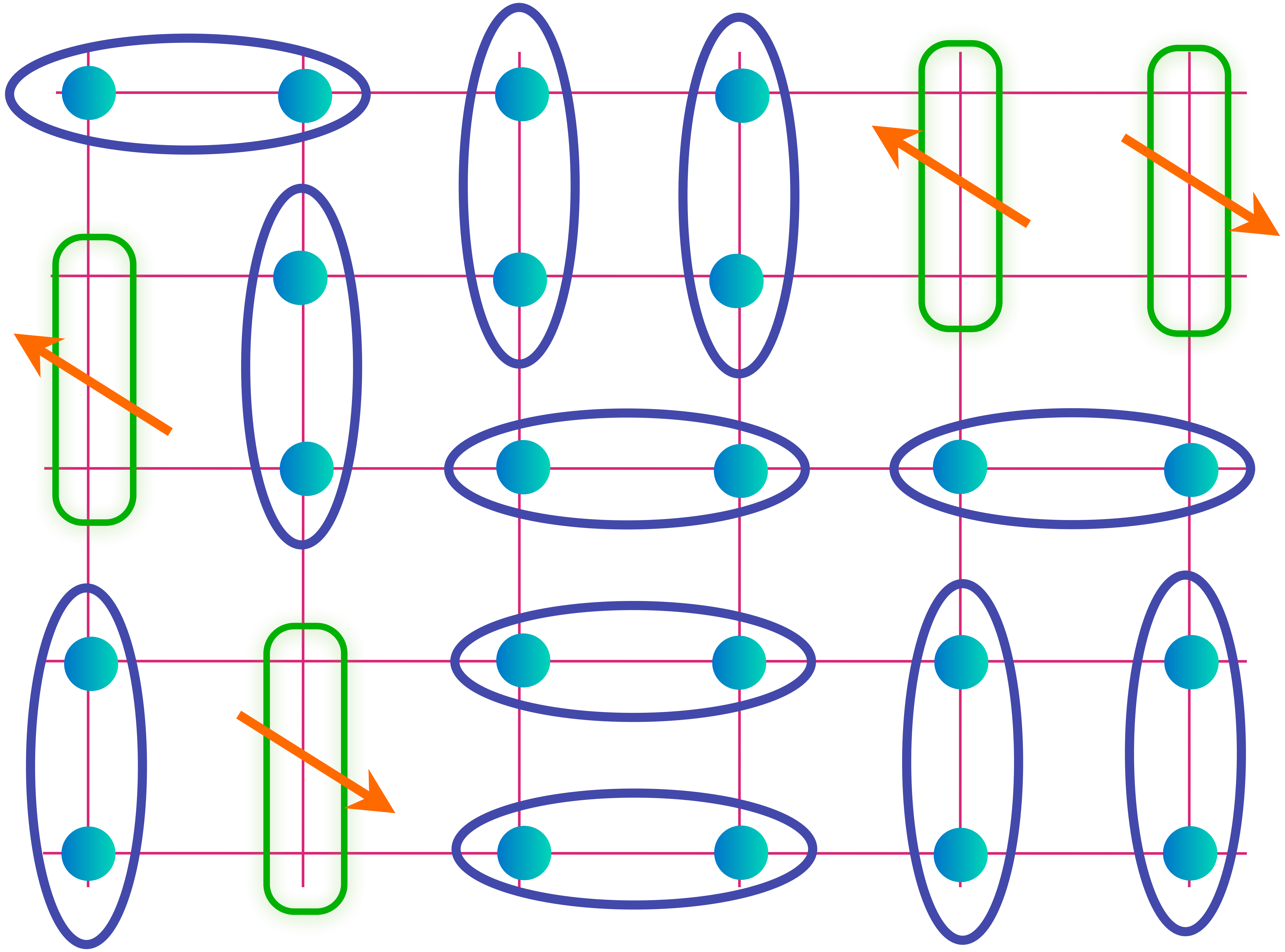

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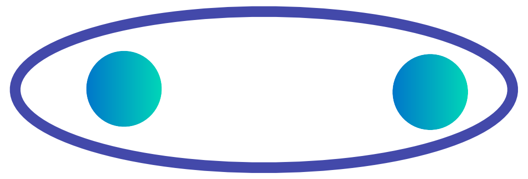

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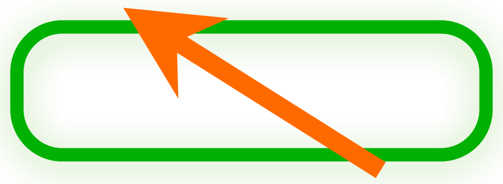

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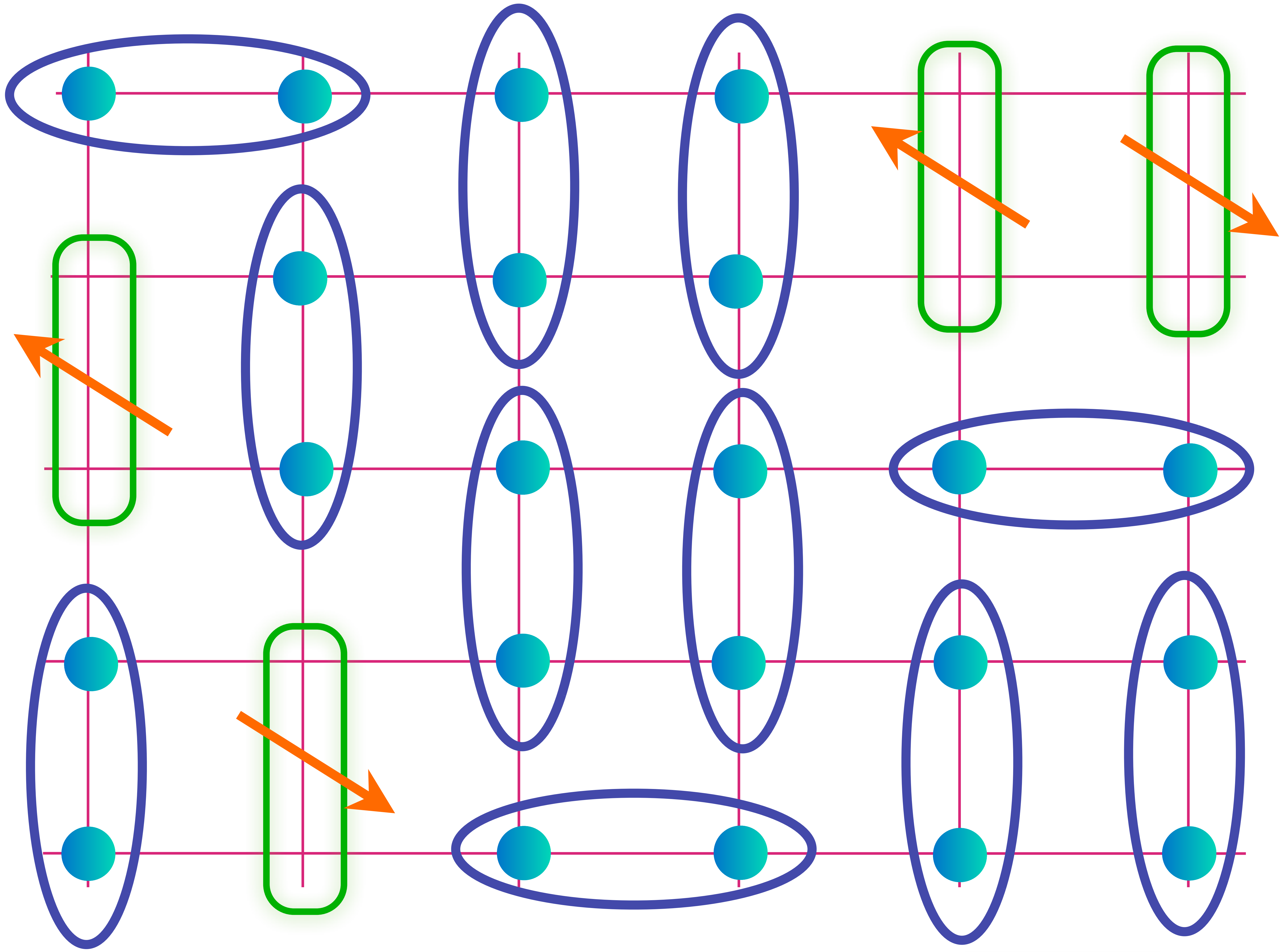
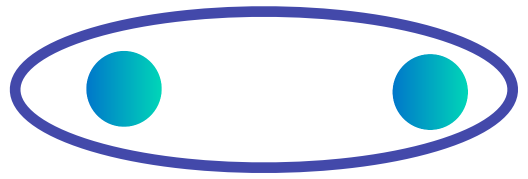
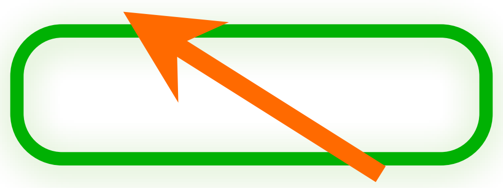


Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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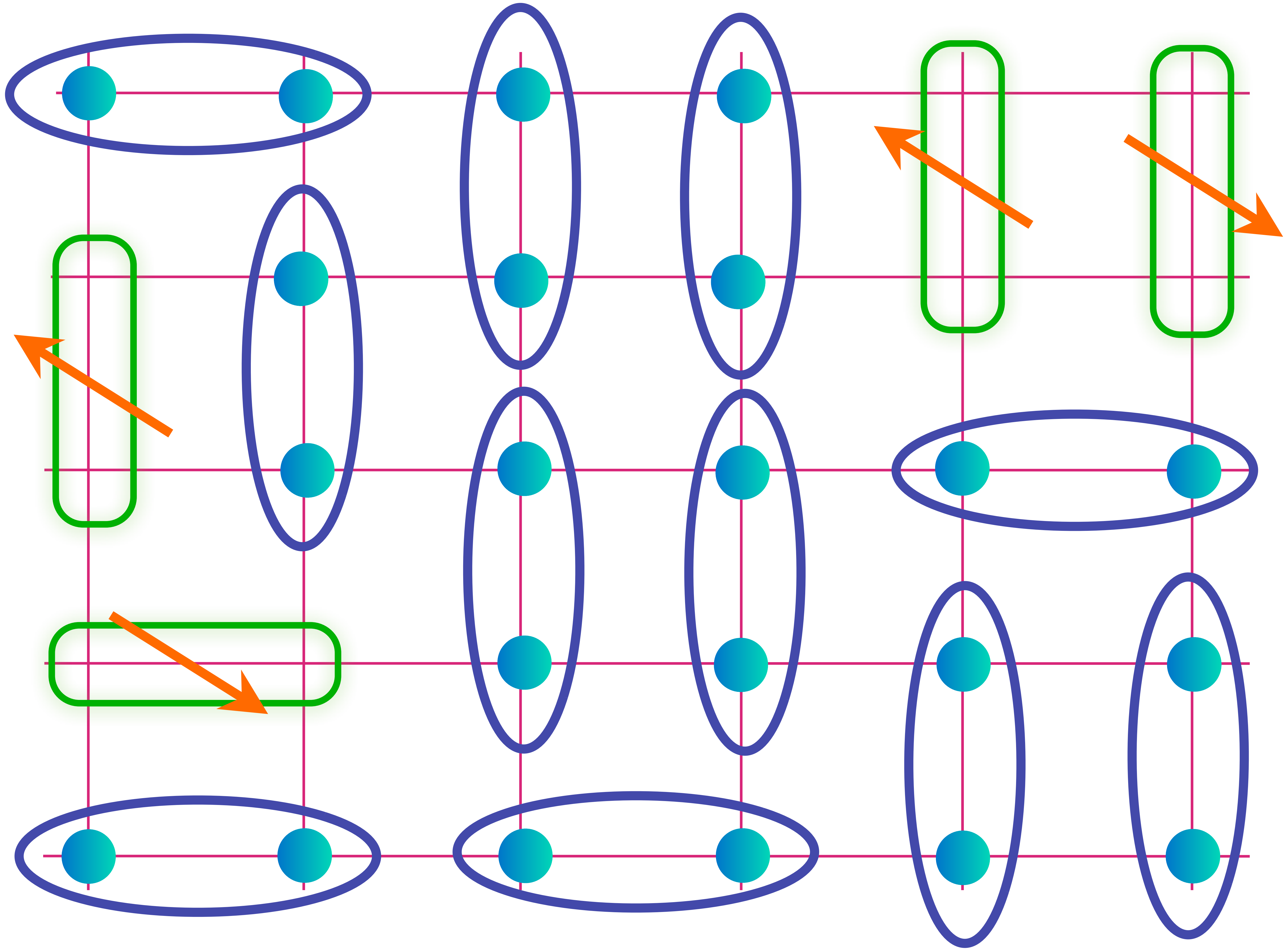
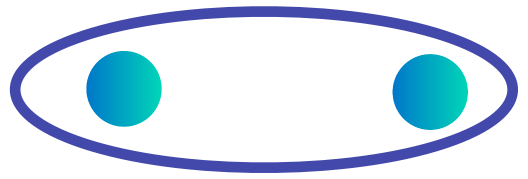
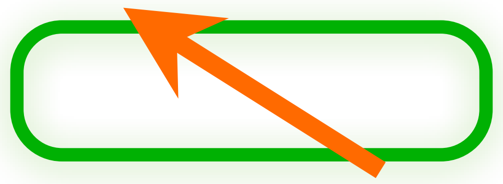
Kaul, Kolezhuk, Levin, Sachdev, and T. Senthil, *Phys. Rev. B* **75**, 235122 (2007)



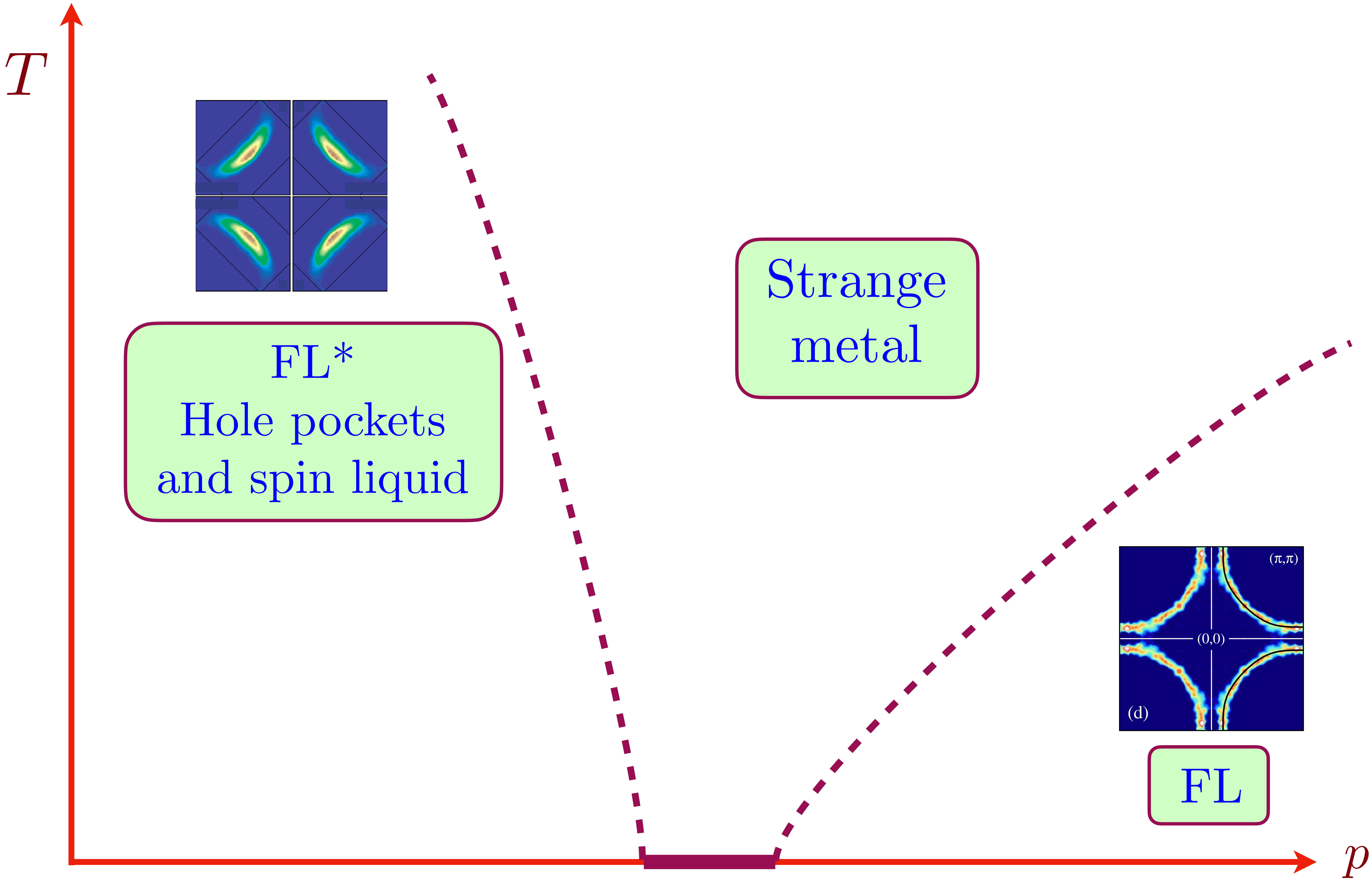
Metal with electron-like quasiparticles on a Fermi surface of size p, and emergent gauge fields

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

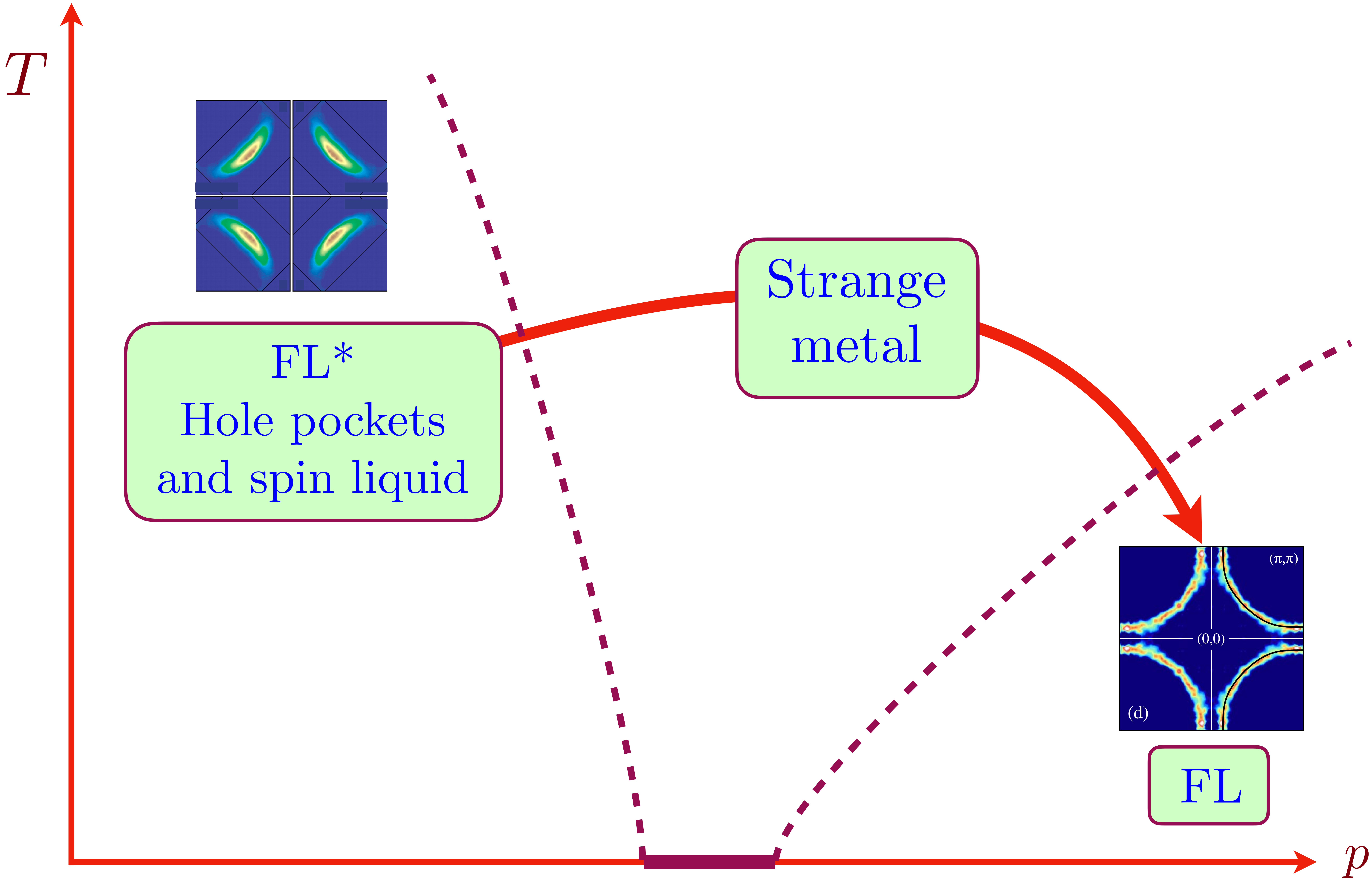
 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

Moon and Sachdev, *Phys. Rev. B* **83**, 224508 (2011); Punk, Allais, and Sachdev, *Proc. Nat. Acad. Sci.* **112**, 9552 (2015)

From FL*
to
a cuprate phase diagram



T
FL*
Hole pockets
and spin liquid
Strange
metal
(π,π)
(0,0)
(d)
FL
p



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Strange
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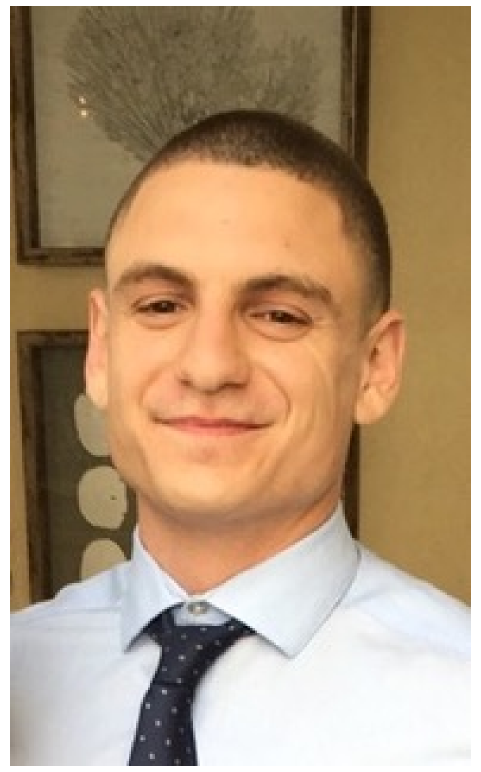
FL
p

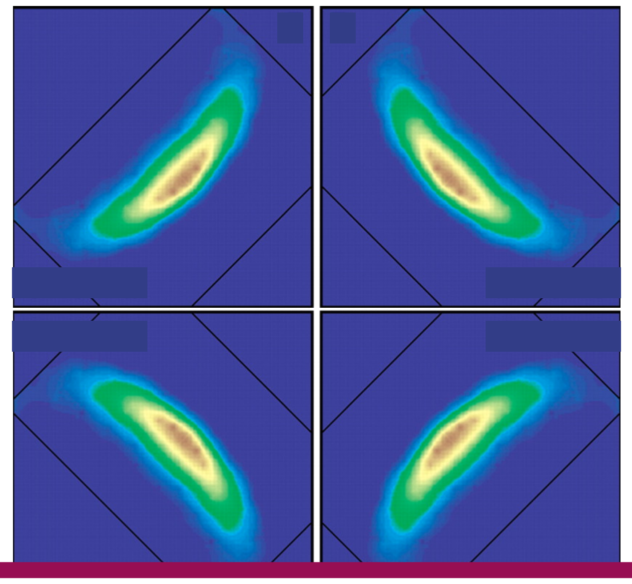
PHYSICS

Science **381**, 790 (2023)

Universal theory of strange metals from spatially random interactions

Aavishkar A. Patel[1,2], Haoyu Guo[3,4,5], Ilya Esterlis[4,6], Subir Sachdev[4,7]*



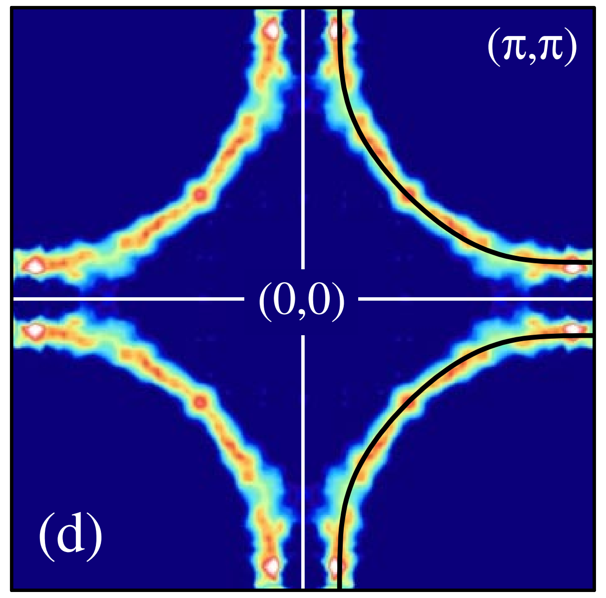


we consider two-dimensional metals of fermions coupled to quantum critical scalars, the latter representing order parameters or fractionalized particles. We show that at low temperatures (*T*), such metals generically exhibit strange metal behavior with a *T*-linear resistivity arising from spatially random fluctuations in the fermion-scalar Yukawa couplings about a nonzero spatial average.

T

FL*
Hole pockets
and spin liquid

Strange
metal



FL

p

See also Aldape, Cookmeyer, Patel, and Altman, *Phys. Rev. B* **105**, 235111 (2023)

Spatially random interactions!

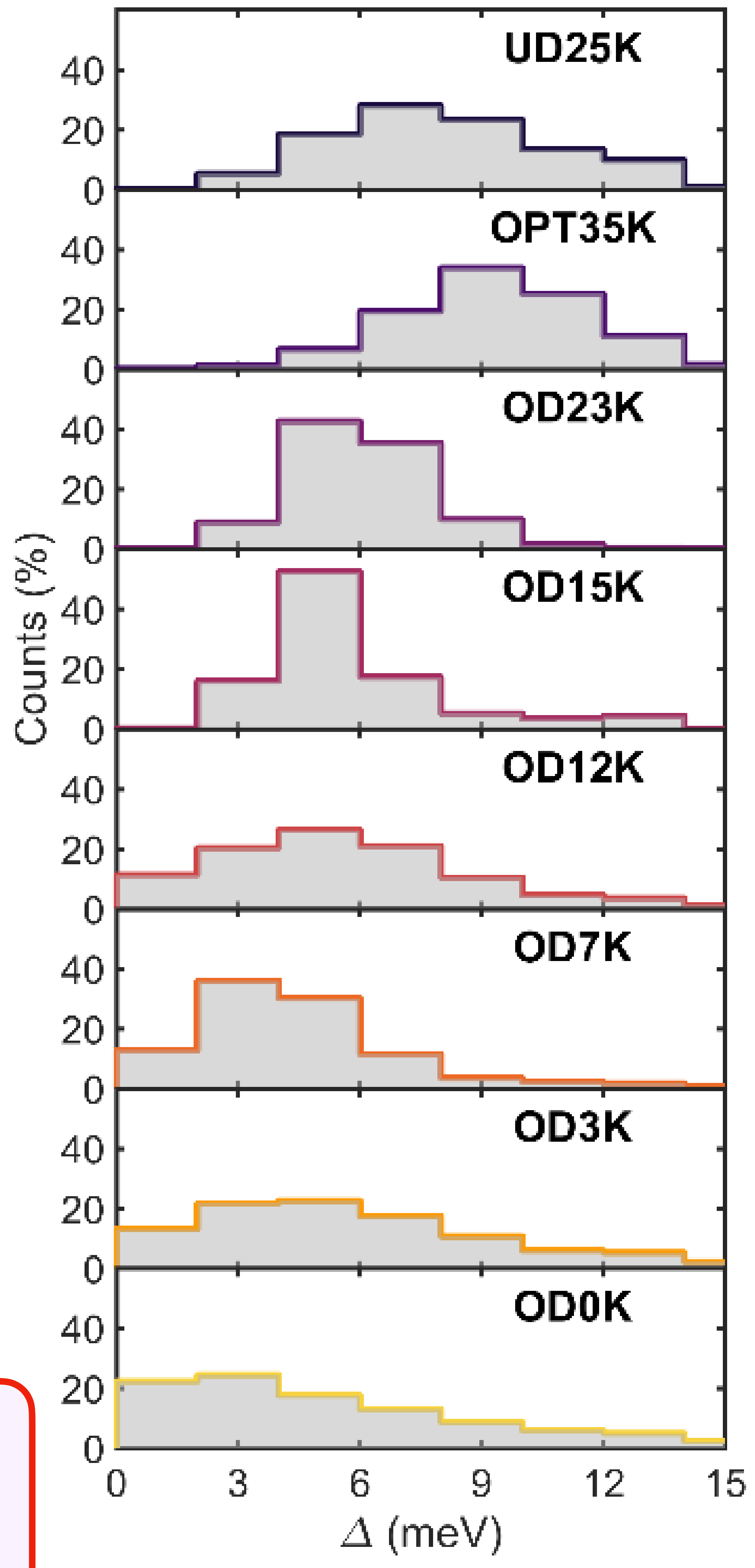
Puddle formation, persistent gaps, and non-mean-field breakdown of superconductivity in overdoped $(Pb,Bi)_2Sr_2CuO_{6+\delta}$

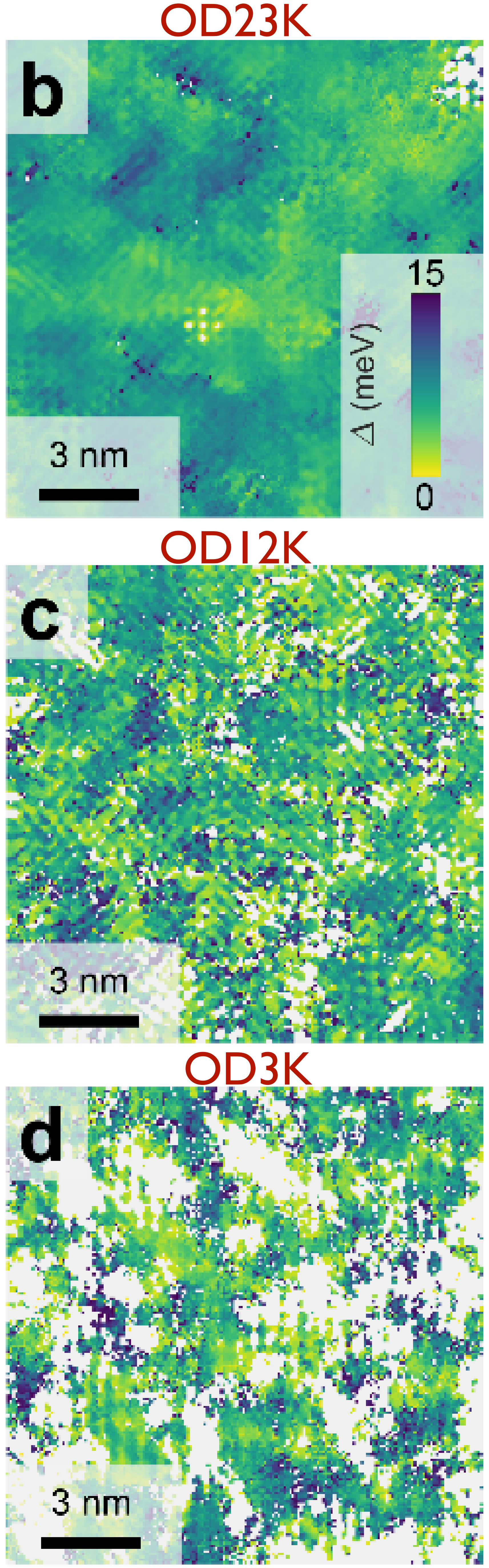
Willem O. Tromp, Tjerk Benschop, Jian-Feng Ge, Irene Battisti, Koen M. Bastiaans, Damianos Chatzopoulos, Amber Vervloet, Steef Smit, Erik van Heumen, Mark S. Golden, Yinkai Huang, Takeshi Kondo, Yi Yin, Jennifer E. Hoffman, Miguel Antonio Sulangi, Jan Zaanen, Milan P. Allan

Our scanning tunneling spectroscopy measurements in the overdoped regime of the $(Pb,Bi)_2Sr_2CuO_{6+\delta}$ high-temperature superconductor show the emergence of puddled superconductivity, featuring nanoscale superconducting islands in a metallic matrix

Nat. Mater. **22**, 703 (2023)

Disorder in t_{ij} induces disorder in $J_{ij} = \dfrac{t_{ij}^2}{U}$





Transport properties of a strange metal:

1. Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$
   and $\rho(T) < h/e^2$ (in $d = 2$).
   Metals with $\rho(T) > h/e^2$ are bad metals.

2. Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*_{\rm trans}(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} \sim |\omega| \Phi_\sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

van Heumen et al., *Phys. Rev. B* **106**, 054515 (2022)
Michon et al., *Nat. Commun.* **14**, 3033 (2023)

Electronic properties of a marginal Fermi liquid:

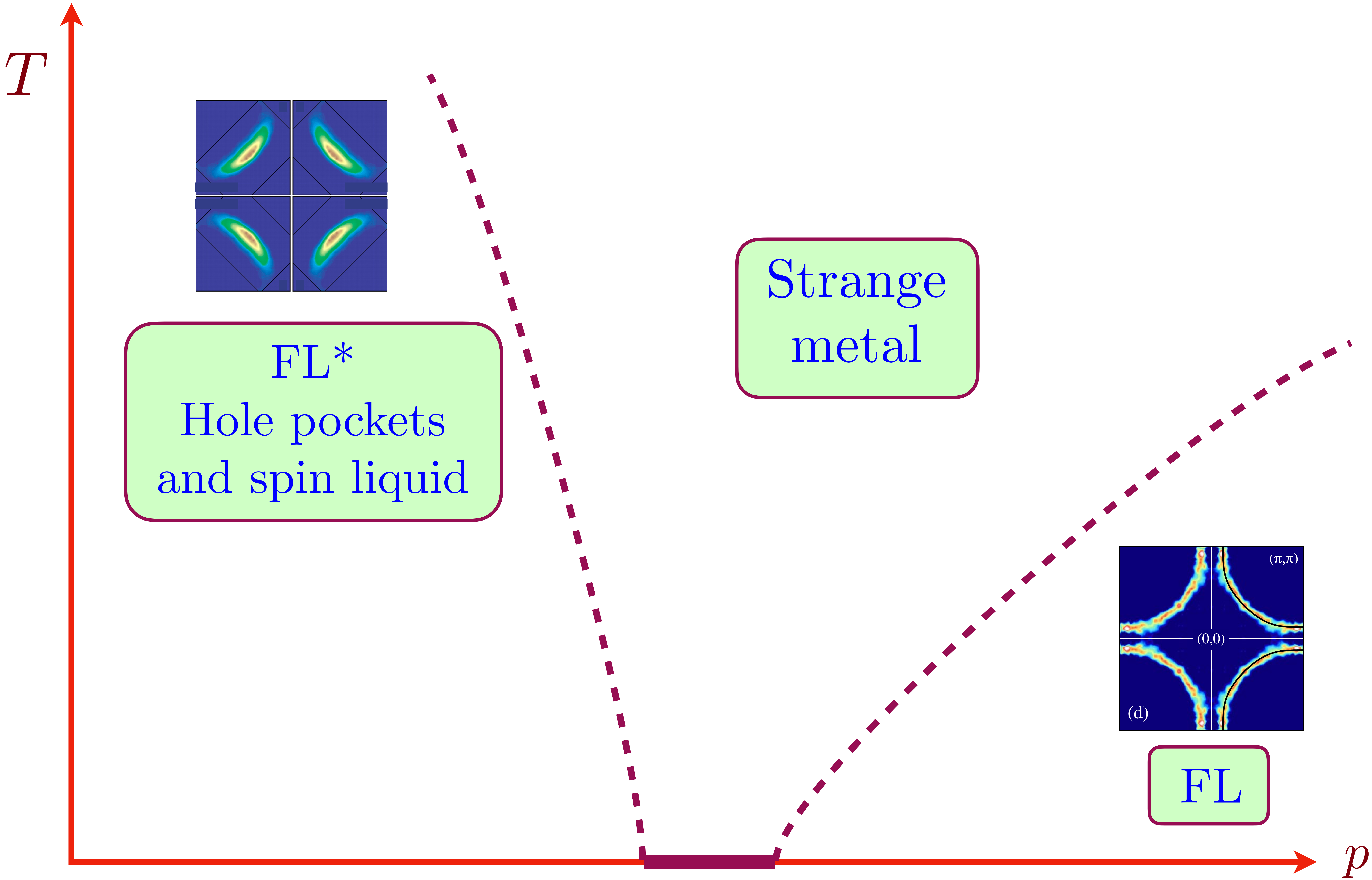
1. Photoemission: nearly marginal Fermi liquid electron spectral density:

$$\text{Im}\Sigma(\omega) \sim |\omega|^{2\alpha} \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right) \quad \text{with } \alpha \approx 1/2 \quad ; \quad \frac{1}{\tau_{\rm in}(\omega)} \sim |\omega| \Phi_\Sigma \left(\frac{\hbar\omega}{k_B T} \right)$$

Reber *et al.*, *Nat. Comm.* **10**, 5737 (2019)

2. Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

Hartnoll and MacKenzie, *Rev. Mod. Phys.* **94**, 041002 (2022)



T
FL*
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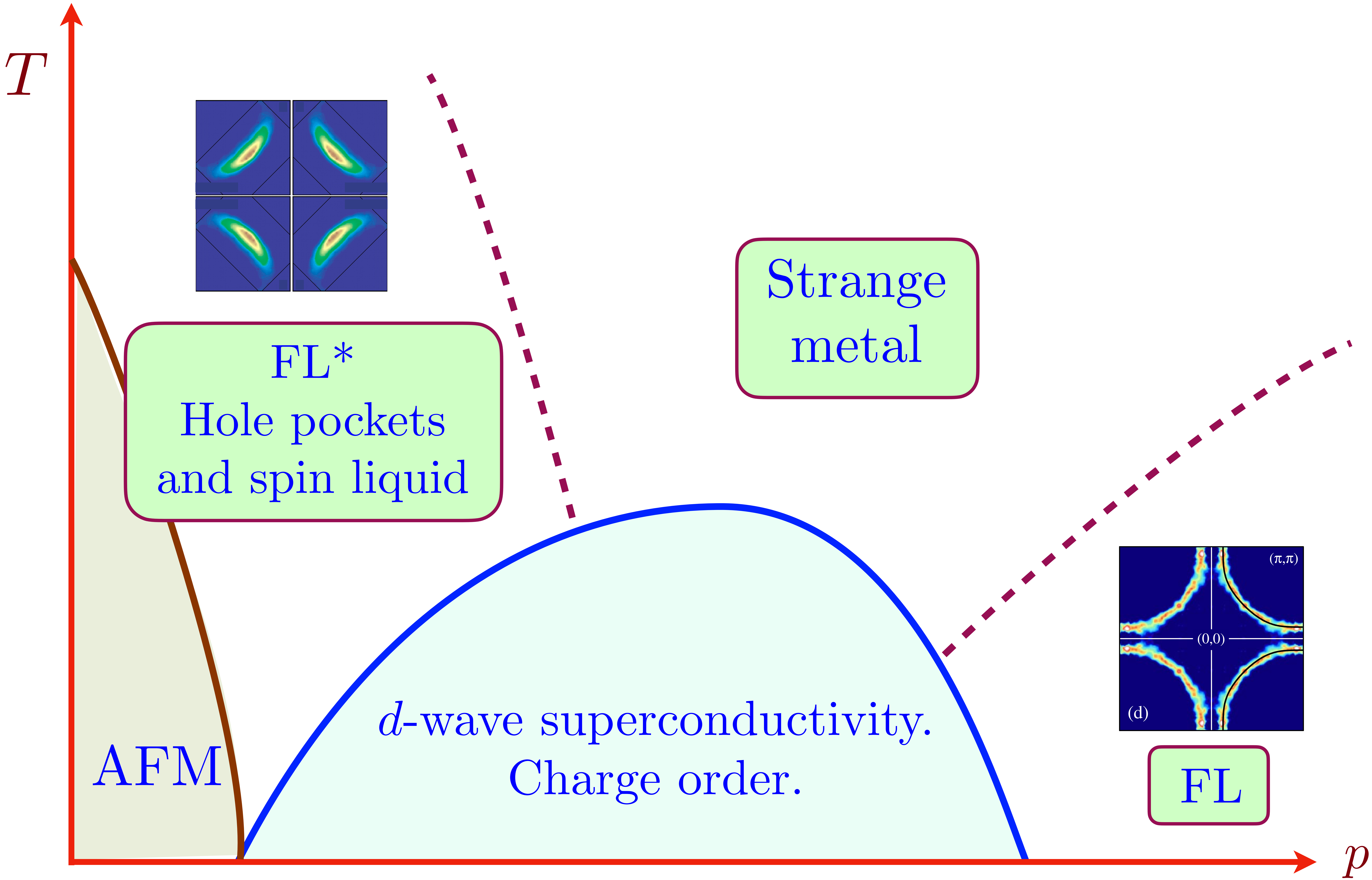
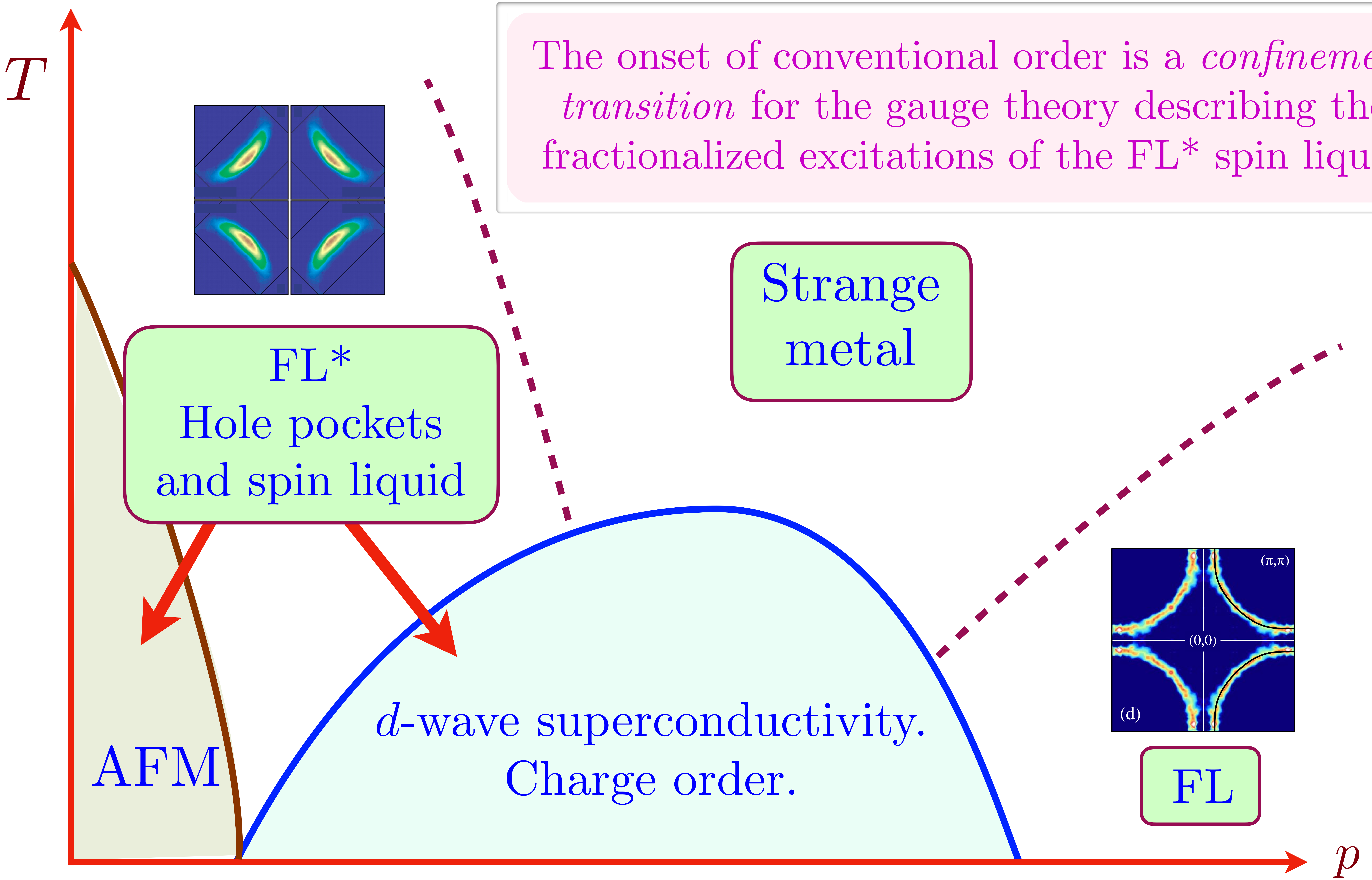
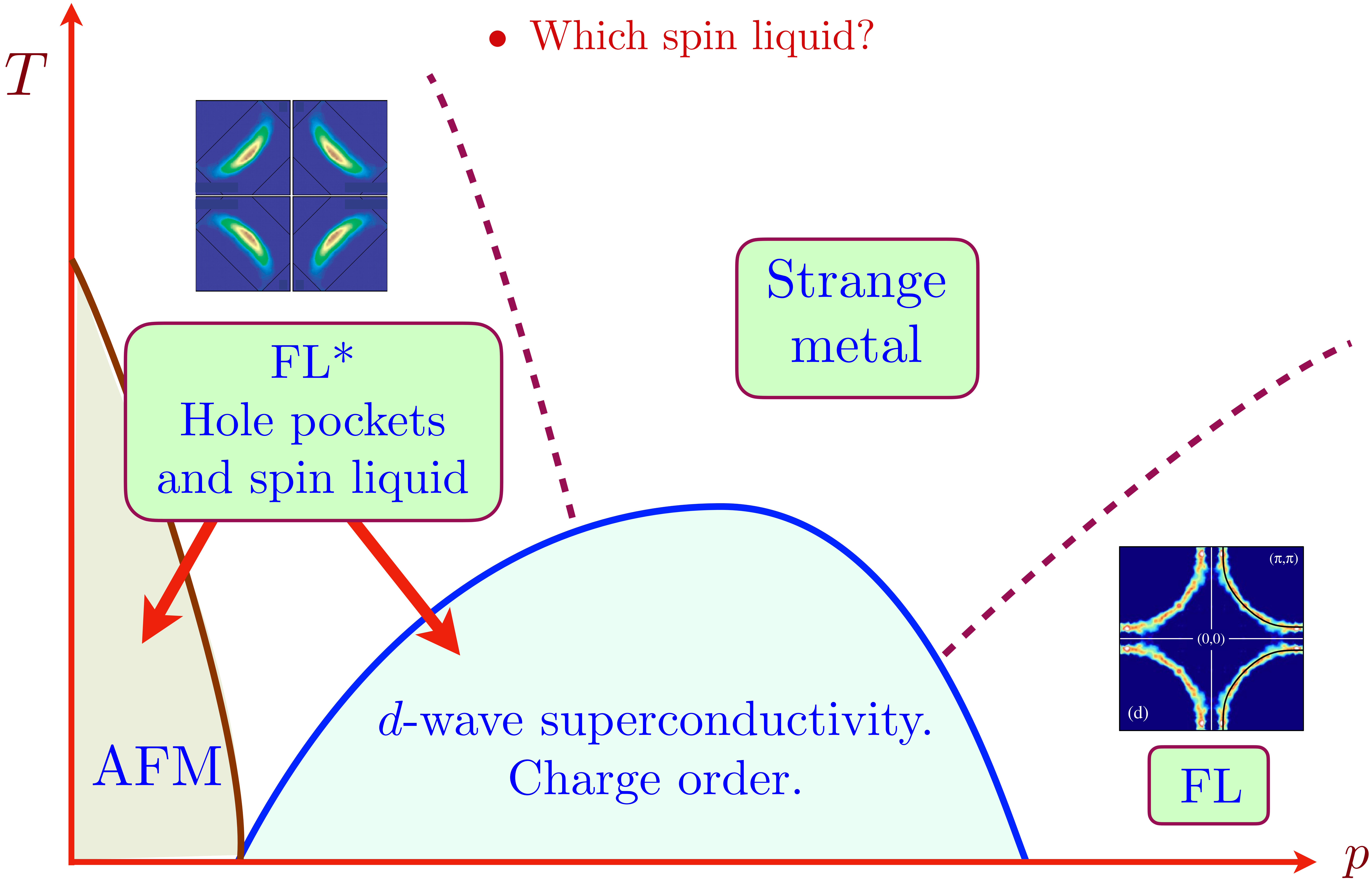


T
FL*
Hole pockets
and spin liquid
Strange
metal
(π,π)
(0,0)
(d)
AFM
d-wave superconductivity.
Charge order.
FL
p

The onset of conventional order is a *confinement transition* for the gauge theory describing the fractionalized excitations of the FL* spin liquid





Which spin liquid?
T
FL*
Hole pockets
and spin liquid
Strange
metal
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p

• Which spin liquid?
• Mean-field theory for FL* in a single-band model?
T
FL*
Hole pockets
and spin liquid
Strange
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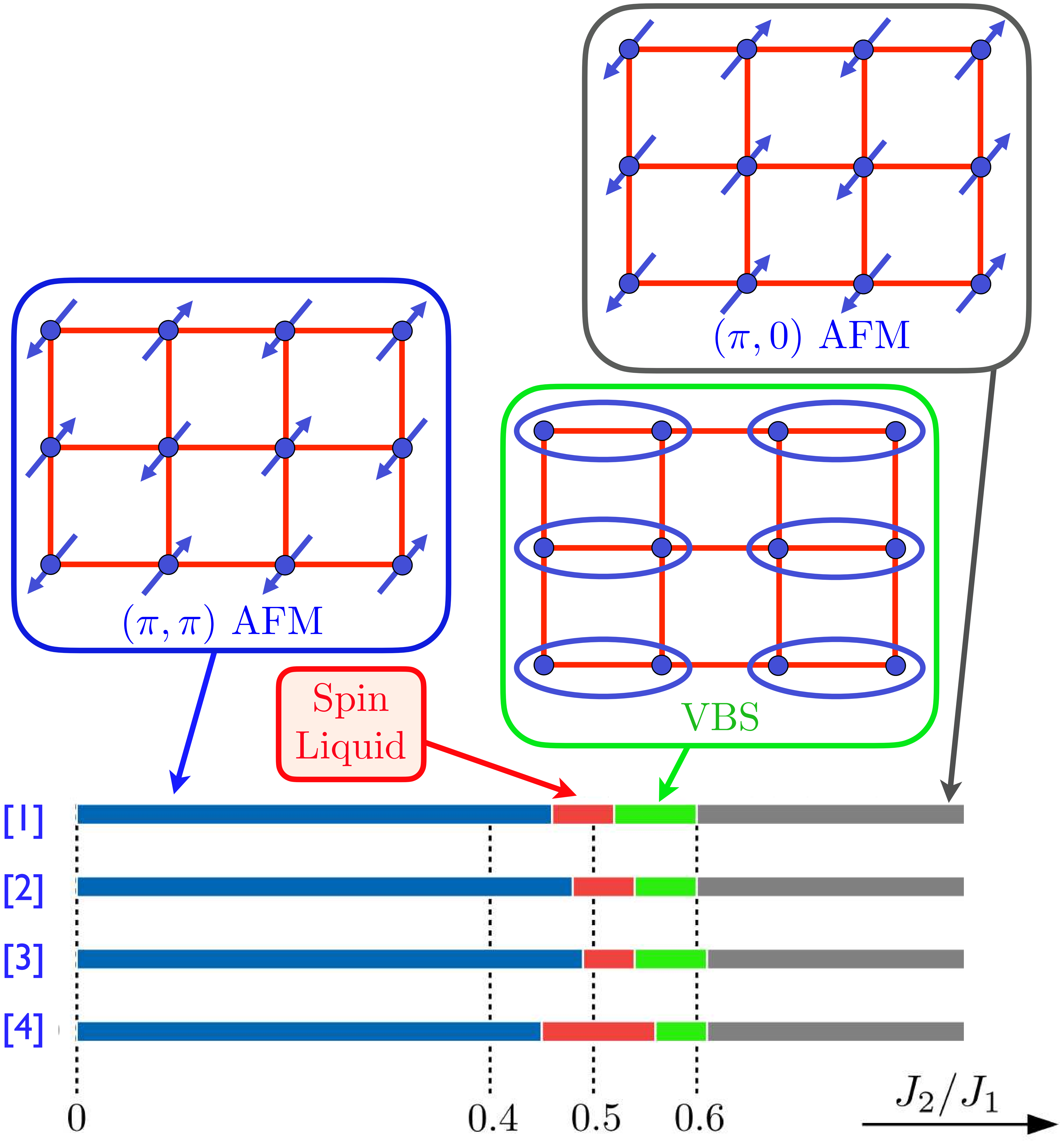
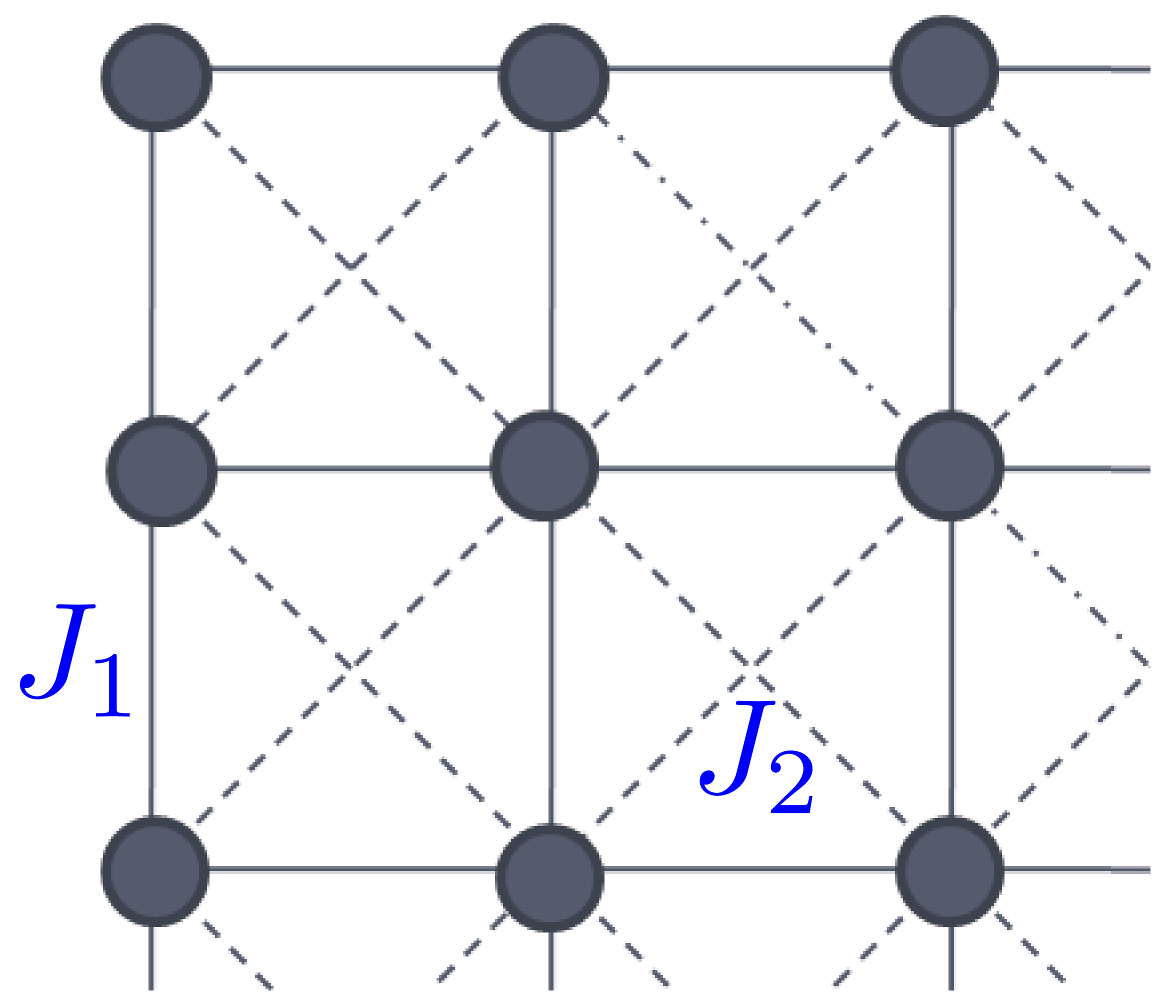
AFM
d-wave superconductivity.
Charge order.
(π,π)
(0,0)
(d)
FL
p

The CP^1/π-flux spin liquid

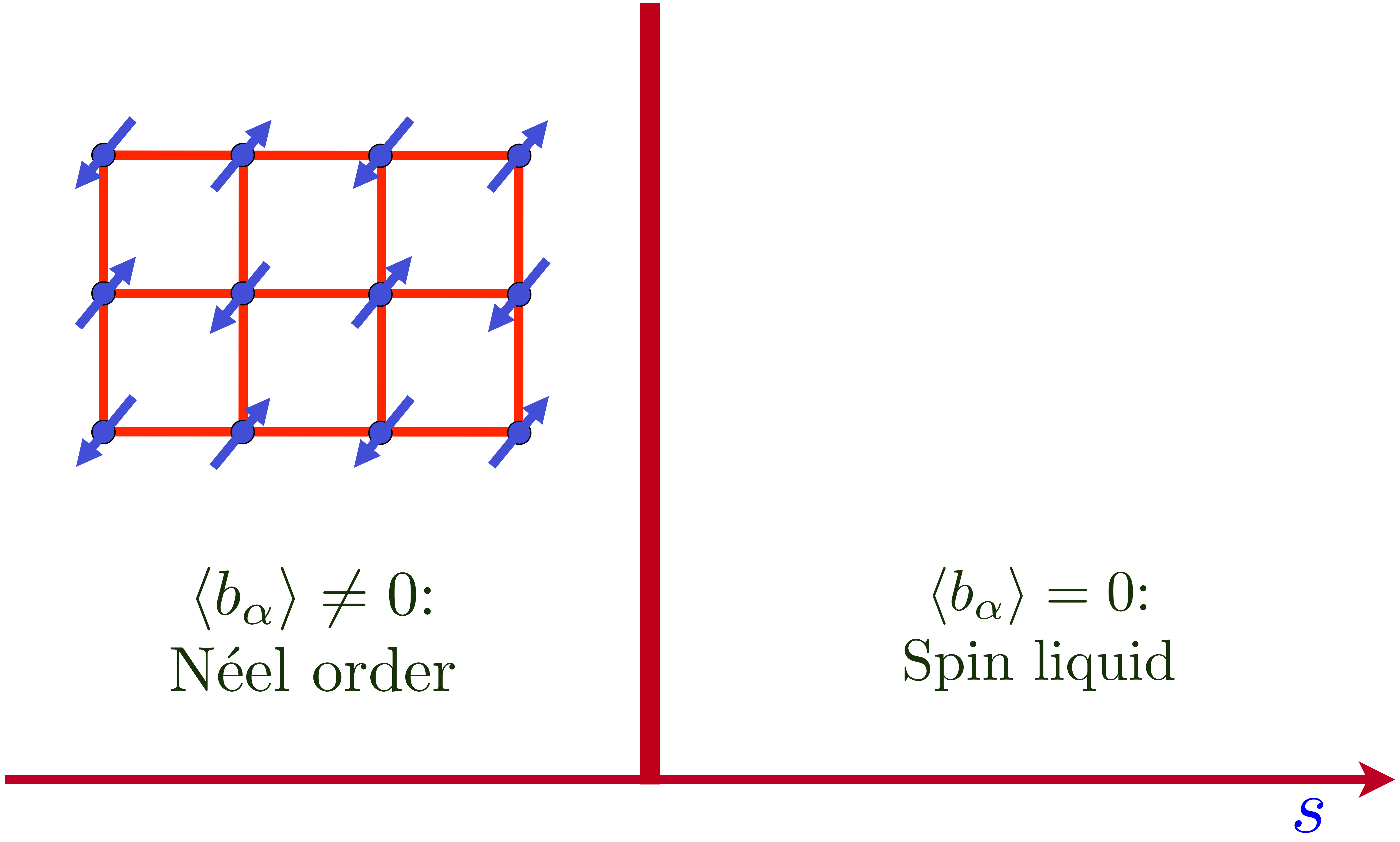
$$H = J_1 \sum_{\langle i,j\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{\langle\langle i,j\rangle\rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$





1. Wang and Sandvik, *Phys. Rev. Lett.* **121**, 107202 (2018)
2. Ferrari and Becca, *Phys. Rev. B* **102**, 014417 (2020)
3. Nomura and Imada, *Phys. Rev. X* **11**, 031034 (2021)
4. Liu, Gong, Li, Poilblanc, Chen, and Gu, *Sci. Bull.* **67**, 1034 (2022)

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

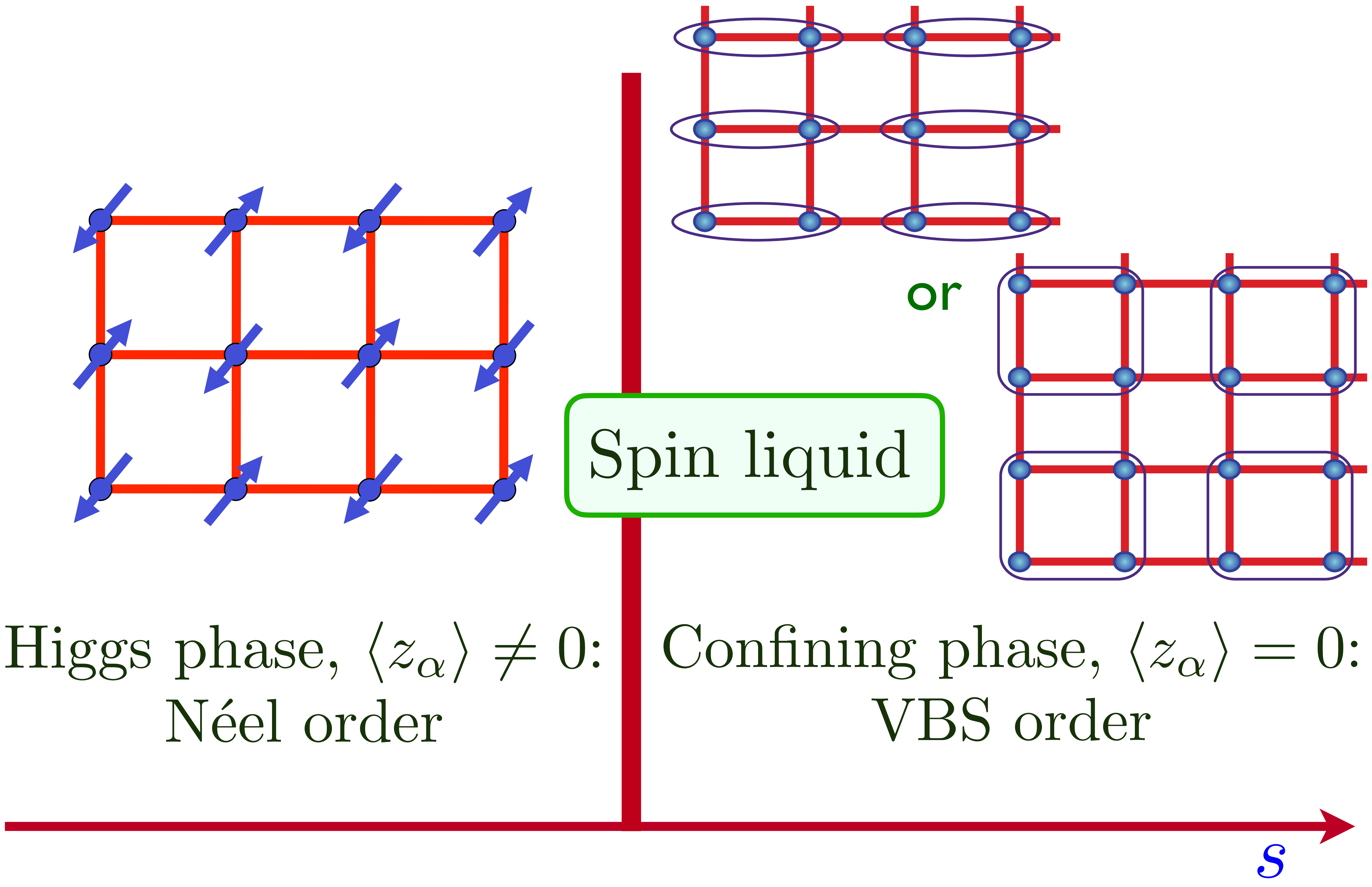
Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \qquad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^\dagger b_{i\alpha} = 1$$

Mean-field spin liquid
with gapped bosonic spinons.

Arovas and Auerbach, *Phys. Rev. B* **38**, 316 (1988)

Insulating S=1/2 antiferromagnet



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Mean-field spin liquid
with gapped bosonic spinons.

Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\rm monopole}$$

Read and Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

Insulating S=1/2 antiferromagnet

$\mathbb{CP}^1$ U(1) gauge theory

Read and Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

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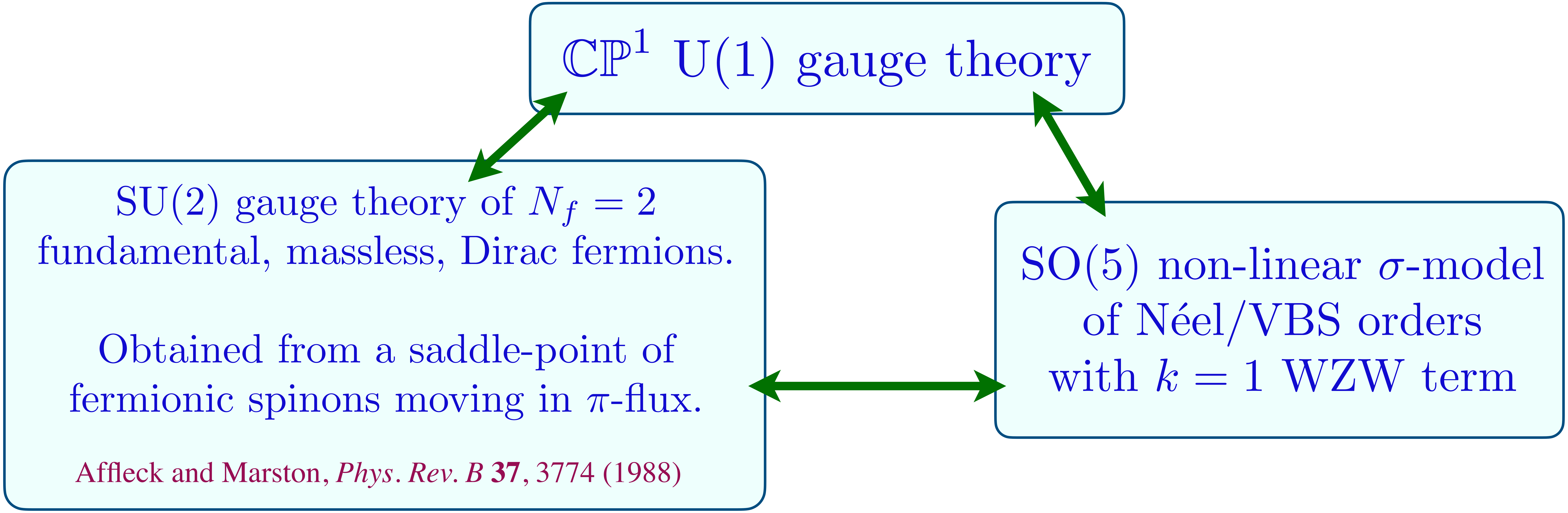


Tanaka and Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); Senthil and Fisher, *Phys. Rev. B* **74**, 064405 (2006); Ran and Wen, arXiv:cond-mat/0609620 (2006); Wang, Nahum, Metlitski, Xu, and Senthil, *Phys. Rev. X* **7**, 031051 (2017)

Insulating $S=1/2$ antiferromagnet

$\mathbb{CP}^1$ U(1) gauge theory

SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

Affleck and Marston, *Phys. Rev. B* **37**, 3774 (1988)

SO(5) non-linear σ-model of Néel/VBS orders with $k=1$ WZW term

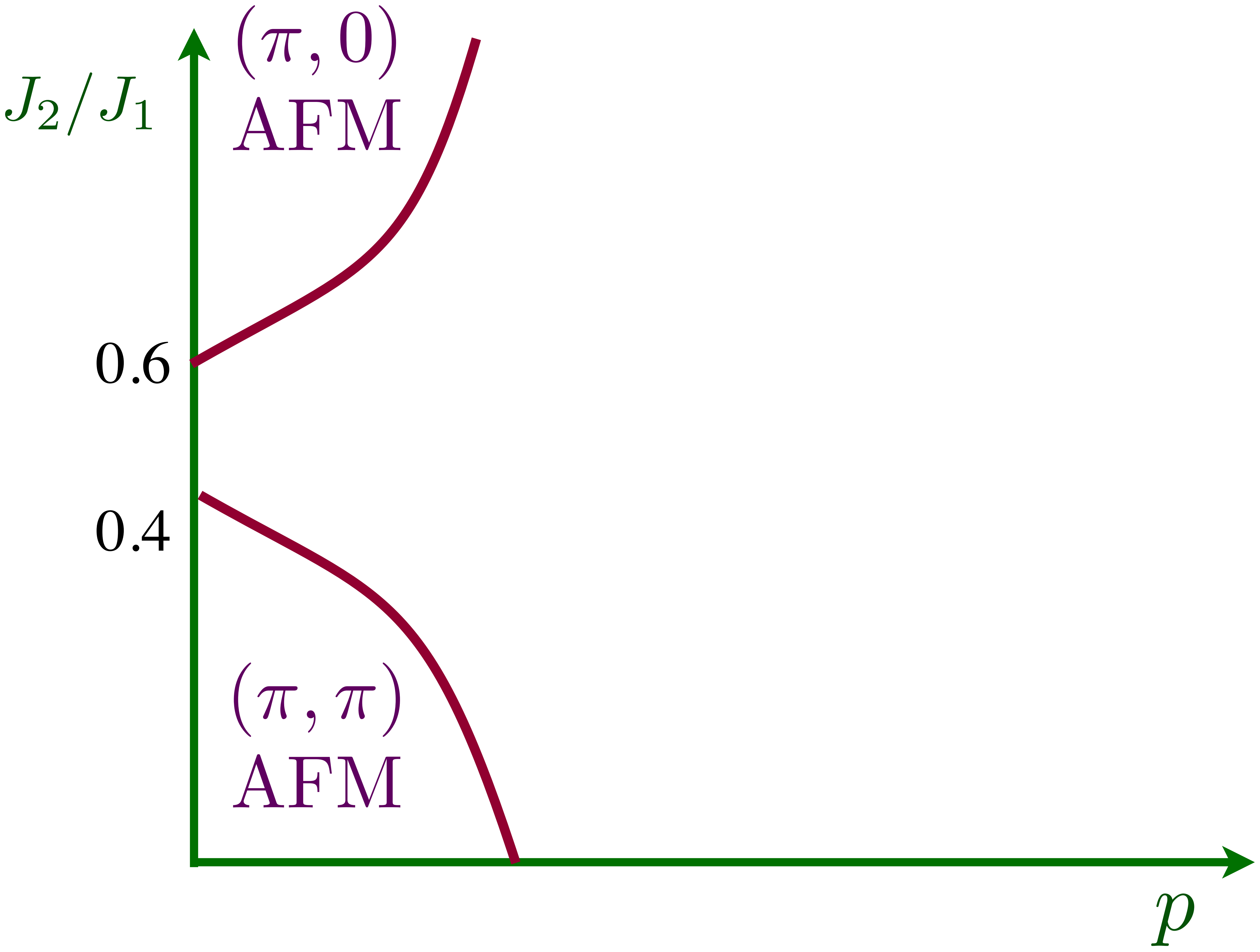
Many numerical works show that deconfined critical theory applies over a substantial length scale, but ultimately confines at the longest distances.

Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007)
Nahum, Serna, Chalker, Ortuño, and Somoza, *Phys. Rev. Lett.* **115**, 267203 (2015)
Zhou, Hu, Zhu, and He, arXiv:2306.16435 (2023)

High Temperature Superconductivity in a Lightly Doped Quantum Spin Liquid

Hong-Chen Jiang[1,*] and Steven A. Kivelson[2]

Phys. Rev. Lett. **127**, 097002 (2021)



Superconducting valence bond fluid in lightly doped 8-leg *t-J* cylinders
Jiang, Kivelson, and Lee, *Phys. Rev. B* **108**, 054505 (2023)

Upon increasing the cylinder width from 4 to 8, we observed a significant strengthening of the quasi-long-range superconducting correlations, and a dramatic suppression of any "competing" charge-density-wave order. Extrapolating from the observed behavior of the width 8 cylinders, we speculate that the system has a nodeless d-wave superconducting ground-state in the 2D limit.

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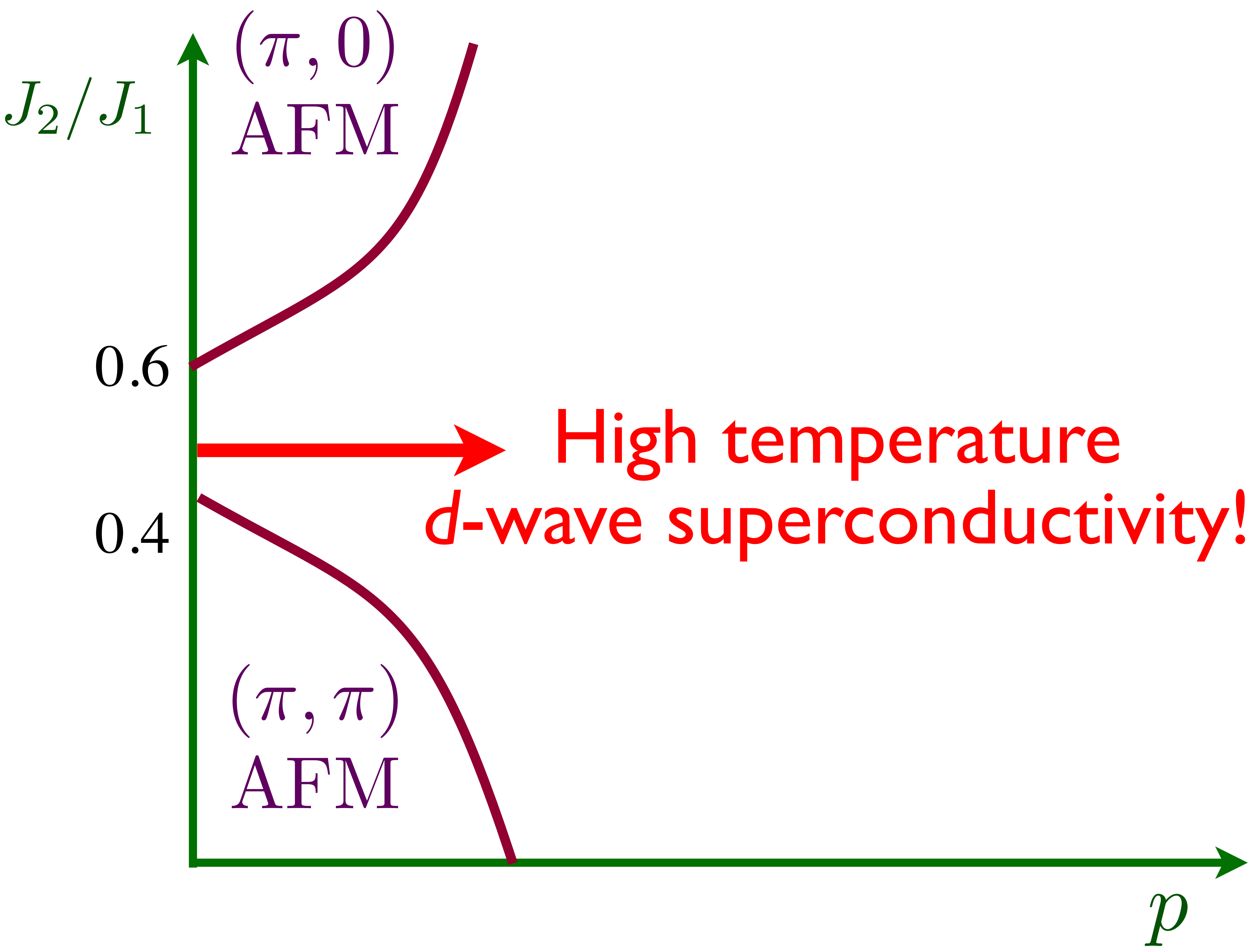

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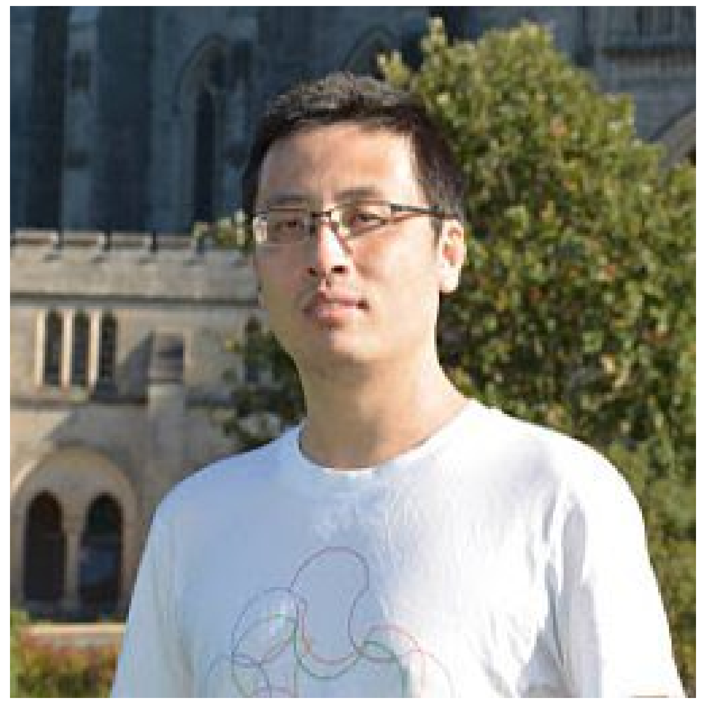
Ancilla theory of FL*
in a single-band model



Ya-Hui Zhang

Paramagnon theory of the Hubbard model

$$H = -\sum_{i<j} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) - \mu \sum_i c_{i\sigma}^{\dagger} c_{i\sigma}$$

We use the operator equation (valid on each site i):

$$U \left(n_{\uparrow} - \frac{1}{2} \right) \left(n_{\downarrow} - \frac{1}{2} \right) = -\frac{2U}{3} \boldsymbol{S}^2 + \frac{U}{4}$$

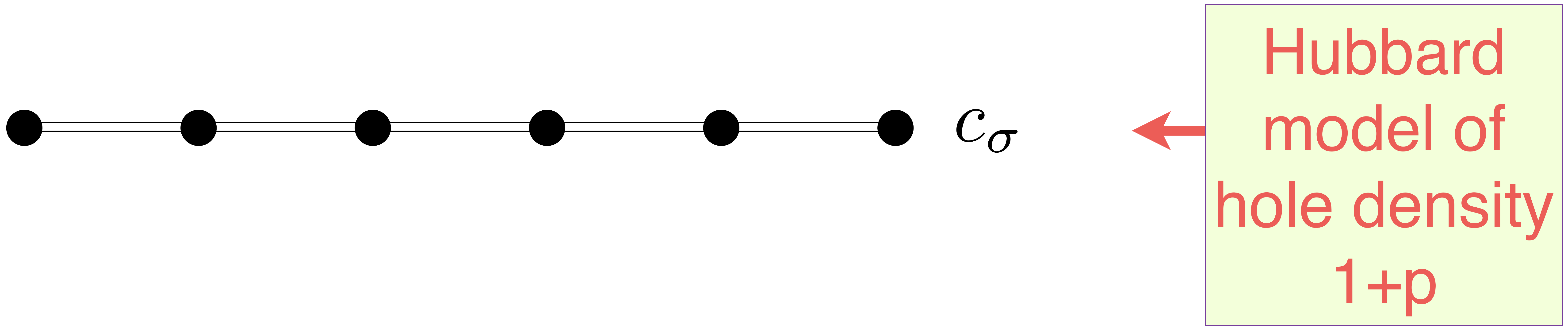
Then we decouple the interaction via

$$\exp \left(\frac{2U}{3} \sum_i \int d\tau \boldsymbol{S}_i^2 \right) = \int \mathcal{D} \boldsymbol{\Phi}_i(\tau) \exp \left(- \sum_i \int d\tau \left[\frac{3}{8U} \boldsymbol{\Phi}_i^2 - \boldsymbol{\Phi}_i \cdot c_{i\sigma}^{\dagger} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \right] \right)$$

This yields the 'Scalapino-Pines-Chubukov-Schmalian...' theory for a 'paramagnon quantum rotor' $\boldsymbol{\Phi}_i$ coupled to otherwise free fermions $c_{i\sigma}$.

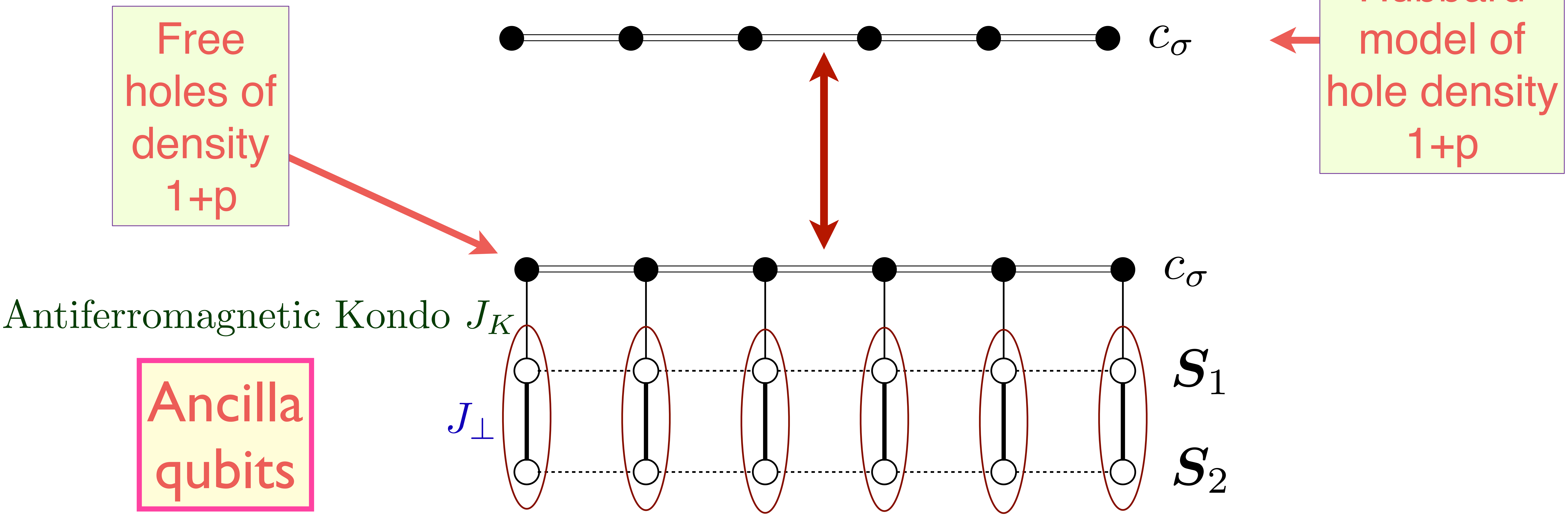
$\boldsymbol{\Phi}_i$ is the creation/annihilation operator for charge 0, spin $S = 1$ particle.

Ancilla theory of the Hubbard model



Ancilla theory of the Hubbard model

Zhang and Sachdev,
Phys. Rev. Res. **2**, 023172 (2020)



$$\mathcal{H}_{\rm ancilla} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma} + J_K \sum_i c_{i\sigma}^\dagger \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

Ancilla theory of the Hubbard model

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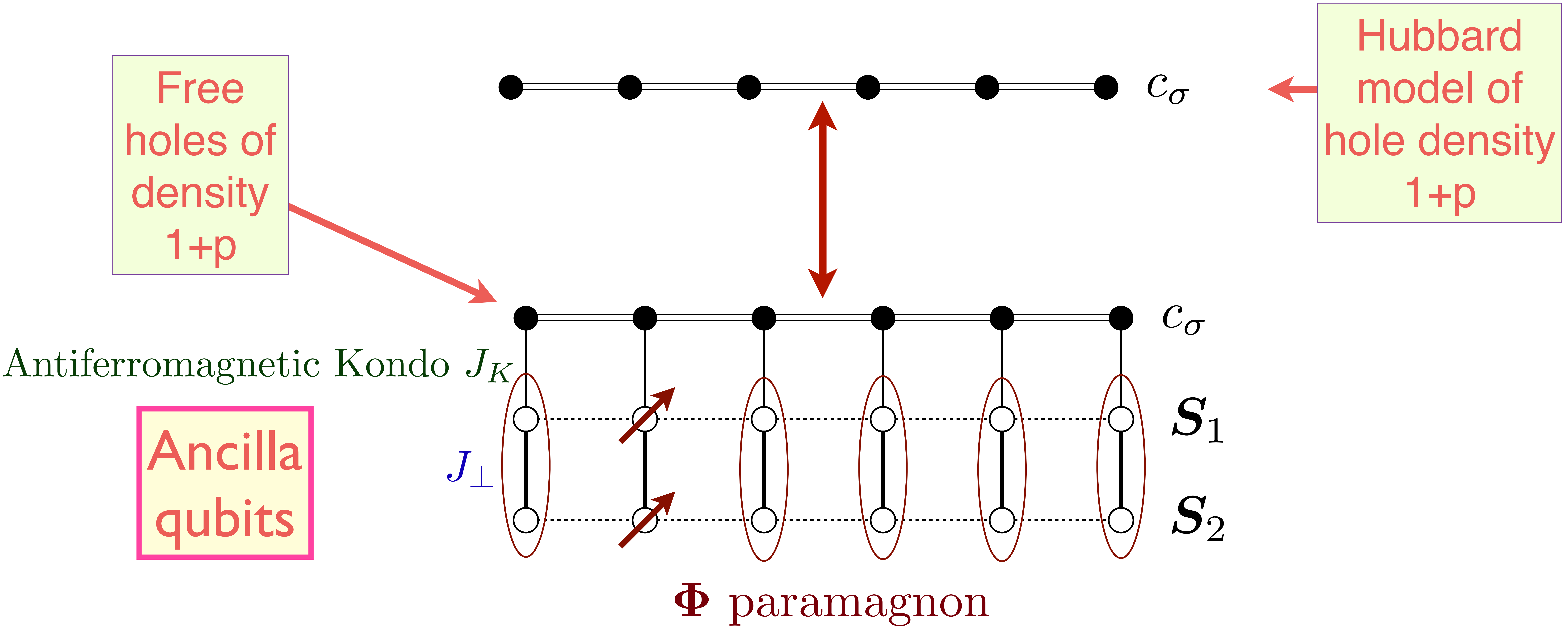
Phys. Rev. Res. **2**, 023172 (2020)



$$\mathcal{H}_{\text{ancilla}} = \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + J_K \sum_i c^\dagger_{i\sigma} \frac{\boldsymbol{\tau}_{\sigma\sigma'}}{2} c_{i\sigma'} \cdot \boldsymbol{S}_{1i} + J_\perp \sum_i \boldsymbol{S}_{1i} \cdot \boldsymbol{S}_{2i}$$

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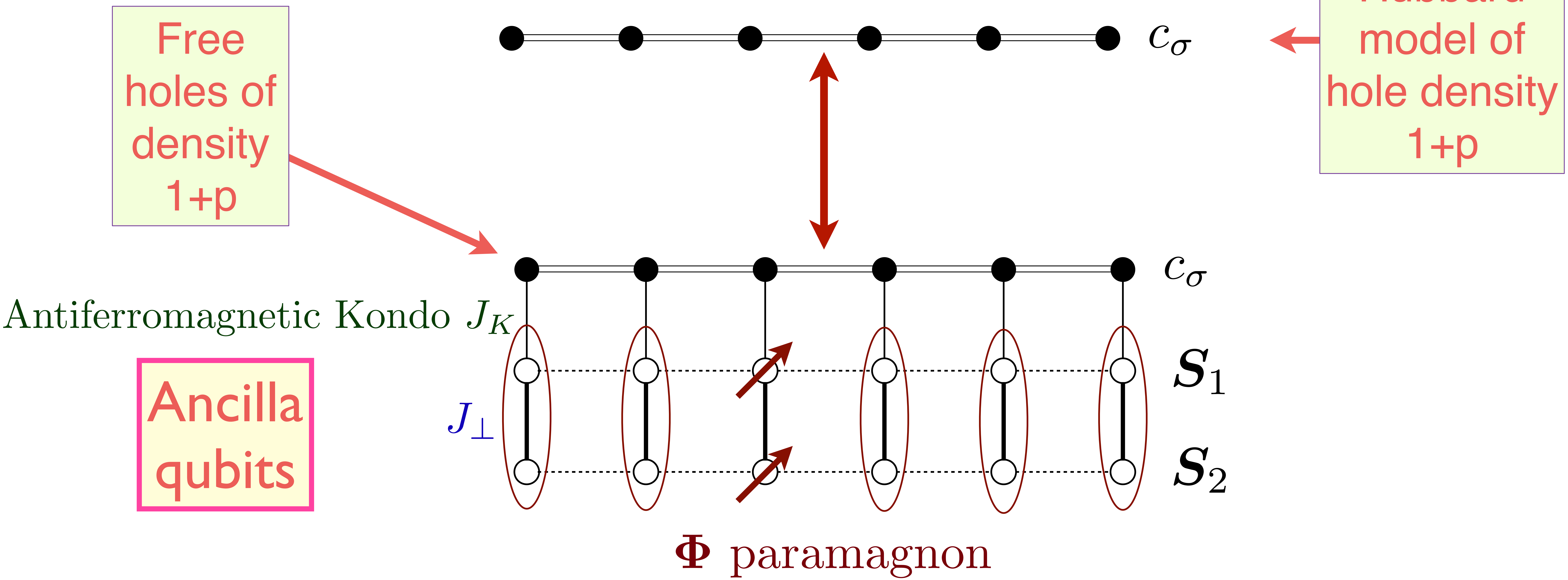

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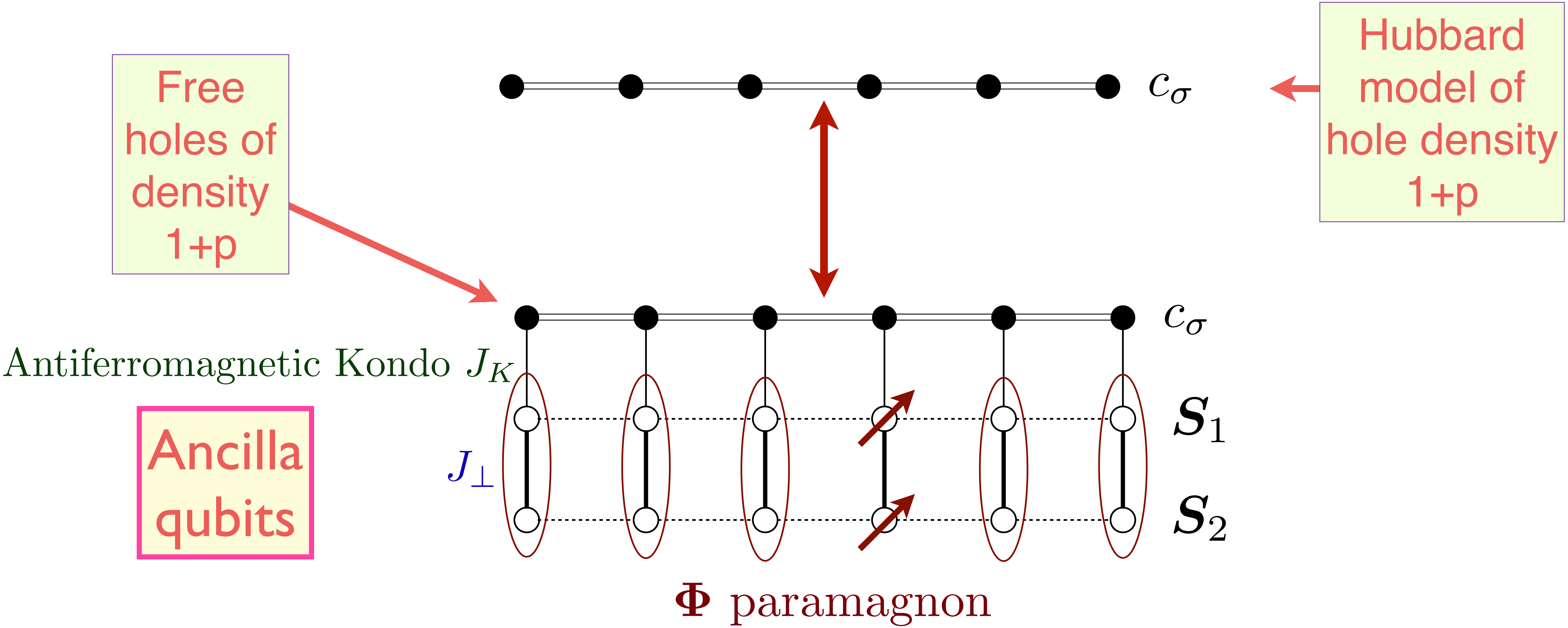

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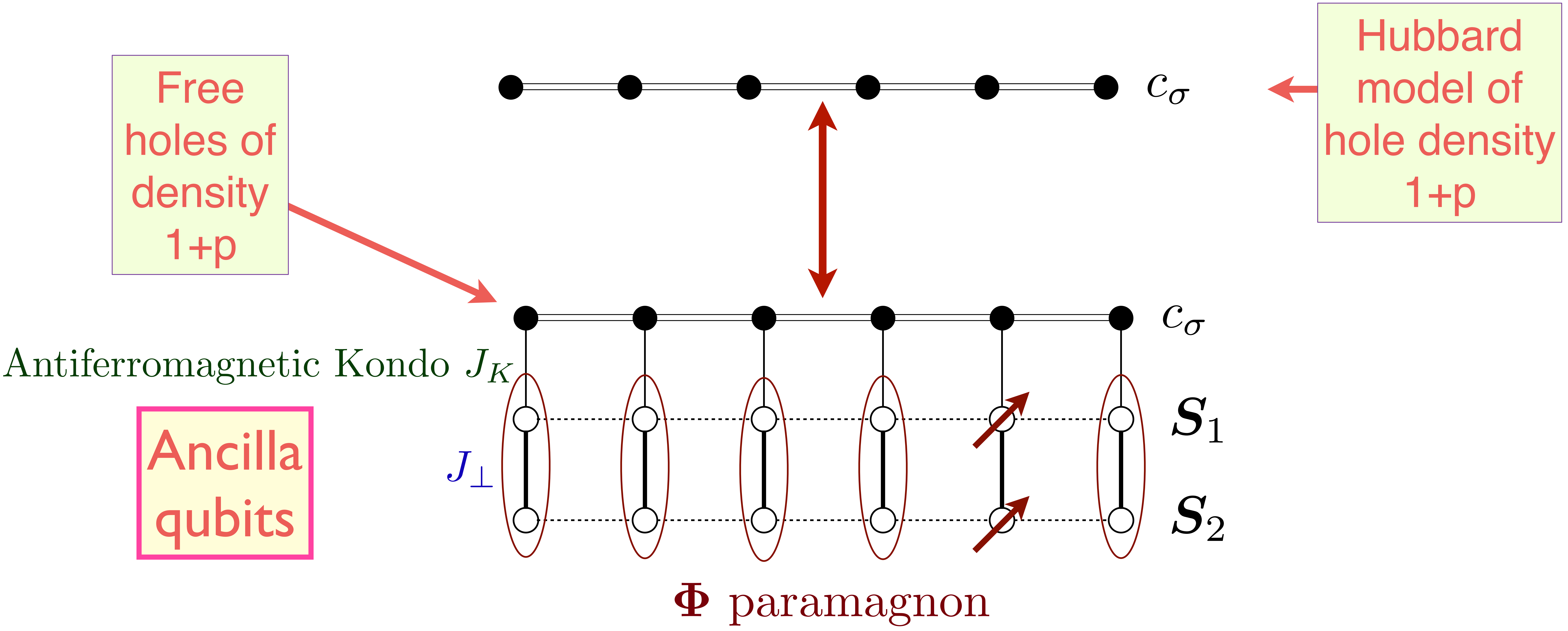

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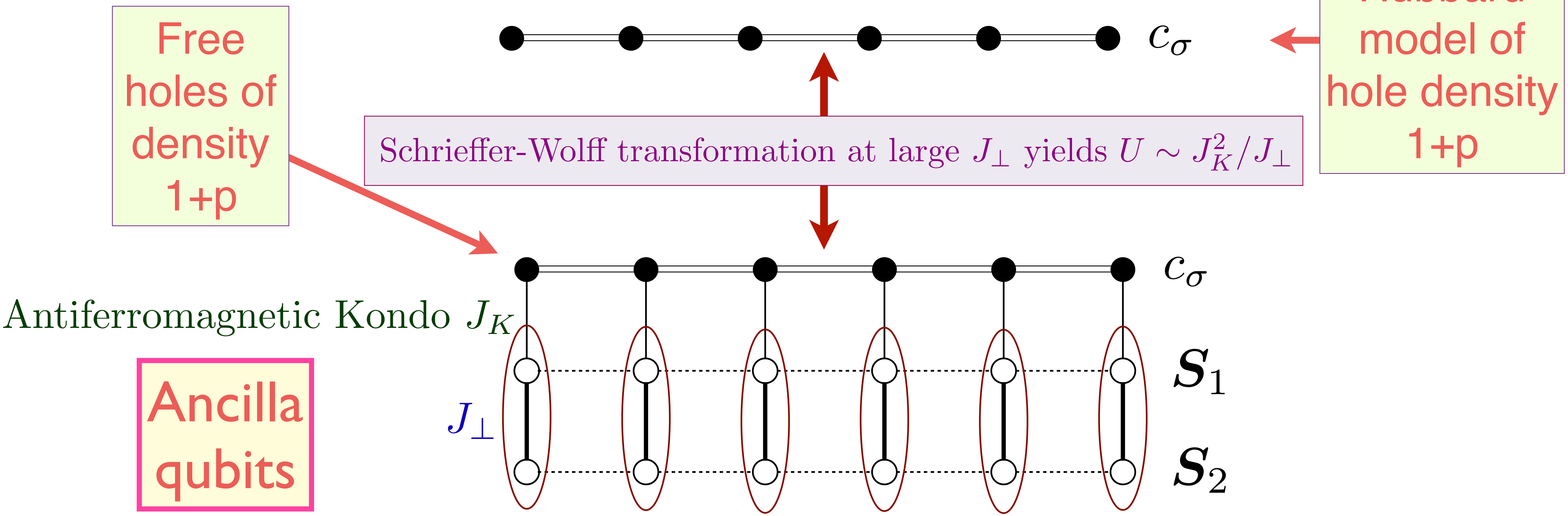

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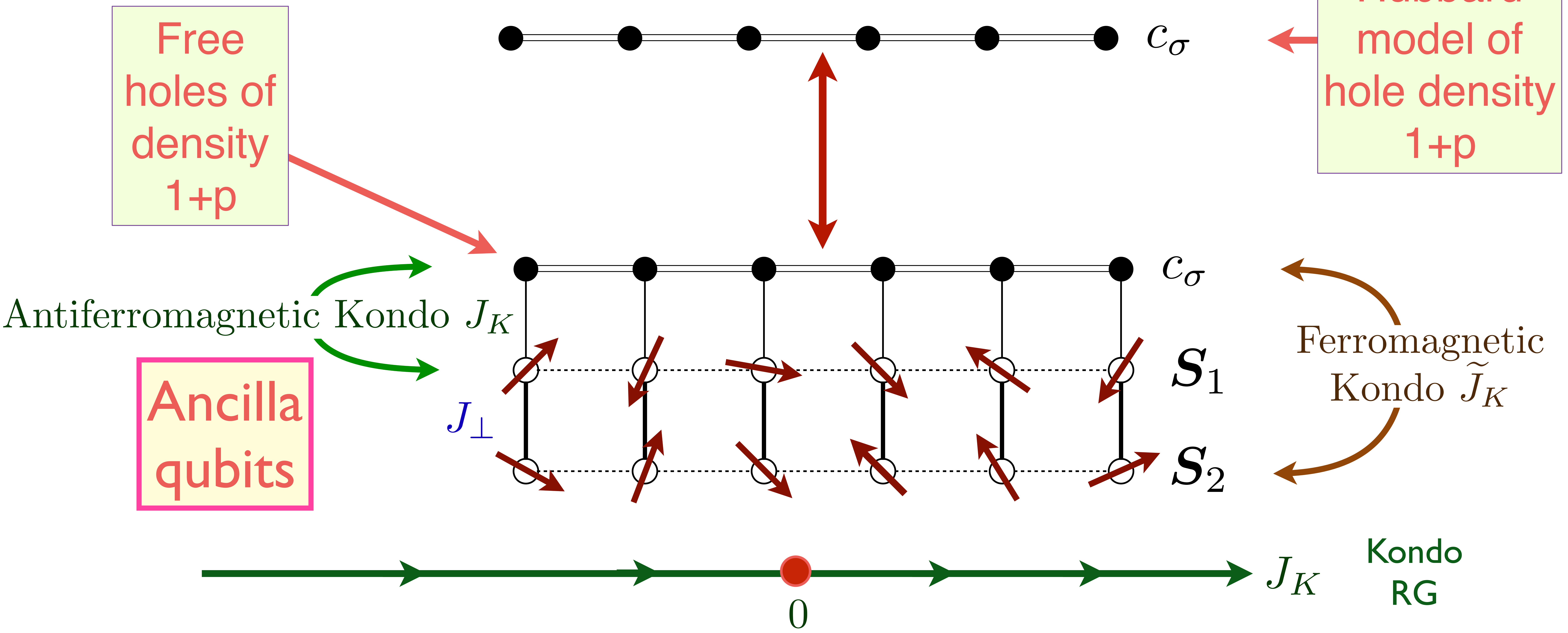

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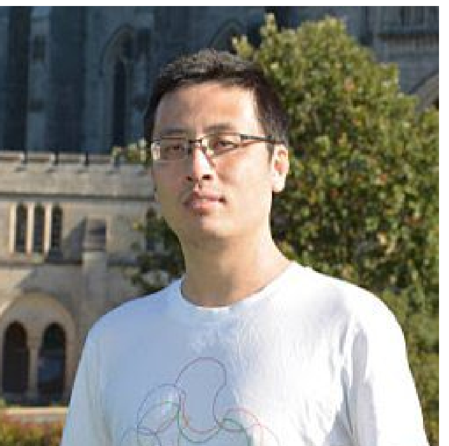
Ya-Hui
Zhang

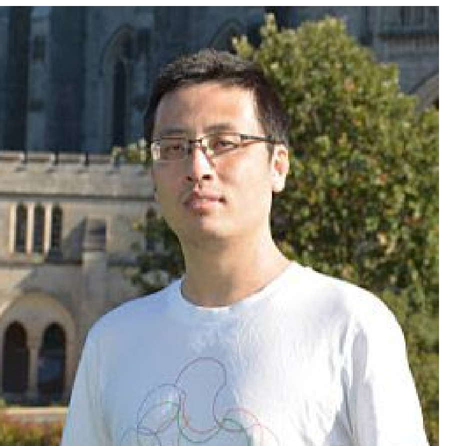


Ancilla theory of the Hubbard model

Zhang and Sachdev, *Phys. Rev. Res.* **2**, 023172 (2020)

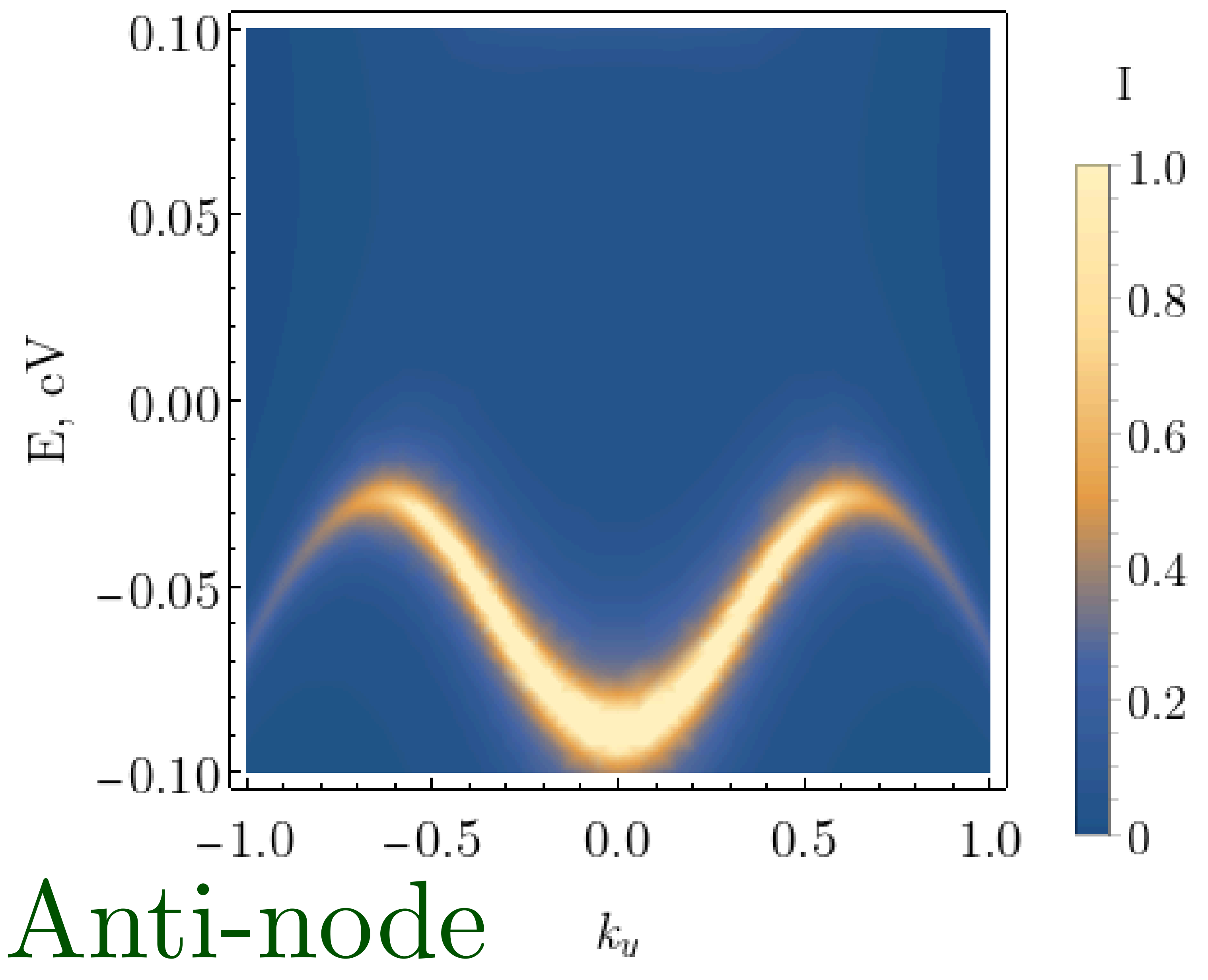
Pseudogap metal =
Kondo Lattice Heavy
Fermi Liquid
$\oplus$
Spin Liquid

Ya-Hui Zhang

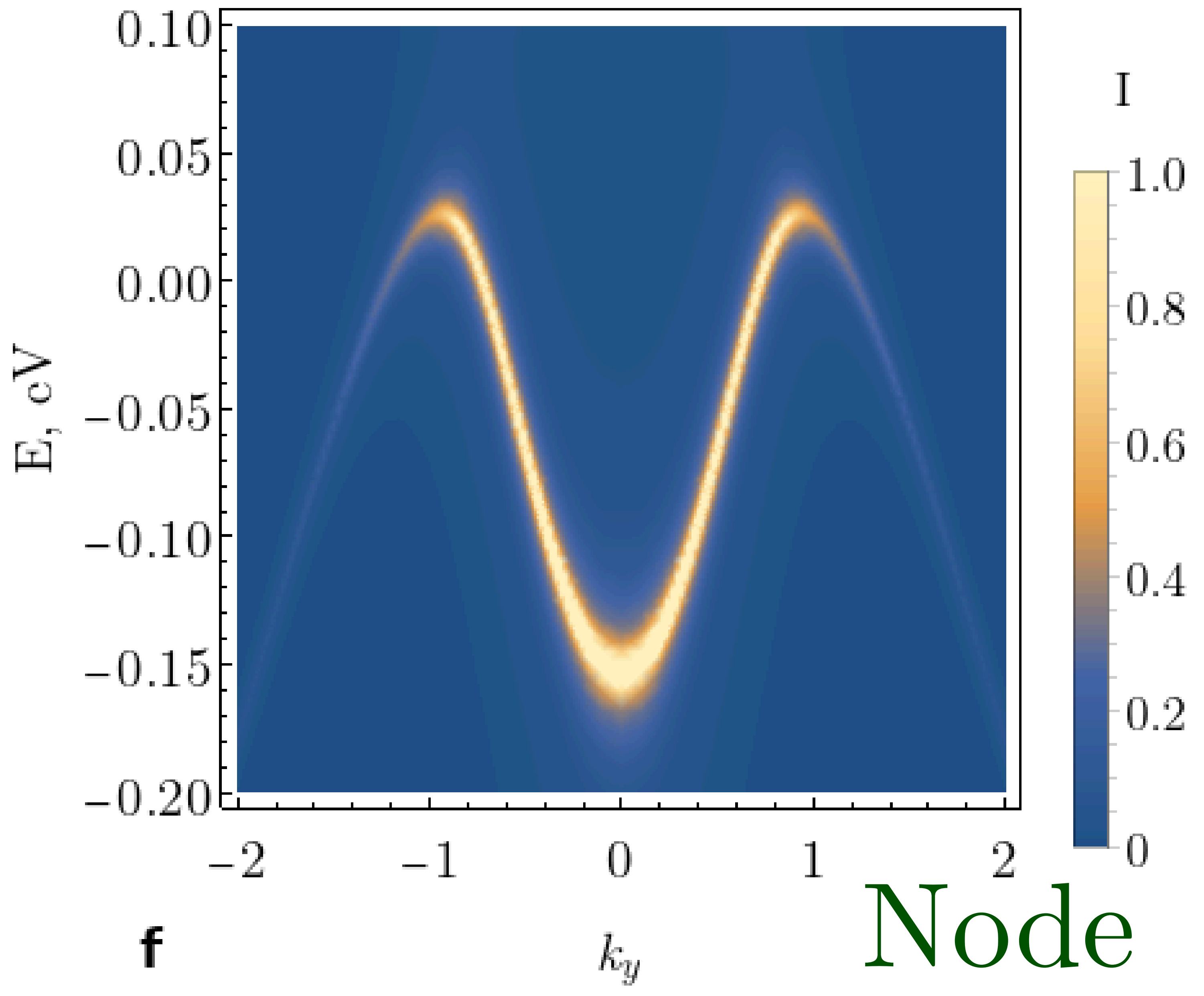


Mascot, Nikolaenko, Tikhanovskaya, Zhang, Morr, and Sachdev, *Phys. Rev. B* **105**, 075146 (2022)

FL* in a *one-band* model

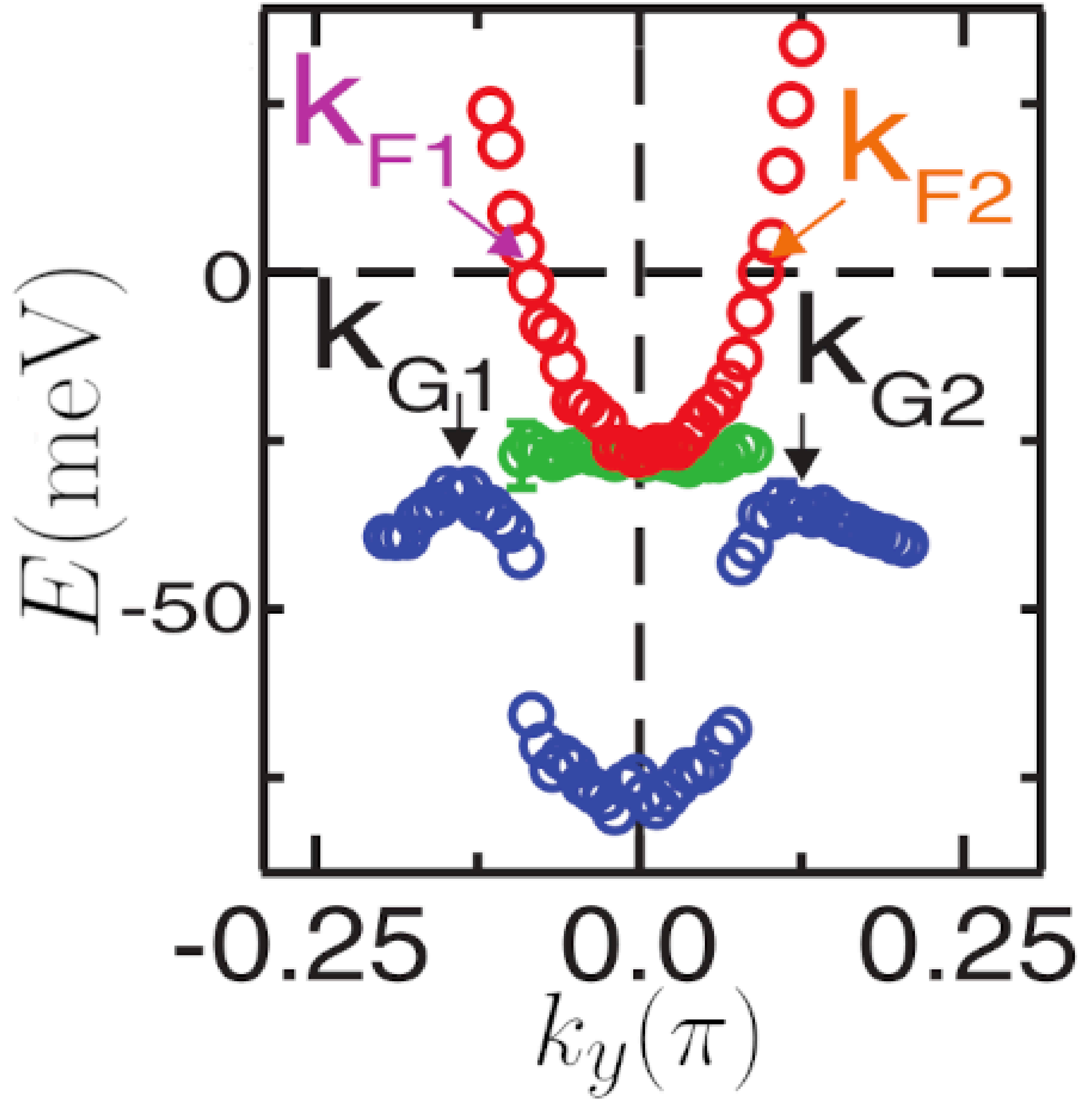


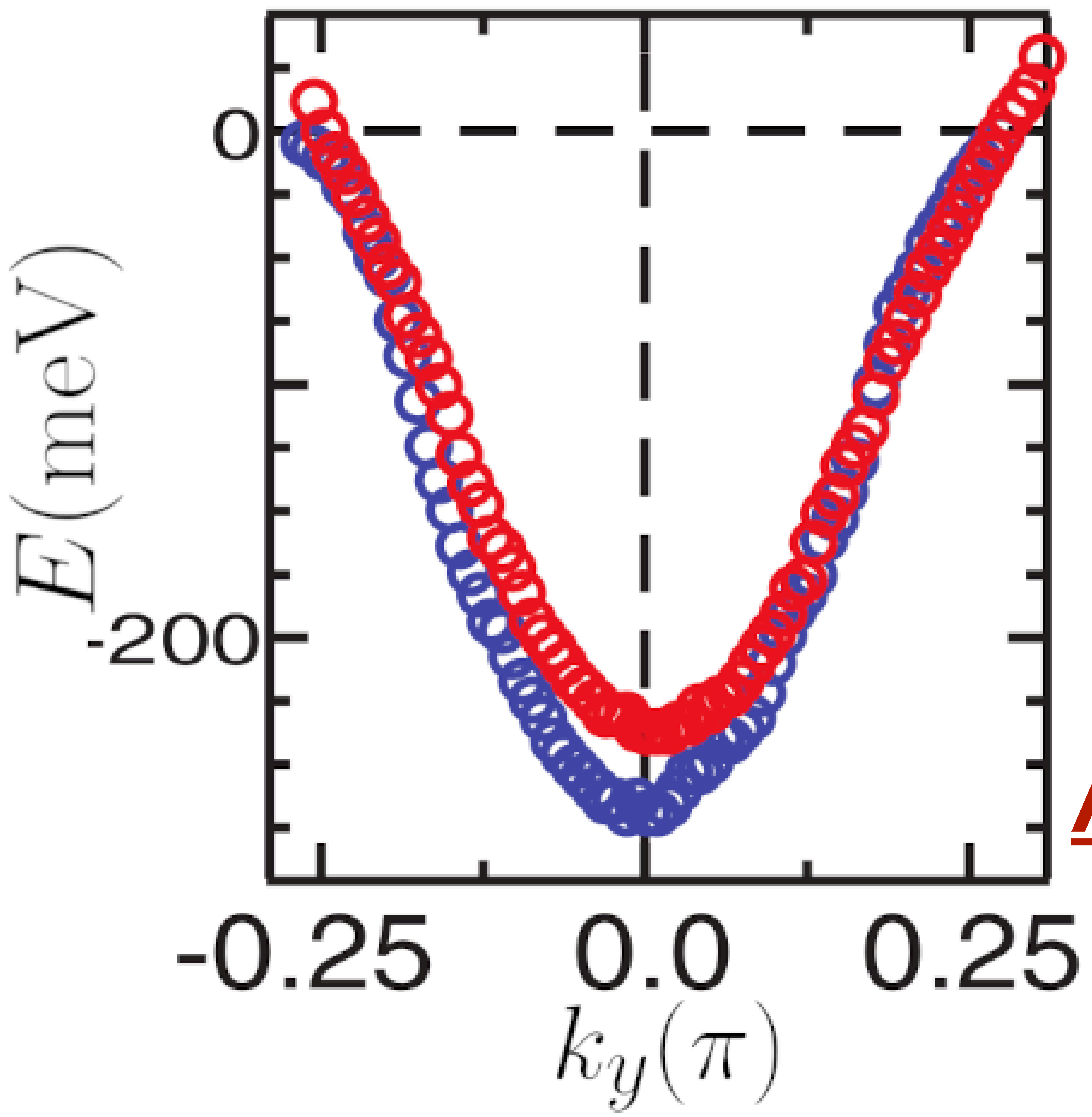
Anti-node



Node

Broadening by second ancilla layer is needed to describe MDC and EDC





He, Hashimoto, Karapetyan, Koralek, Hinton, Testaud, Nathan, Yoshida, Yao, Tanaka, Meevasana, Moore, Lu, Mo, Ishikado, Eisaki, Hussain, Devereaux, Kivelson, Orenstein, Kapitulnik, and Shen, *Science* **331**, 1579 (2011)

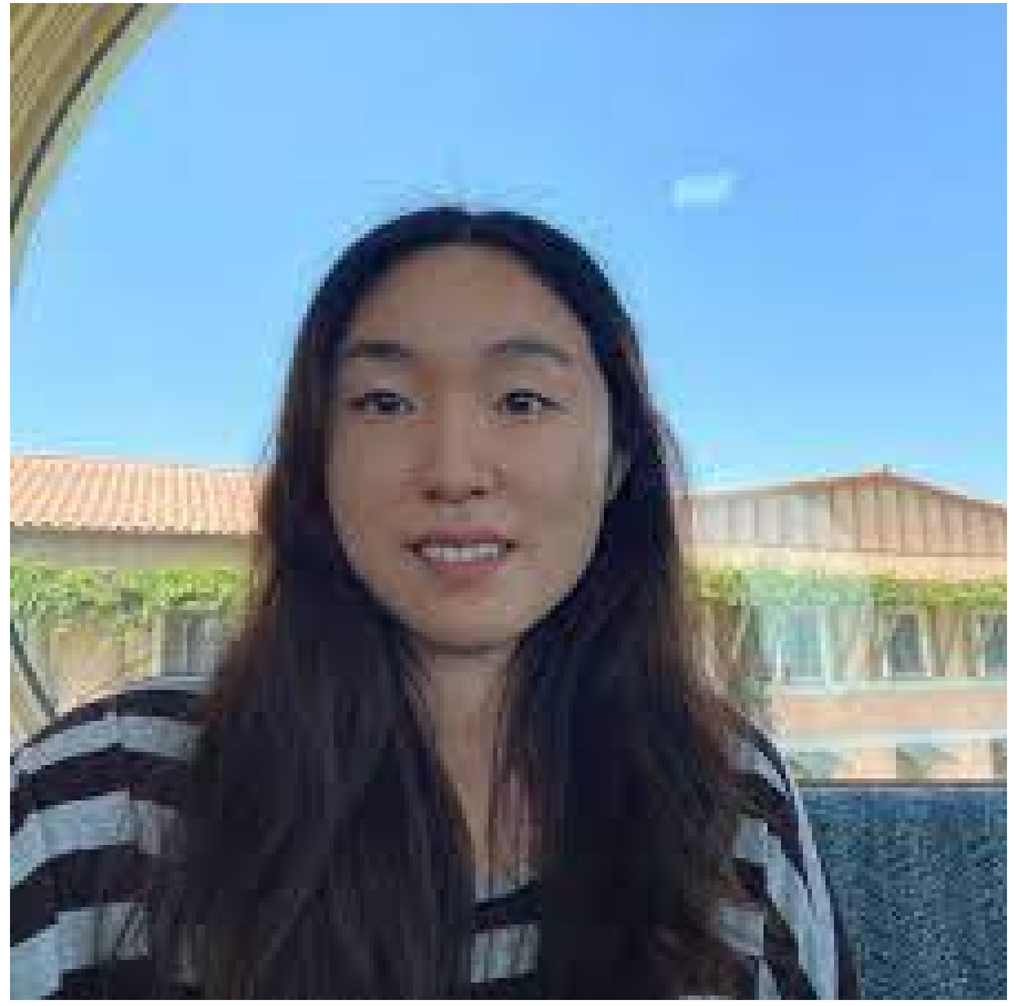
ARPES on Bi2201

Similarities to theories, which do not have explicit reference to spin liquid on second ancilla layer

Yang, Rice, and Zhang, *Phys. Rev. B* **73**, 174501 (2006)
Sakai, Motome, and Imada, *Phys. Rev. Lett.* **102**, 056404 (2009)

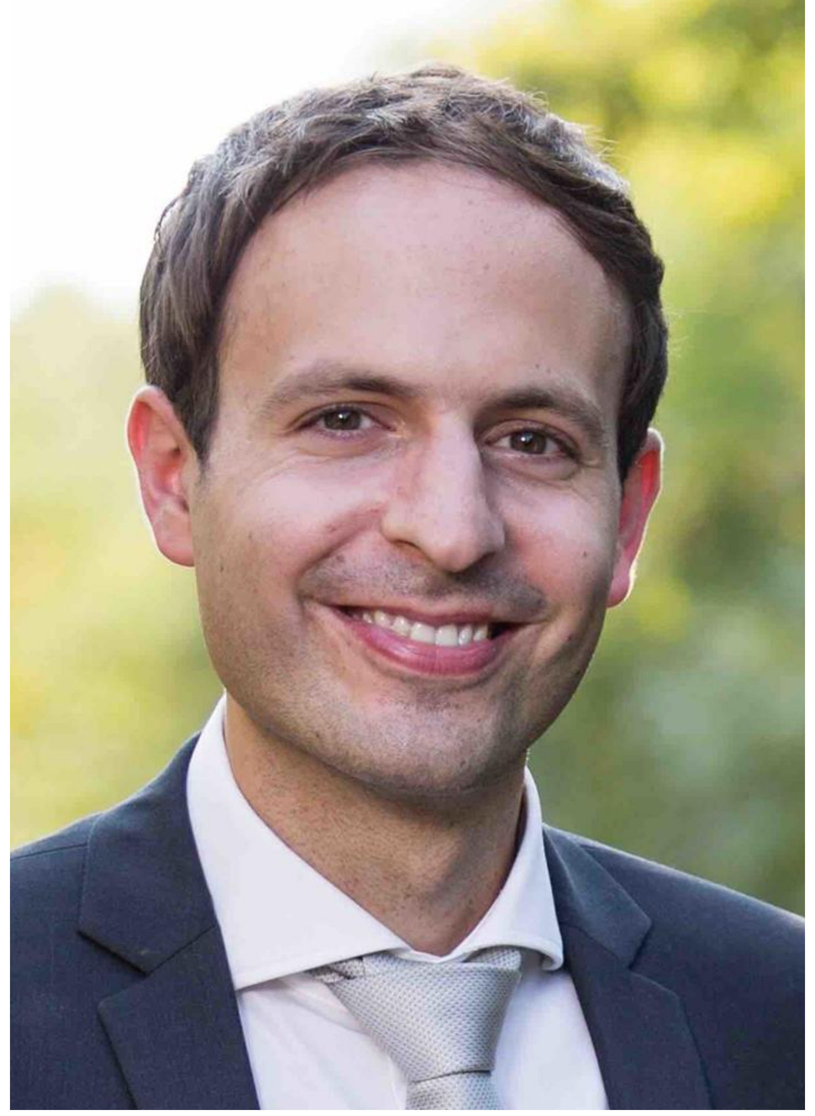
From CP^1/π-flux FL*
to
d-wave superconductivity

Maine Christos

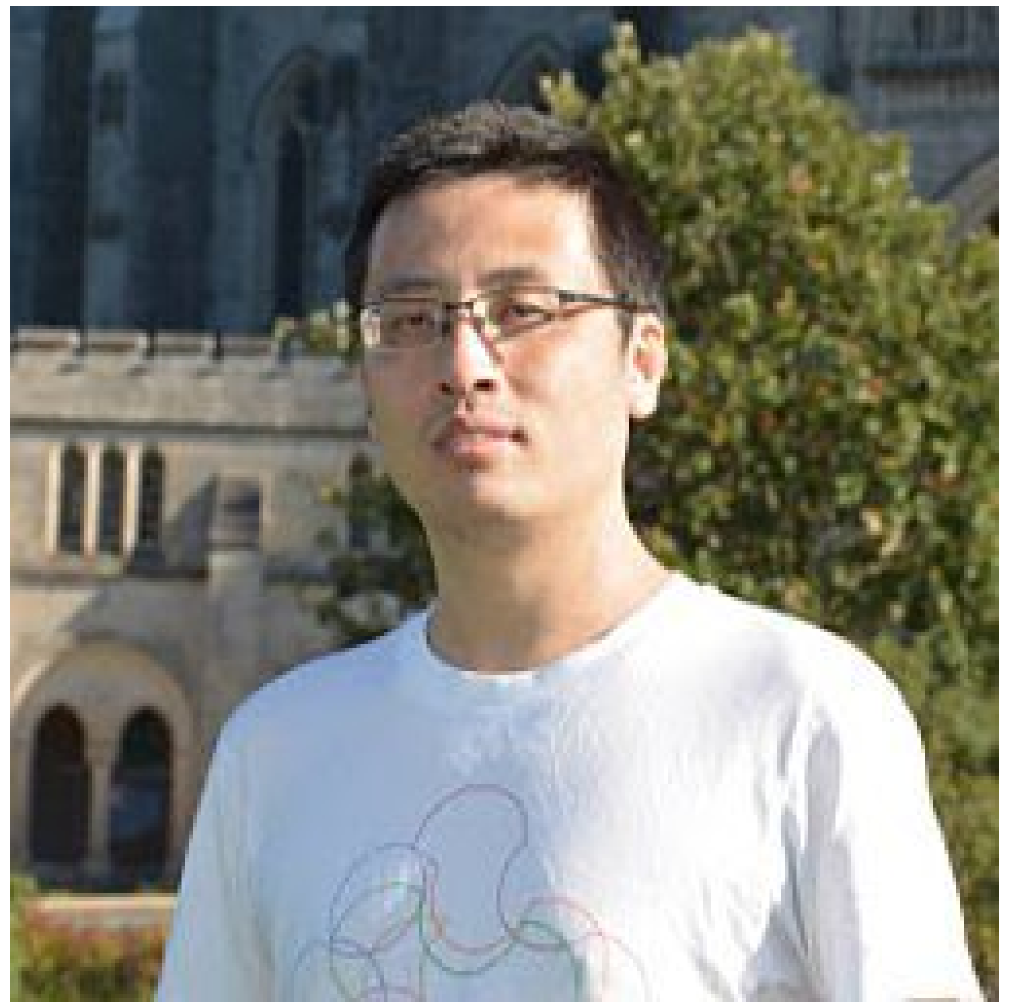


Zhu-Xi Luo
→GA Tech

Henry
Shackleton

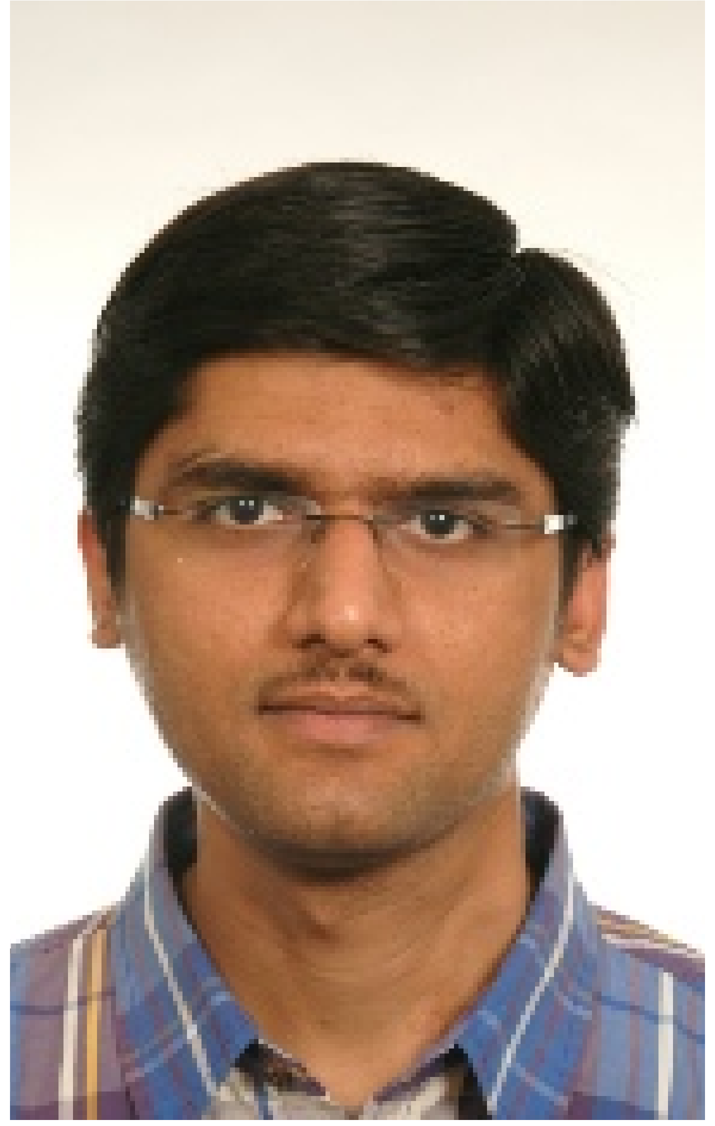


Mathias Scheurer
Innsbruck→Stuttgart



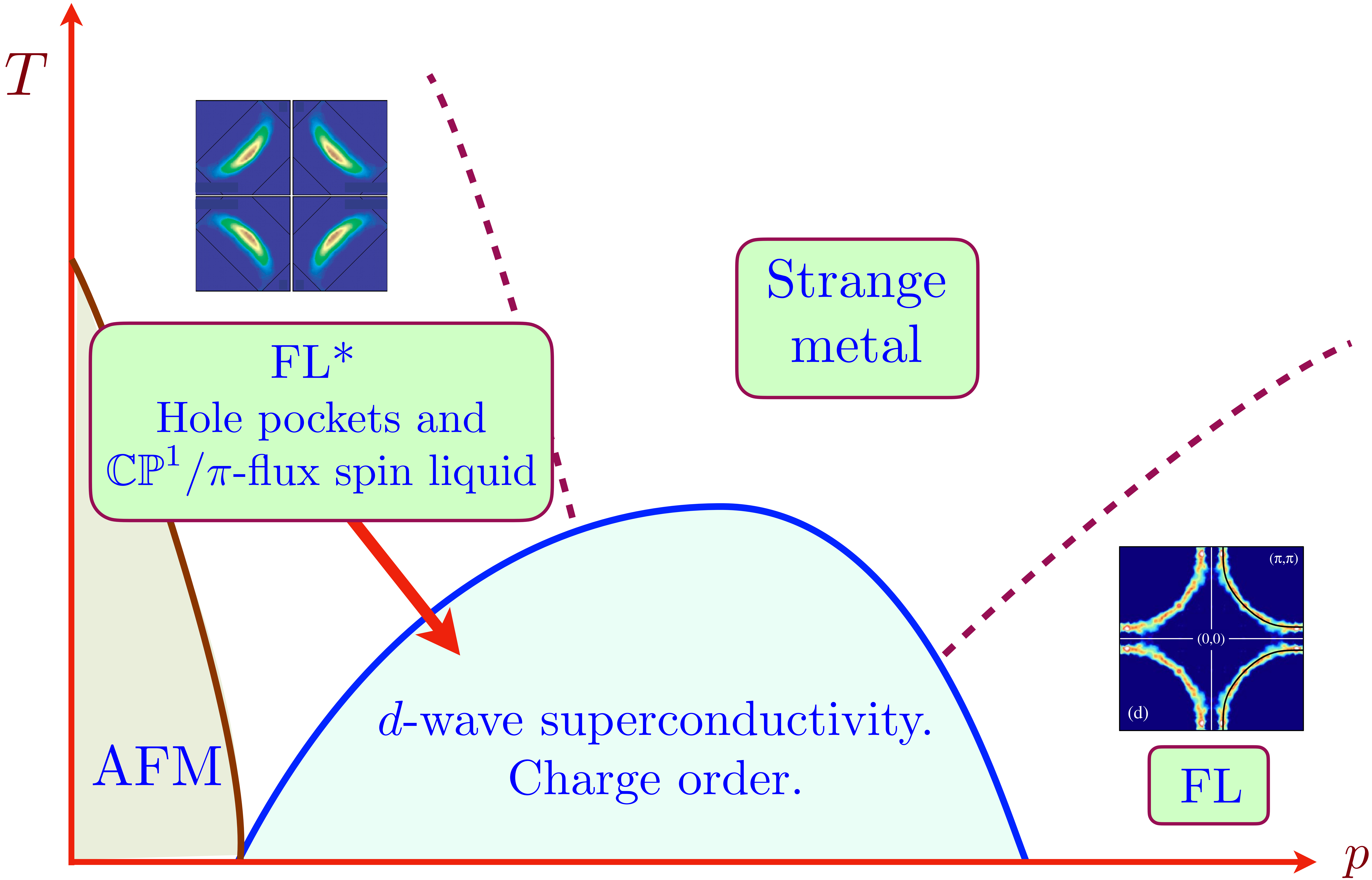
Ya-Hui Zhang
Johns Hopkins

Alexander
Nikolaenko



Darshan Joshi
TIFR Hyderabad

Jonas von Milczewski



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FL
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Ancilla theory of the Hubbard model

Ancilla theory of the Hubbard model

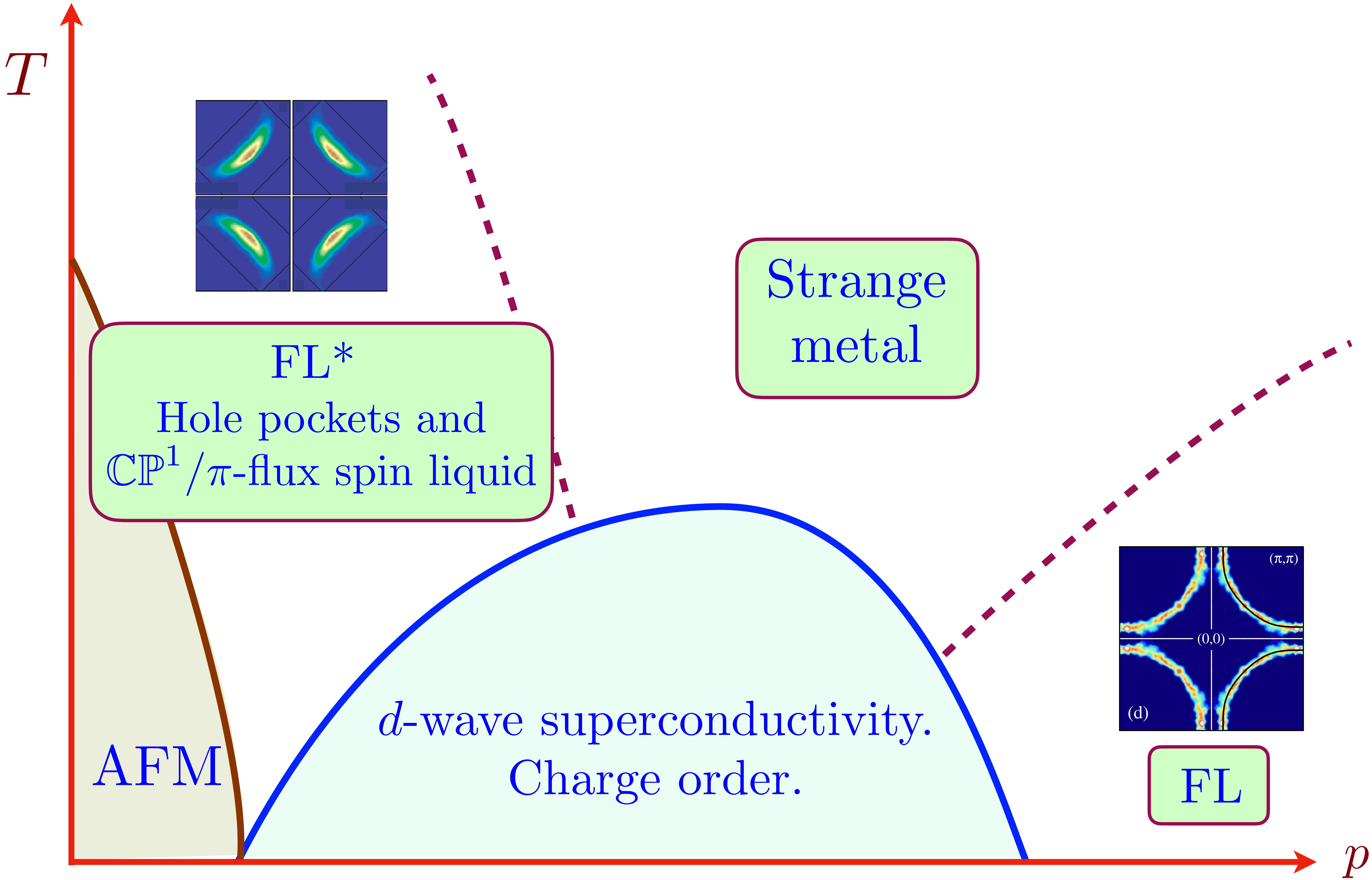
Higgs field Φ drives FL*-strange metal-FL

Ancilla theory of the Hubbard model

B has same quantum numbers as boson in X.-G. Wen and P.A. Lee, *Phys. Rev. Lett.* **76**, 503 (1996)

Ancilla theory of the Hubbard model

Higgs field B drives FL*-dSC/CDW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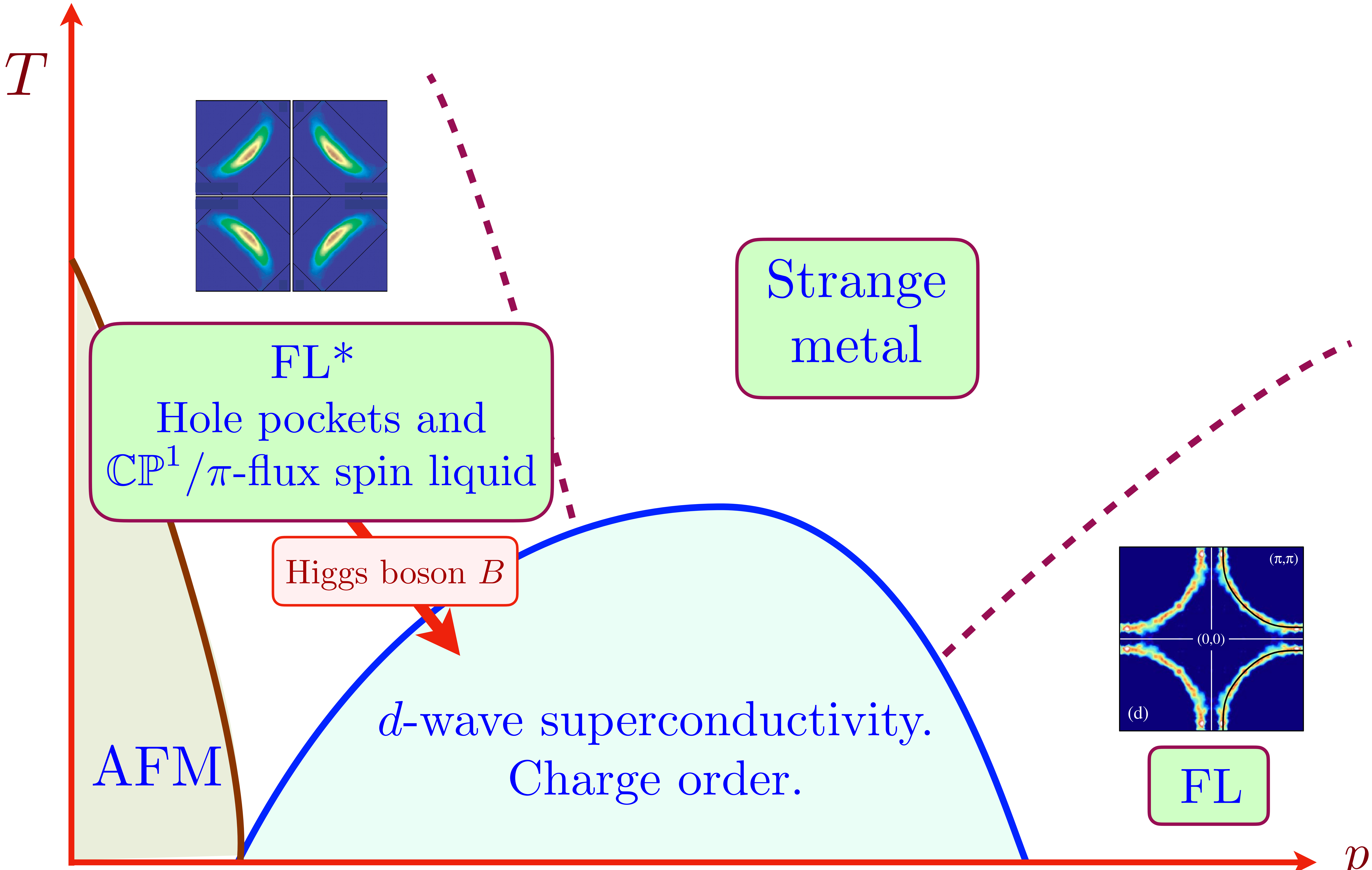

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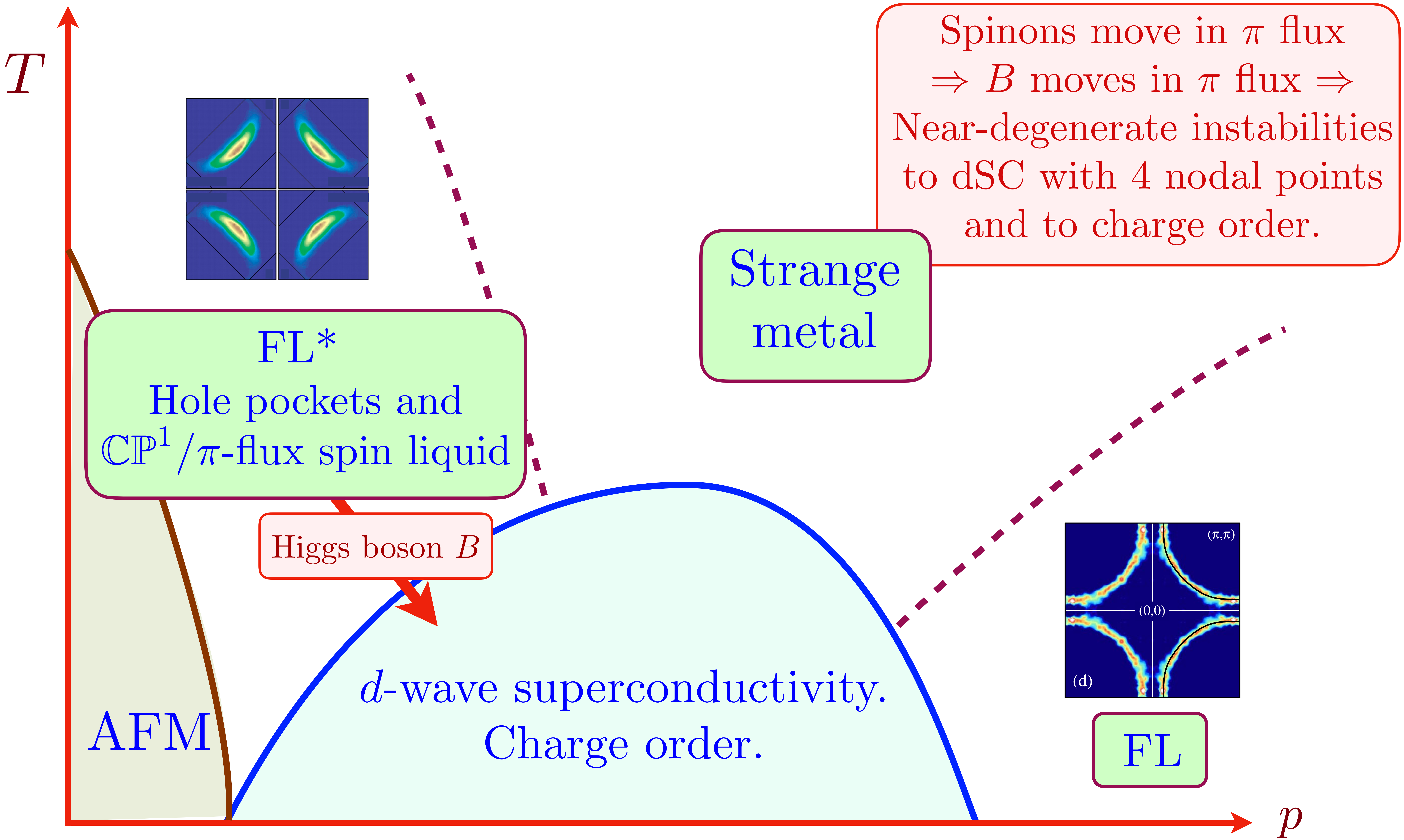
Spinons move in π flux $\Rightarrow$ B moves in π flux $\Rightarrow$ Near-degenerate instabilities to dSC with 4 nodal points and to charge order.
T
Strange metal
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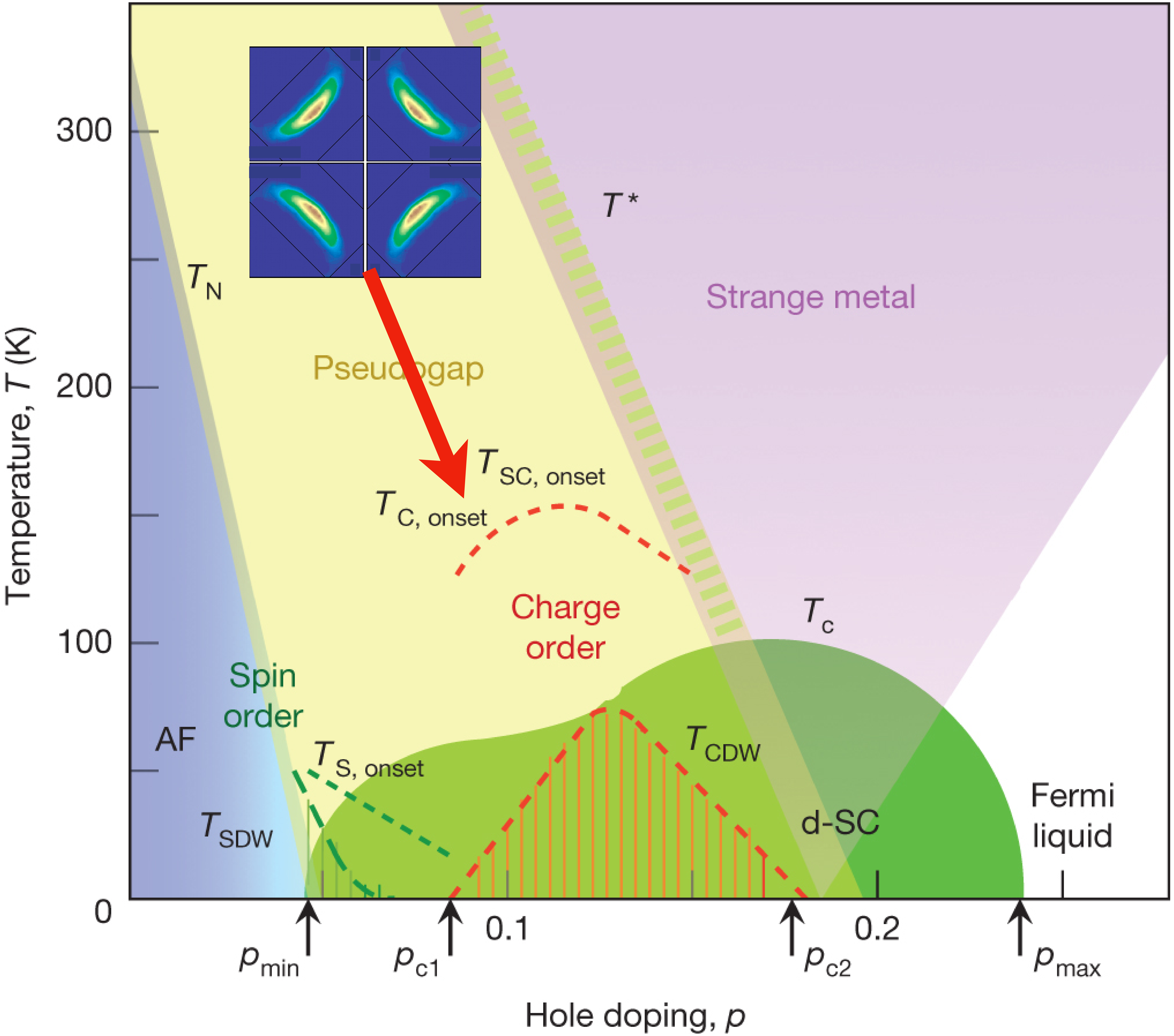
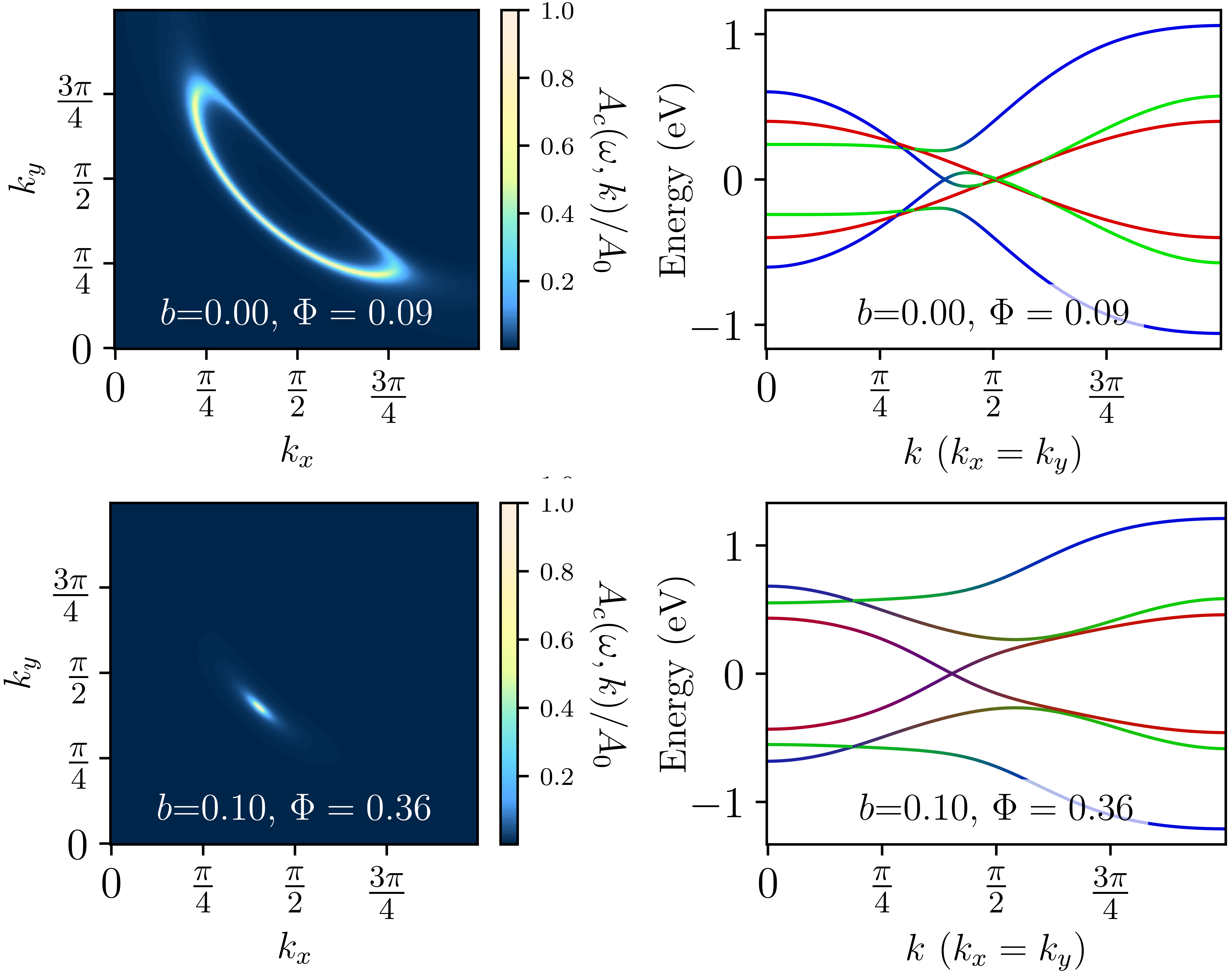
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



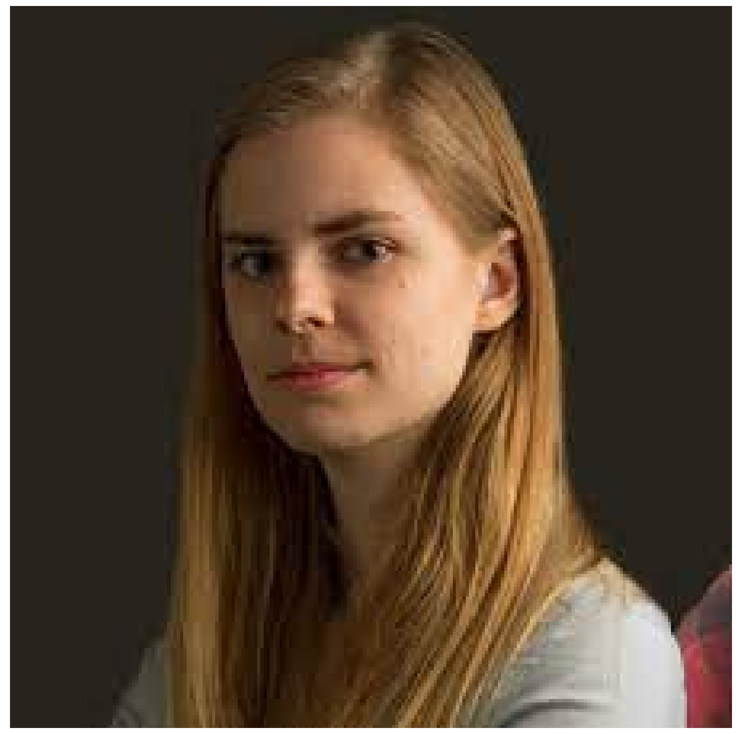
The onset of conventional order is a *confinement transition* for the emergent gauge theory describing the fractionalized excitations of the spin liquid in the FL* state.

Electron spectral density in hole-doped cuprates



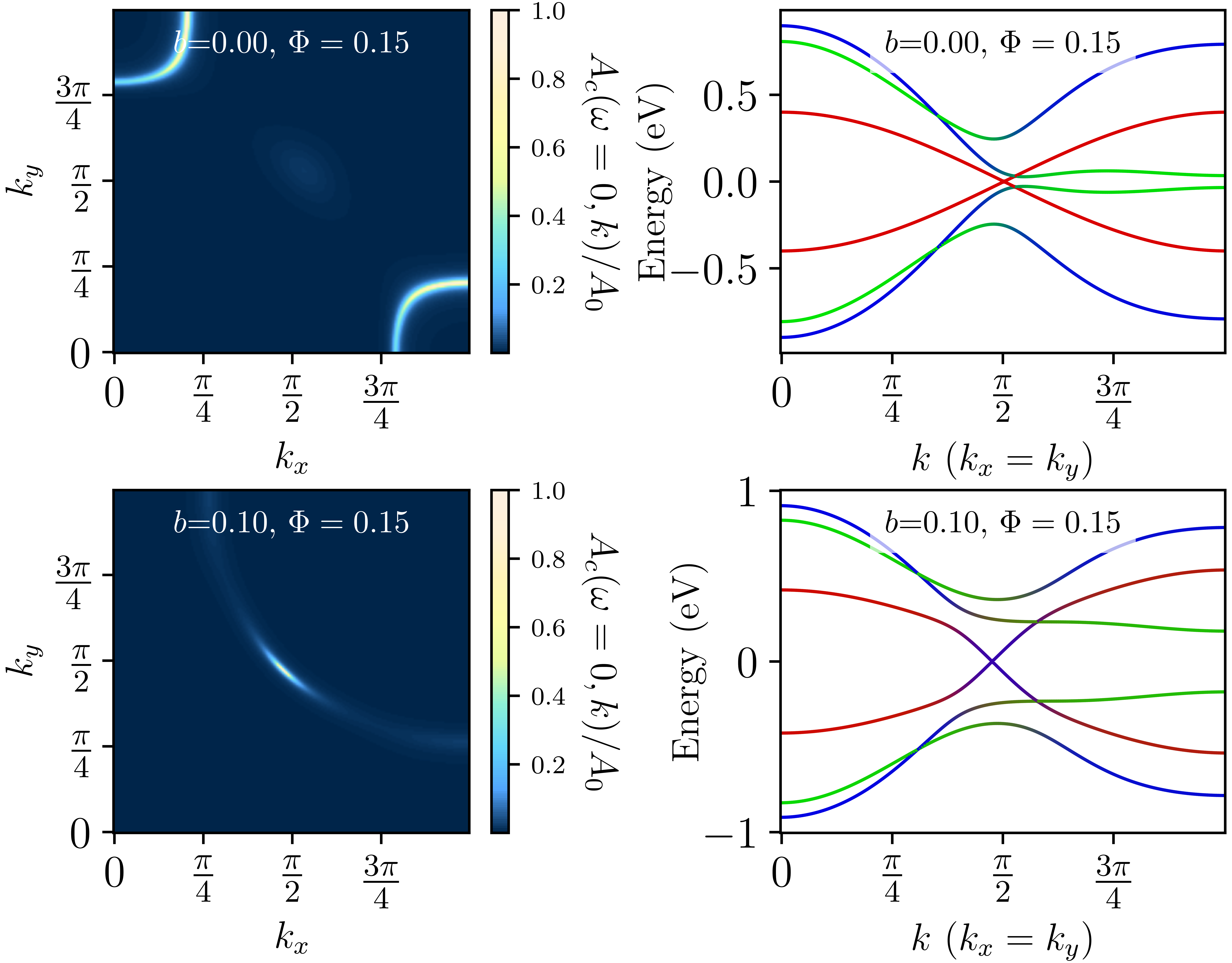
FL*

dSC



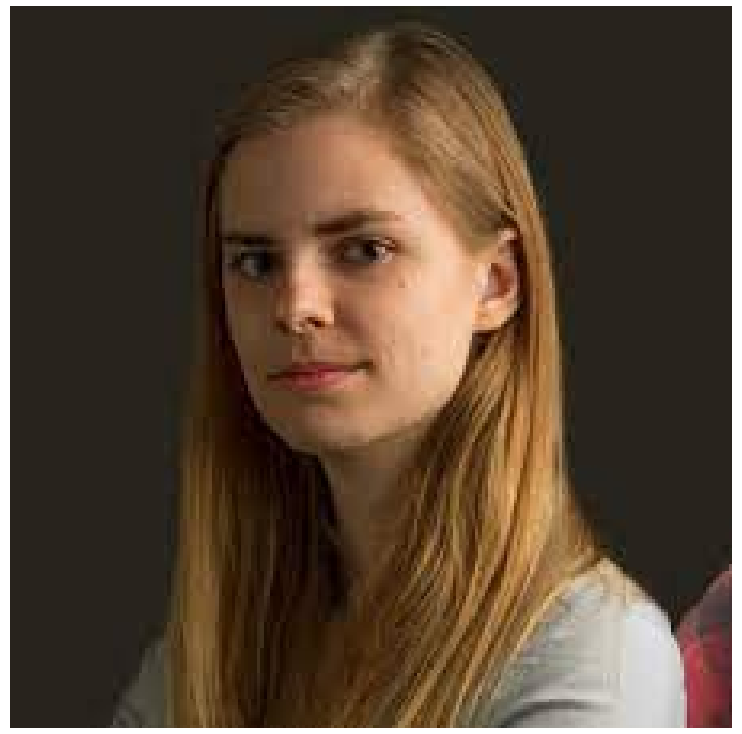
Christos and Sachdev, arXiv:2308.03835 (2023)

Electron spectral density in electron-doped cuprates



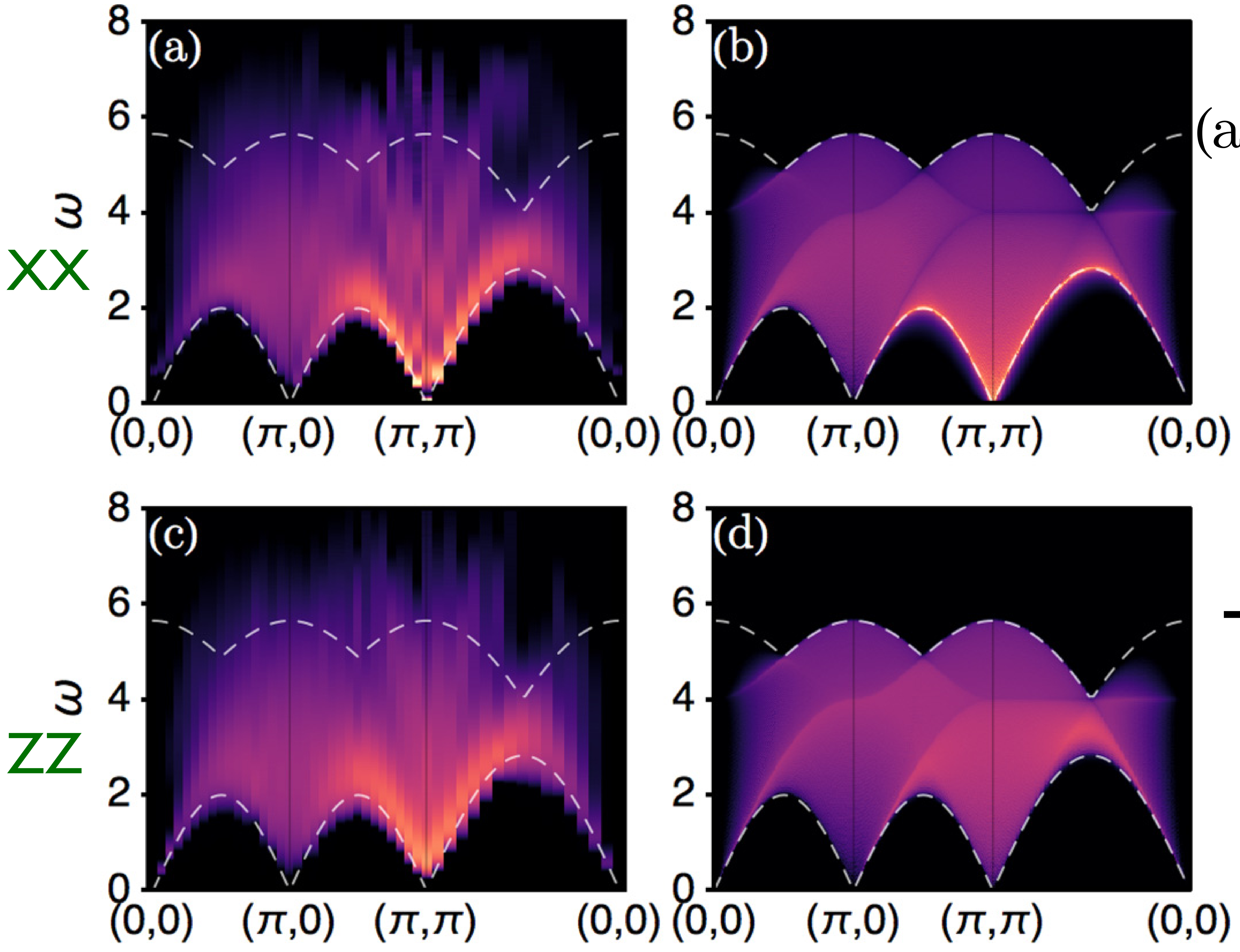
FL*

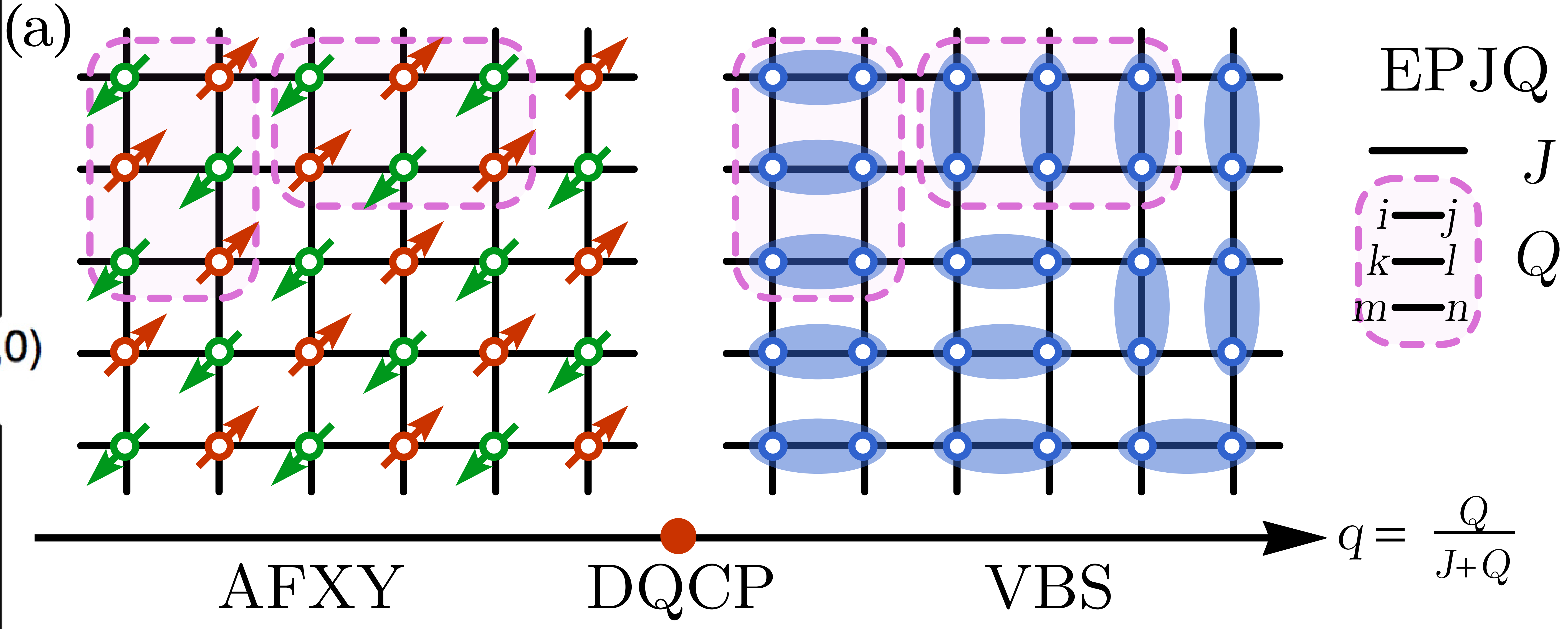
dSC



Christos and Sachdev, arXiv:2308.03835 (2023)

Observable by neutron scattering in pseudogap ?





Ma, Sun, You, Xu, Vishwanath, Sandvik, and Meng, *Phys. Rev. B* **98**, 174421 (2018)

Summary

Insulating S=1/2 antiferromagnet

$\mathbb{CP}^1$ U(1) gauge theory

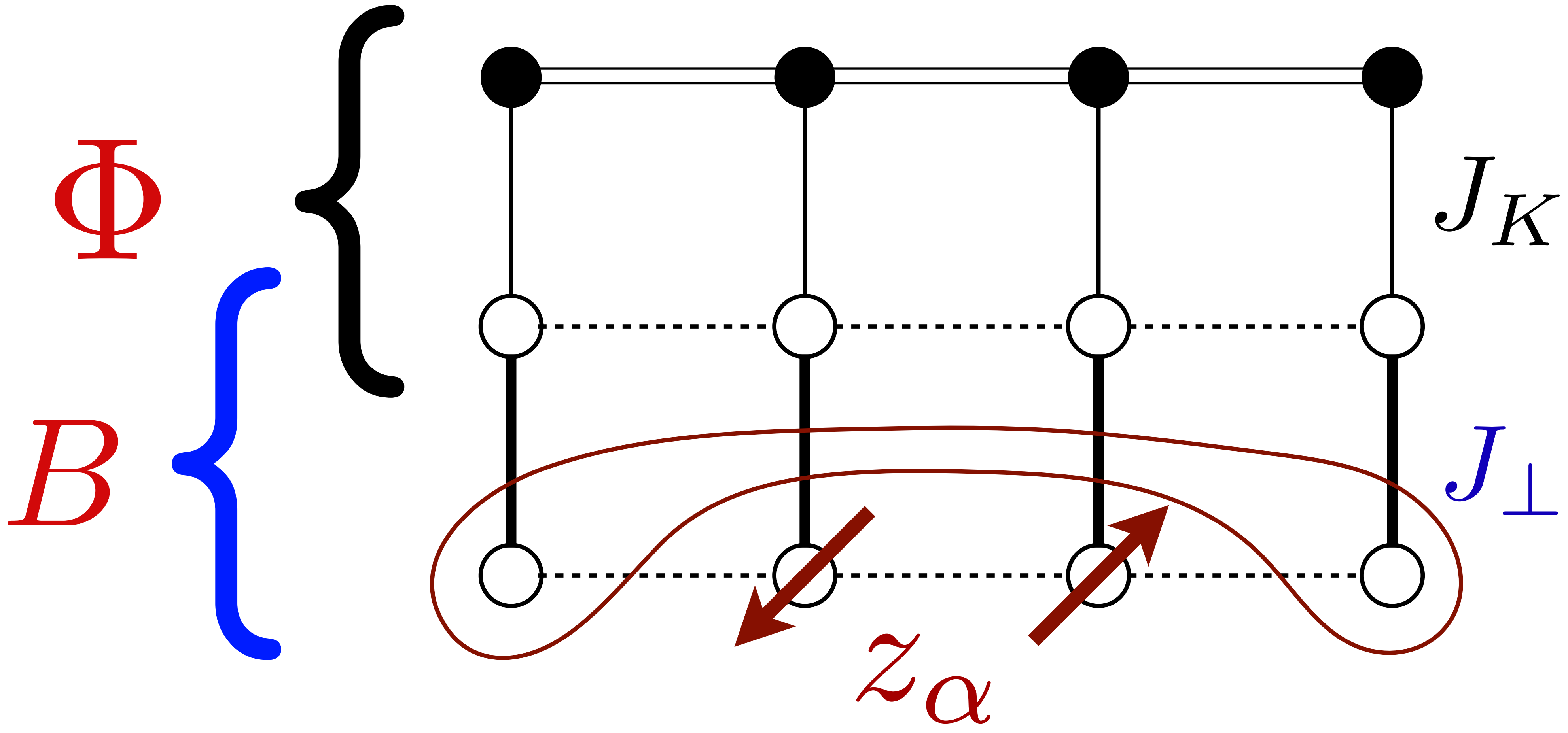
SU(2) gauge theory of $N_f = 2$ fundamental, massless, Dirac fermions.

Obtained from a saddle-point of fermionic spinons moving in π-flux.

Affleck and Marston, *Phys. Rev. B* **37**, 3774 (1988)

SO(5) non-linear σ-model of Néel/VBS orders with $k = 1$ WZW term

Tanaka and Hu, *Phys. Rev. Lett.* **95**, 036402 (2005); Senthil and Fisher, *Phys. Rev. B* **74**, 064405 (2006);
Ran and Wen, arXiv:cond-mat/0609620 (2006); Wang, Nahum, Metlitski, Xu, and Senthil, *Phys. Rev. X* **7**, 031051 (2017)



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T
FL*
Hole pockets
and spin liquid
Strange
metal
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J_K
$J_\perp$
z_α
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(0,0)
(d)
FL
AFM
d-wave superconductivity.
Charge order.
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