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A multiple access channel game with users implementing throughput and latency metrics

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Abstract

We consider a multiple access channel (MAC) problem where several users communicate with a base station and in which the users may have different applications or communication purposes for using the network, which is reflected via associated communication metrics. Specifically, we use throughput as the metric to reflect regular data transmission purposes, and latency, modeled by the inverse throughput, is used to reflect data transmission speed as another metric. The problem is formulated as a non-zero sum game. The equilibrium is derived in closed form. Stability in communication for such a heterogeneous network is established by proving the uniqueness of the equilibrium, except for particular cases where stability still can be maintained via cooperation of users with throughput metric or their switching to latency metric.

Keywords: Multi-user; Multi-access channel; Throughput; Latency; Equilibrium

1. Introduction

Problems involving mobile devices communicating wirelessly with a base station (BS) and allocating resources in a decentralized manner are multi-objective by their nature, and have been studied widely under a game theoretic framework [1]. For example, game theory has been used in [2–4] to study a fading MAC scenario, and, in [5–7], to study an orthogonal frequency division multiplexing (OFDM) scenario. In all of these works [2–7], the user's communication utility (metric) is throughput. Meanwhile, in [8], latency, modeled by the inverse signal-to-interference-plus-noise ratio (SINR), was considered as communication metric. Throughout all of the above papers, communication networks were considered homogeneous in the sense that all users have the same communication metric.

Motivation of this research. First note that due to the open access nature of wireless networks the users might differ in its communication purposes or implemented applications, and hence the users may need to address different communication metrics, for example, throughput and latency as metrics to reflect regular and emergency communication purposes, respectively. In this sense all of the above papers deal with multi- user communication where all the users have a similar purpose or application, which may not correspond to a realistic network scenario. The other example implementing different metrics might be IoT (Internet of Things) sensor nodes which are usually widespread in the environment and some of them might be located in places difficult to serve (say, for empty battery replacement). Then, throughput metric might suit the nodes easy to be served meanwhile latency metric might suit the nodes that are difficult to be served since such metric allows to control idle mode, i.e., the mode where the node does not spend resources. The goal of this paper is to study such a heterogeneous multi-user network by throughput and latency metrics implemented by different users.

Contribution of the paper. A heterogeneous MAC problem, where multiple users being administered by a base station can use either throughput or latency communication metrics, is modeled in a game-theoretic framework. To the best knowledge of the authors such heterogeneous MAC networks have not been considered in literature. The studied multi-user networks in literature are homogeneous by implemented communication metric by users (either each of them implements throughput or latency metric only). Thus, these studied

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networks are the boundary cases of the suggested generalized model. Specifically, in this paper the network is a heterogeneous one, where some of the users implement throughput metric, meanwhile the others implement latency metric. To solve this problem we propose a novel approach to design an equilibrium in a such heterogeneous network and, in particular: (a) it is proven that each user in the heterogeneous network implementing latency metric always has a unique equilibrium strategy that reflects stability in their communication, (b) it is proven that in such a heterogeneous network the users implementing throughput metric might have multiple equilibrium strategies. Moreover, the conditions, when they are unique, are established. Also, it is shown that even in such particular bifurcation cases where multiple equilibrium strategies arise, stability in communication can be maintained via either cooperation of such users or their switching to latency metric.

The most related works on MAC problems to this research are [4,8]. In [4], for a single cell multi-user CDMA (code-division multiple access) system where each user has throughput as communication metric equilibrium strategies are found in closed form and their uniqueness is proven under assumption that the spreading gain coefficient of the system is greater than one. In [8], for homogeneous network where each user has latency as communication metric modeled by the inverse SINR, equilibrium strategies are found in closed form and their uniqueness is proven.

The organization of this paper. In Section 2, multi-user communication model is formulated as a non-zero sum game, and existence of equilibrium is proven. In Section 3, dependence of each user's equilibrium strategy on the total power of the interference generated by all users implementing equilibrium strategies is established. In Section 4, auxiliary notations and results to support the derivation of the equilibrium are introduced. In Section 5, equilibrium strategies are found in closed form. In Section 6, numerical illustrations on a joint use of latency and throughput metrics in multi-user communication are provided. Finally, in Section 7, conclusions are offered.

2. Communication model

We consider a time-slotted flat-fading MAC in a single cell network [4], in which each mobile terminal (user) from the set $\mathcal{N} \triangleq \{1, \dots, n\}$ of n users is simultaneously sending data to a BS. Let the strategy of user i be its transmit power level P_i, with $P_i \in [0, \overline{P}_i]$ and $\overline{P}_i$ is the maximal feasible power level. Let (P_i, P_{-i}) be the ith users' strategy *profile*, with $P_{-i} \triangleq (P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_n)$. Let $\mathcal{N}_{-i}$ denote the set of all users except user i.

In this paper we consider heterogeneous communication involving the users belonging to a BS having different communication purposes reflected by different metrics. Specifically, regular and fast data transmission communication purposes will be modeled by throughput and latency metrics, respectively.

The throughput of user i at the BS is given as follows:

$$T_i(P_i, P_{-i}) = \ln\left(1 + \frac{h_i P_i}{N + \sum_{j \in \mathcal{N}_{-i}} h_j P_j}\right), \tag{1}$$

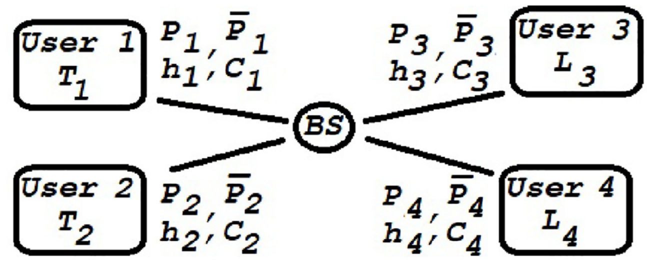


Fig. 1. The communication model between four users, i.e., $\mathcal{N} = \{1, 2, 3, 4\}$, and the BS where throughput and latency metrics are implemented by two users, respectively, i.e., $\mathcal{N}_T = \{1, 2\}$ and $\mathcal{N}_L = \{3, 4\}$.

where h_i is the path gain of user i to the BS, and N is the ambient noise in the network.

Latency as a communication metric for user i is modeled here by the inverse throughput [9], i.e.,

$$L_i(P_i, P_{-i}) = 1/T_i(P_i, P_{-i}). \tag{2}$$

Let $\mathcal{N}_T$ and $\mathcal{N}_L$ be subsets of users who has throughput and latency as communication metrics, and they consist of n_T and n_L users, respectively (Fig. 1), i.e.,

$$n_T + n_L = n \text{ and } \mathcal{N}_T \cup \mathcal{N}_L = \mathcal{N}. \tag{3}$$

By (3), without loss of generality we can assume that

$$\mathcal{N}_T = \{1, \dots, n_T\} \text{ and } \mathcal{N}_L = \{n_T + 1, \dots, n\}. \tag{4}$$

The user with throughput metric, *which we call an TM user*, i.e., user i, $i \in \mathcal{N}_T$, faces a trade-off between an increase in throughput for the signal received by the BS, and the price that the user pays for using a specific amount of power that causes interference in the system which is a linear function of the user's power level [4], i.e., $C_i P_i$ for user i, with $C_i > 0$ being the price per unit power level. The payoff to the user i is defined as

$$V_i(P_i, P_{-i}) = T_i(P_i, P_{-i}) - C_i P_i \text{ for } i \in \mathcal{N}_T. \tag{5}$$

The user with latency metric, *which we call an LM user*, i.e., user i, $i \in \mathcal{N}_L$, faces a trade-off, reflected by payoff given in (6) below, between a decrease in latency for the signal received by the BS, and the price that user pays for using a specific amount of power that causes interference in the system

$$V_i(P_i, P_{-i}) = -L_i(P_i, P_{-i}) - C_i P_i \text{ for } i \in \mathcal{N}_L. \tag{6}$$

Each user wants to maximize its payoff. Thus, we look for a (Nash) equilibrium. Recall that $(P_1, \dots, P_n)$ is a (Nash) equilibrium if and only if each of these strategies is the best response to the others, i.e., the following relations hold

$$P_i = \operatorname{argmax}\left\{V_i(\tilde{P}_i, P_{-i}) : \tilde{P}_i \in [0, \overline{P}_i]\right\} \text{ for } i \in \mathcal{N}. \tag{7}$$

Denote this non-zero sum game by Γ.

Proposition 1. *In game Γ there exists an equilibrium.*

Proof. For $i \in \mathcal{N}_T$, by (1) and (5), we have that

$$\frac{\partial^2 V_i(P_i, P_{-i})}{\partial P_i^2} = -h_i^2 \Big/ (N + \sum_{j\in\mathcal{N}} h_j P_j)^2 < 0, \tag{8}$$

meanwhile, for $i \in \mathcal{N}_L$, by (2) and (5), we have that

$$\frac{\partial^2 V_i(P_i, P_{-i})}{\partial P_i^2} = -\frac{h_i^2}{\ln^3\left(1 + \dfrac{h_i P_i}{N + \sum_{j\in\mathcal{N}_{-i}} h_j P_j}\right)} \times \frac{\left(2 + \ln\left(1 + \dfrac{h_i P_i}{N + \sum_{j\in\mathcal{N}_{-i}} h_j P_j}\right)\right)}{(N + \sum_{j\in\mathcal{N}} h_j P_j)^2} < 0. \tag{9}$$

Thus, payoff $V_i(P_i, P_{-i})$ of user i, $i \in \mathcal{N}$, is concave on P_i. Then, since the set of feasible strategies of each user is compact, there exists at least one equilibrium [10]. ■

3. Equilibrium strategies and the total power of the interference generated by all users

In this section we establish the dependence of the users' equilibrium strategies on the total power of the interference generated by all users implementing equilibrium strategies. First, to avoid bulkiness in the formulas, let us introduce an auxiliary notation for the ratio of fading gain and power cost as follows:

$$H_i \triangleq h_i / C_i \quad \text{for } i \in \mathcal{N}. \tag{10}$$

Proposition 2. *Let $(P_1, \dots, P_n)$ be an equilibrium and let $\mathcal{P}$ be the total power of the interference generated by all users implementing these equilibrium strategies, i.e.,*

$$\mathcal{P} = \sum_{j\in\mathcal{N}} h_j P_j. \tag{11}$$

Then, for each LM user, i.e., for each user i with $i \in \mathcal{N}_L$, the following relation holds:

$$P_i = \mathbb{P}_i(\mathcal{P}) \triangleq \min\left\{F(N + \mathcal{P}, H_i)/h_i, \overline{P}_i\right\}, \tag{12}$$

where

$$F(x, a) \triangleq x\left(1 - \exp\left(-\sqrt{a/x}\right)\right). \tag{13}$$

Proof. Since $i \in \mathcal{N}_L$, by (1), (2) and (6), we have that

$$\frac{\partial V_i(P_i, P_{-i})}{\partial P_i} = \frac{h_i/(N + \sum_{j\in\mathcal{N}} h_j P_j)}{\ln^2\left(1 + h_i P_i \Big/ (N + \sum_{j\in\mathcal{N}_{-i}} h_j P_j)\right)} - C_i. \tag{14}$$

Then,

$$\partial V_i(P_i, P_{-i})/\partial P_i \text{ is decreasing on } P_i \tag{15}$$

such that

$$\lim_{P_i\downarrow 0} \frac{\partial V_i(P_i, P_{-i})}{\partial P_i} = \infty \text{ and } \lim_{P_i\uparrow\infty} \frac{\partial V_i(P_i, P_{-i})}{\partial P_i} = -C_i. \tag{16}$$

Thus, payoff $V_i(P_i, P_{-i})$ with $i \in \mathcal{N}_L$ achieves its maximum in $[0, \infty)$ at the unique $P_i > 0$ such that

$$\partial V_i(P_i, P_{-i})/\partial P_i = 0. \tag{17}$$

By (10) and (14), Eq. (17) is equivalent to

$$\ln^2\left(\frac{N + \sum_{j\in\mathcal{N}} h_j P_j}{N + \sum_{j\in\mathcal{N}_{-i}} h_j P_j}\right)\left(N + \sum_{j\in\mathcal{N}} h_j P_j\right) = H_i. \tag{18}$$

By (11), we have that

$$\sum_{j\in\mathcal{N}_{-i}} h_j P_j = \mathcal{P} - h_i P_i. \tag{19}$$

Substituting (11) and (19) into (18) implies:

$$\ln^2\left(\frac{N + \mathcal{P}}{N + \mathcal{P} - h_i P_i}\right)(N + \mathcal{P}) = H_i. \tag{20}$$

Solving this equation on P_i implies that $P_i = F(N + \mathcal{P}, H_i)/h_i$ with F given by (13). This and (15)–(17) imply that $V_i(P_i, P_{-i})$ achieves its maximum in $[0, \overline{P}_i]$ at P_i given by (12), and the result follows. ■

Proposition 3. *Let $(P_1, \dots, P_n)$ be an equilibrium. Then for each TM user, i.e., for each user i with $i \in \mathcal{N}_T$, the following relations hold with $\mathcal{P}$ given by (11):*

(a) if $H_i < \mathcal{P} + N$ then $P_i = 0$,

(b) if $\mathcal{P} + N < H_i$ then $P_i = \overline{P}_i$,

(c) if $\mathcal{P} + N = H_i$ then each user from users' subset $\mathcal{I}_i \triangleq \{j \in \mathcal{N}_T : H_j = H_i\}$ can have any feasible strategies such that

$$\sum_{j\in\mathcal{I}_i} h_j P_j = H_i - N - \sum_{j\notin\mathcal{I}_i} h_j P_j. \tag{21}$$

Proof. Since $i \in \mathcal{N}_T$, by (1) and (5), we have that

$$\frac{\partial V_i(P_i, P_{-i})}{\partial P_i} = h_i \Big/ \left(N + \sum_{j\in\mathcal{N}} h_j P_j\right) - C_i. \tag{22}$$

Then, by (22), for a fixed P_{-i} function $V_i(P_i, P_{-i})$ achieves its maximum in $[0, \overline{P}_i]$ at P_i given as follows:

(i) if

$$h_i \Big/ \left(N + \sum_{j\in\mathcal{N}_{-i}} h_j P_j\right) \le C_i \tag{23}$$

then $P_i = 0$,

(ii) if

$$C_i \le h_i \Big/ \left(N + h_i \overline{P}_i + \sum_{j\in\mathcal{N}_{-i}} h_j P_j\right) \tag{24}$$

then $P_i = \overline{P}_i$,

(iii) if

$$\frac{h_i}{N + h_i\overline{P}_i + \sum_{j\in\mathcal{N}_{-i}} h_j P_j} < C_i < \frac{h_i}{N + \sum_{j\in\mathcal{N}_{-i}} h_j P_j} \quad (25)$$

then $P_i \in (0, \overline{P}_i)$ is such that

$$h_i \Bigg/ \left(N + h_i P_i + \sum_{j\in\mathcal{N}_{-i}} h_j P_j\right) = C_i. \quad (26)$$

By (10) and (11), Eq. (26) is equivalent to $N + \mathcal{P} = H_i$. This, jointly with (i) and (ii), imply the result. ■

4. Auxiliary notations and results

In this section we introduce auxiliary notations and results to support the derivation of the equilibrium in closed form in Section 5.

First note that, by Proposition 3, the TM users' equilibrium strategies depend on ratio H_i. Based on this observation, let us arrange the TM users, without loss of generality, in increase order by this ratio, and then split such way arranged TM users' set $\mathcal{N}_T$ into K subsets $\mathcal{N}_1, \dots, \mathcal{N}_K$ consisting of $m_1, \dots, m_K$ users, respectively, such that each of these subsets consists of the users with equal ratio H_i. Then,

$$H_{m_1} < H_{m_2} < \cdots < H_{m_K}, \quad (27)$$

with $H_{m_0} \triangleq 0$ and $H_{m_{K+1}} \triangleq \infty$. Thus, $\mathcal{N}_T = \cup_{k\in\mathcal{K}}\mathcal{N}_k$ with $\mathcal{K} \triangleq \{1, \dots, K\}$, and for each $k \in \mathcal{K}$ we have that

$$H_i = H_{\tilde{i}} \text{ for all } i \in \mathcal{N}_k \text{ and } \tilde{i} \in \mathcal{N}_k \text{ such that } i \neq \tilde{i}. \quad (28)$$

Let ρ_k be total fading power gain of users from set $\mathcal{N}_k$, and R_k be total fading power gain of users from sets $\mathcal{N}_k, \dots, \mathcal{N}_K$ implementing the maximal powers, i.e.,

$$\rho_k \triangleq \sum_{i\in\mathcal{N}_k} h_i\overline{P}_i \quad \text{and} \quad R_k \triangleq \sum_{i=k}^{K} \rho_i \quad \text{for } k \in \mathcal{K}, \quad (29)$$

and $R_0 \triangleq \infty$ and $R_{K+1} \triangleq 0$.

Let

$$\Phi(x) \triangleq \sum_{i\in\mathcal{N}_L} \min\left\{F(x, H_i), h_i\overline{P}_i\right\} \quad (30)$$

with F given by (13).

Note that the intuition behind the function $\Phi(x)$ is that, by (12), $\Phi(N + \mathcal{P})$ is the total fading power gain of all LM users. In the following proposition we establish monotonicity properties of this function.

Proposition 4. *Function $\Phi(x)$ is continuous and concave in $[0, \infty)$. Moreover, $\Phi(0) = 0$, $\Phi(x)$ is strictly increasing in $[0, \overline{x}]$ and $\Phi(x) = \sum_{i\in\mathcal{N}_L} h_i\overline{P}_i$ for $x \geq \overline{x}$ with $\overline{x} \triangleq \max\left\{F^{-1}\left(h_i\overline{P}_i, H_i\right) : i \in \mathcal{N}_L\right\}$, where for each fixed $a > 0$, $F^{-1}(\cdot, a)$ denotes the inverse function to $F(\cdot, a)$.*

Proof. By (13), for $x > 0$ we have that

$$\frac{\partial^2 F(x, a)}{\partial x^2} = -\frac{a + \sqrt{ax}}{4x^2}\exp\left(-\sqrt{a/x}\right) < 0, \quad (31)$$

$$\frac{\partial F(x, a)}{\partial x} = \psi\left(\sqrt{a/x}\right)\exp\left(-\sqrt{a/x}\right), \quad (32)$$

where $\psi(t) \triangleq \exp(t) - 1 - t/2$.

By (31), $F(x, a)$ is concave on x. Note that $\psi(0) = 0$ and $d\psi(t)/dt = \exp(t) - 1/2 > 0$ for $t \geq 0$. Thus, $\psi(t) > 0$ for $t > 0$, and, so, by (32), $F(x, a)$ also is increasing on x. Moreover, by (13), we have that

$$F(0, a) = 0 \text{ and } \lim_{x\uparrow\infty} F(x, a)/\sqrt{x} = \sqrt{a}. \quad (33)$$

Thus, for fixed $a > 0$ and $y > 0$ there is the unique root of equation $F(x, a) = y$. This and (30) imply the result. ■

Finally, let us introduce a sequence

$$\Xi_0 \triangleq 0,\ \Xi_k \triangleq H_{m_k} - \Phi(H_{m_k}) \text{ for } k \in \mathcal{K} \text{ and } \Xi_{K+1} \triangleq \infty. \quad (34)$$

Proposition 5. *R_k is decreasing on k.*

Proof. The result follows from (29). ■

5. Equilibrium

In this section we derive equilibrium strategies in closed form.

Theorem 1. *In game Γ each LM user has the unique equilibrium strategy. The TM users also have the unique equilibrium strategies except the only case (b-ii-3) below where a continuum of equilibrium strategies arise. Moreover,*

(a) the LM users' equilibrium strategies $P_i, i \in \mathcal{N}_L$ are given as follows:

$$P_i = \mathbb{P}_i(\mathcal{P}_*) \quad \text{for } i \in \mathcal{N}_L \quad (35)$$

with $\mathbb{P}_i$ given by (12) and $\mathcal{P}_*$ uniquely given by (38) and (40);

(b) the TM users' equilibrium strategies $P_i, i \in \mathcal{N}_T$ are given by (b-i) and (b-ii) with k_* given by (36) and (39):

(b-i) if

$$\Xi_{k_*-1} < N + R_{k_*} < \Xi_{k_*} \quad (36)$$

then

$$P_i = \begin{cases} 0, & i \in \cup_{k=1}^{k_*-1}\mathcal{N}_k, \\ \overline{P}_i, & i \in \cup_{k=k_*}^{K}\mathcal{N}_k, \end{cases} \quad (37)$$

and $\mathcal{P}_*$ is the unique root in $(H_{m_{k_*-1}} - N, H_{m_{k_*}} - N)$ of the following equation

$$\mathcal{P}_* = \Phi(\mathcal{P}_* + N) + R_{k_*}. \quad (38)$$

This root can be found via the bisection method;

(b-ii) if

$$N + R_{k_*} < \Xi_{k_*-1} \leq N + R_{k_*-1} \quad (39)$$

then

$$\mathcal{P}_* = H_{m_{k_*-1}} - N \quad (40)$$

and

(b-ii-1) if $i \in \cup_{k=1}^{k_*-2}\mathcal{N}_k$ then $P_i = 0$,

(b-ii-2) if $i \in \cup_{k=k_*}^{K}\mathcal{N}_k$ then $P_i = \overline{P}_i$,

(b-ii-3) each user i such that $i \in \mathcal{N}_{k_*-1}$ can have any feasible strategies such that

$$\sum_{j\in\mathcal{N}_{k_*-1}} h_j P_j = \Xi_{k_*-1} - R_{k_*} - N. \tag{41}$$

Theorem 1 implies that each LM user always has a unique equilibrium strategy. The TM users also have unique equilibrium strategies except the subset $\mathcal{N}_{k_*-1}$ of users in case (b-ii-3) when this set consists of at least two users. Although in this case a continuum of equilibrium strategies arises, such users still can maintain stability in communication via cooperation implementing strategies $P_i = (\Xi_{k_*-1} - R_{k_*} - N)/h_i$ with $i \in \mathcal{N}_{k_*-1}$ which makes the throughput of each TM user of subset $\mathcal{N}_{k_*-1}$ to be equal to each other.

Proof of Theorem 1. Let $(P_1, \dots, P_n)$ be an equilibrium, and $\mathcal{P}$ be given by (11). Also, let

$$\mathcal{P}_L \triangleq \sum_{i\in\mathcal{N}_L} h_i P_i \quad \text{and} \quad \mathcal{P}_T \triangleq \sum_{i\in\mathcal{N}_T} h_i P_i. \tag{42}$$

Then, by (3), (11) and (42), we have that

$$\mathcal{P} = \mathcal{P}_T + \mathcal{P}_L. \tag{43}$$

Multiplying both sides of Eq. (12) by h_i and summing up these equations by $i \in \mathcal{N}_L$,(30), (42) and (43) imply

$$\mathcal{P}_L = \Phi(N + \mathcal{P}). \tag{44}$$

Note that, by Proposition 3, (27) and (28), there is a k_* such that two cases arise to consider:

$$H_{m_{k_*-1}} < \mathcal{P} + N < H_{m_{k_*}}, \tag{45}$$

$$\mathcal{P} + N = H_{m_{k_*-1}}. \tag{46}$$

(I) Let (45) hold. Then, by Proposition 3, (27) and (28), equilibrium strategies P_i for $i \in \mathcal{N}_T$ have to be given by (37). Then, by (29), we have

$$\mathcal{P}_T = \sum_{k=k_*}^{K} \sum_{i\in\mathcal{N}_k} h_i \overline{P}_i = R_{k_*}. \tag{47}$$

By (43) and (47), we have that

$$\mathcal{P} = R_{k_*} + \mathcal{P}_L. \tag{48}$$

Adding up $\mathcal{P}_T$ to both sides of Eq. (44), by (43), (47) and (48), we have that

$$\mathcal{P} = \Phi(N + \mathcal{P}) + R_{k_*}. \tag{49}$$

Note that left-side of Eq. (49) is the identity function $\mathcal{P}$. Meanwhile, by Proposition 4, right-side of Eq. (49) is an upper-bounded non-decreasing and concave function which is positive at the initial point $\mathcal{P} = 0$. Thus, Eq. (49) has a root in interval $(H_{m_{k_*-1}} - N, H_{m_{k_*}} - N)$ given by (45), and, it is a unique one, if and only if

$$\mathcal{P} < \Phi(N + \mathcal{P}) + R_{k_*} \text{ for } \mathcal{P} = H_{m_{k_*-1}} - N, \tag{50}$$

$$\mathcal{P} > \Phi(N + \mathcal{P}) + R_{k_*} \text{ for } \mathcal{P} = H_{m_{k_*}} - N. \tag{51}$$

Substituting $\mathcal{P}$, given by conditions in (50) and (51), into the corresponding them inequalities, implies that (50) and (51) are equivalent to

$$H_{m_{k_*-1}} - \Phi\left(H_{m_{k_*-1}}\right) < R_{k_*} + N < H_{m_{k_*}} - \Phi\left(H_{m_{k_*}}\right). \tag{52}$$

By (34), (52) is equivalent to (36), and (b-i) follows.

(II) Let (46) hold. Then, by Proposition 3 and (27)–(29), equilibrium strategies P_i for $i \in \mathcal{N}_T \backslash \mathcal{N}_{k_*-1}$ have to be given by (b-ii-1) and (b-ii-2). Meanwhile, for $i \in \mathcal{N}_{k_*-1}$ the following relation has to hold:

$$\mathcal{P}_T = R_{k_*} + \rho \text{ with } \rho \triangleq \sum_{j\in\mathcal{N}_{k_*-1}} h_j P_j. \tag{53}$$

By (29), we have that

$$0 < \rho \le \rho_{k_*-1}. \tag{54}$$

Adding up $\mathcal{P}_T$ given by (53) to both sides of Eq. (44), by (43) and (46), we have that

$$H_{m_{k_*-1}} - N = \Phi(H_{m_{k_*-1}}) + R_{k_*} + \rho. \tag{55}$$

Solving this equation by ρ implies

$$\rho = H_{m_{k_*-1}} - N - \Phi(H_{m_{k_*-1}}) - R_{k_*}. \tag{56}$$

Combining (54) and (56) implies that such ρ exists if and only if the following inequalities hold

$$N + R_{k_*} < H_{m_{k_*-1}} - \Phi(H_{m_{k_*-1}}) < N + R_{k_*} + \rho_{k_*-1} \tag{57}$$

By (29), $R_{k_*} + \rho_{k_*-1} = R_{k_*-1}$. This and (34) imply that (57) is equivalent to (39), and (b-ii) follows. (a) follows from (b) and Proposition 2. ■

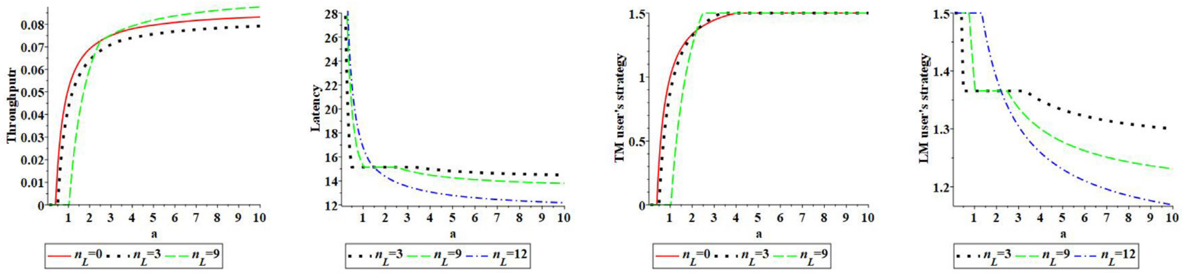
6. Numerical illustration

Here, we use a numerical example to illustrate a joint use of latency and throughput metrics in multi-user communication. Suppose that the total number of users is $n = 12$, i.e., $n_L + n_T = 12$.

Note that the boundary case $n_L = 0$ (i.e., $n_T = 12$) corresponds the network consisting only of the TM users, i.e., the network where each user has throughput as communication metric (please, see, Fig. 2(a) and Fig. 2(c) with $n_L = 0$ for throughput and TM user strategy). This is a limiting case of the problem studied in [4] with spreading gain tending to one. Moreover, in this limit case, the game is the potential one [11], and, so, the equilibrium strategies also can be found via the best response strategy algorithm.

The other boundary case $n_L = 12$ (i.e., $n_T = 0$) corresponds the network consisting only of the LM users, i.e., the network where each user has latency as communication metric (please, see, Fig. 2(b) and Fig. 2(d) with $n_L = 12$ for latency and LM user strategy). This case has been studied in [8] for latency modeled by the inverse SINR.

Let the ambient noise be $N = 10$ and the LM and TM users differ by power cost, and be symmetric by maximal power



(a) (b) (c) (d)

Fig. 2. (a) Throughput of TM users, (b) latency of LM users, (c) strategies of TM users and (d) strategies of LM users.

resources and fading gains. Specifically, let power costs for the LM and TM users be equal to $C_L = 10$ and $C_T = 0.05$, respectively, and fading gain of each user, h, is taken as the mean of a Rayleigh (fading) distribution with probability density function $\rho(x, a) \triangleq \exp(-x^2/(2a^2))x/a^2$, $x \geq 0$, where $a > 0$ is the (Rayleigh) scale parameter. Let the maximal power level of each user be $\overline{P} = 1.5$. We observe, substituting these data in Theorem 1, that a degradation in communication condition, caused by a decrease in the fading gain reflected by a decrease in Rayleigh scale parameter leads to a decrease in throughput of the TM users (Fig. 2(a)) and an increase in latency in communication of the LM users (Fig. 2(b)). In spite on such degradation in quality of communication, the LM users' communication always is uninterrupted, and it is maintained by an increase in their transmission efforts (Fig. 2(d)), meanwhile, the TM users' communication can be interrupted, and it is reflected by a decrease in transmission efforts which could vanish finally (Fig. 2(c)). An increase in the number of LM users makes the TM users apply the maximal power level for larger range of the Rayleigh scale parameter, and it reflects a decrease in sensitivity of the TM users' strategies to network parameters. Presence of the TM users also could reduce sensitivity of the LM users' strategies, which is reflected by their intermediate flat segments (see, Fig. 2(c), (35) and (40)).

7. Conclusions

In this paper a heterogeneous MAC problem, where multiple users being administered by a base station can use either throughput and latency communication metrics, has been studied in a game-theoretic framework. A communication protocol based on the derived equilibrium strategies maintaining stability in communication in such heterogeneous network is suggested and illustrated. In particular, it is shown that the users might benefit from communication in such heterogeneous network due to a decrease in sensitiveness of their equilibrium strategies to the network parameters. A goal of our future research is to generalize the suggested approach for multi-carrier communication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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