Differential Game Analysis of Energy Efficiency for Satellite Communication Subsystems

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***Abstract*—A satellite transponder's communication channel is studied in this paper. The multiple terminal users in this channel compete for limited radio resources to satisfy their own data rate needs. Because inter-user interference limits the transponder's performance, it is beneficial for the transponder's power-control system to coordinate all users in its channel to reduce interference and to improve performance. By the special properties of channel gain in this type of channel, a non-cooperative Differential Game (DG) is set up to study the competition in a transponder's channel. Each user's utility is a trade-off between transmission data rate and power consumption. Nash Equilibrium (NE) is defined to be the solution of the DG model. The optimality condition of NE is derived to be a system of Differential Algebraic Equations (DAE). An algorithm based on minimizing all users' Hamiltonian is developed to solve the DAE system. The numerical solution of the NE provides the transponder's power control system with each user's power-control strategy at the equilibrium.**

Keywords— Spectrum and Power Allocation, Energy-Efficiency, Satellite Communication, Differential Game

I. Introduction

Many satellite communications transmissions use C-band. The C-band communication satellites typically have 24 radio transponders. Within a 36-MHz bandwidth channel, each transponder can handle an enormous amount of information by using different multiple-access schemes, so each channel contains many pairs of senders and receivers [1], [2]. Each pair is assumed to be selfish to maximize its own performance by a specific power-allocation scheme in the study of this paper. The interference from other pairs through this transponder affects the channel performance [3]. Furthermore, the C band's heavier use leads to more interference. Shifting satellite communication to higher frequencies is one effective way to minimize interference, but crowding and interference problems still exist, which motivates us to develop a technique that increases bandwidth efficiency and signal-caring capacity, and decreases interference of satellite communication subsystems.

This paper models a transponder's communication channel as an interference channel with aim to optimize the trade-off between transmission data rate and power consumption. Section II reviews a transponder's communication channel and static energy-efficient power control games. Section III models the power-allocation optimization problem for all users in a transponder as a Differential Gaussian Interference Channel Game (DGICG) based on the special properties of satellite wireless communications. Section IV and Section V derive and analyze the DGICG model's optimality condition, and develop a numerical method to solve the optimality condition of NE and then solve the model. The numerical solution from the model provides the power control system of transponder with all users' NE power-allocation scheme.

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II. Preliminaries

A. Satellite Communications Subsystem

A transponder implements a wideband communication channel, in which there exist many simultaneous one-to-one communication transmissions [1], so it can be modelled as a multiuser interference channel [4], [5]. This interference channel in Fig. 1 is an M-to-M network where a one-to-one correspondence exists between senders and receivers such that each sender communicates information only to its corresponding receiver [4], [6]. Each pair of sender-receiver in a transponder channel is regarded as a user and a player in a game in this study. The interference limits the system's performance. Interference cancellation is an option when the interference signal is sufficiently strong, but its implementation is complex, requiring prior knowledge of users' transmission schemes is accessible by other users [5], [7]. This study assumes that each player applies power to affect the cross-coupling gain, and then reduce interference but all players do not apply any interference cancellation techniques.

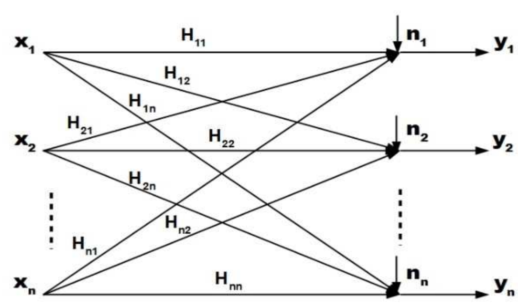

Fig. 1: Multiuser Interference Channel

B. Static Power Control Game

A static energy-efficient power control game on a distributed multiple-access channel with a finite number of users is set up by Goodman and Mandayam [8]. The channel model is given by

$$y(n)=\sum_{i=1}^{K}H_i(n)x_i(n)+\sigma(n) \tag{1}$$

, where K is the number of users, $x_i(n)$ is the symbol transmited by sender i at time n, $\sigma(n)$ is a Gaussian random variable with zero-mean and variance σ^2. Each user in the channel chooses its own power control policy p_i to maximize its energy-efficiency $u_i = \frac{R_i f(SINR_i)}{p_i}$, where R_i is the information transmission rate in bit/s for user i, and f is an efficiency function representing the block success rate, which is assumed to be sigmoidal and identical for all the users [8], [9]. This game is static because it assumes that the users transmit data over quasi-static or block-fading channels at the same time and in the same frequency band, and assumes each channel gain $H_i(n)$ to be constant over each block. Furthermore, each user applies a fixed power policy, once per block, to maximize its utility. With assumption of complete information and rationality, the existence of Nash Equilibrium is guaranteed by Debreu-Fan-Glicksberg existence theorem [10]:

- For all $i \in \kappa$, the feasible region for control $[0, P_i^{max}]$ is convex and compact.
- For $i \in \kappa$, and $\lim_{p_i \to 0} u_i(p_i, p_{-1}) = 0$, u_i is continuous in control variables $\bar{p} = (p_1, \cdots, p_K)$ over the feasible control region.
- $u_i(\bar{p})$ is quasi-concavity with respect to p_i. There are two important efficiency functions: $f(x) = (1 - e^{-x})^M$, which is from Goodman and Mandayam [8] and $f(x) = e^{-\frac{2^{R-1}}{x}}$, where R is the outage-probability target rate, which is from Belmega and Lasaulce [11].

The Nash Equilibria are found by solving a set of equations:

$$\frac{\partial u_i}{\partial p_i}(\bar{p}) = 0 \tag{2}$$

, where $i \in \kappa$. The static power game has unique pure Nash Equilibrium, which is studied by Yates [12], and Saraydar [13].

Besides the energy-efficient game for communication channel, there are other types of noncooperative games [14], [15] constructed for different utility, which are generally called Gaussian Interference Games (GIGs). The water-filling algorithm also solves for Nash Equilibrium of GIG without the need for centralized control [15]. Amir Leshem applied cooperative game theory to analyze interference channels [16]. Wei Wan [17] created a cooperative static game for a transponder's centralized power control to maximize overall channel data transmission rate.

III. Differential Game For a Transponder

For long-distance wireless communication like satellite communication, channel gain varies with time, and its modulus is usually assumed to be in a compact set $|H_i|^2 \in [\eta_i^{min}, \eta_i^{max}]$. Thus, a variable power policy is expected to be designed to control channel gain. This paper studies a transponder's communication channel, which is modelled as a multiuser interference channel in Fig. 1. Each pair of $(x_i, y_i), i \in \kappa$ is defined to be a user, and be a player in the game. All users simultaneously choose their power-control policy before establishing communication. This implies an open-loop power control policy, which is only a function of time. Each user's communication is through N sub-frequency channels simultaneously, and each user applies independent power control policy in each sub-frequency channel. Furthermore, each user divides its power consumption into two uses: the first is to improve its own channel gain, and the second is to decrease interference. The major variables are defined as follows:

$H_{ii}^f(t)$: the direct channel gain from the transmitter to the receiver of user i over frequency f at time t.

$H_{ji}^f(t)$: the cross-coupling gain from the transmitter j to the receiver of user i over frequency f at time t.

$p_i^f(t)$: the transmit power spectrum density used by user i over frequency f at time t.

τ_i^f: the fixed constant over frequency f for user i, which stands for the proportion of user i's $p_i^f(t)$, used by user i to decrease cross-coupling channel gain.

$\sigma_i^f(t)$: the noise power spectrum density at user i over frequency f at time t.

Construction of objective function: Since the first and most interesting objective for each user in this transponder is to optimize the trade-off between the achievable data rate and energy consumption. With an assumption of no channel interference cancellation, the interference from other users is consequently noise. Then, the achievable rate for player i at time t over frequency (f_1, f_2) is as follows [5], [3]:

$$\begin{aligned} R_i(t) &= \int_{f_1}^{f_2} log_2\left(1 + \frac{p_i^f(t)|H_{ii}^f(t)|^2}{\sigma_i^f(t) + \sum_{j \neq k} P_j^f(t)|H_{ji}^f(t)|^2}\right) df \\ &\cong log_2\left(1 + \frac{p_i^f(t)|H_{ii}^f(t)|^2}{\sigma_i^f(t) + \sum_{j \neq k} P_j^f(t)|H_{ji}^f(t)|^2}\right) \Delta f \end{aligned} \tag{3}$$

, where approximation assumes the variables to be constant over small bands. The energy efficiency for user $i, i \in \kappa$ over time $[0, T]$ is

$$\int_0^T \sum_{f=1}^N \left[R_i(t) - c_i^f (p_i^f)^2\right] dt \tag{4}$$

, which is the log transformation of ratio of information bits that are transmitted without error per unit time to the transmit power. It is to be maximized. The second goal of transponder power control is for the direct channel gain to reach a certain channel-capacity level and also to reduce the cross-coupling gain to certain level. This second objective is to minimize the following expression:

$$\sum_{f=1}^N w_1^{(f,i)}(|H_{ii}^f(T)|^2 - r_{ii}^f \eta_{ii}^f)^2 + w_2^{(f,i)}(|H_{ji}^f(T)|^2 - r_{ji}^f \eta_{ji}^f)^2 \tag{5}$$

, where $w_1^{(f,i)}, w_2^{(f,i)}$ are weights between different objectives; η_{ii}^f, η_{ji}^f are constants, and upper bounds of $|H_{ii}^f(T)|^2, |H_{ji}^f(T)|^2$; and r_{ii}^f, r_{ji}^f are targeted channel-gain levels.

Construction of dynamics: Generally, $|H_{ii}^f(t)|^2, |H_{ij}^f(t)|^2$ belong to a compact set $[\eta_i^{min}, \eta_i^{max}]$, and can be approximated by Kronecker's delta function [18]. In satellite wireless communication, satellite transponders can apply energy $p_i^f(t)$ to impact and control channel gain. The analysis in this paper assumes that the growth rate is proportional to power consumption. Thus, logistic growth with carrying capacity is adopted to approximate the dynamics of $|H_{ii}^f(t)|^2$:

$$\frac{d|H_{ii}^f(t)|^2}{dt} = \alpha_i^f(1-\tau_i^f)p_i^f(t)\left(\eta_{ii}^f - |H_{ii}^f(t)|^2\right) \qquad (6)$$

, where $(1-\tau_i^f)$ is the fixed constant over frequency f for user i, which stands for the proportion of $p_i^f(t)$, used by user i to increase channel gain. Furthermore, when user i applies $p_i^f(t)$ to improve the channel gain $|H_{ii}^f(t)|$, it also increases the cross-coupling gain $|H_{ij}^f(t)|$. Furthermore, user j is able to cost power $\tau_j^f p_j^f(t)$ to decrease interference brought by $p_i^f(t)$. At last, because of threshold effects existing in channel gain, cross-coupling gain has a lower bound. Thus, the dynamics of $|H_{ij}^f(t)|^2$ is approximated by:

$$\frac{d|H_{ij}^f(t)|^2}{dt} = \beta_{ij}^f\left(p_i^f(t) - \tau_j^f p_j^f(t)\right)\left(\eta_{ij}^f - \left|H_{ij}^f(t)\right|^2\right)\left(\left|H_{ij}^f(t)\right|^2 - \xi_i^f\right) \qquad (7)$$

where $i \neq j$. Thus, the DGICG model $(\kappa, \{p_i^f\}_{i\in\kappa}, \{J_i\}_{i\in\kappa})$ is set up as follows:

$$\begin{cases} J_1 = \min_{p_1^f(t)} \int_0^T \left\{\sum_{f=1}^N c_1^f (p_1^f)^2 - \log_2\left(1 + \frac{p_1^f(t)|H_{11}^f(t)|^2}{\sigma_1^f(t) + \sum_{j\neq 1} P_j^f(t)|H_{j1}^f(t)|^2}\right) dt\right\} \\ + \sum_{f=1}^N \left[w_1^{(f,1)}(|H_{11}^f(T)|^2 - r_{11}^f\eta_{11}^f)^2 + \sum_{j\neq 1} w_2^{(f,1)}(|H_{j1}^f(T)|^2 - r_{j1}^f\eta_{j1}^f)^2\right] \\ \vdots \\ J_K = \min_{p_K^f(t)} \int_0^T \left\{\sum_{f=1}^N c_K^f (p_K^f)^2 - \log_2\left(1 + \frac{p_K^f(t)|H_{KK}^f(t)|^2}{\sigma_K^f(t) + \sum_{j\neq k} P_j^f(t)|H_{jK}^f(t)|^2}\right) dt\right\} \\ + \sum_{f=1}^N \left[w_1^{(f,K)}(|H_{KK}^f(T)|^2 - r_{KK}^f\eta_{KK}^f)^2 + \sum_{j\neq K} w_2^{(f,K)}(|H_{jK}^f(T)|^2 - r_{jK}^f\eta_{jK}^f)^2\right] \end{cases}$$

$$s.t. \quad \begin{cases} \frac{d|H_{ii}^f(t)|^2}{dt} = \alpha_i^f(1-\tau_j^f)p_i^f(t)\left(\eta_{ii}^f - |H_{ii}^f(t)|^2\right) \\ \frac{d|H_{ij}^f(t)|^2}{dt} = \beta_{ij}^f\left(p_i^f(t) - \tau_j^f p_j^f(t)\right)\left(\eta_{ij}^f - \left|H_{ij}^f(t)\right|^2\right)\left(\left|H_{ij}^f(t)\right|^2 - \xi_{ij}^f\right), \quad j\neq i \\ \sum_{f=1}^N P_i^f(t) \leq \boldsymbol{P_i^{max}} \\ |H_{ij}^f(0)|^2 \, given \quad i,j = 1,\ldots,K. \quad f = 1,\ldots,N. \end{cases} \qquad (8)$$

IV. Analysis of Optimality of Equilibrium

The solution of above non-Cooperative DGICG among all users in one satellite transponder communication channel is defined by a Nash Equilibrium (NE).

Definition 1 [19]**:** Suppose $J_i(u_1, u_2, \cdots, u_i, \cdots, u_n)$ are utility function for player i, where u_i is control policy for player i. The control policy $(u_1^*, u_2^*, \ldots, u_n^*)$ is Nash Equilibrium (NE) if

$$J_i(u_1^*, u_2^*, \cdots, u_i, \cdots, u_n^*) \preccurlyeq J_i(u_1^*, u_2^*, \cdots, u_i^*, \cdots, u_n^*) \qquad (9)$$

for all i.

The control policy at NE is optimal in the sense that if one of the player deviates from NE, then its utility will be reduced. The necessary condition for controls $\{p_i^f(t), i = 1, \ldots, K\}$ to be NE of differential game model is derived from Pontryagin's Minimum Principle [20], [21], which is composed of following system (In the following expressions, we set $x_{ij}^f(t) = |H_{ij}^f(t)|^2$):

- Player i's Hamiltonian

$$\begin{aligned} H_i = &\sum_{f=1}^N \left[c_i^f\left(p_i^f(t)\right)^2 - \log_2\left(1 + \frac{p_i^f(t)x_{ii}^f(t)}{\sigma_i^f(t) + \sum_{j\neq i} p_j^f(t)x_{ji}^f}\right)\right] + \\ &\sum_{f=1}^N \sum_{i=1}^K \lambda_{ii}^f\left[\alpha_i^f(1-\tau_j^f)p_i^f(t)\left(\eta_i^f - x_{ii}^f(t)\right)\right] + \\ &\sum_{f=1}^N \sum_{i\neq j} \lambda_{ij}^f(t)[\beta_{ij}^f\left(p_i^f(t) - \tau_j^f p_j^f(t)\right)\cdots \\ &\cdots\left(\eta_{ij}^f - x_{ij}^f(t)\right)\left(x_{ij}^f(t) - \xi_{ij}^f\right)] \end{aligned} \qquad (10)$$

, which is to be minimized by player i's control $p_i^f(t)$.

- A System of State equations is

$$\begin{cases} \frac{dx_{ii}^f}{dt} = \alpha_i^f(1-\tau_j^f)p_i^f(t)\left(\eta_{ii}^f - x_{ii}^f(t)\right) \\ \frac{dx_{ij}^f}{dt} = \beta_{ij}^f\left(p_i^f(t) - \tau_j^f p_j^f(t)\right)\cdots \\ \cdots\left(\eta_{ij}^f - x_{ij}^f(t)\right)\left(x_{ij}^f(t) - \xi_{ij}^f\right), j\neq i \\ \sum_{f=1}^N P_i^f(t) \leq \boldsymbol{P_i^{max}} \\ x_{ij}^f(0) \; given. \quad i,j = 1,\ldots,K. \quad f = 1,\ldots,N. \end{cases} \qquad (11)$$

- A System of Co-state equations is

$$\begin{cases} \frac{d\lambda_{ii}^f}{dt} = -\frac{dH_i}{dx_{ii}^f} \\ = \frac{1}{\ln 2}\frac{p_i^f(t)}{\sigma_i^f + \sum_{j\neq i} p_j^f(t)x_{ji}^f(t) + p_i^f(t)x_{ii}^f(t)} + \lambda_{ii}^f(t)\alpha_i^f(1-\tau_j^f)p_i^f(t) \\ \frac{d\lambda_{jj}^f}{dt} = -\frac{dH_i}{dx_{jj}^f} = \lambda_{jj}^f(t)\alpha_j^f(1-\tau_j^f)p_j^f(t), j\neq i \\ \frac{d\lambda_{ij}^f}{dt} = -\frac{dH_i}{dx_{ij}^f} \\ = \lambda_{ij}^f(t)\beta_{ij}^f\left(p_i^f(t) - \tau_j^f p_j^f(t)\right)\left(2x_{ij}^f(t) - \eta_{ij}^f - \xi_{ij}^f\right), j\neq i \\ \frac{d\lambda_{ji}^f}{dt} = -\frac{dH_i}{dx_{ji}^f} \\ = \frac{-1}{\ln 2}\frac{p_i^f(t)x_{ii}^f(t)p_j^f(t)}{\left(\sigma_i^f + \sum_{j\neq i} p_j^f(t)x_{ji}^f(t)\right)^2 + p_i^f(t)x_{ii}^f(t)(\sigma_i^f + \sum_{j\neq i} p_j^f(t)x_{ji}^f(t))} \\ +\lambda_{ji}^f(t)\beta_{ji}^f\left(p_j^f(t) - \tau_j^f p_j^f(t)\right)\left(2x_{ji}^f(t) - \eta_{ji}^f - \xi_{ji}^f\right) \end{cases} \qquad (12)$$

and has the following boundary condition:

$$\begin{cases} \lambda_{ii}^f(T) = \frac{dh_i}{dx_{ii}^f} = 2w_1^{(f,i)}(x_{ii}^f(T) - r_{ii}^f\eta_{ii}^f) \\ \lambda_{jj}^f(T) = \frac{dh_i}{dx_{jj}^f} = 0 \\ \lambda_{ij}^f(T) = \frac{dh_i}{dx_{ij}^f} = 0 \\ \lambda_{ji}^f(T) = \frac{dh_i}{dx_{ji}^f} = 2w_1^{(f,i)}(x_{ji}^f(T) - r_{ji}^f\eta_{ji}^f) \end{cases} \qquad (13)$$

, where $i, j = 1, \ldots, K; f = 1, \ldots, N$.

- The candidates for player i's control policy at NE are those $p_i^f(t)$ that make $\frac{dH_i}{dp_i^f}$ to vanish by assuming other players are using their NE control policies:

$$\begin{cases} \frac{dH_i}{dp_i^f} = 2c_i^f p_i^f(t) \\ -\frac{1}{ln2}\frac{x_{ii}^f(t)}{\sigma_i^f(t)+\sum_{j\neq i} p_j^f(t)x_{ji}^f(t)+p_i^f(t)x_{ii}^f(t)} \\ +\lambda_{ii}^f(t)\alpha_i^f(1-\tau_j^f)\left(\eta_{ii}^f - x_{ii}^f(t)\right) \\ +\lambda_{ij}^f(t)\beta_{ij}^f\left(\eta_{ij}^f - x_{ij}^f(t)\right)\left(x_{ij}^f(t)-\xi_{ij}^f\right) \\ -\lambda_{ji}^f(t)\beta_{ji}^f\tau_j^f\left(\eta_{ji}^f - x_{ji}^f(t)\right)\left(x_{ji}^f(t)-\xi_{ji}^f\right) = 0 \end{cases} \quad (14)$$

V. NUMERICAL SOLUTION OF DGICG MODEL

In nonlinear system (14), the optimal control policy cannot be solved explicitly. Thus, solving the n players' open-loop DGICG is equivalent to solving $n^2 + 2n$ Differential-Algebraic Equation (DAE) system (11)-(14). The design of following algorithm is based on the fact that each player optimizing its own Hamiltonian with its own control by assuming all other players have adopted optimal control policy is necessary for its control to reach NE. This implies searching NE could be achieved by each player searching to optimize its own Hamiltonian simultaneously. The procedure of following iterative algorithm starts with a randomly generated discretized control for each player, and then solved the DAE system by these control policies. Next, each player updates its control in its own Hamiltonian's steepest descent direction, that is each player optimizes its own Hamiltonian simultaneously in its own space of controls. Finally, the algorithm terminates when all players' $\{\frac{dH_i}{dp_i^f}\}$ vanish, or there is little improvement of objective utilities.

Algorithm 1:

Step 1: Generate randomly discrete approximations to controls $\{p_i^f(t)\}$ over $t \in [0, T]$, which satisfy the constraint of control:

$$p_i^{f,n}(l) = p_i^{f,n}(t), t \in [t_l, t_{l+1}), l = 1, \cdots, M \quad (15)$$

, where $i \in 1, \cdots, K$. $f = 1, \ldots, N$, and n stands for nth iteration, and M stands for partitioning $[0, T]$ into M subintervals.

Step 2: Use discretized controls (15) to integrate the state equations (11) over $[0, T]$ forward by Runge-Kutta (RK4) method with given initial condition $x_{ij}^f(0)$.

Step 3: Evaluate the terminal value of costate variables in (13) by terminal value of state variables $x_{ij}^f(T)$, and then integrate costate equations (12) backward by RK4.

Step 4: Evaluate each player's objective function $J_i(n)$ by the discrete values of state and costate variables from Step 2 and Step 3.

Step 5: If $|J_i(n+1) - J_i(n)| < \epsilon_1$ or $\left\|\frac{dH_i}{dp_i^f}\right\| < \epsilon_2$ for all i, then the iterative procedure terminates and output the optimal controls and state equations. Otherwise, new piecewise constant controls are generated by following equations for each player:

$$p_i^{f,n+1}(l) = p_i^{f,n}(l) - s_i^f \frac{dH_i}{dp_i^f}(l) \quad (16)$$

, where $-\frac{dH_i}{dp_i^f}$ provides search direction, and step length s_i^f are computed independently for each player by inexact line search procedure to guarantee all objective values $\{J_i, i = 1, \ldots, K\}$ decrease at each iteration.

VI. NUMERICAL EXPERIMENTS

A two-player DGICG over one sub-frequency channel is solved by Algorithm 1. The numerical experiment aims to study the effects of cost of power on Nash Equilibrium. The values of parameters are chosen as follows. Comparing the values of parameters in Table I and II, these two players are symmetric except for the cost of power, where $c_1 > c_2$ implies player 1's cost of power is more expensive than player 2.

TABLE I. PARAMTERS OF OBJECTIVE FUNCTIONS

Player 1		Player 2	
c_1	6	c_2	4
σ_1	0.2	σ_2	0.2
$w_1^{(1)}$	2	$w_1^{(2)}$	2
$w_2^{(1)}$	1	$w_2^{(2)}$	1
$r_{11}^{(1)}$	0.9	$r_{22}^{(2)}$	0.9
$r_{21}^{(1)}$	0.3	$r_{12}^{(2)}$	0.3

TABLE II. PARAMTERS OF DYNAMICS

Player 1		Player 2	
α_1	6	α_2	6
β_{12}	3	β_{21}	3
τ_1	0.5	τ_2	0.5
η_{11}	1	η_{22}	1
η_{12}	0.5	η_{21}	0.5
ξ_{12}	0.001	ξ_{21}	0.001
$x_{11}(0)$	0.1	$x_{22}(0)$	0.1
$x_{12}(0)$	0.02	$x_{21}(0)$	0.02

Convergence of the algorithm is shown by convergence of objective functions in Fig. 2, where $|J_1(n+1) - J_1(n)| \approx 1 * 10^{-6}$, $|J_2(n+1) - J_2(n)| \approx 1 * 10^{-8}$, and the vanishing of $\{\frac{dH_i}{dp_i^f}, i = 1, \ldots, K\}$, where $\left\|\frac{dH_1}{dp_1^1}\right\| \approx 5.3 * 10^{-6}$, $\left\|\frac{dH_2}{dp_2^1}\right\| \approx 1.1 * 10^{-4}$. The total number of iterations is 16.

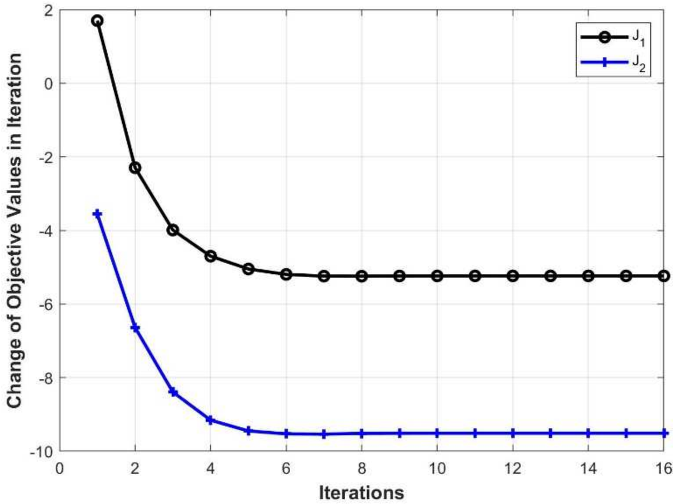

Fig. 2. Convergence of Player 1 and 2's Objective Function Values

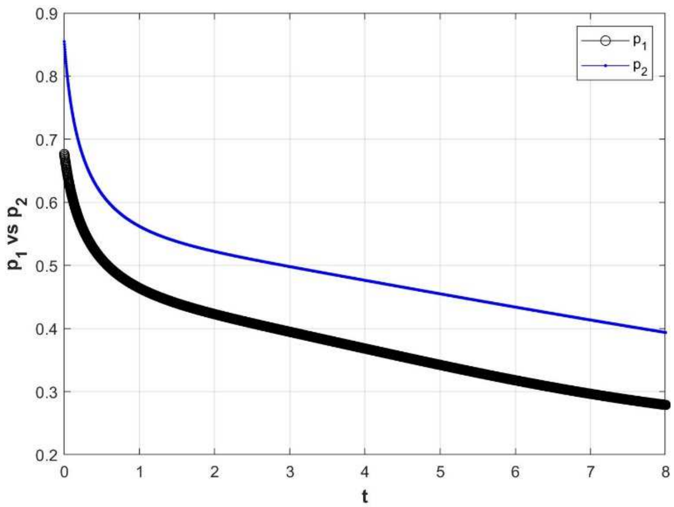

Fig. 3. Trajectories of Optimal Control $p_1(t)$ and $p_2(t)$ at NE

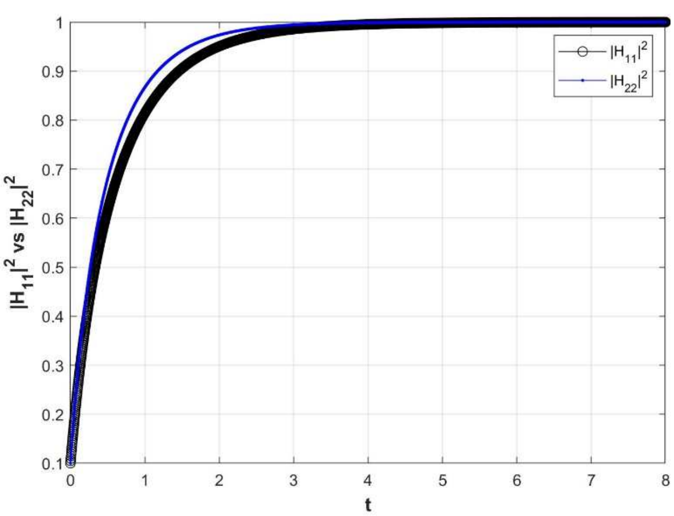

Fig. 4. Trajectories of Direct Channel Gain $|H_{11}^{f}|^2$ and $|H_{22}^{f}|^2$ at NE

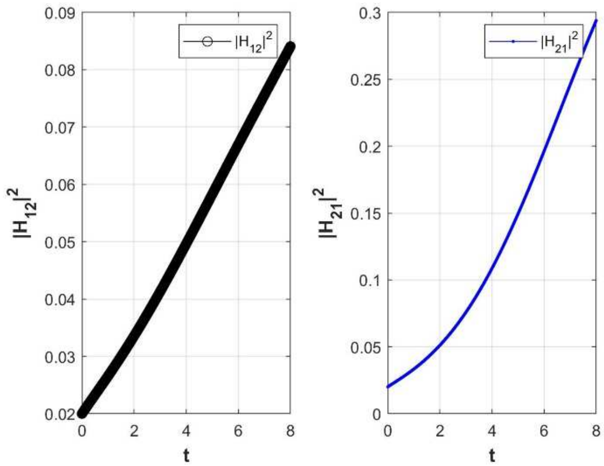

Fig. 5. Trajectories of Cross-coupling Gain $|H_{12}^{f}|^2 and\ |H_{21}^{f}|^2$ at NE

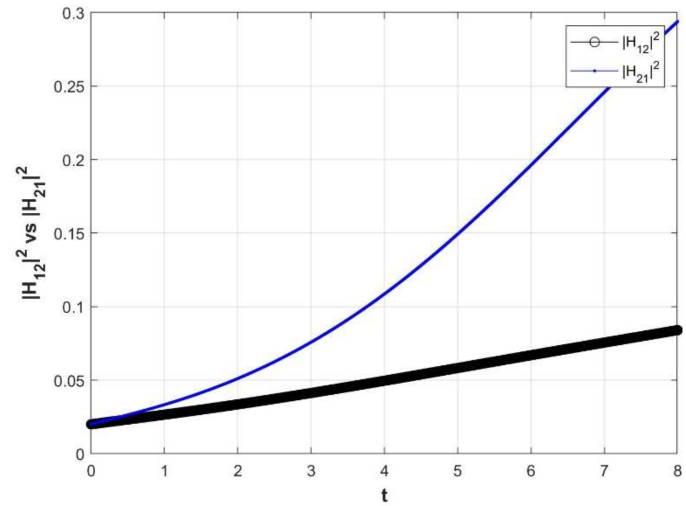

Fig. 6. Comparison of $|H_{12}^{f}|^2$ and $|H_{21}^{f}|^2$ at NE

Two players' optimal controls at Nash Equilibrium are given in Fig. 3. The most important feature of optimal controls is that both players compete intensely at the beginning of the game, and reduce competition level gradually over time. Furthermore, player 1's competition level is always lower than player 2. This is expected since the cost of player 1's control is higher than player 2.

Two players' direct channel gain at Nash Equilibrium behave similar and approach to the channel carrying capacity (Fig. 4). The player 2's direct channel gain level is slightly higher than Player 1's. It is also expected that the cost of player 2's control is cheaper with other parameters of these two players being at the same level.

In the end, it is interesting to observe the cross-coupling gain of these two players behave different (Fig. 5, 6). Player 1's interference to player 2 ($|H_{12}^{f}|^2$) is increasing slightly, but $|H_{21}^{f}|^2$ is increasing sharply over time. It could be understood since the cost of player 2 is cheaper, and player 2 is able to apply more power to reduce player 1's interfering to player 2.

VII. Summary and Conclusion

This paper models a satellite transponder's communication channel as a multiuser interference channel and focuses on its power allocation to improve energy efficiency. In satellite communication subsystems, the performance of each pair of transmitters and receivers depends not only on its own power allocation, but also on the other pairs'. Each user in the transponder's channel would be competing for limited radio resources to meet their selfish data rates with less energy consumption. Another feature of satellite communication is its long-distance, so the channel gain is not constant. Thus, each user is able to apply energy to improve its own channel gain and reduce interference. This paper introduced a non-cooperative DGICG model for all users in one transponder's communication channel. In this game model, each user's energy efficiency is redefined, and logistic growth is adopted to approximate the changing of channel gain under specific energy consumption. The objective function of each user is a weighted sum of energy efficiency and targeted channel gain level. The optimality condition for Nash Equilibrium of the game model is derived to be DAE system. An algorithm is developed to solve the DAE for Nash Equilibrium. The design of algorithm is based on a steep-descent method and optimizes all players' Hamiltonian simultaneously. This algorithm is especially efficient to solve differential game model even if the optimal controls are not able to be solved explicitly.

The goal of power allocation design for a transponder is to optimize the energy efficiency of the whole communication channel by coordinating all users. The numerical solution of the game model can be used to support designing power allocation scheme of transponders. In the end, one limitation of research work in this paper is proof of uniqueness of Nash Equilibrium. Generally, a game model could have multiple Nash Equilibria, and Nash Equilibrium may not be Pareto optimal. Mathematical analysis will be expected in the next step of research. Furthermore, the differential game model in this paper assumes that all users in the channel make decision at the same time, but in practice there exists some users who have priority to communicate, so multi-level game analysis is also expected in the near future research work.

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