In Memory of Steve Zelditch

Coordinated by Bernard Shiffman and Jared Wunsch

Figure 1.

Steve Zelditch, our beloved and admired colleague and a major figure in spectral theory, semiclassical analysis, quantum chaos, and Kähler geometry, died on September 11, 2022, at the age of 68. Steve is survived by his wife, Ursula Porod and their two sons, Benjamin and Phillip.

He died during a hurriedly organized Zoom conference with a star-studded speaker lineup and enormous attendance, celebrating his achievements and their impact on a huge range of analysis and geometry. Steve, who had an unquenchable thirst for mathematics, was present at this meeting and discussing as much math as he could, all through the first three days of talks. He died on the night before the fourth and final day, when the grief-stricken attendees had to carry on without him. The outpouring of admiration, sadness, and appreciation of Steve's mathematical and human dimensions has been overwhelming, and a small sample is provided below.

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Steve grew up in Palo Alto. He was an undergraduate at Harvard, where his initial ambition was to become a novelist; he got distracted by mathematics along the way. (His love for literature stayed with him, however, and he was incredibly well read and opinionated on literature of all kinds.) Steve received his PhD from Berkeley in 1981 under the direction of Alan Weinstein. He was subsequently a Ritt Assistant Professor at Columbia, then joined the faculty at Johns Hopkins in 1995. In 2010, Steve moved to Northwestern, where he was the Wayne and Elizabeth Jones Professor of Mathematics. Steve was an ICM speaker in 2002, won the Stefan Bergman Prize in 2013, and was a Fellow of the AMS.

You can read about many aspects of Steve's research in the tributes that follow, but for those who didn't know the breadth of his work, it is perhaps instructive to point to one central theme and four main threads within that theme.

Steve loved asymptotics. Any problem phrased in terms of asymptotic expansions lit his enthusiasm. He had a remarkable capacity for seeing common features in seemingly disparate asymptotic questions, and in particular, for finding ways that "semiclassical" asymptotics, expressing the relationship of quantum mechanics and classical mechanics as Planck's constant is allowed to tend to zero, could be employed in surprising new areas.

Steve's first great success was the story of *quantum ergodicity*, describing how in a quantum system whose underlying classical dynamics are chaotic, the energy eigenfunctions must be correspondingly scrambled up in both position and momentum. This became a major area of mathematics, with Zelditch as its unquestionable leader.

Steve was the first to systematically use semiclassical tools in Kähler geometry, where the "Tian–Yau–Zelditch" expansion dictates how pulling back the Fubini–Study metric under the Kodaira embeddings via powers of an ample line bundle can approximate any Kähler metric. The TYZ expansion has become a key tool in Kähler geometry. In addition, Steve developed a new area of mathematics,

Figure 2. Steve Zelditch with his sons, Phillip and Ben, in 2002.

Figure 3. Jared Wunsch and Steve Zelditch at the New Chair Investiture Ceremony, 2012.

"stochastic Kähler geometry" to which he further applied asymptotics.

Mark Kac famously asked "Can one hear the shape of a drum?" By work of Gordon–Webb–Wolpert, we know the answer to be "no." But one can hear a lot *about* the drum, and Steve's positive inverse-spectral results (on analytic domains with symmetry, and on nearly circular ellipses) are the best known.

Finally, somewhat more loosely, we remark on a large body of Steve's work involving asymptotics of randomized objects, sometimes studied for their own sake, and sometimes as a proxy for deterministic objects (like individual Laplace eigenfunctions) that are too elusive to cope with directly. Some of this work has turned out to have a rich relationship with problems in the theory of string vacua.

We have just scratched the surface here: MathSciNet currently lists 184 publications, and Steve had many active collaborations at the time of his death. He was, moreover, interested in *everything*, in mathematics and in life. We miss him.

Nalini Anantharaman

I feel shy about taking up the pen to write about my friend and collaborator Steve Zelditch, in a language that is not my mother tongue. Steve enjoyed words and literature, his conversation was full of savor and he liked to play with the American language—you could guess when he was the referee of one of your papers. He was eager to help you improve your style, both in mathematics and in English.

Steve is famous, among other things, for a large body of articles concerning "quantum ergodicity." After Alexander Shnirelman, in 1974, stated a theorem relating classical ergodicity of a Hamiltonian flow to the equidistribution of eigenfunctions of the associated Schrödinger operator, Steve Zelditch developed a pseudodifferential calculus on hyperbolic surfaces that allowed him to give the first full proof of the theorem. Interestingly, he told me that his work aroused no interest in the US at the time, but received quick recognition in France. Steve developed the subject in all possible directions, he showed how rich a subject this is, and he is largely responsible for the popularity of the subject nowadays: quantum ergodicity for Laplacian eigenfunctions on Riemannian manifolds without and with boundary, for eigenfunctions of Dirac operators, quantum ergodicity for abstract $\mathbb{C}^*$ dynamical systems, for restrictions of Laplacian eigenfunctions to hypersurfaces, relations between quantum ergodicity and counting of nodal domains.

Steve made fundamental contributions to several other areas, such as zeroes of random polynomials, random Kähler geometry, and inverse spectral problems.

It strikes me that I never heard Steve criticize a colleague or a mathematical result, based on anything other than scientific grounds. He helped me a lot when I was preparing my Bourbaki talk about random nodal domains and was struggling to compare the contributions of various teams: instead of describing the various contributions in terms of competition, he tried to explain to me the vision and merits of each author. He liked talking and I liked listening, which was both fun and tiring, as he could become enthusiastic about all sorts of unexpected things. Because of, or

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maybe "thanks to" him, I bought CDs of Green Day, System of a Down, Arcade Fire—which I never really listened to afterwards....

When you lose a friend who lives on the other side of the Atlantic and whom you used to see about once a year, it's easy to imagine he's still there. These days, when I go through moments of great intellectual enthusiasm, I think of him and how intense his intellectual life was, I think "this is a moment that Steve would prize"—and I suddenly recall that he is lost forever.

Michael R. Douglas

My relationship with Steve Zelditch began with mathematical physics, and quickly grew into friendship. In early 2002, I was studying what would soon be dubbed "the string landscape," the set of solutions of superstring and M theory which might describe our universe considered not one by one (as was usual in physics) but as a totality. The original example is the set of three-dimensional Calabi–Yau (Ricci-flat Kähler) manifolds, candidates for the "hidden" dimensions of superstring theory. Over the years many more solution sets were proposed, and the goal of describing them was extended to defining a probability distribution over solutions, called the "measure factor" in quantum cosmology. A basic problem of the string landscape is to find and study natural random distributions over algebraic geometric objects: varieties, vector bundles, sections, their zeroes, and so on.

With this in mind, I started poring over the mathematical literature, and sometime in 2002 I ran into a paper of Bleher–Shiffman–Zelditch [BSZ00], which studied zeroes of random sections of a holomorphic line bundle L. Since these spaces of sections are finite dimensional and linear, the normal distribution is well defined, and one can ask for the distribution of simultaneous zeroes of $n = \dim M$ sections. The basic result of [BSZ00] was that, considering a sequence of bundles $L, L^2, \ldots, L^N$, as $N \to \infty$ the limiting normalized distribution is the n-th power of the curvature of L. Now one of the physics problems I was looking at was to find critical points of a "random flux superpotential," a holomorphic section drawn from a finite-dimensional linear space. These are simultaneous zeroes of the components of the covariant derivative, very similar to the zeroes studied in [BSZ00]. Even better, the techniques (such as the Kac–Rice formula) were familiar from random matrix theory.

I was delighted to learn that Zelditch would be at ICM 2002, and we arranged a meeting there. He explained his work, and I asked him whether they had considered doing the same for critical points of a random section. Indeed they had considered it, but they had thought that nobody would be interested. So that is where our collaboration (with Bernie Shiffman as well) began. This led to [DSZ04, DSZ06b, DSZ06a], which solved my problem, and many subsequent works. It also led to many interactions with geometric probabilists at workshops Steve invited me to. I should also mention Steve's long collaboration with my student Semyon Klevtsov which further deepened his impact on physics.

My interactions with Steve made a great impression on me, going far beyond these specific works. As every mathematician who has worked with physicists knows, even when you are talking about the same mathematical objects, and even when the language barriers have been overcome, there are great differences in how you think. You ask different questions, and you can have very different opinions about when they have been answered. Steve had very broad interests and was flexible in what he would consider, but at the end of the day uncompromising. And in these difficult days for fundamental physics, lacking much experimental guidance, rigorous standards are all the more valuable to keep us on track.

Steve had a great love of life which made him a pleasure to be around, and I have wonderful memories of times together, in particular of a wine tour in central California we took with Bernie. I will greatly miss him.

Boris Hanin

The first time I met Steve we were both new to Northwestern. I was a first year PhD student and he had just moved from Johns Hopkins. We got to talking over lunch (faculty would sometimes eat lunch in the common room of Lunt Hall) about a curious relationship between zeros and critical points of high degree polynomials.

I had no idea that Steve was in the middle of writing an influential series of articles, mainly with Bernie Shiffman, studying zeros and critical points of random polynomials and holomorphic sections. He immediately reframed the result I mentioned into a question that could be approached using Bergman kernel expansions and suggested that after a few weeks of reading his papers I could find an answer.

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In this way, Steve was a magician. His mind would key into unexpected mathematical facts and, seemingly out of thin air, he'd produce a connection to something he knew. This was often followed by a question and a plan of attack.

After around a year of work, I excitedly came to Steve's office to explain my new theorem on zeros and critical points. His answer: "I don't believe the result." Not knowing that such a response was even possible, I replied "but I have a proof." Steve went on to assure me both that my proof was certainly wrong and that this wasn't the issue. Results—especially unexpected ones—demand conceptual explanation.

In this way, Steve was a purist. He insisted on understanding mathematics in a manner so deep that the technical and the intuitive merged. Finding an explanation that Steve found satisfying took me several months and taught me what it's really like to understand my own work. Far from being frustrated by me, when I finally came to him with a simple heuristic derivation, Steve was overjoyed and arranged for me to speak about it at a conference on random geometry in Montreal the following summer.

I had never attended a conference before, and I still remember Steve asking virtually every speaker questions, with follow-ups in the breaks and even during dinners. That image of Steve sitting in the front row, engaging with the content of the summer schools, workshops, and conferences we both attended over the years is how I'll most remember him.

In this way, Steve was a mentor. He taught me to see opportunities for growth as a mathematician, but didn't prescribe how to use them. He also encouraged me to learn new fields, even when I became fascinated by neural networks as a postdoc and probably should have been writing more articles on spectral asymptotics.

Over the past few years it was my great pleasure to continue working with Steve, and we submitted the revisions for our final joint paper a few days before he passed away. Though he was quite unwell by that point, he was still intent on pursuing mathematics, both insisting that we include certain oscillatory integral estimates in our revision and, in the same breath, asking me to send him a recent paper on quantum computing that I had gotten excited about.

I miss him dearly.

Andrew Hassell

I first met Steve when I was a graduate student in the early 90s, at various spectral theory/microlocal analysis conferences in the US. Steve stood out in such gatherings: he would ask many questions both during and after talks, and because of his remarkable breadth of knowledge, he could ask searching questions on seemingly any topic that came up. He was as comfortable with representation theory, geometric quantization, complex geometry or random matrices as he was with more "core" topics such as Fourier integral operators or eigenvalue counting functions. Steve continued his lively (and loud—he had a penetrating voice) questions and discussions during breaks and over meals. His questioning style was intense, passionate and sometimes verging on aggressive! He was generous with ideas, throwing out numerous questions to groups of participants arising from the talks or from his own research. A junior mathematician could do very well for him or herself by listening to Steve! It was certainly a good way to acquire research questions, and if things developed well, it sometimes led to a joint publication with Steve. Over the course of my career, at least eight research articles were influenced by or were the result of answering questions posed by Steve. I'll describe some of these in the remainder of this note.

Isospectral problems. Steve was interested in isospectral problems (popularized by Kac's famous article [Kac66]) throughout his career, see for example [Zel92, Zel00, Zel09, HZ22]. When I was a postdoc, knowing I was interested in scattering theory, Steve asked what we could say about a class of *isophasal* domains — the scattering theoretic analogue of isospectrality. Could we show that such a class is compact, similar to the famous result of Osgood–Phillips–Sarnak? We worked on this when Steve visited Brisbane and Canberra in 1997, and showed this in our first joint paper [HZ99].

Quantum ergodicity. Steve's seminal result on quantum ergodicity [Zel87] was obtained early in his career. Yet it took some time for its importance to be appreciated. In fact, MathSciNet lists no citations of this paper until 1997 (it now has 249, at the time of writing). Steve was fascinated by the equidistribution of eigenfunctions and researched aspects of this question throughout his career. This includes questions on the rate of quantum ergodicity, C^*-algebraic aspects of quantum ergodicity, ergodicity of billiards, quantum ergodicity of boundary values and restrictions to hypersurfaces, quantum mixing and quantum variance. I worked with Steve on quantum ergodicity of boundary values. The paper arose from Steve sending me what he called an "embryo," that is, an unfinished manuscript that contained his attempt (usually a very significant attempt) to prove the result, with a detailed strategy and much preparatory work. I was fortunate to receive several embryos from Steve over the years. We investigated the microlocal distribution of boundary values of eigenfunctions

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of the Laplacian on bounded Euclidean domains, for various different self-adjoint boundary conditions [HZ04].

In 2008, I found a simple way to prove the widely conjectured statement that stadium billiards are not quantum unique ergodic (QUE) (for almost every aspect ratio of the central rectangle). This work was not in collaboration with Steve but was informed by an earlier paper of his [Zel04b] together with several conversations we'd had over the years. In 2008, during an MSRI program, he asked whether the method, which involves studying the spectral flow as one varies the aspect ratio, could be adapted to show non-QUE for a system with classically KAM dynamics. Almost a decade later, I gave this problem to my graduate student Sean Gomes. Sean was able to show not just non-QUE but non-QE for 1-parameter perturbations of completely integrable systems, again for almost every value of the perturbation parameter. Shortly afterward, when Sean was a postdoc at Northwestern, we showed that a stronger statement could be made in the case of two-dimensional KAM systems.

Semiclassical asymptotics of scattering matrices. In the late 1990s, Steve asked me what one could say about the semiclassical asymptotics of the spectrum of a scattering matrix, say for the Schrödinger operator $h^2\Delta + V(x) - E$, where V is a C_c^∞ potential and E is a positive energy level. It was motivated by equidistribution results that he obtained for quantized contact transformations [Zel97]. Fifteen years later, I started working on this problem, initially with Datchev, Gell-Redman, and Humphries in the centrally symmetric case. When Steve saw the result he realized that this could be combined with his ideas in his early paper on quantized contact transformations. Steve, Jesse Gell-Redman and I showed that the spectrum can be divided into two parts, one of which lies very close to 1 on the unit circle, and the other is equidistributed [GRHZ15].

Steve was a great supporter of early career researchers. He loved discussing mathematics, and he gave people in his audience equal respect whether they were legendary mathematicians or lowly PhD students. He always believed that he could learn from whoever he was talking to, and was never happier than when suggesting research problems to his audience. Personally, I felt very encouraged by Steve in my first few years post-PhD and my mathematical life was greatly enriched by interacting with Steve. I miss him severely.

Hamid Hezari

I feel privileged to call myself a former PhD student and collaborator of Steve Zelditch. In fact, I became his student in a fascinatingly lucky way. What later became the key to connect me to Steve was an Iranian math magazine, given to me on the first day of my undergraduate education, which contained a translation into Farsi of Kac's famous 1966 paper, "Can one hear the shape of a drum?" The catchy title caught my attention. I tried to read it, but understood close to nothing of the mathematical content and methods except that the author raised the question of whether one can find the shape of a (not necessarily circular) drum from its frequencies of vibrations, and showed that one can actually hear the shape of a perfectly round drum (a disk). Later in 2004, when I was admitted to the PhD program of Johns Hopkins University, Steve was in charge of the graduate analysis course I was taking. My intention was to pursue number theory. Steve's generosity was striking, both in terms of his time and his mathematical ideas. At that time, he was focused on the inverse spectral problem for analytic domains, and after one of his lectures, he openly shared with me the challenges he was facing. Steve's contagious enthusiasm quickly got me interested in the inverse problem. In 2007, after a series of three long and technical papers, Steve proved that generic analytic plane domains with one axial symmetry are distinguishable from each other by their sound frequencies. This theorem still stands as one of the strongest results in the subject. A natural problem was to extend this result to higher dimensions. In 2008, in my first joint work with Steve, we proved an analogous inverse result for generic analytic domains in $\mathbb{R}^n$, $n \geq 3$, under the condition that they are symmetric with respect to all coordinate axes. Removing the symmetry assumptions, even one of them, still remains as a big challenge.

Steve's next mission was to investigate the inverse problem for smooth plane domains. One of Steve's main approaches in doing mathematics was to always work out a simple, and at the same time important, example first. For our case, ellipses were a natural choice because of their unique billiard dynamical properties that seem to characterize them amongst other planar smooth domains. The famous Birkhoff conjecture states that ellipses are the only completely integrable billiard tables. One then asks whether ellipses are unique from the quantum mechanical point of view; i.e., are the eigenvalues of the Laplacian associated to an ellipse (with Dirichlet or Neumann boundary conditions) unique amongst all smooth domains? This is

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Figure 4. From left to right: Yanir Rubinstein, Steve Zelditch, Hamid Hezari. Baltimore, Spring 2009.

a difficult problem and remains open in this generality. In 2011, we proved a partial result about the spectral rigidity of ellipses within the class of smooth domains with two axial symmetries. A big development was subsequently announced in 2014 by Avila, de Simoi, and Kaloshin, who proved a version of Birkhoff's conjecture for nearly circular ellipses. I remember that Steve got very excited about this result, knowing that there was something valuable for us to use. Indeed, in 2019, we managed to show that one can hear the shape of a nearly circular ellipse among all smooth domains. This is a strong result, but raises the question: what about ellipses of arbitrary eccentricity? In fact, this was a problem we investigated until the last few months of Steve's life.

Without a doubt, Steve was the most influential person in my life. He taught me how to do, read, write, and speak mathematics, and even how to fully live life. While I did not learn to his standards, what I could absorb helped me enormously with my career and personal life, for which I am forever thankful. He is greatly missed by his entire mathematical community.

Semyon Klevtsov

I met Steve in January of 2009 at a conference on random geometry in Quebec that he co-organized. I was finishing my thesis under Mike Douglas, providing another derivation of the celebrated Tian–Yau–Zelditch–Catlin expansion of the Bergman kernel, using a quantum mechanical path integral parametrix. I was looking forward to talking to Steve about holomorphic sections, balanced metrics, and Kähler geometry. We started talking right from the moment we met. Steve was easily approachable, very friendly, and always eager to discuss math. I immediately felt "on the same wavelength" with him after just a first few minutes of the conversation. During the conference dinner Steve sketched an idea of random Bergman metrics. (It must have been done on a napkin as my memory tells me it was definitely during the dinner.) I liked the idea a lot and we started working together.

Our collaboration, which also included Frank Ferrari for a while, and our regular meetings ran until his last days and it was the most exciting intellectual adventure in my scientific life. I moved to Europe after finishing my PhD and Steve and I would work by Skype and then meet between semesters or at conferences. As transatlantic collaborations go, the first days of in-person meetings consistently began with the jet-lagged person trying not to doze off in front of the blackboard. At some point into our collaboration I realized that holomorphic sections can be used to describe quantum Hall states. I then branched out part-time to develop this subject with Steve's continued support and influence. We even organized a conference together on geometric aspects of the quantum Hall effect in 2015.

Figure 5. Steve Zelditch, Antwerp, 2014.

One of Steve's deep and original contributions to modern mathematics is that he was one of the pioneers of "marrying" probability and geometry, often via the ideas from quantum field theory and quantum gravity. In fact, as he told me a few times, he decided early on that he would work on math related to quantum theory. In his grad school days he even took Richard Feynman's quantum mechanics class, although apparently that experience turned out be somewhat disappointing. Steve definitely was a top-notch expert in all things quantum. I would guess that his ideas about random geometry stem from his earlier work in spectral theory and quantum chaos. Later on, with Bernie Shiffman they launched a very successful area of random holomorphic sections—one of the first random geometry models. My later work with Steve on random Kähler metrics continued this line of thought.

Together with several of Steve's friends and colleagues, we long planned to organize a conference in his honor. Covid interfered and we finally got to do this only when we learned about his illness. It is quite extraordinary that about 500 people signed up to participate despite very short notice. This and all the outpouring of emotions following his passing away on the last day of the conference

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are testaments to his influence and lasting impact on so many people in very diverse areas of mathematics.

I am truly blessed to have known Steve, worked with him, and enjoyed his friendship and mentorship. He was and is a role model, not only scientifically, but also as a human being. He will be dearly missed.

William P. Minicozzi II

I was fortunate to overlap with Steve Zelditch for about fifteen years at Johns Hopkins University. Steve was a great colleague and a remarkable mathematician—technically powerful, broadly knowledgable and always curious. He had virtually limitless energy and enthusiasm, he was great to talk math with, and he was enormously fun to be around (his protracted discussions causing me to miss my bus to the train station too many times).

Steve had very broad interests, mathematically and more generally, but he had a particular interest in eigenfunctions. Fourier analysis describes the spectral theory of the circle of radius one: the eigenvalues are square integers k^2, and the eigenfunctions are $\sin kx$ and $\cos kx$. For compact manifolds, the spectral theory of elliptic operators gives a complete basis of eigenfunctions with eigenvalues going to infinity. There are some universal features, but the eigenfunctions behave very differently depending on the geometry of the manifold M.

One of the themes in Steve's work is the mysterious analogy between classical and quantum mechanics. In the quantum perspective, for each $x \in M$, the value $u^2(x)$ is the probability density of the quantum particle being at x. This theme appears early in his influential 1987 paper on hyperbolic surfaces [Zel87], where he showed that the (quantum) eigenfunctions become uniformly distributed just like the (classical) geodesic flow.

This perspective leads to natural questions. For example, on which spaces do the eigenfunctions "concentrate" the most? One way to measure concentration is to look at the ratios of various L^p norms. If we normalize the L^2 norms to be one, then how large can each L^p norm be and on which spaces is this achieved? Steve and Chris Sogge proved beautiful results in this direction with geometry and dynamics playing key roles; see, e.g., their results on L^∞ norms in [SZ02]. Their ideas generated a lot of activity and this continues to be an important area of research.

Instead of looking at the places most likely to find the quantum particle, what can we say about the places least likely to find it? These are the points where the eigenfunction vanishes; this zero set is known as the *nodal set* and it has been studied for hundreds of years. Steve made a number of important contributions to this problem and to related questions in complex geometry. A personal favorite is the paper [SZ11] inspired by a conjecture of Yau.

Steve Zelditch was truly one of a kind. He brought energy and life to the department and the community. He will be sorely missed.

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Duong H. Phong

I am heartbroken to write these lines in memory of Steve Zelditch, who passed away so suddenly on September 11, 2022. I can only share the grief of Steve's entire family, and especially his wonderful wife Ursula, who extended the warm hospitality of their home to me so many times. In this context, I can't even count the number of times when Steve confided to me how happy he was, and how lucky he was to have met Ursula.

I am probably among those mathematicians and colleagues who have known Steve the longest, since 1981 when he joined Columbia University as a Ritt Assistant Professor. Mathematics and life are long and hard journeys, and it was a privilege for me to travel much of it in his company, often side by side, and always, I believe, in communal spirit. He graduated with a thesis on Schrödinger equations and microlocal analysis, and I witnessed first hand his growing interest in dynamical systems, and the emergence of his foundational paper on geometric quantum chaos. While we were not in as close contact after his departure from Columbia for Johns Hopkins in 1985, I followed his regular great works as well as I could after this early period, including the asymptotic expansion of the Bergman kernel, the many amazing applications he found for which he won the Bergman Prize, practically the creation of a whole new field of random complex geometry, and the first positive advance for decades on M. Kac's famous question on whether one can hear the shape of a drum. Steve was the undoubted master of semiclassical analysis, transfigured with insights from other fields. From this vantage point, he would cast a new and unexpected light into a wide area of mathematics, including complex geometry, probability, dynamical systems, mathematical physics, and reveal phenomena that even seasoned experts in these areas would not suspect had existed.

Steve excelled in every intellectual enterprise which he set his mind to. One example is the speed with which he built the Northwestern Mathematics department into the powerhouse in complex geometry which it is today. This can probably be traced in large part to the enormous

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influence which he had on all his friends and those around him. While very much aware of other people's opinions, he always had to form his own, which he would base on careful thought and study. He was always fair, and never overlooked the positive aspect of things. Even though referee reports of mathematics papers are anonymous, I suspect that anyone who got a report from Steve would instantly recognize it as such: it was always detailed, positive, informed, and raised unexpected interesting questions. And those who have served on the same National Science Foundation panels with him can vouch for his lucidity and eloquence. I myself learned a lot from him, from topics that I did not know at all before to subjects where I had had some familiarity, such as semiclassical analysis. Since Steve's opinions were so well thought out to begin with, it was not easy to get him to change them. So it is with a bit of childish pride that I can report one instance when he came to me and said, "You know, after all these years, I have now come to your view that Richard Gere is a very bad actor."

It is terribly sad for me to think that Steve and I won't be discussing mathematics, literature, and movies again, or be taking rides in his car listening to Armenian duduk music, or simply be arguing, as close friends are prone to, from the most mundane topics to the ones that we care most about. But his work will live on in mathematics, and his memory will be with me always.

Yanir A. Rubinstein

I realize that in this type of memorial, exaggerations frequently happen. But to stay true to Steve's legacy, I will do my best to say things as straight as I can.

Steve Zelditch was my postdoc advisor for the academic year 2008–2009. Surprisingly, there was something of a mutual first in that relationship. I was Steve's first NSF postdoc mentee. In 2007, when I asked Steve if he would agree to sponsor my application, he told me that if successful, I would be his first such mentee. I was astonished. To me, it was already shocking that Steve was not, say, at MIT or Stanford. When I first asked Steve about this, he shrugged it off and gave me the "one day you'll understand, boy" reaction. On one occasion I pressed him hard on the issue as I felt it was unjust—at the time I was young and idealistic and believed that belonging to a top university was decided purely on the basis of the level of one's mathematical originality and production. But it is not. And Steve explained that to me thoroughly.

Steve was honest down to his very last bone. Early on, he chose very hard problems that did not always have elegant solutions. Unwittingly, sometimes his results and choices created rivals. Also, it did not help that it took quite some decades for some of Steve's most foundational work to become mainstream and appreciated for its pioneering value. Finally, it did not help that Steve was obsessive about mathematics. He could talk mathematics for hours on end, oftentimes interrupting his interlocutor repeatedly.

Luckily for me, I came to know of Steve through his work on Kähler geometry, and the Kähler community accepted Steve with open arms almost immediately following his much-cited 1998 paper on the asymptotic expansion of the Szegő kernel. In a community with some rivalries and big egos, Steve was a soothing presence, universally appreciated and admired in the community. Steve didn't take sides and often served as an ambassador making crucial connections.

Steve loved talking to me about Kähler geometry, in which I was supposed to be an expert. In retrospect, I understood that his making me the purported expert actually somewhat contributed to my becoming one—it pushed me to deliver the answers he wanted and uphold that image. When he spoke about microlocal analysis, his true passion, I felt he really lit up. On the other hand, in Kähler geometry he put on the student gown, which he loved as well. Either way, regardless of the discussion topic, Steve always seemed to have unbelievable levels of energy—like nothing I have ever seen then or since. During my postdoc, I was in my 20's while he was in his 50's. Yet, that would be impossible for an outside observer to tell judging from his energy. It was the same in our collaborations. He would work on a problem by collecting books on a topic and putting dozens of papers in a folder, many of which he had read quite thoroughly. I imagined that his bullet reading skill came from his early life as an English major, but maybe it was just his genius. He then wrote several notes in another folder about different aspects of the theory, either summarizing results from the first folder, or trying to work out ideas on his own. He had many such projects at any given moment. It was breathtaking for me over the years to see him venture into completely new fields, from Kähler geometry to probability and mathematical physics and biology, certainly an inspiration for me, as I have also been a happy mathematical nomad throughout my career.

In our last conversation, shortly before his passing, I expressed my deep admiration and gratitude to him. He listened but then insisted on emphasizing to me that "these relationships are very much mutual." It was one of the most moving things a mentor has ever told me. He also told me, "you have truly surpassed yourself, Yanir." I share

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this with the readers not to compliment myself but rather to try to communicate the magnitude of greatness and generosity of Steve, who was not thinking of himself, even as he was dying, and tragically since he was in possession of his full intellectual forces and in the midst of one of the most creative periods of his career.

Let me share another recollection from about two months prior. Steve first mentored me in grant writing. As a small tradition, I would call Steve whenever I got an NSF grant. When I called him in June 2022, I felt something was off. He didn't sound quite like himself. I asked him what was the matter. He shared that he was undergoing some tests and had some health issues. I asked him if there was anything I could do for him, which he dismissed. When it turned out that the disease was terminal, he wrote to remind me that I had asked what I could do, saying that there actually was something: take care of one of his famous folders, and see to it becoming published papers. In the following two months up until his death we spoke a few times about that folder. He fervently cared about mathematics and wanted to make sure those ideas got worked out. For him, each one of his folders had a life of its own, much beyond whether his name appeared on it.

Three more things are unforgettable to me about Steve. First, his idealism; second, his relentless support of young mathematicians; third, his keen dislike of "declaring victory." He sought to tackle hard problems that often did not have a nice and beautiful solution, but required many long, ingenious computations. It was not easy to write a paper with Steve because of his extremely high standards.

Others will undoubtedly talk about Steve's sense of humor, which deserves its own separate essay. For my part I will end with the following anecdote from our last conversation. I told Steve that he had achieved more than most mathematicians achieve in two lifetimes. He replied, "Well, I wish I had a third." I already miss Steve dearly and doing mathematics will never be the same for me.

Bernard Shiffman

I've known Steve Zelditch since he came to Johns Hopkins in 1985 as an Assistant Professor. When I called Steve to offer him a job at Hopkins (as I was on the hiring committee), I knew that he was an outstanding hire, but didn't expect that I would ever work with him, since Steve hadn't done Kähler geometry and I knew very little about microlocal analysis. I didn't know then how Steve would latch onto and learn about almost any subject he knew nothing about and rapidly become an expert.

Steve jumped headfirst into complex geometry with his 1998 seminal paper on what is now called the

Figure 6. Steve Zelditch and Bernie Shiffman, Santa Barbara, 2005.

Tian–Yau–Zelditch asymptotic expansion of the Bergman kernel for powers of a positive line bundle on compact Kähler manifolds. He was able to see connections between different areas of mathematics that others wouldn't notice—our collaboration began when he heard a talk I gave on complex dynamics and saw a connection to quantum ergodicity. Then over the next 24 years, Steve developed a new area of mathematics, "stochastic Kähler geometry," with the help of numerous collaborators including myself. Stochastic Kähler geometry involves the asymptotics of probabilistic invariants such as distribution and correlation functions of zeros and of critical points of random holomorphic sections of line bundles. Recently, together with Ferrari and Klevtsov, Steve began the study of random "Bergman metrics."

Steve was inspirational to his students and postdocs, as we know, and also to his colleagues and collaborators. He was generous with his ideas, in fact too generous, as the number of ideas he would come up with in one afternoon could take up many years. Steve was also a very gracious host and was generous with his time.

Working with Steve was not only inspiring, but also fun. As all his colleagues know, Steve could talk entertainingly for hours—when you got in a conversation with Steve, on the phone or in person, you could expect the discussion to last two or more hours. His loquaciousness wasn't only with mathematics. Steve would discourse at length on myriad topics, from stamp collecting to literature to whatever.

Steve didn't do anything halfway, from setting up an aquarium in his house, which at one time was the only furniture in his living room, to being a wine connoisseur, to following politics—in 2000 Steve convinced me to go with him to a rally in Washington for Ralph Nader, who was then the Green Party candidate for president.

When working on a paper with Steve, he wouldn't want to stop after obtaining the desired result, but would keep pushing the result to more settings and generalizations, until I had to insist we stop and submit the paper.

I last spoke on Zoom with Steve on August 3, 2022, after his cancer had progressed beyond treatment. He began the conversation by saying that he couldn't talk long and that I would have to do most of the talking—he then talked for an entire hour. I miss Steve. His passing leaves a large void.

Chris Sogge

I first met Steve Zelditch at a microlocal analysis conference in Irsee, Germany, in the summer of 1990. He made a big impression on me. Even though, at the time, we were at the beginning of our careers and had much different backgrounds, we really hit it off and started a professional relationship and friendship that was one of my most important ones and would grow over the next three decades.

A few years later, in 1996, I moved from UCLA to Johns Hopkins University. I was happy at UCLA and my career was going well, but Steve was always a master of persuasion. The move to Baltimore was great for me and my family, and I especially loved the 14 years that we overlapped until Steve left for Northwestern University. There are so many fond memories.

Our families quickly became very close and we spent much time together, either at each other's house or at our children's sporting events. Our two youngest children are the same age as the Zelditch children, and they became very good friends.

The family dinners would always end the same way. Steve would snag me away from everyone else and attempt to spend hours either talking about mathematics or his latest obsession. The worst was during his stamp collecting phase. Steve's long soliloquies about Grauert tubes, Austrian stamps, Georgian wines, 1930s Shanghai music,…, would be interrupted (usually to my relief) by family members wanting to go home.

When our children competed against each other in a sporting event, such as soccer, Steve would always be working out a calculation on a pad of paper sitting in his fold-up Home Depot chair. It was always remarkable how he was able to look up at exactly the right time to cheer on one of his sons louder than any of the rest of us parents. Steve really was great at multitasking.

I really grew as a mathematician through my collaborations and many discussions with Steve over the years, attempting to become a practitioner of what Steve liked to call "Global Harmonic Analysis." During the time he was at Johns Hopkins he tended to bring out the best in us, and

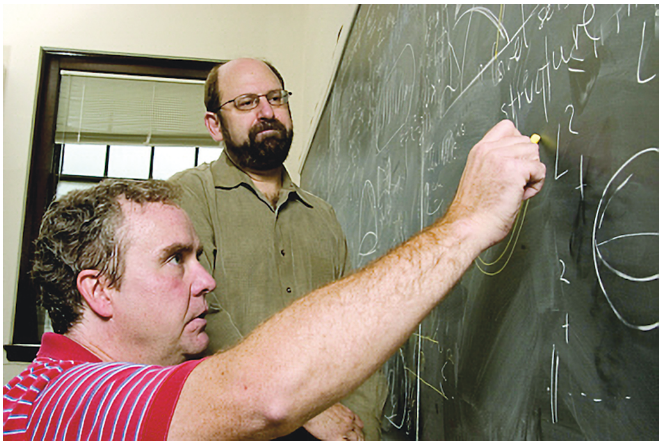

Figure 7. Steve Zelditch and Chris Sogge in 2004.

he perked up the department in so many ways. He was a very impactful and successful department chair (1999–2002), who, among many other things, was instrumental in starting our very important and thriving J. J. Sylvester Assistant Professor (postdoc) program. Steve was somebody who rarely lost arguments, and I am sure that this was the case in his negotiations with the JHU administration. Also, even though he was so loquacious, Steve could be a great listener. This was especially true during seminar talks when his multiple interruptions would inevitably force the speaker to really tell us what he or she was attempting to say. Steve's encyclopedic knowledge could be intimidating, but his charm and sense of humor would always result in a smile after a couple of well-directed questions.

Steve really was a force of nature and, without a doubt, the most interesting person I have had the pleasure of knowing throughout my career. Two days before Steve sadly passed away I was honored to speak in the amazing online conference, "Global Harmonic Analysis," which was quickly but skillfully and lovingly put together by several young mathematicians whom Steve had impacted. I ended my talk with a quote from our friend and former colleague, Bill Minicozzi: *"Steve is a unicorn. Unique on the planet."*

Joel Spruck

In 1990, while I was at the University of Massachusetts at Amherst, I was contacted by the hiring committee of the mathematics department at JHU and asked if I was interested in a senior position. I learned that Steve Zelditch was the only "hard analyst" in a department dominated by homotopy theory, algebraic geometry, and number theory. I

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had never met Steve as we moved in different circles. I visited the department soon thereafter and I gave a talk on my recent joint work with Craig Evans on the level set mean curvature flow. Steve must have liked my talk because I quickly received an offer.

I joined the department in the 1992–1993 academic year. Steve quickly became my favorite person to talk to and have lunch with because he was so magnetic. I came to learn over the ensuing years that Steve was perfectly comfortable with the abstract high-powered algebraic side of the department because he knew and understood so much mathematics. I don't think Steve fully realized that other math people were not as broad as he was. This could sometimes be frightening to students when he was on their oral exam committees.

Steve was, apart from being a brilliant mathematician, a wonderful and delightful person who was devoted to his wife Ursula and sons Benjamin and Phillip. He was endlessly curious and talkative and made you smile inside and out. Steve was also what in Yiddish is called a *mensch*, roughly translated as an honorable person, someone full of integrity. We will all miss him dearly.

Alexander Strohmaier

When I was a young postdoc I became interested in the intriguing relation between eigenvalues of the Laplacian and the geodesic flow. After working out some consequences of spectral properties of the geodesic flow on the clustering of eigenvalues, I was going to give a talk about this at a conference in Montreal in June 2004. Talking to the other participants I quickly learned that what I had done was contained in the work of Steve Zelditch. It was at this conference that I first met Steve. He was mathematically firm whilst very kind on a personal level. He told me that these things happen all the time and I should not be discouraged.

I would like to describe here the simple and beautiful correspondence between the spectral measures of the Laplacian and the spectral measure of the geodesic vector field on the unit tangent bundle. The spectrum of the Laplacian $-\Delta$ on a closed Riemannian manifold (M, g) has been of interest to mathematicians for a long time. Let λ_j be the positive roots of the eigenvalues and ϕ_j the corresponding eigenfunctions. The formula $\omega_j(A) = \langle A\phi_j, \phi_j\rangle$ defines a state ω_j on the $*$-algebra of pseudodifferential operators of order zero and therefore on its norm completion $\mathcal{A}$, which is a C^*-algebra. Any weak-$*$-limit point descends to a state on $\mathcal{A}/\mathcal{K} \cong C(S^*M)$. A theorem by Helton from 1977 links the clustering properties of eigenvalues to the geodesic flow. Namely, the existence of a single nonclosed geodesic implies that the set $\{\lambda_j - \lambda_k \mid k, j \in \mathbb{N}\}$ is dense in $\mathbb{R}$. This theorem can be made more precise and follows from a trace formula that originates from the work of Helton and Zelditch, which I would like to explain here. Let Z be the geodesic vector field on S^*M. Then $\mathrm{i}Z$ generates a unitary group on $L^2(S^*M)$ and therefore defines a self-adjoint operator. Next, one can define the Riesz means of the spectral measures $\mu_{A,j}$ associated with the Gelfand–Naimark–Segal representation of the states ω_j. It has been noted by Zelditch in [Zel96], that the sequence $\mu_{A,k}(f)$ converges to the measure $\langle \mathrm{d}\tilde{E}_\lambda a, a\rangle$, where $a \in C(S^*M)$ is the principal symbol of A, and $\mathrm{d}\tilde{E}_\lambda$ is the spectral measure of the generator of the geodesic flow. This shows the following trace-formula

$$\frac{1}{k}\sum_{j=1}^{k}\langle \mathrm{d}E_{\lambda-\lambda_j} A\phi_j, A\phi_j\rangle \to \langle \mathrm{d}\tilde{E}_\lambda a, a\rangle$$

with convergence in the weak-$*$-sense as $k \to \infty$, where $\mathrm{d}E_\lambda$ is the operator-valued spectral measure of the root $\sqrt{-\Delta}$ of the Laplacian. This immediately implies that any point in the spectrum of $\mathrm{i}Z$ must be a cluster point of $\{\lambda_j - \lambda_k\}$. This gave rise to a finer analysis of the interplay between the geodesic flow and quantum ergodicity, much of which owes to Zelditch.

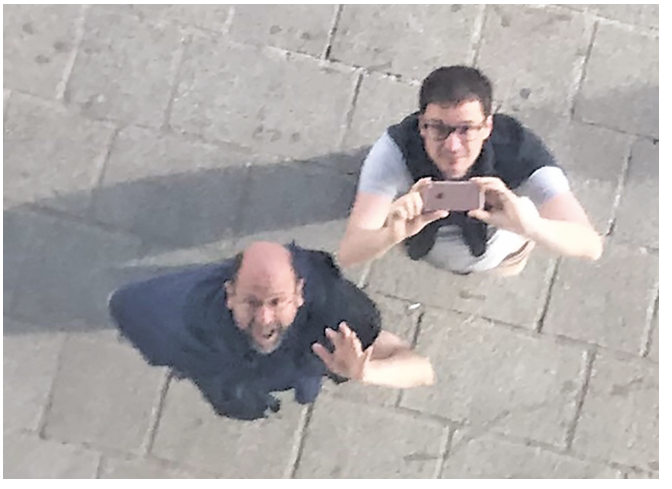

Figure 8. Steve Zelditch and Alexander Strohmaier, seen looking at their reflection.

Steve Zelditch and I have recently collaborated on several projects related to the generalization of spectral theory of the Laplacian to the more general relativistic situation of stationary spacetimes. The Gutzwiller–Duistermaat–Guillemin trace formula was shown in this context in [SZ21]. Without going into details, the framework set up in [Zel96] is very general and it is therefore likely that

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Helton's observation holds for stationary spacetimes with compact Cauchy surfaces. This would imply spectral clustering if there exists a single nonperiodic null geodesic in the space-time.

Steve has had a profound influence on the field with many different results, ranging from nodal sets and restriction theorems to complex properties of eigenfunctions. He was able to describe in a single sentence the essence of a paper. It is this type of interaction that is so important amongst mathematicians. I owe him a lot and he will very much be missed.

Jacob Sturm

I met Steve in 1981 at Columbia. He was just starting his postdoc; I just finished mine and had recently moved to Johns Hopkins. But I missed NYC terribly, and although I had just met Steve and he barely knew me, we somehow hit it off and he suggested that I could stay with him on weekends in his two-bedroom apartment on 113th Street between Amsterdam and Broadway, so that's what I did. I have very fond memories of those days. At that time, I was a number theorist (a student of Goro Shimura) and Steve was interested in geodesic flows on compact Riemann surfaces, so we didn't have much in common except for the upper half plane. Nevertheless, we had a lot to talk about, both mathematically and about "life." We went to parties, movies, bars, restaurants, hosted dinners, etc., and just enjoyed the exuberance that living in NYC inspires. Little did I know at the time that 20 years later, when I switched fields to complex differential geometry, that his work would have a profound influence on my own. The Tian–Yau–Zelditch theorem was first recognized as a very powerful tool in Kähler geometry by Simon Donaldson, who used it in several papers to prove some marvelous theorems. After Donaldson's work, many other researchers in the area took notice of Steve's work and applied it to great advantage. TYZ says that Kähler metrics, which are rather transcendental sorts of objects, can be approximated in a very precise sense by Bergman metrics, which are "algebraic." Phong and I realized how one could use TYZ to show that geodesics in the space of Kähler metrics can be approximated by Bergman geodesics and how geodesic rays could be approximated by test configurations. We wrote several papers about this topic, and Steve was very interested. He invited me to Hopkins a couple of times, and then he and Jian Song worked out in beautiful detail (obtaining much more precise results) the geodesic theorems for toric varieties. I think they wrote three papers on this subject. So, it ended up that we influenced each other, which made me very happy.

Many people have said that Steve was not just a brilliant mathematician—he was just plain brilliant, and I couldn't agree more. I remember a dinner I once had with Steve and John Morgan and Phong at the Lion's Head (a famous Greenwich Village bar/restaurant, now defunct, that was a favorite hangout for journalists and writers like Jimmy Breslin, Norman Mailer, etc). Steve, John, and I had (separately) seen a recently released movie and were discussing it over drinks. John and I were saying the sorts of things that people often say: "the plot was formulistic," "the acting was great," "I didn't like the ending," but Steve's take was at a completely different level. He spoke for an uninterrupted 15 minutes or so, comparing it to other films by the same director, pointing out subtle symbolism, the role played by the history of the setting, the influence of Greek mythology, ... It was as if he had written a detailed film review and was reading it aloud! All of this delivered without pretension, in fact seeming unaware of his own brilliance. I think we were all a bit awestruck.

Steve was a lot of fun to be around: he had a great sense of humor and could talk about virtually anything. At the end of a long math day at a conference or during a visit, the group would often go out for drinks, usually wine, usually pinot noir. Steve was fond of saying "Pinot noir isn't just a wine. It's a way of life." Once, during a dinner at Pasha, a Turkish restaurant in NYC with a limited selection, I told him that I enjoyed the Kendall Jackson pinot noir that we had ordered. He told me that there was a lot to be experienced, and that KJ was just scratching the surface. He went on to say something like "Toric varieties are probably the Kendall Jackson of Kähler geometry. But if one doesn't start scratching the surface, how does one get deeper into things?" Quintessential Steve.

Steve often talked about his family—one instance I recall was a workshop at Park City in July of 2013 where Steve was giving a minicourse on eigenfunctions of the Laplacian. We spent a lot of time hanging out together, and Steve seemed a bit homesick. The fact that he could connect with Ursula at many levels (including mathematics) meant a great deal to him. He was very proud of Benny who was a top student, an award-winning guitarist, and highly motivated. And Philly was Steve's great buddy with whom, despite the fact that Philly was only 14 years old at the time, he was able to have long, stimulating intellectual discussions.

I was planning to call Steve after the September 2022 conference in his honor, not realizing how far his illness had progressed. Now I regret that I didn't call him earlier when I first learned he was sick. I miss him a great deal.

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John A. Toth

I first met Steve Zelditch while I was a graduate student in the early 1990s. Steve organized a workshop at Johns Hopkins on Birkhoff normal forms and the computation of the associated spectral invariants. I distinctly recall Steve's infectious enthusiasm for the field and his unusual generosity in sharing his ideas on a wide variety of topics, and in inviting me to visit. Our collaboration, which began in the late 1990s and lasted more than two decades, was a continuous source of inspiration for me, and his emails, sometimes multiple in a single day or night, reflected his enthusiasm that continued unabated through the years.

Our joint projects all dealt with the semiclassical asymptotics of eigenfunctions of self-adjoint elliptic operators on compact manifolds in various settings. Starting in around 1997, we began working on the asymptotic properties of joint eigenfunctions of quantum completely integrable (QCI) systems. On a compact n-manifold, these systems are characterized by the existence of a family of n self-adjoint pseudodifferential operators $P_j; j = 1, \dots, n$ with the property that $[P_i, P_j] = 0; i \neq j$. Given the associated principal symbols $p_j = \sigma(P_j)$ one forms the associated classical moment map $\mathcal{P} := (p_1, \dots, p_n) : T^*M \to \mathcal{B} \subset \mathbb{R}^n$. Given a regular value $b \in \mathcal{B}_{reg}$, by the Liouville–Arnold theorem, the invariant sets $\mathcal{P}^{-1}(b)$ are a finite union of Lagragian tori. However, given a singular value $b \in \mathcal{B}_{sing}$, these tori can degenerate in a rather complicated fashion. The associated semiclassical blow-up properties of the joint eigenfunctions of the P_j's are closely linked to the properties of the singular leaves of the moment map via the quantum Birkhoff normal form associated with the commuting operators. In the period between 1997 and 2006, Steve and I wrote several papers on the concentration properties of QCI eigenfunctions and their link to the singular leaves of the moment map.

Around 2008, in a discussion with Steve at a conference in Austria, he raised the question of whether the celebrated QE theorem of Shnirelman, Zelditch, and Colin de Verdière extends to generic hypersurfaces $H^{n-1} \subset M^n$. A few years earlier, Hassell and Zelditch [HZ04] and, independently, Burq had answered this question in the affirmative for Neumann (or Dirichlet) eigenfunctions in the special case where H was the boundary of a piecewise-smooth domain with ergodic billiards. Steve and I began working on the general question in 2009 and published a series of papers proving that the QE theorem was indeed true for a full density of restrictions of QE Laplace eigenfunctions under a generic microlocal asymmetry property on the hypersurface. Specifically, given a Riemannian manifold (M, g) with ergodic geodesic flow $G^t : S^*M \to S^*M$ there is a density one subsequence of QE Laplace eigenfunctions $\{\phi_{\lambda_{j_k}}\}, k \in \mathbb{N}$ such that given any zeroth-order pseudodifferential operator $A \in \Psi^0(H)$ and provided H is microlocally asymmetric with respect to the geodesic flow, the Dirichlet data $\phi^H_{\lambda_{j_k}} := \phi_{\lambda_{j_k}}|_H$ satisfies $\lim_{k\to\infty} \langle A\phi^H_{\lambda_{j_k}}, \phi^H_{\lambda_{j_k}} \rangle = \int_H \sigma(A) d\mu_H$, where $d\mu_H$ denotes the restriction of Liouville measure to $S^*_H M$. This is the quantum ergodic restriction (QER) theorem that we proved in the papers [TZ12, TZ13] both on manifolds with or without boundary. The corresponding result for general Schrödinger operators was proved by Dyatlov and Zworski. Steve and I together with Hans Christianson also proved a companion QER theorem for Cauchy data $(\phi_j|_H, \partial_\nu \phi_j|_H)$ in [CTZ13].

Over the last decade, most of our joint work dealt with applications of eigenfunction restriction results (including QER) to upper bounds on the Hausdorff measures of intersections of eigenfunction nodal sets with general hypersurfaces in the real-analytic setting. We proved sharp upper bounds on the measure of such nodal intersections first for piecewise-analytic bounded planar domains and then in the general analytic setting in arbitrary dimension in [TZ21].

Our work together was just one of many different collaborations that Steve fueled with his enormous energy and his wide and deep mathematical interests. He was an extraordinary mathematician and a force of nature as a person. I will miss him immensely.

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Ben Weinkove

I met Steve when I was a graduate student in the early 2000s, giving a talk at Johns Hopkins. Steve had recently brought powerful new techniques into Kähler geometry, in his proof of the Tian–Yau–Zelditch expansion. Later I learned that this was typical of Steve's style. His knowledge of many disparate areas of mathematics gave him a large tool box which he exploited in whatever problem sparked his interest.

I was surprised and flattered by Steve's interest in my work. Again, this was classic Steve. It didn't matter if a graduate student or a Fields Medalist was giving a talk, Steve wanted to understand it, and would keep asking questions until he did. I admired this attitude which represented to me the best of mathematical culture: interest driven by genuine curiosity, not credentials.

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About a decade later I became Steve's colleague at Northwestern as part of a group in geometric analysis, including Valentino Tosatti and Aaron Naber, built with his help. Steve had a huge presence in our department. In seminars, the question, "Does anyone have any questions for the speaker?" often became, "Steve: do you have any more questions?"

Steve had a strong sense of responsibility toward the discipline and the department. He loved to teach graduate functional analysis so much he even offered to teach it for free. In hiring matters, Steve offered well informed opinions on candidates in almost every field. Not content with merely reading letters, Steve scrutinized the papers of candidates, and often had specific questions to follow up with them. Steve's devotion to mathematics continued even as his health was failing. He still met with students, wrote reference letters and took the time to write preliminary exam problems.

Steve's untimely passing was a terrible blow to us all and to me personally. He had been a large part of my mathematical life from the beginning of my career. He was an inspiration to me. Thank you, Steve—it was an honor to have known you.

Alan Weinstein

Steve Zelditch was one of my early PhD students, and it was clear almost immediately that he was exceptional. Many students later, I still found him one of the best.

Shortly after Steve finished his PhD, his thesis inspired me to write a paper on a symbol calculus for Schrödinger operators on $\mathbb{R}^n$. The analysis in the paper was essentially that of the thesis, to which I added geometric interpretation. In the introduction, I wrote, "I would like to thank Steven Zelditch for many stimulating discussions concerning [his thesis], and frequent reassurance that integrating by parts would work whenever I needed it to." After that paper, since I figured that my kind of microlocal analysis was in very good hands (an assumption which turned out to be absolutely correct), I took a long break from the subject to pursue other interests. Of course, Steve went on to deepen and broaden his work to encompass many aspects of the theory of eigenvalues and eigenfunctions of differential operators.

To the personal sadness of his passing to those close to him is added the loss of someone still in his scientific prime. In recent years, in his mid-to-late 60s, he continued to produce excellent work, with many papers on MathSciNet, one of them having appeared in the *Annals of Mathematics*.

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Richard A. Wentworth

Steve Zelditch was an exceptional individual; intensely smart, gregarious, energetic, funny, and with an insatiable appetite for intellectual engagement. There was almost no part of mathematics, or science more generally, that he didn't find interesting. His scholarship and vast knowledge were exemplary, and his enthusiasm was infectious. His presence left such an indelible impression on all who knew him that his disappearance is difficult to comprehend.

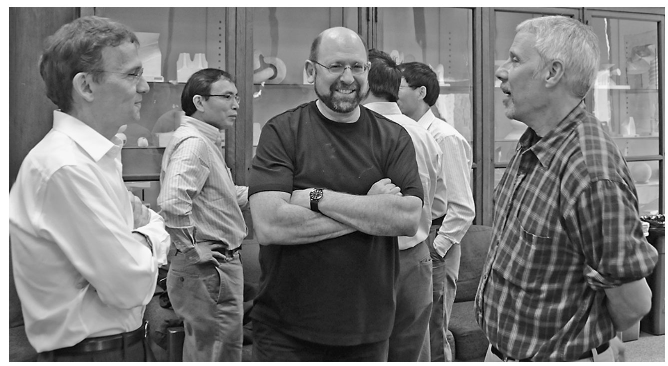

Figure 9. Steve Zelditch in 2012 with Jacob Sturm (right) and Richard Wentworth (left).

Steve approached mathematics as a scientist, in a way that I always found unique and inspiring. He would often say that what he looked for in a person's work was whether they were "discovering new phenomena." His lectures were replete with references to quantum mechanics and pictures of Chladni plates. While having a clear perspective from his own expertise in microlocal analysis, he was fearless in incorporating whatever new methods might be needed for the problem at hand. I once heard someone describe his research as resembling a "big truck rumbling down the street."

On any topic, Steve was a formidable debater and a master of dispassionate discourse. He frustrated his opponents by taking apart the logic of their arguments and exposing inconsistencies, all in a calm yet persistent way. He was also a remarkable judge of character and human nature, and with this came an understanding of and a compassion for people. Similarly, despite his intense focus, what I think everyone remembers about Steve was his terrific sense of humor. His talks were invariably a blend of

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scholarly prowess and self-deprecating jokes, always well-timed and always well-received.

I likely first met Steve at an AMS meeting in Ann Arbor in the early 1990s, where he, Lizhen Ji, and I discussed spectral problems on surfaces with cone singularities. While Steve and I never formally collaborated, he gave me crucial input on several projects related to determinants of elliptic operators, and he vastly broadened my knowledge of what was happening in the mathematical world. He was always generous in sharing his wisdom and experience. I remember and use to this day many examples of his sound advice on how to approach all aspects of our profession. He was a great colleague.

In talking to people who didn't know him I was always in the habit of describing Zelditch as "the most remarkable person I have ever met." This somewhat Gurdjieffian formulation, one that I hope Steve would have appreciated, was meant only half in jest. With his far-too-early passing, indeed, it seems to me truer than ever.

Shing-Tung Yau

I have known Steve for more than thirty years. He was at Johns Hopkins and I was very impressed by his deep insight in geometry and in modern analysis. Both he and I are fond of eigenvalues and eigenfunctions of the Laplacian. I was pleasantly surprised to find out that he graduated from Berkeley and was a student of Alan Weinstein, whom I knew well.

About 40 years ago, I proposed a program to construct Kähler–Einstein metrics on Fano manifolds. I was convinced that their existence is related to stability of the manifold in the sense of Mumford's geometric invariant theory. The first step was to show that the Kähler–Einstein metric can be recovered from the Fubini–Study metrics obtained from embeddings into projective space. I assigned this problem to Gang Tian for his thesis. I suggested using the ideas of peak sections in my work with Siu on holomorphic isometric embeddings. Tian was able to do so, and the higher regularity was accomplished by my other student Ruan. The Fubini–Study metric is actually the Bergman metric for the projective embedding, and it turned out that Steve was able to look at this problem from the point of view of the asymptotic expansion of the Bergman kernel. Several mathematicians followed his insight and made important contributions to my original conjecture on the existence of Kähler–Einstein metrics on Fano manifolds. The idea contributed in a key manner to the solution of my conjecture provided by Donaldson et al.

There were of course many other important contributions made by Steve in geometry and in analysis. A notable contribution was his beautiful work on spectral rigidity for a large number of domains in Euclidean space.

Steve was so direct on revealing his insight to other people that sometimes I thought he was arrogant—but each time he proved to be correct. I admired him. I tried several times to nominate him to be elected to the National Academy of Sciences. Although we did not have much chance to communicate, I believe that Steve liked me, because he insisted on listening to my talk in honor of his birthday even while he was dying. I tried my best. But I was giving the talk in Beijing through Zoom. I could not tell his response to my talk. He passed away a short time after my talk.

I lost a good friend. But I am glad that I gave my talk in his honor right before he passed away. All of us will remember his deep contribution to mathematics and his friendship.

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Maciej Zworski

Steve Zelditch spent part of the academic year 1987–1988 at MIT and that is where we met for the first time. I was a third-year grad student working with Richard Melrose while Steve, who was already at Johns Hopkins, was visiting as an NSF postdoctoral fellow. He very quickly became a strong and irresistible presence in my mathematical life, which had perhaps been all too comfortable till then. He talked about everything and asked questions about everything. He was particularly aggressive in trying to find out from me if the Lax–Phillips semigroup was a Fourier integral operator. I was lost and he felt somewhat uncharacteristically guilty, apologizing that if "one comes from the Guillemin-style school, one would ask that kind of question about your father and mother." Not long afterward he invited me to Johns Hopkins and while sitting in Baltimore harbor he was talking about the Langlands program. I understood nothing but when I heard the words meromorphic continuation I mumbled if "it isn't something like Lax–Phillips automorphic scattering." Steve turned to me and bellowed: "You see this skyscraper, you see this water hydrant—that is how the two compare!." This type of passion mixed with humor (half self-deprecating, half wicked) has been a sometimes endearing, sometimes infuriating, force for good in my mathematical life and that of many others.

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Figure 10. Lizhen Ji, Steve Zelditch, and Maciej Zworski.

A few years later, when thanks to Steve's good offices I was also at Johns Hopkins, he introduced me to one of his favorite subjects, and one which he pioneered in the West: *quantum ergodicity*. Ten years before, Steve had discovered an announcement by Shnirelman stating that for a compact Riemannian manifold M with an ergodic geodesic flow, almost all eigenfuctions equidistribute. That means that if $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \cdots$ is the complete list of eigenvalues with eigenfunctions, u_j, $-\Delta u_j = \lambda_j u_j$, $\int_M |u_j|^2 = 1$, then there exists a density one subsequence u_{j_k} such that

$$\int_M \varphi |u_{j_k}|^2 \to |M|^{-1} \int_M \varphi, \quad k \to \infty, \tag{1}$$

for all $\varphi \in C^\infty(M)$. In fact, this equidistribution is valid in a stronger position and momentum sense. Steve Zelditch provided a proof in the constant negative curvature case, including the finite volume noncompact case. (When in addition, a surface is arithmetic, a celebrated work of Lindenstrauss later showed that the sequence of u_j's can be chosen so that there is no need for a subsequence—the case of so-called *unique* quantum ergodicity). Colin de Verdière then gave a proof for closed manifolds. He recalls how Steve, without any prior arrangements, drove up to Institut Fourier in Grenoble, found him in his office, explained his work on quantum ergodicity and drove off. While at Johns Hopkins, we generalized this work to the case of compact manifolds with piecewise smooth boundaries.

Many advances, a lot of which are by Steve and his collaborators, have been made since and it is impossible to survey them here. I conclude with a very recent one: if in (1) we take $\varphi \geq 0$ to be equal to 1 on a nonempty open set Ω, then there exists a constant $c(\Omega) > 0$ such that $\int_\Omega |u_{j_k}|^2 > c(\Omega)$. Dyatlov, Jin, and Nonnenmacher (building on earlier work of Anantharaman, Bourgain–Dyatlov, and Dyatlov–Jin) showed that for negatively curved surfaces $\int_\Omega |u_j|^2 > c(\Omega)$ where u_j is *any* sequence of eigenfunctions. We do not know if all u_j's are equidistributed but at least there cannot be any holes in their supports, uniformly as $\lambda_j \to \infty$.

Figure 11. Phillip Zelditch, Ben Zelditch, and Ursula Porod at the Northwestern University memorial for Steve, October 2022.

References

[BSZ00] Pavel Bleher, Bernard Shiffman, and Steve Zelditch, *Universality and scaling of correlations between zeros on complex manifolds*, Invent. Math. **142** (2000), no. 2, 351–395, DOI 10.1007/s002220000092. MR1794066

[CTZ13] Hans Christianson, John A. Toth, and Steve Zelditch, *Quantum ergodic restriction for Cauchy data: interior que and restricted que*, Math. Res. Lett. **20** (2013), no. 3, 465–475, DOI 10.4310/MRL.2013.v20.n3.a5. MR3162840

[DSZ04] Michael R. Douglas, Bernard Shiffman, and Steve Zelditch, *Critical points and supersymmetric vacua. I*, Comm. Math. Phys. **252** (2004), no. 1-3, 325–358, DOI 10.1007/s00220-004-1228-y. MR2104882

[DSZ06a] Michael R. Douglas, Bernard Shiffman, and Steve Zelditch, *Critical points and supersymmetric vacua. II. Asymptotics and extremal metrics*, J. Differential Geom. **72** (2006), no. 3, 381–427. MR2219939

[DSZ06b] Michael R. Douglas, Bernard Shiffman, and Steve Zelditch, *Critical points and supersymmetric vacua. III. String/M models*, Comm. Math. Phys. **265** (2006), no. 3, 617–671, DOI 10.1007/s00220-006-0003-7. MR2231684

[GRHZ15] Jesse Gell-Redman, Andrew Hassell, and Steve Zelditch, *Equidistribution of phase shifts in semiclassical potential scattering*, J. Lond. Math. Soc. (2) **91** (2015), no. 1, 159–179. MR3335243

[HZ99] Andrew Hassell and Steve Zelditch, *Determinants of Laplacians in exterior domains*, Internat. Math. Res. Notices **18** (1999), 971–1004. MR1722360

[HZ04] Andrew Hassell and Steve Zelditch, *Quantum ergodicity of boundary values of eigenfunctions*, Comm. Math. Phys. **248** (2004), no. 1, 119–168. MR2104608

[HZ10] Hamid Hezari and Steve Zelditch, *Inverse spectral problem for analytic $(\mathbb{Z}/2\mathbb{Z})^n$-symmetric domains in $\mathbb{R}^n$*, Geom. Funct. Anal. **20** (2010), no. 1, 160–191. MR2647138

[HZ22] Hamid Hezari and Steve Zelditch, *One can hear the shape of ellipses of small eccentricity*, Ann. of Math. (2) **196** (2022), no. 3, 1083–1134. MR4502596

[Kac66] Mark Kac, *Can one hear the shape of a drum?*, Amer. Math. Monthly **73** (1966), no. 4, part II, 1–23. MR201237

[SZ02] Christopher D. Sogge and Steve Zelditch, *Riemannian manifolds with maximal eigenfunction growth*, Duke Math. J. **114** (2002), no. 3, 387–437, DOI 10.1215/S0012-7094-02-11431-8. MR1924569

[SZ11] Christopher D. Sogge and Steve Zelditch, *Lower bounds on the Hausdorff measure of nodal sets*, Math. Res. Lett. **18** (2011), no. 1, 25–37, DOI 10.4310/MRL.2011.v18.n1.a3. MR2770580

[SZ21] Alexander Strohmaier and Steve Zelditch, *A Gutzwiller trace formula for stationary space-times*, Adv. Math. **376** (2021), Paper No. 107434, 53. MR4178908

[TZ12] John A. Toth and Steve Zelditch, *Quantum ergodic restriction theorems. I: Interior hypersurfaces in domains wth ergodic billiards*, Ann. Henri Poincaré **13** (2012), no. 4, 599–670, DOI 10.1007/s00023-011-0154-8. MR2913617

[TZ13] John A. Toth and Steve Zelditch, *Quantum ergodic restriction theorems: manifolds without boundary*, Geom. Funct. Anal. **23** (2013), no. 2, 715–775, DOI 10.1007/s00039-013-0220-0. MR3053760

[TZ21] John A. Toth and Steve Zelditch, *Nodal intersections and geometric control*, J. Differential Geom. **117** (2021), no. 2, 345–393, DOI 10.4310/jdg/1612975018. MR4214343

[Zel87] Steven Zelditch, *Uniform distribution of eigenfunctions on compact hyperbolic surfaces*, Duke Math. J. **55** (1987), no. 4, 919–941. MR916129

[Zel92] Steven Zelditch, *Isospectrality in the FIO category*, J. Differential Geom. **35** (1992), no. 3, 689–710. MR1163455

[Zel96] Steven Zelditch, *Quantum ergodicity of C^* dynamical systems*, Comm. Math. Phys. **177** (1996), no. 2, 507–528. MR1384146

[Zel97] Steven Zelditch, *Index and dynamics of quantized contact transformations*, Ann. Inst. Fourier (Grenoble) **47** (1997), no. 1, 305–363. MR1437187

[Zel99] Steve Zelditch, *Spectral determination of analytic bi-axisymmetric plane domains*, Math. Res. Lett. **6** (1999), no. 3-4, 457–464. MR1713144

[Zel00] Steve Zelditch, *Spectral determination of analytic bi-axisymmetric plane domains*, Geom. Funct. Anal. **10** (2000), no. 3, 628–677. MR1779616

[Zel04a] Steve Zelditch, *Inverse spectral problem for analytic domains. I. Balian-Bloch trace formula*, Comm. Math. Phys. **248** (2004), no. 2, 357–407. MR2073139

[Zel04b] Steve Zelditch, *Note on quantum unique ergodicity*, Proc. Amer. Math. Soc. **132** (2004), no. 6, 1869–1872. MR2051153

[Zel09] Steve Zelditch, *Inverse spectral problem for analytic domains. II. $\mathbb{Z}_2$-symmetric domains*, Ann. of Math. (2) **170** (2009), no. 1, 205–269. MR2521115

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