Source Localization with Spatially Distributed Active and Passive Sensors

Yifan Liang and Hongbin Li
Department of Electrical and Computer Engineering
Stevens Institute of Technology
Hoboken, NJ 07030, USA

***Abstract*—We consider solving the source localization problem by exploiting measurements collected from both active and passive sensors. We first briefly review some existing least squares approaches that utilize only one type of measurement (active or passive), and further establish a hybrid objective function that includes active and passive measurements simultaneously in the sense of squared least squares. We propose two different methods, Newton's method and the semidefinite relaxation method, to efficiently solve the optimization problem. Simulation results indicate that the source location estimates given by the proposed hybrid methods are superior to the peer methods that only utilize one type of measurement. The performance difference between Newton's method and the semidefinite relaxation method is also investigated.**

***Index Terms*—source localization, active and passive sensors, hybrid measurements, nonconvex optimization, least squares.**

I. Introduction

The problem of locating a source from range measurements or from range-difference measurements collected using a network (or array) of active or passive sensors has received significant attention in the signal processing literature owing to its importance to many applications including teleconferencing, wireless communications, surveillance, navigation, and geophysics [1]–[16]. The first type of source localization systems, which are based on range measurements from active sensors, employ the time of arrival (TOA). The distance can be directly calculated from the TOAs as signals travel with a known velocity. TOA data from two sensors will narrow a position to a position circle; data from a third sensor is required to resolve the precise position to a single point. Many positioning systems, including GPS, use TOA [2]–[4].

Compared to the first type, a time difference of arrival (TDOA) localization system, which is based on range-difference measurements from passive sensors, determines the difference in the source of interest's distance to pairs of sensors at known fixed locations [5], [6]. For one sensor pair, the distance difference results in an infinite number of possible subject locations that satisfy the TDOA. When these possible source locations are plotted, they form a hyperbolic curve. It relies on multiple TDOAs to locate the exact source's position along that curve. For two dimensions, a second TDOA, involving a different pair of sensors (typically one sensor is a member of both pairs, so only one sensor is new), will produce a second curve, which intersects with the first. When the two curves are compared, a small number of possible source locations (typically two) are revealed.

Existing work in [7]–[10] gave detailed discussions about locating sources through TOA/TDOA systems. Efficient algorithms have been proposed for the source localization problem based on maximum likelihood or least squares estimation. Source location and velocity estimation using both TDOA and frequency difference of arrival (FDOA) measurements were examined in [11], [14], [15]. The primary limitation of TOA lies in its stringent requirement for precise time synchronization between the signal source and the receiver; any discrepancy in timekeeping can significantly impair the accuracy of the positioning. This dependency on synchronized clocks introduces complexity, especially in decentralized systems or those with limited infrastructure for time calibration. In contrast, TDOA, while alleviating the need for strict synchronization, introduces its own set of challenges. The accuracy of TDOA is heavily contingent on the precise knowledge of the relative locations of multiple receivers, as the technique is predicated on measuring the time difference as the signal arrives at these disparate points. Furthermore, the computational complexity in TDOA is typically higher than in TOA, as it involves more intricate algorithms to resolve the position from the time differences.

To improve the accuracy and reliability of localization and reduce dependency on system complexity, we try to develop a hybrid approach to locate the source by simultaneously exploiting both active TOA measurements and passive TDOA measurements, which can be solved by two different methods. The development that we consider here is based on the assumption that the sensor network includes both active and passive sensors and can be utilized to obtain (noisy) range or range-difference measurements. Simulation results show that the exploitation of both TOA and TDOA measurements markedly improves the localization performance and outperforms several peer methods in various setups.

II. Problem Formulation

We consider a distributed sensor network that consists of m active sensors respectively located at $\mathbf{a}_i \in \mathbb{R}^2, i = 1, \dots, m$, and n passive sensors located at $\mathbf{c}_j \in \mathbb{R}^2, j = 1, \dots, n$. Let $\mathbf{x} \in \mathbb{R}^2$ denote the source coordinates. The active sensors are capable of proactively detecting the distance between

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themselves and a source, and the range measurement between the source with the i-th active sensor can be expressed as

$$r_i = \|\mathbf{x} - \mathbf{a}_i\| + \varepsilon_i, \quad i = 1, \dots, m, \tag{1}$$

where ε_i denotes the noise term. In contrast, passive sensors can only discern the difference in distance to the source by comparing the variations in the arrival time of the received signals. Suppose there exists an additional reference sensor (sensor 0) located at the coordinate origin and the range-difference measurements obtained by comparing the jth passive sensor and sensor 0 are written as

$$d_j = \|\mathbf{x} - \mathbf{c}_j\| - \|\mathbf{x}\| + \varepsilon_j, \quad j = 1, \dots, n. \tag{2}$$

In this paper, we assume all the unknown noise terms ε in (1) and (2) are independent and identically distributed, following a Gaussian distribution with zero mean and standard deviation σ. Both range information of (1) and (2) can be utilized for source localization. In this work, we propose to concurrently incorporate range measurements and range-difference measurements into a single *hybrid* optimization problem, contributing synergistically to accurate target localization. In the following section, we will briefly review some of the existing methodologies that focus on utilizing only active or passive measurements and further introduce our hybrid algorithm.

III. Localization using active or passive measurements

A. Range-Based Least Squares (R-LS)

Using active sensor measurements, one common approach to obtaining the estimated source location is via establishing the following optimization problem

$$\text{(R-LS):} \quad \min_{\mathbf{x}} \sum_{i=1}^{m} \left(\|\mathbf{x} - \mathbf{a}_i\| - r_i\right)^2 \tag{3}$$

which is a maximum likelihood estimator with Gaussian measurement noise. The solution to (3) is called the *range-based least squares* (R-LS) estimate. Note (3) is a nonconvex problem. There are no existing methods to acquire the exact solution [12]. Semidefinite relaxation (SDR) was used in [12] to solve (3), though the global optimum is not guaranteed.

B. Squared-Range-Based Least Squares (SR-LS)

Another way to exploit active measurements is to apply the least squares methodology to the squared range measurements, leading to the *squared-range-based least squares* (SR-LS) estimate [7]

$$\text{(SR-LS):} \quad \min_{\mathbf{x}} \sum_{i=1}^{m} \left(\|\mathbf{x} - \mathbf{a}_i\|^2 - r_i^2\right)^2 \tag{4}$$

which is suboptimal compared to the maximum-likelihood estimates obtained from (3). Note that (4) is still a nonconvex problem. However, an efficient approach is available to find the global solution of SR-LS, as well as an unconstrained SR-LS solution denoted by *USR-LS*, as shown in [7].

C. Squared-Range-Difference-Based Least Squares (SRD-LS)

To utilize the range-difference measurements from passive sensors for the source localization problem, square both sides of (2) to yield

$$-2d_j\|\mathbf{x}\| - 2\mathbf{c}_j^\top \mathbf{x} = d_j^2 - \|\mathbf{c}_j\|^2, \quad j = 1, \dots, n. \tag{5}$$

The objective function can be derived as

$$\min_{\mathbf{x}} \sum_{j=1}^{n} \left(-2\mathbf{c}_j^T \mathbf{x} - 2d_j\|\mathbf{x}\| - (d_j^2 - \|\mathbf{c}_j\|^2)\right)^2. \tag{6}$$

The solution to the above problem is called the *squared-range-difference-based least squares* (SRD-LS) estimate, which is also a suboptimal solution compared with the maximum likelihood estimate. The global optimum can be efficiently solved by a numerical algorithm while an unconstrained version of the SRD-LS estimate is denoted by *USRD-LS*, which were reported in [7].

IV. Proposed Hybrid Localization Approach

In this section, we propose a hybrid approach that uses both active (range) and passive (range-difference) measurements simultaneously in the sense of squared least squares to solve the source localization problem. Two methods are employed to solve the hybrid estimation.

A. Squared-Hybrid Least Squares (SH-LS)

Based on (1) and (2), the hybrid objective function is formulated as

$$\begin{aligned} \min_{\mathbf{x}} \sum_{i=1}^{m} \left(\|\mathbf{x} - \mathbf{a}_i\|^2 - r_i^2\right)^2 + \\ \sum_{j=1}^{n} \left((\|\mathbf{x}\| + d_j)^2 - \|\mathbf{x} - \mathbf{c}_j\|^2\right)^2. \end{aligned} \tag{7}$$

Note that the optimization problem (7) is a nonconvex problem [17], [18]. To find the global optimal solution, we transform (7) into a constrained minimization problem by using the substitution $\mathbf{y} = \left(\mathbf{x}^\top, \|\mathbf{x}\|^2, \|\mathbf{x}\|\right)^\top$

$$\begin{aligned} &\text{(SH-LS):} \quad \min_{\mathbf{y}} \|\mathbf{A}\mathbf{y} - \mathbf{b}\|^2 \\ &\text{s.t.} \quad \mathbf{y}^\top \mathbf{C}\mathbf{y} + 2\mathbf{f}^\top \mathbf{y} = 0, \quad \mathbf{y}^\top \mathbf{D}\mathbf{y} + 2\mathbf{f}^\top \mathbf{y} = 0 \end{aligned} \tag{8}$$

where

$$\mathbf{A} = \begin{pmatrix} -2\mathbf{a}_1^\top & 1 & 0 \\ \vdots & \vdots & \vdots \\ -2\mathbf{a}_m^\top & 1 & 0 \\ -2\mathbf{c}_1^\top & 0 & 2d_1 \\ \vdots & \vdots & \vdots \\ -2\mathbf{c}_n^\top & 0 & 2d_n \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} r_1^2 - \|\mathbf{a}_1\|^2 \\ \vdots \\ r_m^2 - \|\mathbf{a}_m\|^2 \\ d_1^2 - \|\mathbf{c}_1\|^2 \\ \vdots \\ d_n^2 - \|\mathbf{c}_n\|^2 \end{pmatrix}$$

and

$$\mathbf{C} = \begin{pmatrix} \mathbf{0}_{(n+1)\times(n+1)} & \mathbf{0}_{(n+1)\times 1} \\ \mathbf{0}_{1\times(n+1)} & 1 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 2\mathbf{I}_n & \mathbf{0}_{n\times 1} & \mathbf{0}_{n\times 1} \\ \mathbf{0}_{1\times n} & 0 & 0 \\ \mathbf{0}_{1\times n} & 0 & -1 \end{pmatrix}$$

$$\mathbf{f} = \begin{pmatrix} \mathbf{0}_{n\times 1} \\ -0.5 \\ 0 \end{pmatrix}.$$

One way to obtain the approximation solution of (8), is to discard the quadratic constraints. This gives rise to the *unconstrained squared-hybrid least squares* (USH-LS) problem

$$\text{(USH-LS):} \quad \min_{\mathbf{y}} \|\mathbf{A}\mathbf{y} - \mathbf{b}\|^2. \tag{9}$$

The closed-form solution of (9) is given by

$$\hat{\mathbf{y}} = \left(\mathbf{A}^\top \mathbf{A}\right)^{-1} \mathbf{A}^\top \mathbf{b} \tag{10}$$

and the corresponding estimate of $\mathbf{x}$ is the vector comprised of the first two components of $\hat{\mathbf{y}}$.

The exact solution of (8) can also be efficiently found [19], and the resulting method is called *constrained squared-hybrid least squares* (SH-LS) estimator. The Lagrangian function of (8) is

$$\begin{aligned}\Phi(\mathbf{y}, \lambda_1, \lambda_2) = \mathbf{y}^\top \mathbf{A}^\top \mathbf{A}\mathbf{y} - \mathbf{b}^\top \mathbf{A}\mathbf{y} - \mathbf{y}^\top \mathbf{A}^\top \mathbf{b} + \\ \lambda_1 \left(\mathbf{y}^\top \mathbf{D}\mathbf{y} + 2\mathbf{f}^\top \mathbf{y}\right) + \lambda_2 \left(\mathbf{y}^\top \mathbf{C}\mathbf{y} + 2\mathbf{f}^\top \mathbf{y}\right).\end{aligned} \tag{11}$$

The partial derivative of $\mathbf{y}$ should be equal to 0

$$\begin{aligned}\frac{\partial \Phi}{\partial \mathbf{y}} =& 2\mathbf{y}^\top \left(\mathbf{A}^\top \mathbf{A} + \lambda_1 \mathbf{D} + \lambda_2 \mathbf{C}\right) \\ &- 2\mathbf{b}^\top \mathbf{A} + 2\lambda_1 \mathbf{f}^\top + 2\lambda_2 \mathbf{f}^\top.\end{aligned} \tag{12}$$

Therefore $\mathbf{y}$ can be expressed in λ_1 and λ_2 as:

$$\mathbf{y}(\lambda_1, \lambda_2) = \left(\mathbf{A}^\top \mathbf{A} + \lambda_1 \mathbf{D} + \lambda_2 \mathbf{C}\right)^{-1} \left(\mathbf{A}^\top \mathbf{b} - \lambda_1 \mathbf{f} - \lambda_2 \mathbf{f}\right). \tag{13}$$

Let

$$\begin{aligned}\mathbf{G} &= \mathbf{A}^\top \mathbf{A} + \lambda_1 \mathbf{D} + \lambda_2 \mathbf{C} \\ \mathbf{p} &= \mathbf{A}^\top \mathbf{b} - \lambda_1 \mathbf{f} - \lambda_2 \mathbf{f}.\end{aligned} \tag{14}$$

Then the two constraints (8) can be expressed as:

$$f_1(\lambda_1, \lambda_2) = \mathbf{p}^\top \mathbf{G}^{-\top} \mathbf{D} \mathbf{G}^{-1} \mathbf{p} + 2\mathbf{f}^\top \mathbf{G}^{-1} \mathbf{p} \tag{15}$$

$$f_2(\lambda_1, \lambda_2) = \mathbf{p}^\top \mathbf{G}^{-\top} \mathbf{C} \mathbf{G}^{-1} \mathbf{p} + 2\mathbf{f}^\top \mathbf{G}^{-1} \mathbf{p}. \tag{16}$$

Newton's method can be utilized to find the optimal λ_1 and λ_2 in the following form to force f_1 and f_2 equal to 0. Let $\boldsymbol{\lambda}^{(0)} = \left[\lambda_1^{(0)}, \lambda_2^{(0)}\right]^\top$ and $\mathbf{h}^{(0)} = \left(f_1^{(0)}, f_2^{(0)}\right)^\top$, where $f_1^{(0)} = f_1(\lambda_1^{(0)}, \lambda_2^{(0)})$ and $f_2^{(0)} = f_2(\lambda_1^{(0)}, \lambda_2^{(0)})$. The problem is solved by implementing the iteration starting from $k = 0$

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} - \left(1 - \alpha^k\right) \left[\frac{\partial \mathbf{h}^{(k)}}{\partial \boldsymbol{\lambda}^{(k)}}\right]^{-1} \mathbf{h}^{(k)} \tag{17}$$

where

$$\frac{\partial \mathbf{h}^{(k)}}{\partial \boldsymbol{\lambda}^{(k)}} = \begin{bmatrix} \frac{\partial f_1^{(k)}}{\partial \lambda_1^{(k)}} & \frac{\partial f_1^{(k)}}{\partial \lambda_2^{(k)}} \\ \frac{\partial f_2^{(k)}}{\partial \lambda_1^{(k)}} & \frac{\partial f_2^{(k)}}{\partial \lambda_2^{(k)}} \end{bmatrix} \tag{18}$$

$$\mathbf{h}^{(k)} = \left(f_1^{(k)}, f_2^{(k)}\right)^\top \tag{19}$$

$$f_1^{(k)} = f_1(\lambda_1^{(k)}, \lambda_2^{(k)}) \tag{20}$$

$$f_2^{(k)} = f_2(\lambda_1^{(k)}, \lambda_2^{(k)}). \tag{21}$$

α is the learning rate of Newton's method and is usually set between 0 and 1. The iteration is stopped when the values of (15) and (16) become smaller than a threshold η. In this work, the elements of the initial $\boldsymbol{\lambda}^{(0)}$ are randomly chosen between 0 and 1.

B. Semidefinite Relaxation (SDR)

Another way to solve the SH-LS problem is by applying semidefinite relaxation to (7). Here we keep using the substitution $\mathbf{y} = \left(\mathbf{x}^\top, \|\mathbf{x}\|^2, \|\mathbf{x}\|\right)^\top$. Let

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}\mathbf{y}^\top & \mathbf{y} \\ \mathbf{y}^\top & 1 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{D} & \mathbf{f} \\ \mathbf{f}^\top & 1 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{A}^\top \mathbf{A} & -\mathbf{A}^\top \mathbf{b} \\ -\mathbf{b}^\top \mathbf{A} & \mathbf{b}^\top \mathbf{b} \end{bmatrix}$$

where $\mathbf{Y}$ is a 5×5 semi-definite symmetric matrix, and

$$\begin{aligned} \text{(SDR):} \quad & \min_{\mathbf{Y}} \operatorname{tr}(\mathbf{Y}\mathbf{S}) \\ \text{s.t.} \quad & \mathbf{Y}_{5,5} = 1 \\ & \operatorname{tr}(\mathbf{Y}_{1:2,1:2}) = \mathbf{Y}_{4,4} \\ & \operatorname{tr}(\mathbf{Y}\mathbf{M}) = 1 \\ & \operatorname{rank}(\mathbf{Y}) = 1. \end{aligned} \tag{22}$$

The SDR problem can be efficiently solved by using the SeDuMi toolbox in Matlab by dropping the rank constraint. The SDR solution exactly leads to the global optimum of the SH-LS problem if the solution is strictly restricted as in (22). However, the matrix $\mathbf{Y}$ solved by SeDuMi may have a rank larger than 1. In these situations, *singular value decomposition* (SVD) can be used to obtain a rank-one approximation of $\mathbf{Y}$. The SVD of $\mathbf{Y}$ is written as

$$\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \tag{23}$$

where $\mathbf{U}$ is the matrix consisting of the left-singular vectors, and $\boldsymbol{\Sigma}$ is a diagonal matrix with diagonal entries of $\boldsymbol{\Sigma}$ equal to the singular values of $\mathbf{Y}$. If the sum of the four smaller singular values of $\mathbf{Y}$ is close to zero, the solution matrix $\mathbf{Y}$ is considered a *tight* case [7]. In tight cases, we can get a good rank-one approximation result, by simply adjusting the first left-singular vector that corresponds to the largest singular value. We will discuss more details in the next section.

C. CramrRao Lower Bound (CRLB)

Denote the true source location by $\boldsymbol{\theta} = (\theta_1, \theta_2)^\top \in R^2$. The covariance matrix of the unbiased estimator $\hat{\boldsymbol{\theta}}$ satisfies [20]

$$\mathrm{Cov}(\hat{\boldsymbol{\theta}}) \geqslant I^{-1}(\boldsymbol{\theta}) \tag{24}$$

where $I(\boldsymbol{\theta})$ denotes the Fisher Information

$$I(\boldsymbol{\theta}) = \begin{bmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{bmatrix}. \tag{25}$$

The individual component of the Fisher information is given by

$$I_{ij} = E\left[\frac{\partial}{\partial \theta_i} \ln f(\mathbf{w}; \boldsymbol{\theta}) \frac{\partial}{\partial \theta_j} \ln f(\mathbf{w}; \boldsymbol{\theta})\right] \tag{26}$$

where the observation vector $\mathbf{w}$ contains all active and passive measurements and $f(\mathbf{w}; \boldsymbol{\theta})$ denotes the likelihood function. According to (1) and (2),

$$\mathbf{w}(\boldsymbol{\theta}) = [r_1, \ldots, r_m, d_1, \ldots, d_n]^\top. \tag{27}$$

For the typical problem form (7), we can obtain

$$I(\boldsymbol{\theta}) = \frac{1}{\sigma^2}\left(\frac{\partial \mathbf{w}}{\partial \boldsymbol{\theta}}\right)^\top \left(\frac{\partial \mathbf{w}}{\partial \boldsymbol{\theta}}\right). \tag{28}$$

The CRLB is given by $I^{-1}(\boldsymbol{\theta})$.

V. Numerical Results

In this section, Monte-Carlo simulation results are presented to compare the localization performance of our proposed methods with different methods that are mentioned previously.

Consider a sensor network of $m = 5$ active sensors and $n = 10$ passive sensors. The locations of all sensors are randomly generated in a 100×100 square area centered at $(0, 0)$. Without loss of generality, the coordinates of the source are randomly generated in another 100×100 square area centered at $(\beta, 0)$. Therefore, $|\beta|$ denotes *the distance between the two areas*. We compare the performance of SR-LS, SRD-LS, and SH-LS, as well as their unstructured versions USR-LS, USRD-LS, and USH-LS, by using the mean squared error (MSE). Let $\hat{\mathbf{x}}_l$ denote the estimate of the true source location $\mathbf{x}_l$ in the lth simulation. The MSE is given by

$$\mathrm{MSE} = \frac{1}{L}\sum_{l=1}^{L} \|\hat{\mathbf{x}}_l - \mathbf{x}_l\|^2 \tag{29}$$

where L denotes the total simulation number.

First, we investigate the localization accuracy of different methods when $\beta = 0$. An example of the randomly generated sensors and source is illustrated in Fig. 1. In such a case, the source is located in the same generation area as the active and passive sensors and is thus surrounded by them. The simulation results averaged on 800 independent runs are shown in Fig. 2. The target and sensor locations are all randomly generated in each trial. The proposed SH-LS estimate outperforms the other methods that exploit only active or passive measurements; furthermore, SH-LS is close to CRLB at all considered noise levels. The two solutions,

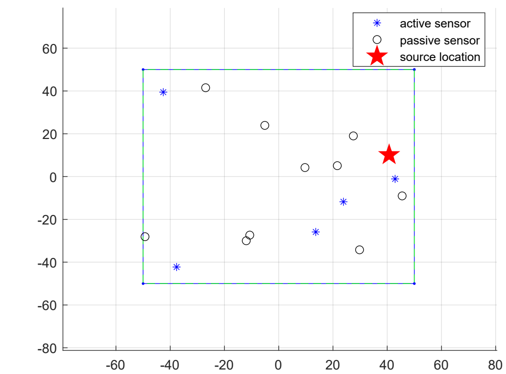

Fig. 1. An example of the distributed sensors and the source location with $\beta = 0$. The two generation areas are respectively denoted by the blue and green dashed squares that overlap when $\beta = 0$.

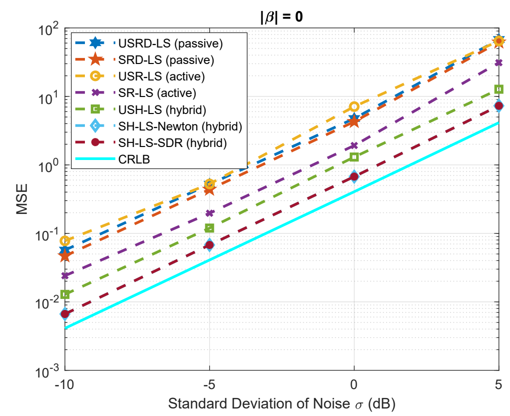

Fig. 2. MSE of different methods with noise level from -10dB to 5dB. $\beta = 0$.

respectively solved by SDR and Newton's method, give almost the same localization accuracy across the 800 runs.

Then we look into the case with $|\beta| = 100$, which means the source is located in a different generation area, not surrounded by the active/passive sensors anymore. One instance of the distribution is given in Fig. 3. With $|\beta| = 100$, SH-LS still leads to better localization accuracy than the other peer methods. While Newton's method works well for all simulations, the SDR solution suffers from poor estimates of the source locations in some rare cases. As mentioned in Section IV-B, the solution matrix $\mathbf{Y}$ usually has a higher rank than 1, which violates one of the constraints in (22), and we can employ SVD to impose the constraint on the solution matrix $\mathbf{Y}$ provided by the SDR method. As the distance between the target and the sensors increases, our simulations have revealed that some trials are not tight, i.e. the sum of four small singular values of the SDR solution matrix $\mathbf{Y}$ is not close to zero (here we consider larger than 10^{-3} as "not close"). These trials cause poor estimates of the source location even after the rank-one approximation process. For example, with the setup $|\beta| = 100$, 11 trails among the total 800 simulations were observed not

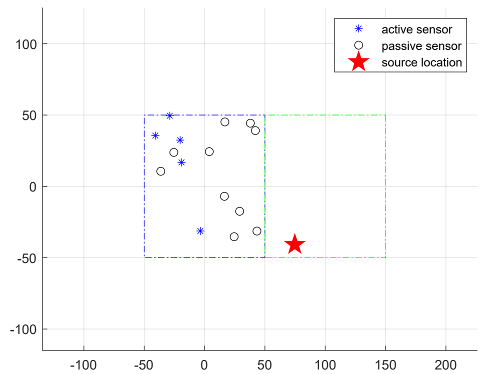

Fig. 3. An example of the distributed system with $\beta = 100$. Compared to Fig. 1, the blue and green areas do not overlap when $\beta = 100$.

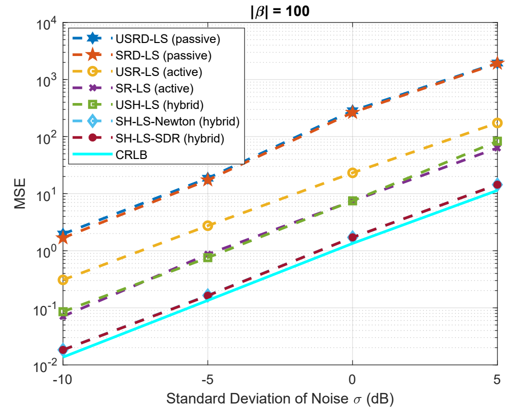

Fig. 4. MSE of the methods with $\beta = 100$.

TABLE I
TIGHT CASES OF SDR AMONG 800 INDEPENDENT SIMULATIONS

β	**-10dB**	**-5dB**	**0dB**	**5dB**
0	800 (100%)	800 (100%)	800 (100%)	800 (100%)
100	790 (98.75%)	791 (98.88%)	790 (98.75%)	789(98.62%)
200	729 (91.2%)	747 (93.4%)	739 (92.4%)	740 (92.6%)
300	473 (59.2%)	435 (54.4%)	451 (56.4%)	473 (59.2%)

tight with $\sigma = 5$dB, and the SDR's MSE of these "not tight" simulations are several orders of magnitude larger than the MSE given by Newton's method. We show how the number of tight cases changes with the increasing β in Table I.

After eliminating the influence of non-tight cases from the SDR results, the MSE comparison for the methods with $\beta = 100$ is illustrated in Fig. 4. It is observed that for the remaining tight cases, SDR provides the same estimation accuracy as Newton's method. The SR-LS (using active measurements only) and SRD-LS (using passive measurements only) are sensitive to the increased distance between the source location and the sensor network. The proposed SH-LS, however, is more robust to the distance $|\beta|$. It achieves a better localization performance than the peer methods by utilizing both active and passive measurements and stays close to CRLB at all noise levels.

VI. CONCLUSION

In this paper, we proposed a hybrid SH-LS localization approach that takes advantage of both active and passive measurements. Two efficient solutions, Newton's method, and the SDR method, are proposed to find the SH-LS location estimate. Simulation results indicate that the proposed SH-LS estimate outperforms other peer methods and is close to CRLB. Among the two SH-LS methods, SDR is more sensitive to the distance between the source and the sensors and the increasing distance can cause extremely poor performance of the SDR solution in some cases, whereas Newton's method is more robust and provides more reliable estimates of the true source location.

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