Improving High School Math Engagement with Circuit and Transistor Examples

Andrew Ash and John Hu
School of Electrical and Computer Engineering, Oklahoma State University
Stillwater, OK, USA
Emails: {andrew.ash, john.hu}@okstate.edu

***Abstract*—This summer, Oklahoma State University began a Research Experience for Teachers (RET) site focused on circuit design and semiconductors. Our work with math educators led to the development of lesson plans incorporating semiconductors and circuits to offer real-world examples in place of more abstract word problems. Through this effort we expect to improve math engagement of high school students and introduce them to valuable semiconductor topics. The inclusion of circuit and semiconductor topics also begins the workforce development needed to fill the jobs projected to be created by the CHIPS Act. Real-world examples developed focus on diode and MOSFET models as applications of piecewise functions in Algebra I and inductor and capacitor circuits as applications of the Fundamental Theorem of Calculus.**

***Index Terms*—high school education, math engagement, circuits, transistors, semiconductor workforce development**

I. Introduction

Students tend to struggle with engagement and motivation in math courses particularly in high school [1] [2]. This disconnect with math can push students away from STEM degrees as they start college and those who select STEM degrees may require remedial math courses to gain the expected math proficiency for their major [3]. There are many solutions to help students studying electrical engineering gain missing math proficiency once in college [4] [5], but solutions that improve math skills earlier in the educational journey may enhance math proficiency in time for college studies and encourage more students to select a STEM major.

With the passing of the CHIPS Act in 2022, there is a great need for a large skilled workforce ready to enter the semiconductor industry [6]. By 2030, there will be a projected 67,000 unfilled jobs for technicians, engineers, and computer scientists in the semiconductor industry [7]. To fill this gap, more secondary school students must pursue certifications and degrees in STEM, but current education practices do not yet support this changing need in the workforce.

To begin preparing educators to broaden the STEM pathways for the semiconductor industry, Oklahoma State University started an NSF Research Experience for Teachers (RET) site in 2023 focused on introducing semiconductor and circuit design topics in high school classrooms. In this first cohort, we had three math educators we worked with to develop lesson plans that bring real-world electrical engineering concepts into their math courses with the goal of helping students see how abstract math topics relate to the world around them. These real-world examples are expected to help improve issues of math engagement in secondary school.

The rest of the paper is organized as follows. Section II reviews the problem of math engagement and solutions to improve engagement. Sections III and IV explain math lessons developed around specific circuit topics discussed at the RET site. Section V outlines the evaluation plan. Section VI summarizes the current work.

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II. Background

A. The Problem of Math Engagement

The problem of math engagement is not a new one. George Pólya dedicated substantial efforts to improving the problem of math engagement throughout his career in the mid 20^{th} century; he noted that mathematics teachers in primary and secondary education have the power to excite their students and fuel a lifelong interest in math or diminish potential for future math engagement [8]. Even with decades of improvements, math engagement in secondary school is still a common struggle [1] [2]. The reduced math skills students have when starting college can have a negative impact on performance in electrical engineering coursework and may push students away to other fields of study [5] [9]. Although modifications to courses designed for first year college students help with math skills and engagement in post-secondary education, there may be students who choose not to study engineering before beginning college. To impact more students' math engagement and increase potential enrollment in STEM majors, changes should be made earlier in the educational journey.

B. Replacing Abstract Problems with Real-World Examples

Studies have shown that one way to improve engagement with math is through the use of problems based on real-world scenarios. In [10], researchers, education experts, and industry experts worked together to create real-world word problems for the math classroom that emphasize the importance of uniting the math theory of the classroom with the math used in technical workplaces. While [11] outlined a project developed in an Indiana school district that united students in Algebra I with other student groups to make an autonomous robot incorporating key concepts from each of their classrooms. By showing students how math can actually apply to their

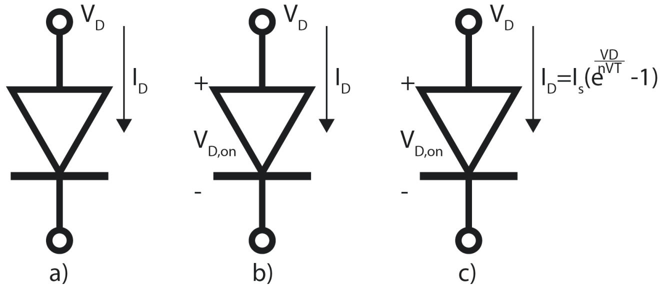

Fig. 1. a) Simple diode model. b) Diode model with turn-on voltage, $V_{D,on}$. c) Exponential diode model.

future careers, they become more interested in developing a strong math foundation. Replacing abstract word problems with examples from STEM allows the concepts taught in the math classroom to become more relevant and approachable.

With this goal in mind, math educators at the RET site were introduced to a variety of circuit topics starting from a mathematical perspective. Connecting these new topics to familiar mathematical modeling principals improved educator understanding and helped them justify including concrete examples typically reserved for physics courses into their classrooms. The teachers were excited to develop examples based on circuit topics that would give their students an answer to questions about where math is relevant to daily life.

III. Introducing Circuit Concepts in Algebra

For some students algebra is the beginning of their high school math journey while others take it shortly before graduation. 85% of US students will take this class in high school [12], making it an ideal target for improving math engagement and introducing more students to basic electrical engineering topics. Piecewise functions are a common algebra topic [13] [14]; Diodes and MOSFETs are central to circuit design and use models that bring real-world relevancy to lessons on piecewise functions.

A. Diode Models

A simple starting point for diode-based problems uses (1). Where V_D is the anode voltage and I_D is the current through the diode (see Fig. 1a). With this basic model, students can be easily introduced to interpreting piecewise functions with simple questions to determine if a diode is conducting given V_D is -1 V or 1 V for example.

$$I_D \begin{cases} = 0 & \text{for } V_D < 0 \\ \geq 0 & \text{for } V_D \geq 0 \end{cases} \tag{1}$$

LTspice is a lightweight, free SPICE simulator distributed by Analog Devices [15]. During their time at the RET site this summer teachers were trained on how to do basic circuit analysis using LTspice to help them create more interactive lessons and examples. After introducing the students to the basic diode model, teachers can use LTspice to simulate a simple diode circuit and lead a discussion on mathematical

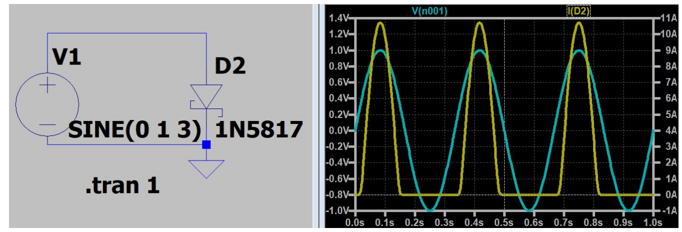

Fig. 2. LTspice simulation of 1N5817 diode. Voltage across the diode is shown in blue and current through the diode in yellow.

modeling strategies. Fig. 2 shows a simple LTspice transient simulation of a 1N5817 diode. A lesson could focus on increasingly complex models allowing for more accurate results at the cost of more difficult calculations. Students can easily see the diode does not begin conducting as soon as V_D is greater than zero, instead it must surpass a certain voltage ($V_{D,on}$). The teacher can then introduce (2), a slightly more complex piecewise model that better represents the diode's behavior. Students should now be able to determine whether a diode is conducting based on a given V_D and $V_{D,on}$.

$$I_D \begin{cases} = 0 & \text{for } V_D < V_{D,on} \\ \geq 0 & \text{for } V_D \geq V_{D,on} \end{cases} \tag{2}$$

Although the behavior of the diode is better approximated with (2), students should be able to identify there is behavior missing from the model. At this point, (3) can be introduced where n is the emission coefficient (assumed to be 1), V_T is the thermal voltage (near 26 mV at room temperature), and I_s is the saturation current [16]. Students must now be given values for I_s, V_D, and $V_{D,on}$ to find the current through the diode. Students must first use boundary conditions of the piecewise function and then calculate the current through the diode with the appropriate part of the model.

$$I_D = \begin{cases} 0 & \text{for } V_D < V_{D,on} \\ I_s e^{\frac{V_D}{nV_T}} & \text{for } V_D \geq V_{D,on} \end{cases} \tag{3}$$

After completing the full exercise students have seen increasingly complex piecewise functions that all model the behavior of the same device. The teacher can now discuss the pros and cons of different levels of complexity when using math to model the behavior of real-world phenomena.

B. MOSFET Modeling

While diodes make for a good first introduction to piecewise functions, they only have one boundary condition. MOSFETs allow for a deeper dive into piecewise functions due to their two boundary conditions shown in (4) [16]. Using Fig. 3 as a starting point, teachers can show V_{GD}, V_{GS}, and V_{DS} are voltages drops between the gate, drain, and source terminals of the MOSFET. Teachers must also explain that values for the width to length ratio of the MOSFET (W/L), the oxide capacitance (C_{ox}), the electron mobility (μ_N), and the threshold voltage (V_{th}) are material properties of the MOSFET that will be provided as constants.

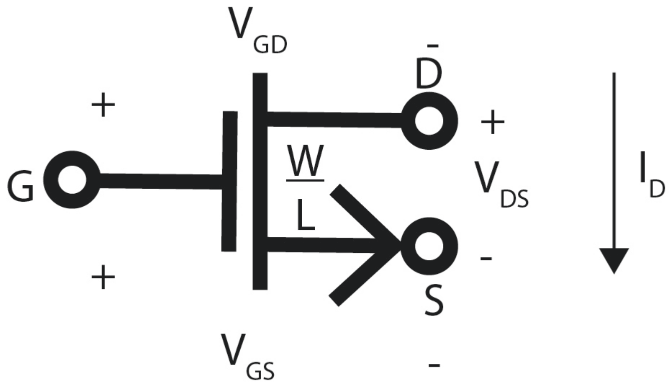

Fig. 3. Diagram of MOSFET.

$$
\begin{aligned}
\text{Cut-off:} \quad & I_D = 0 \\
& V_{GS} < V_{th} \\
\text{Triode:} \quad & I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{th}) V_{DS} - \frac{V_{DS}^2}{2} \right] \\
& V_{GS} \geq V_{th},\ 0 \leq V_{DS} < V_{GS} - V_{th} \\
\text{Saturation:} \quad & I_D = \mu_n C_{ox} \frac{W}{L} \frac{(V_{GS} - V_{th})^2}{2} \\
& V_{GS} \geq V_{th},\ V_{DS} \geq V_{GS} - V_{th}
\end{aligned} \tag{4}
$$

Two boundary conditions in the piecewise function makes for a larger range of practice problems and examples. Easy questions will provide values for V_{GS} and V_{DS} then ask for the current flowing through the MOSFET. Students must first check the given voltage values to determine which portion of the piecewise function defines the current. After this it is relatively simple to plug in the given values for constants and voltages to find the current. A slight variation on the problem would give voltage values at the gate, drain, and source and require students to start by calculating V_{GS} and V_{DS}.

In a more challenging problem students will be given values for I_D and V_{DS} and be asked to determine the value of V_{GS} and the associated operating region. Students will have to state the piecewise functions in terms of V_{GS} and plug in the given values to find possible answers in the triode and saturation regions. Then these values can be checked against the boundary conditions of the piecewise function to determine which answer is correct. These more challenging questions will give students practice with piecewise functions, help improve skills in algebraic manipulation, and develop students' ability to determine if their answer makes sense given the constraints of the problem.

IV. Introducing Circuit Concepts in Calculus

During calculus, in the transition from derivatives to integrals, students are introduced to (5), the Fundamental Theorem of Calculus, that ties the relationship between the two together. This is an important turning point in calculus curriculum [17]

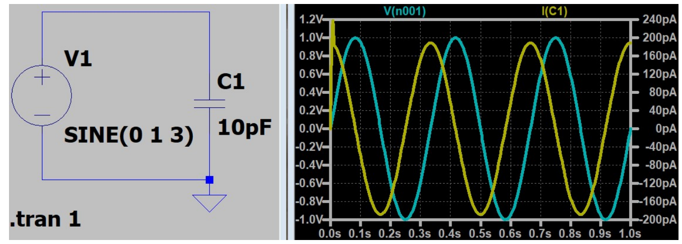

Fig. 4. LTspice simulation of an ideal capacitor driven by a sine wave voltage. Capacitor voltage is shown in blue and capacitor current in yellow.

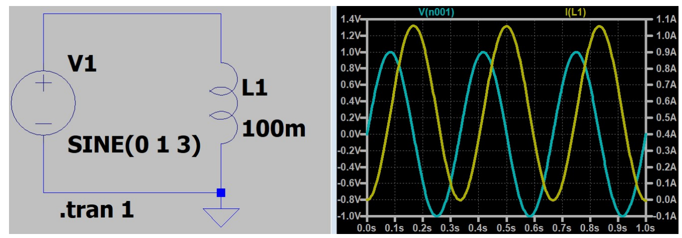

Fig. 5. LTspice simulation of an ideal inductor driven by a sine wave voltage. Inductor voltage is shown in blue and inductor current in yellow.

[18], having real-world examples to help students understand this concept is valuable.

$$\int_a^b f(x)\,dx = F(b) - F(a) \tag{5}$$

$$\text{where: } F(x) + C = \int f(x)\,dx$$

A fantastic example to help solidify this relationship comes in the form of capacitors and inductors. The relationship between voltage (V), current (I), and capacitance (C) can be seen in (6) and (7) while the relationship with inductance (L) can be seen in (8) and (9). In these equations V_0 and I_0 are the initial conditions.

$$I(t) = C \frac{dV(t)}{dt} \tag{6}$$

$$V(t) = \frac{1}{C} \int I(t)\,dt + V_0 \tag{7}$$

$$V(t) = L \frac{dI(t)}{dt} \tag{8}$$

$$I(t) = \frac{1}{L} \int V(t)\,dt + I_0 \tag{9}$$

When introducing students to these equations an excellent first exercise will be to simply combine (6) and (7) to show the final result is equal on both sides. Students may then work through the similar inductor equations on their own to practice moving between derivatives and integrals using the Fundamental Theorem of Calculus.

With practice in the basic theory, students should be ready to do more concrete examples using sinusoidal and linear inputs. Once again, LTspice [15] becomes an excellent tool

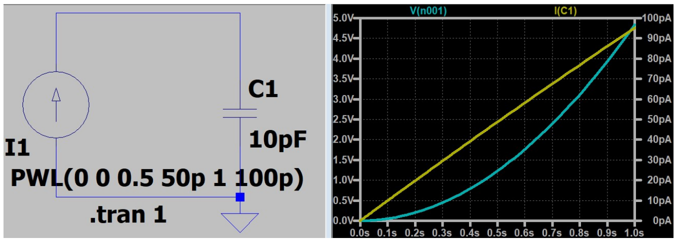

Fig. 6. LTspice simulation of an ideal capacitor driven by a ramp current. Capacitor voltage is shown in blue and capacitor current in yellow.

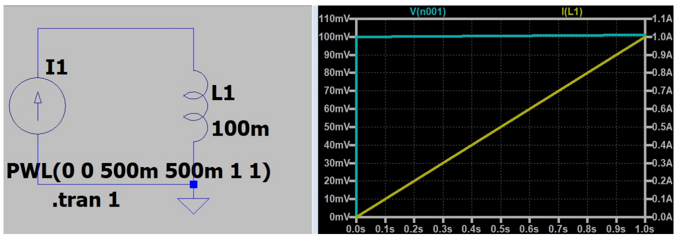

Fig. 7. LTspice simulation of an ideal inductor driven by a ramp current. Inductor voltage is shown in blue and inductor current in yellow.

for helping to connect math theory to reality. Teachers can quickly and easily create circuits using an ideal source and either an inductor or capacitor to demonstrate the expected behavior of a device. In Fig. 4 and Fig. 5 the instructor is able to demonstrate the simple derivative and integral relationship between sine and cosine. Students can approximate the slope of the voltage in Fig. 4 and the area under the voltage curve in Fig. 5 to estimate the value of the current for each circuit. As an additional example to further cement the Fundamental Theorem of Calculus the teacher can create simulations shown in Fig. 6 and Fig. 7. From Fig. 6 students can identify a conversion from a linear input current to a quadratic voltage using (7) while Fig. 7 shows the same linear input becoming a constant voltage with the inductor using (8).

At the RET site teachers were trained on the ADALM1000 to demonstrate physical circuits in their classrooms. The ADALM1000 (shown in Fig. 8) simultaneously acts as a two-channel signal generator and oscilloscope each capable of 100,000 samples per second [19]. To show the Fundamental Theorem of Calculus in action, teachers can measure simple L and C circuits on the ADALM1000. Results can be compared to LTspice simulations leading to discussion on mathematical modeling of real-world scenarios. With both simulations and real-world measurements, students should be ready to work calculus examples based on inductor and capacitor circuits.

Introductory problems give a value for the capacitor/inductor as well as either a function for voltage or current. Students must then set up the appropriate equation to find the missing value. More complex equations can set an initial condition alongside the other provided information to give students practice with a constant of integration. Teachers can challenge students by providing them with the change in voltage, the starting and ending time values of the change,

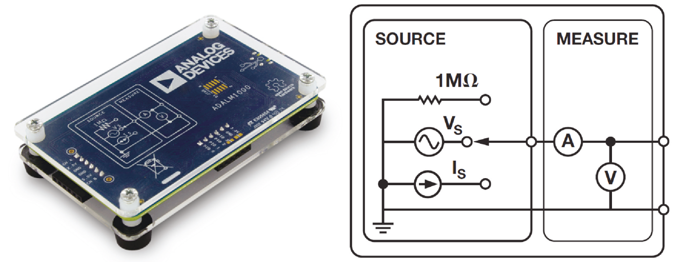

Fig. 8. The Analog Devices Active Learning Module (ADALM1000) and a basic schematic of the sourcing/measuring circuit [19].

an equation for current, and any initial conditions then asking the students to find the value and type of circuit element that made these conditions possible. Through these calculus-based circuit examples students will be able to practice using the Fundamental Theorem of Calculus with derivation and integration, graphical interpretation, and see the benefits of mathematical modeling in software tools.

V. Evaluation Plan

During the Spring 2024 semester we will visit the teachers who participated in the RET site to assess the results of their experience. During visits we will conduct interviews with the teachers to learn their perspective on student responses to the semiconductor and circuit examples used in math curriculum. We will also have students fill out surveys on their thoughts regarding the semiconductor and circuit concepts they have learned with questions on their enjoyment of the lessons, their mastery of the math topics related to the lessons, and their interest in learning more about circuits and semiconductors because of the lessons. The results of these visits will be used to refine the content and structure of the developed curriculum.

VI. Conclusion

Math engagement in high school is a long-standing problem leading to student's requiring remedial math courses if they select a STEM major or potentially avoiding STEM studies altogether. In addition, the need for skilled workers in the semiconductor industry will greatly increase due to investments from the CHIPS Act. Efforts to increase math engagement in secondary schools by introducing circuit and semiconductor concepts as real-world applications of math topics progresses goals of both educators and industry. Towards these goals, Oklahoma State University's new RET site is working with teachers to broaden STEM education pathways.

The semiconductor and circuit examples outlined in this paper are initial results from teachers during the RET program that ran from June to July of 2023. Lesson plans developed during the experience will be used in classrooms this year. Feedback from teachers and students will aid in the development of further lessons plans with future groups of teachers.

REFERENCES

[1] S. Schukajlow, K. Rakoczy, and R. Pekrun, "Emotions and motivation in mathematics education: theoretical considerations and empirical contributions," *ZDM*, vol. 49, no. 3, pp. 307–322, Jun. 2017. [Online]. Available: https://doi.org/10.1007/s11858-017-0864-6

[2] H. Gaspard, I. Häfner, C. Parrisius, U. Trautwein, and B. Nagengast, "Assessing task values in five subjects during secondary school: Measurement structure and mean level differences across grade level, gender, and academic subject," *Contemporary Educational Psychology*, vol. 48, pp. 67–84, Jan. 2017. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0361476X16300418

[3] K. Saxe and L. Braddy, "A Common Vision for Undergraduate Mathematical Sciences Programs in 2025," 2015. [Online]. Available: https://maa.org/sites/default/files/pdf/CommonVisionFinal.pdf

[4] M. M. Jamali, S. Arbabi, H. Hosseini, and H. Aryal, "Addressing Retention and Improving Performance in Gateway Engineering Courses," in *2022 IEEE International Symposium on Circuits and Systems (ISCAS)*, May 2022, pp. 2137–2141, iSSN: 2158-1525. [Online]. Available: https://ieeexplore.ieee.org/document/9937803

[5] L. S. DeBrunner and V. DeBrunner, "Engaging Students in an Introductory Circuits Course," in *2023 IEEE International Symposium on Circuits and Systems (ISCAS)*, May 2023, pp. 1–5, iSSN: 2158-1525.

[6] NIST, "CHIPS Workforce Development Planning Guide," Mar. 2023. [Online]. Available: https://www.nist.gov/system/files/documents/2023/03/30/CHIPSWorkforceDevelopmentPlanningGuide(1).pdf

[7] S. I. Association, "Chipping Away: Assessing and Addressing the Labor Market Gap Facing the U.S. Semiconductor Industry," Jul. 2023. [Online]. Available: https://www.semiconductors.org/wp-content/uploads/2023/07/SIA_July2023_ChippingAway_website.pdf

[8] G. Polya, *How to Solve It: A New Aspect of Mathematical Modeling*, 2nd ed. Princeton, NJ: Princeton University Press, 1957, http://www.im.ufrj.br/˜monica/funcoes/Polya.pdf.

[9] N. Lohgheswary, E. Zakaria, Z. M. Nopiah, and A. A. Aziz, "Innovative Learning in Engineering Mathematics," in *2017 7th World Engineering Education Forum (WEEF)*, Nov. 2017, pp. 768–772. [Online]. Available: https://ieeexplore.ieee.org/document/8467063

[10] Z. Kohen and D. Orenstein, "Mathematical modeling of tech-related real-world problems for secondary school-level mathematics," *Educational Studies in Mathematics*, vol. 107, no. 1, pp. 71–91, May 2021. [Online]. Available: https://doi.org/10.1007/s10649-020-10020-1

[11] V. Tan, C. Nicholas, J. A. Scribner, and D. C. Francis, "enhancing STEM learning through an interdisciplinary, industry-generated project," *Technology and Engineering Teacher*, vol. 79, no. 1, pp. 26–31, Sep. 2019. [Online]. Available: https://www.proquest.com/docview/2309764378/abstract/60F95C760499409DPQ/1

[12] N. C. for Education Statistics, "Condition of Education - High School Mathematics and Science Course Completion," 2022. [Online]. Available: https://nces.ed.gov/programs/coe/indicator/sod/high-school-courses

[13] O. D. of Education, "Oklahoma Academic Standards for Mathematics," Feb. 2022. [Online]. Available: https://sde.ok.gov/sites/default/files/documents/files/OklahomaAcademicStandardsforMathematics2022.pdf

[14] C. C. S. S. Initiative, "Common Core State Standards for Mathematics." [Online]. Available: https://learning.ccsso.org/wp-content/uploads/2022/11/ADA-Compliant-Math-Standards.pdf

[15] A. Devices, "LTspice," Wilmington, MA, Oct. 2023. [Online]. Available: https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html

[16] A. Sedra and K. Smith, *Microelectronic Circuits*, 7th ed. New York, NY: Oxford University Press, 2015.

[17] G. D. of Education, "Georgia's K-12 Mathematics Standards," 2021. [Online]. Available: https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/Mathematics/Georgia-K12-Mathematics-Standards/Georgia-HS-Calculus-Mathematics-Standards.pdf

[18] A. D. of Education, "Arkansas Academic Standard: Calculus Content Standards," 2016. [Online]. Available: https://dese.ade.arkansas.gov/Offices/learning-services/curriculum-support/mathematics-standards-and-courses

[19] A. Devices, "ADALM1000 Evaluation Board," May 2014. [Online]. Available: https://www.analog.com/en/design-center/evaluation-hardware-and-software/evaluation-boards-kits/adalm1000.html