Ion-ion interaction induced non-dispersive dynamics

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We demonstrate both experimentally and using a numerical simulation that, under special conditions, the repulsive Coulomb interaction helps to suppress the emittance growth of an RF-driven bunch of ions in an electrostatic ion beam trap. The underlying mechanisms can be explained by the synchronization of ion motion when nonlinear interactions are present. The surprising effect can help in improving the phase space manipulation of ions and the beam control in storage rings and accelerators and may be applied to other systems with many-body interactions in a periodic potential.

The presence of numerous identical particles interacting with each other leads to a wide array of captivating effects. These include collective excitations observed in nuclei, atoms, and molecules and phenomena like superconductivity [1], and the quantum Hall effect [2]. Even the special case for point-like charged particles interacting through the well-known Coulomb force has led to peculiar emergent phenomena, both at the quantum and classical levels. Examples such as Coulomb crystals [3], which constitute a special class of spatially ordered structures of matter, or evaporative cooling in Penning trap [4] have demonstrated the richness of possible phenomena stemming from a well-known interaction operating in the many-body regime with a well-defined external force applied to the system. Moreover, the study of the ion-ion interaction and phase space manipulation is a very fundamental subject that's common to several fields of physics, such as accelerator physics [5, 6], plasma physics [7], quantum computing [8], cooling of ions [4, 9, 10], and many others. An intriguing (classical) effect related to the collective behavior of point-like charged particles has also been demonstrated when a large number of ions are oscillating between two electrostatic mirrors, such as in the Electrostatic Ion Beam Trap (EIBT) [11]. The EIBT is a uniquely versatile system for studying time-dependent processes in atomic and molecular physics [12], new physics beyond the standard model [13], and study many-body interaction dynamics in a periodic system [14]. Under some specific and well-defined conditions, the stored ions are seen to attract each other, even though the only force acting between them is the repulsive Coulomb force. This self-bunching effect has been well documented and analyzed using analytical and numerical models [15–17].

It is well known that a well-defined bunch of ions, characterized by high density and small emittance, is a basic pre-requisite for accelerator and ion storage/trap devices [18–21]. When a bunch of ions is injected into a storage ring or an ion trap, it will disperse over time due to the ions' energy spread, the different trajectories, and the Coulomb force's inherent repulsive effect on particles. The space charge, or the intrabeam scattering, is the main factor in limiting bunch intensity and size [22–24]. In storage devices, various methods are used for phase space manipulations and to keep the ions in a bunch. One of the most common techniques is RF bunching, where an external time-dependent sinusoidal field is applied with an identical frequency or a high harmonic of the natural oscillation frequency of the ions. The synchronous ion is phase matched (0 or π), with an external RF field, and the other ions oscillate in the longitudinal phase space around the synchronous ion. The oscillation frequency of ions in the longitudinal phase space, often referred to as synchrotron oscillation, depends on the phase offset from synchronous ions. The nonlinearities in the synchrotron oscillations result in emittance growth [25–27]. The suppression of emittance growth is important for a highly intense, well-localized ions bunch but is limited by the repulsive ion-ion interaction in the bunch.

The RF bunching of ions in an EIBT has been studied with great detail [28, 29]. As expected, when bunched, the collective longitudinal oscillations of ions lead to additional peaks in the Fourier-transformed time signal of the ion bunch within the trap providing information about emittance growth in phase space. Recently, a new simulation based on the particle-in-cell (PIC) technique has been developed, which accurately reproduces all the experimental results[30, 31]. The results show that the ion-ion interactions significantly influence the ions' dynamic. With the advancement of simulation techniques, studying the influence of space charge effects on the beam dynamics in the electrostatic ion beam trap has become possible.

In this letter, we demonstrate both experimentally and using the PIC simulation, which includes the complete ion-ion interaction that, in an EIBT, the coupling between ions, which is the repulsive Coulomb interaction, counterintuitively restricts emittance growth within the RF bucket and keeps the bunch localized in phase space. This effect bears similarities to the synchronization of

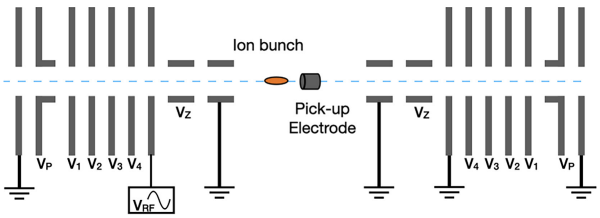

FIG. 1: A schematic of the electrostatic ion beam trap (EIBT). An external time-dependent voltage (V_{RF}) is applied to the innermost electrode of the left mirror. The pickup detector is used to observe the motion of ions in the trap.

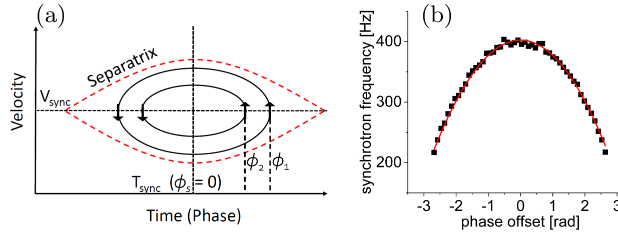

FIG. 2: (a) A schematic of an RF bucket showing the trajectory of motion for two ions at different phase offsets ($\phi_1 > \phi_2$). (b) The phase offset-dependent synchrotron oscillation frequency of ions in an RF bucket. The data points are from the simulation when no ion-ion interaction is included. The red curve is fitting with Equation 1.

ion motion observed in an EIBT when operated in self-bunching mode [15].

The experimental setup is extensively described in [32], and we only provide a brief overview here. As depicted in Figure 1, the ions are confined between the two electrostatic mirrors of the EIBT. The passage of ions in the trap is detected using the induced image charge on the pickup detector. The dynamics of ions in the EIBT can be characterized by the slip factor η, given by $\eta = -(2E_k/f_{osc})(df_{osc}/dE_k)$, where f_{osc} represents the oscillation frequency of ions with energy E_k in the trap. When the slip factor is negative, high-energy ions exhibit a higher oscillation frequency (df/dE is positive), leading to dispersion of the ion bunch over time. Conversely, a positive slip factor results in self-bunching, provided the ion density criterion is met [33]. In the current experiment, the trap is operated in the dispersive mode ($\eta < 0$). The specific mirror potentials for the current configuration are as follows: $V_p = 5.75$ kV, $V_1 = 6.5$ kV, $V_2 = 4.875$ kV, $V_3 = 3.25$ kV, and $V_4 = 1.625$ kV. A bunch of SF_5^+ ions, generated by the Even-Lavie ion source, is accelerated to a kinetic energy of 4.2 keV, focused, and steered using an Einzel lens and an assembly of electrostatic XY deflectors before being injected into the trap. The ion density within the trap can be controlled by adjusting the value of the entrance electrode potential, V_P. The time trace obtained from the pickup detector is analyzed and subjected to Fourier Transform (FT) to determine the oscillation frequency of the ions. In this case, the second harmonic of the ion oscillation frequency (f_{osc}) is approximately 187760 Hz. A time-dependent external voltage, denoted as $V(t) = V_{RF}\sin(2\pi f_{osc}t + \phi)$, is applied to the innermost electrode of the left mirror. Here, V_{RF} represents the amplitude, and ϕ is the phase of the external RF field.

The simulation is based on the particle-in-cell (PIC) technique [34]. The underlying concept of this 2DCylPIC simulation involves the numerical solution of Poisson's equation on a computational grid. This allows for the determination of the electric field at each grid point. The positions and velocities of the simulated ions are then adjusted based on their respective positions on the grid. Subsequently, the updated particle locations are utilized to revise the charge density on the grid, leading to a recalculation of the electric field. This iterative process is repeated for a specified duration. A time step of 5 ns (nanoseconds) was employed in all the simulations discussed here. The simulation technique naturally incorporates the space charge effect, and the ion dynamics in the trap are very well reproduced [30, 31].

In RF bunching, the synchronous ion gains net zero energy while the other ions oscillate in longitudinal phase space around it. The separatrix defines the boundary within which the ions with maximum phase offset from synchronous ions can oscillate in a stable closed orbit. The RF bucket is the phase space area within the separatrix. Figure 2 (a) shows the schematic of the RF bucket. The synchronous ion ($\phi_s = 0$) is located at the center of the bucket. The trajectory of motion for ions at different phase offsets (ϕ_1, ϕ_2) is also shown in this figure. The ions oscillate in longitudinal phase space around the synchronous ion. The oscillation frequency in the RF bucket is nonlinear and depends on the phase offset. Ions located near the center of the bucket exhibit higher oscillation frequencies. As the phase offset increases, the oscillation frequency decreases. For larger phase offsets, the frequency dependency can be approximated by [27]

$$\omega_s(\phi_s) \approx \omega_0\left[1 - \frac{\phi_s^2}{16}\right] \quad (1)$$

where ω_0 is the oscillation frequency of the synchronous ions, and ω_s is the oscillation frequency for the ions at phase offset ϕ_s.

Figure 2(b) shows the simulated phase-dependent distribution of synchrotron frequency in the RF bucket in the EIBT. The synchrotron frequency of the ions at phase ϕ_1 is smaller than ions at phase ϕ_2. This nonlinearity of the synchrotron oscillation within the RF bucket leads to the well-known filamentation of the phase space [25]. The filamentation occurring in the RF bucket can be approximated by emittance growth in the RF bucket. The emittance is a measure of the area occupied by the ions bunch in the RF bucket. The area can be precisely defined for a continuous beam or an infinite number of particles. However, in the case of a finite number of particles, the area can be estimated by the root mean square (rms) emittance in phase space [35].

When two systems oscillate at nearby frequencies, the nonlinear coupling between them results in synchronization. This phenomenon of synchronization in ions motion was observed in the EIBT [15]. The first condition for the synchronization is the positive slip factor (df/dE is negative), that is, high energy ions have low oscillation frequency. Even for the positive slip factor, if we neglect the Coulomb interaction between particles, the ions will disperse and lose their synchronized behavior [30]. For synchronization to happen, the rate of collisional energy redistribution, determined by the ion density, should be much faster than the de-phasing process. As shown in Figure 2b, ions with high phase offset have a smaller synchrotron frequency, resulting in a positive slip factor (negative df/dE) for the RF bucket. Furthermore, the synchrotron frequency within the RF bucket is approximately 400 Hz, corresponding to a period of milliseconds. On the other hand, the characteristic period of ions in the trap is around 10 microseconds, which is much shorter than the oscillation time within the RF bucket. The EIBT exhibits a unique characteristic that sets it apart from other ion traps or storage rings: the ion density oscillates periodically. At the turning point, the ion density can be several orders of magnitude higher than in the field-free region of the trap. This collisional redistribution of energy of ions provides the necessary coupling to keep the ions bunch localized inside the RF bucket.

Experimentally, this behavior can be observed from the Fourier Transform (FT) spectrum of the time signal generated by the ion motion in the EIBT. Consequently, the longitudinal oscillations of ions in the RF bucket lead to the appearance of two side peaks in the FT spectrum [28]. The height of the side peaks relative to the central peak at f_{osc} carries information about the distribution of ions in the RF bucket. If the ions are uniformly distributed in the RF bucket, the side peaks will be suppressed. Figure 3(a) shows the experimentally observed FT spectrum for two different ion densities for approximately 700 ms of storage time. One would expect the repulsive ion-ion interaction to enhance the filamentation and make the distribution uniform in the RF bucket, resulting in the suppression of side peaks for high ion density. Counterintuitively, the side peaks are amplified for the high-ion-density case. This demonstrates that, in a counter-intuitive way, the ions are localized for high ion density, and the filamentation is more pronounced for low ion density. The inset in Figure 3(a), shows the position of side peaks when the bunch is injected at different phase offsets (ϕ_s) for high ion density. Since the ion bunch is localized, the dependence of synchrotron oscillation frequency on phase offset can be seen clearly. The experimental observations follow the same trend as

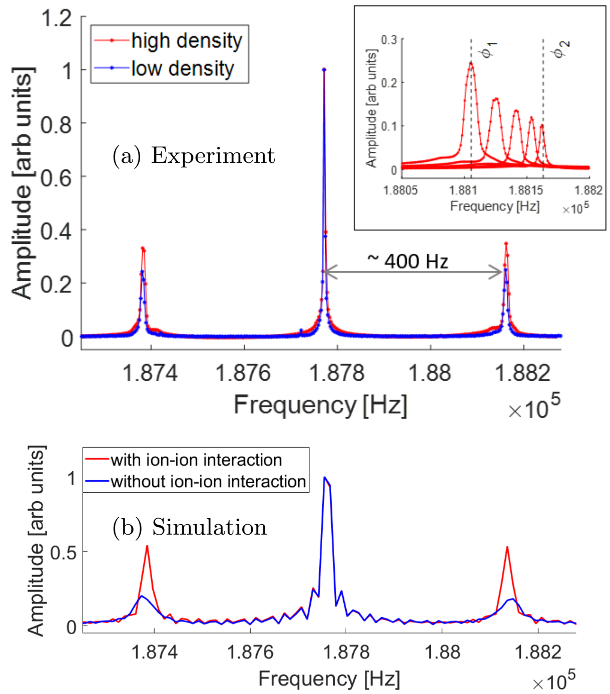

FIG. 3: (a)Experimental FT spectrum of the pick-up electrode signal for bunched ions oscillating in the EIBT for two different ion densities. The inset shows the position of side peaks when the bunch is injected at different phase offsets for increasing (ϕ_s): $\phi_1 > \phi_2$. (b) FT spectrum from the simulation (see details in the text)

shown in Figure 2b, where the synchrotron oscillation frequency (the frequency difference between side peaks and the main peak position) is smaller for high phase offset. As expected, the amplitude of the side peaks decreases when the bunch moves closer to the bucket center. Figure 3(b) shows, for comparison, the simulation results for the FT spectrum for 50 ms of storage time: Since the computing time for 100k ions for 1 ms storage time takes approximately 1.2 hour of CPU time, the simulation is only run for a shorter storage time, and also for a smaller number of ions (20k ions). Charge scaling is used to reduce the computational time [36]. For high ion density, the charge is scaled five times, and to replicate very low ion density, the ion-ion interaction is disabled in the simulation. As can be seen from Figure 3(a) and (b), the dependence of side peaks heights on the ion densities agrees very well with the experimental observations.

Using the simulation, we can now visualize the motions of ions in the phase space. As an example, Figure 4 shows the distribution of ions in the longitudinal phase space using velocity and time for the axis. In this case, the ion bunch is injected slightly offset from the bucket center (phase offset approx -0.8 rad). The top panel shows the phase space of the bunched ions shortly after injection (15 μs after injection). The middle panel shows the

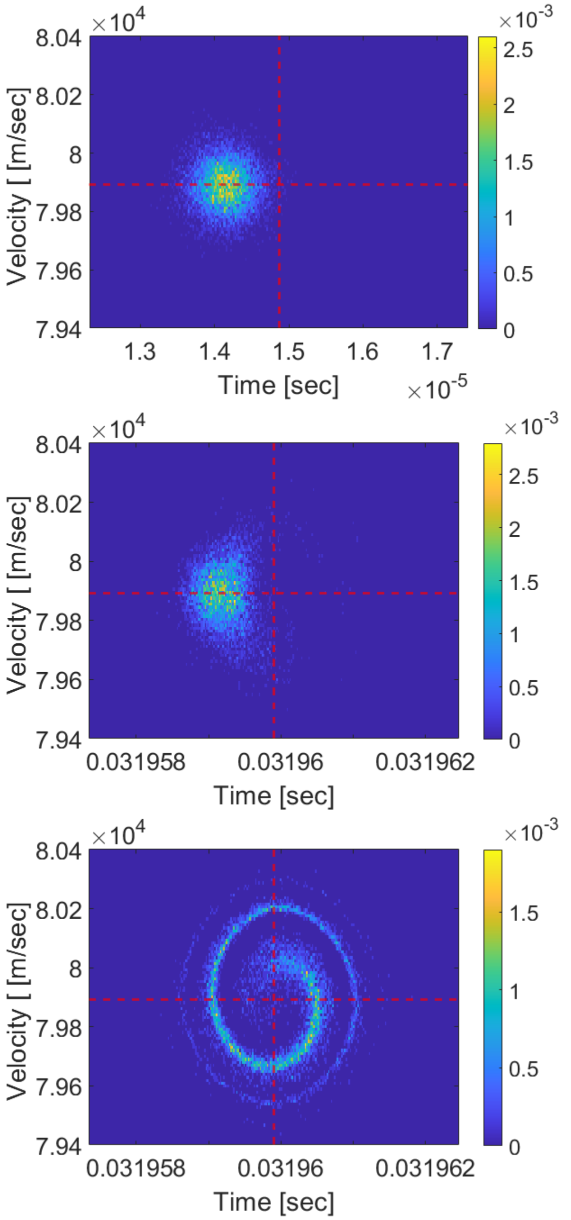

FIG. 4: Motion of ions bunch in phase space shortly after injection (top) and after approx 32 ms with ion-ion interaction (middle) and without ion-ion interaction (bottom). The red lines correspond to the approximate RF bucket center

phase space after the bunched ions have evolved for approximately 32 ms. To demonstrate the effect of the repulsive Coulomb force, the bottom panel is identical to the middle plot, except that the ion-ion interaction has been turned off during the whole storage time (32 ms). The difference is striking, and the filamentation of the ion bunch is clearly visible in the latter case. This demonstrates that the repulsive Coulombic interaction is responsible for keeping the ions localized in the bunch.

A different time-dependent presentation of this effect

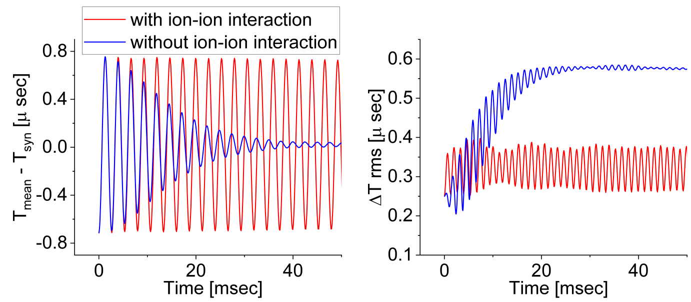

FIG. 5: Simulated Mean position(left) and rms value (right) of the ions bunch in the RF bucket

can be seen in Figure 5. The simulation results for the time difference between the centroid of the ion bunch relative to the bucket center (left panel) and the RMS size of the ion bunch within the RF bucket (right panel) are shown. In the former case, the oscillation represents the periodic motion of ion bunch in longitudinal phase space and has the same frequency as observed in the FT spectrum in Figure 3. Two different cases are shown: In red when the ion-ion interaction is fully taken into account, and in blue without ion-ion interaction. When the ion-ion interaction is neglected, the ions bunch spread out in the RF bucket and the oscillations disappear. Conversely, in the presence of ion-ion interaction, the centroid position keeps oscillating, demonstrating the synchrotron oscillation within the RF bucket. This supports the observed dependence (see Figure 3) of side peaks on ions density. The RMS value (right panel) clearly shows an increase in bunch size when the ion-ion interaction is neglected. On the other hand, when the repulsive ion-ion interaction is taken into account, the RMS stays more or less constant. Here again, it is quite surprising that, despite the high ion density, the ion bunch does not disperse within the RF bucket. These characteristics of the distribution support the experimental observations. A similar trend is reported in [37, 38], where filamentation growth within the RF bucket is inhibited by the high space charge. The ion dynamics within the RF bucket bear similarity with the self-bunching effect in EIBT, where the Coulombic repulsion provides coupling for synchronization.

In summary, we demonstrated that, in an EIBT, the repulsive Coulombic interaction counter-intuitively restricts emittance growth within the RF bucket and keeps the bunch localized. The ion dynamics were studied experimentally, and the phase space evolution was explored using the 2DCYLPIC simulation technique. In an EIBT, the manipulation of the phase space of ions may open up many opportunities, such as the kinematical cooling of ions. The cooling efficiency for mechanisms such as autoresoance dragging of ions can be significantly improved by ion-ion interaction and localization in phase space. Keeping the phase space localized can also help to significantly improve the spectral resolution for collinear laser spectroscopy [39–41]. The EIBT can serve as a beam element in a low energy scale accelerator setup where phase space manipulation, bunch merging, and transfer of bunch from one component to another can be performed in a controlled way. The localization of ions is also very useful for merged beam experiments, where precise control of the position and velocity spread are needed. In general the EIBT can serve as a benchmark for studying many-body interaction and collective phenomena in a periodic system.

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