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Phase jumps in Josephson junctions with time-dependent spin-orbit coupling

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ABSTRACT

Planar Josephson junctions (JJs), based on common superconductors and III-V semiconductors, are sought for Majorana states and fault-tolerant quantum computing. However, with gate-tunable spin-orbit coupling (SOC), we show that the range of potential applications of such JJs becomes much broader. The time-dependent SOC offers unexplored mechanisms for switching JJs, accompanied by the 2π -phase jumps and the voltage pulses corresponding to the single-flux-quantum transitions, key to high-speed and low-power superconducting electronics. In a constant applied magnetic field, with Rashba and Dresselhaus SOC, anharmonic current-phase relations, calculated microscopically in these JJs, yield a nonreciprocal transport and superconducting diode effect. Together with the time-dependent SOC, this allows us to identify a switching mechanism at no applied current bias, which supports fractional-flux-quantum superconducting circuits and neuromorphic computing.

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One of the hallmarks of Josephson junctions (JJs) used in their applications is the current-phase relation (CPR).^{1–3} Instead of being driven by the difference of electrical potentials, the dissipationless supercurrent through a JJ, I , is driven by the phase difference of the superconducting order parameter, φ , across the junction. The dc Josephson effect is commonly given by the CPR, $I(\varphi) = I_c \sin \varphi$, where the critical current, I_c , is the maximum supercurrent in a JJ. The resulting ground-state energy corresponds to $\varphi = 0$ or integer multiples of 2π .

The control of phase jumps or slips in the CPR and different ground states, provides important paths in superconducting electronics and spintronics, neuromorphic computing, advanced qubits, and fault-tolerant quantum computing.^{3–18} A simple example is the π junction¹⁹ with $I(\varphi) = -I_c \sin \varphi = I_c \sin(\varphi + \pi)$, with the minimum JJ energy at $\varphi = \pi$.⁶ In superconductor/ferromagnet/superconductor (S/F/S) junctions, $0 - \pi$ transitions can occur by changing the F-layer thickness.²⁰ Replacing F by a spin valve allows implementing Josephson magnetic random access memory (JMRAM).²¹ Unlike MRAM,²² the readout is not based on sensing the tunneling magnetoresistance,^{23,24} but fast dc-SQUID readout of φ . Switching the

magnetization of a spin valve from parallel to antiparallel orientation allows tunable 0 or π phase.²⁵

In this work, we study planar S/two-dimensional electron gas (2DEG)/S JJs, based on epitaxial Al/InAs heterostructures and motivated by the quest for topological quantum computing.^{26–30} While the experimental detection of $0 - \pi$ phase jump²⁹ confirms topological superconductivity in these JJs,^{31–34} we explore other applications that do not require topological properties, but rely on time-dependent spin-orbit coupling (SOC).

The feasibility of such time-dependent SOC is demonstrated by the quasistatic gate-controlled strength in a 2DEG.^{29,35,36} Instead of the common form, the CPR can acquire an anomalous phase, $\varphi_0 \neq 0, \pi$, which arises from the broken time-reversal and inversion symmetries.^{37–41} This example of anomalous Josephson effect in Al/InAs JJs was shown to be the interplay between SOC and the effective Zeeman field $\mathbf{B}$, with a gate-tunable φ_0 and generally anharmonic CPR.^{29,35} Since the gate switching rates in JJs can exceed the GHz range,¹⁵ the resulting time-dependent electric field offers an opportunity to implement time-dependent SOC.

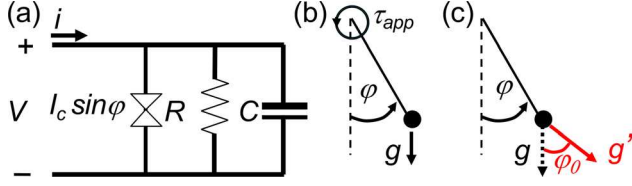


FIG. 1. (a) An equivalent Josephson junction (JJ) circuit, driven by bias current, i , has normal resistance, R , and capacitance, C . (b) A pendulum analogy for (a), driven by the applied torque, τ_{app} , and g is the gravitational force. The displacement angle, φ , is analogous to the superconducting phase difference. (c) With spin-orbit coupling and an effective Zeeman field, the effective gravitational force, g' , is rotated.

A common description of a JJ circuit is given in Fig. 1(a) by a Josephson element, resistor, and capacitor, using the resistively and capacitively shunted junction (RCSJ) model.¹ The bias current through the junction, i , is the sum of the supercurrent $\propto \sin \varphi$ and the quasiparticle current flowing in the resistor and capacitor. The corresponding phase evolution can be understood from a pendulum analog in Fig. 1(b). We generalize it in Fig. 1(c) to also include the anomalous Josephson effect with φ_0 ⁴² and explore different realizations of 2π -phase jumps used in high-speed and low-power superconducting electronics or neuromorphic computing.^{1–3}

For Al/InAs-based JJs in the ballistic regime, their CPR is not simply assumed to be $\propto \sin \varphi$ or $\sin(\varphi + \varphi_0)$. Instead, as we discuss below, the CPR is calculated microscopically from the corresponding Hamiltonian for such junctions and we obtain a generalized RCSJ model,

$$d^2\varphi/dt^2 + (d\varphi/dt)/\sqrt{\beta_c} + I(\varphi, \mu, \mathbf{h}, \alpha, \beta)/I_c = i/I_c, \quad (1)$$

where $\tau = \omega_p t$ is a dimensionless time, expressed using the JJ plasma frequency, $\omega_p = \sqrt{2\pi I_c / \Phi_0 C}$, $\Phi_0 = h/2e$ is the magnetic flux quantum, and C is the capacitance. The damping is given by the Stewart–McCumber parameter, $\beta_c = 2\pi I_c C R^2 / \Phi_0$, where R is the resistance^{43,44} and $Q = \sqrt{\beta_c}$ is the quality factor. The CPR can be modified by the chemical potential μ and $\mathbf{B}$, arising from the applied magnetic field or magnetic proximity effect.⁴⁵ Here, we further show the important dependence of CPR on Rashba and Dresselhaus SOC in the 2DEG,^{46–49} parametrized by their strengths α and β , respectively.

We calculate CPR for S/2DEG/S geometry with an in-plane field, $\mathbf{B}$, as in Fig. 2(a). The resulting Bogoliubov–de Gennes (BdG) equation,²⁰ $H_{BdG}\psi = E\psi$, is expressed using the Hamiltonian, H_{BdG} , and the four-component (in the spin and particle-hole space) wavefunction, ψ , for quasiparticle states with energy E . We solve the discretized BdG equation using a finite-difference method, which recovers our prior results.³³ At the boundary, $x = \pm(W_N/2 + W_S)$, $\psi = 0$. H_{BdG} is given in terms of the single-particle Hamiltonian, H_0 ,

$$H_{BdG} = H_0\tau_z + (g\mu_B/2)\mathbf{B} \cdot \boldsymbol{\sigma} + \Delta(x)\tau_+ + \Delta^*(x)\tau_-, \quad (2)$$

where H_0 includes the kinetic and SOC contribution,

$$H_0 = \frac{\mathbf{p}^2}{2m^*} - \mu + \frac{\alpha}{\hbar}(p_y\sigma_x - p_x\sigma_y) + \frac{\beta}{\hbar}[(p_x\sigma_x - p_y\sigma_y)\cos 2\theta_c - (p_x\sigma_y + p_y\sigma_x)\sin 2\theta_c]. \quad (3)$$

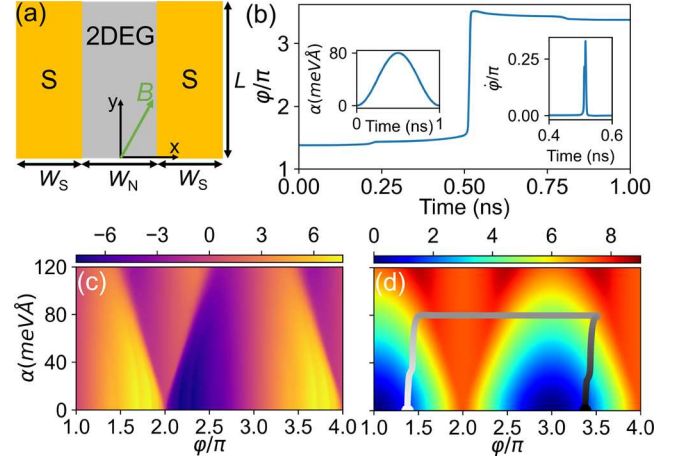


FIG. 2. (a) A JJ schematic. Two superconductors (S) are separated by the uncovered 2DEG region with SOC and a Zeeman field $\mathbf{B}$. (b) Phase evolution, $\varphi(t)$, with sinusoidal Rashba SOC, α (left) and the resulting voltage pulse (right) at $\omega_p = 1$ THz and $\beta_c = 1$. The evolution of (c) CPR, normalized by $I_0 = 2e\Delta/\hbar$, and (d) the JJ energy, normalized by Δ , as a function of φ and α , for chemical potential $\mu = 2.5\Delta$, $B_x = 1$ T, and a critical field $B_c = 3$ T. The white to black path in (d): JJ transition from $\varphi \approx 1.3\pi$ to $\approx 3.3\pi$ from (b).

Here, $\sigma_{x,y,z}$ and $\tau_{x,y,z}$ are the Pauli matrices in spin and particle-hole space, with $\tau_{\pm} = (\tau_x \pm i\tau_y)/2$. $\mathbf{p}$ is the in-plane momentum, m^* is the effective electron mass, g is the g -factor, and the superconducting pair potential is $\Delta(x) = \Delta e^{i\text{sgn}(x)\varphi/2}\Theta(|x| - W_N/2)\Theta(W_S + W_N/2 - |x|)$, where Δ is the superconducting gap. θ_c characterizes the direction of the current (x axis) with respect to the $[100]$ crystallographic direction of the 2DEG.⁴⁷ In our case, we choose $\theta_c = -\pi/4$. We can then obtain the ground-state JJ energy E_{GS} and the CPR, $I(\varphi) \propto \partial E_{GS} / \partial \varphi$.⁵⁰

This framework and the intuition from Figs. 1(b) and 1(c) allows us to examine unexplored methods for JJ switching. We recall that the ac Josephson effect,¹

$$V = (\Phi_0/2\pi)\partial\varphi/\partial t, \quad (4)$$

relates the voltage, V , across the JJ in the resistive state with the phase evolution. With a bias current $i \geq I_c$, creating torque, $\tau_{app} \propto i$, sufficient for a full pendulum rotation in Fig. 1(b), a 2π -phase jump is then accompanied by the voltage pulse corresponding to the area $\int V dt = \Phi_0$ and the switching energy $\approx I_c \Phi_0 \sim 10^{-20}$ J at the characteristic frequency, $\omega_p Q/2\pi$. Unlike CMOS logic and older operation of JJs,² instead of the voltage, the resulting rapid single-flux-quantum (RSFQ) logic employs the presence (absence) of SFQ as logical “1” (0).⁴ In RSFQ logic, a typically large static bias $\sim 0.8I_c$,³ where φ is larger than that in Fig. 1(b), implies that a small time-dependent signal is sufficient for 2π -phase jumps and JJ switching, but requires a large static power dissipation.

Our Fig. 1(c) reveals a different JJ driving mechanism. The time-dependent Rashba SOC, $\alpha(t)$, changed by the gate-controlled electric field, $\mathbf{E}$, rotates the direction of the gravitational acceleration, $\mathbf{g}$ to $\mathbf{g}'$. This could be desirable to reduce the static power dissipation in RSFQ logic from the applied current bias.³ We assume that a gate control primarily changes α , not μ (nor β). Experimentally, this could be realized with dual gating^{51,52} to independently tune the carrier density and the electric field. However, $\alpha(t)$ alone was not sufficient to realize 2π phase

jumps.⁴² Here, we explore how to overcome this limitation, guided by the Al/InAs JJs.^{29,35}

Our findings are illustrated for the JJ in Fig. 2(a), $W_N = 250$ nm, $W_S = 300$ nm, for the normal (N) and S regions, and $L = 2$ μ m. The energies are normalized by Δ and the supercurrent by $I_0 = 2|e\Delta|/\hbar$, where e is the electron charge and $|e\Delta|/\hbar$ is the maximum supercurrent in a single-channel short S/N/S JJ.⁵⁰ We use parameters for high-quality epitaxial InAs/Al-based JJs, $\Delta_{Al} = 0.2$ meV and $g = 10$ for InAs, while its m^* is 0.03, the electron mass.^{29,35} In these JJs, the gate control of Rashba SOC, and thus its magnitude in the range $\alpha \in [0, 200$ meV $\text{\AA}$], has been demonstrated.^{29,35} In III-V 2DEGs, the Dresselhaus SOC, β , can have comparable values to α and be gate tunable.⁵³ Enhanced Al superconductivity is realized near its 2D limit, $\Delta_{Al} = 0.55$ meV.⁵⁴ With strong SOC materials next to Al, its critical magnetic field can be much larger, up to $B_c \approx 9$ T (Refs. 55 and 56) enhancing the JJ parameter space. With these advances, instead of $B_c = 1.45$ T (Ref. 29), we choose a larger $B_c = 3$ T, but retain $\Delta_{Al} = 0.2$ meV and its BCS suppression with $B > 0$.¹

In Figs. 2(b)–2(d), we study JJ dynamics at $B_x = 1$ T. From the calculated CPR and JJ energy, we infer that only a modest energy barrier is preventing 2π -phase jumps with the evolution of $\alpha(t)$, as indicated with the path in Fig. 2(d), where the lighter (darker) shades of gray correspond to the earlier (later) times. Indeed, we find that for E -controlled sinusoidal $\alpha(t) \in [0, 80$ meV $\text{\AA}$], within experimental values, a bias current $i = 0.65I_c$ is needed to realize reproducible 2π -phase jumps. At larger α , $i = 0.35I_c$ is sufficient for 2π -phase jumps. As expected from Eq. (4), these jumps are accompanied by the SFQ voltage pulse shown in the inset of Fig. 2(b).

This scenario, calculated from Eq. (4), corresponds to the combination of the two JJ driving approaches from Figs. 1(b) and 1(c). With $\alpha, B_x, i \neq 0$, from Fig. 1(c) analogy, for this phase evolution, we expect $\varphi_0 \neq 0$, such that the final φ is different from 3π . However, $B_y = 0$ implies that $I(\varphi) = -I(-\varphi)$ and $I(\varphi = 0) = 0$, as shown in Fig. 2(c). The minima in E_{GS} in Fig. 2(d) come in pairs at $\pm\varphi_0$.⁵⁷ The chosen JJ circuit parameters are motivated by those measured in the InAs/Al samples, $I_c \sim 4$ μ A, $R \sim 100$ Ω , and $C \sim 15$ fF, leading to $\omega_p \sim 900$ GHz and the damping $\beta_c \sim 1$, which is also suitable for the RSFQ applications.^{1,3,4} We have verified that the 2π -phase jump is insensitive to the specific $\alpha(t)$ details. Nearly identical results and the SFQ pulse are obtained for a triangular $\alpha(t)$ of the same amplitude.

From the scenario in Fig. 1(c) and the JJ geometry in Fig. 2(a), we can explore other approaches to switching JJs, which could further reduce the bias current for RSFQ. For example, could the rotation of B ensure that $\varphi_0 \neq 0$ (therefore, no minimum E_{GS} at $\varphi = -\varphi_0$) to provide a ratchet effect and a boost for 2π -phase jumps?

With Rashba and Dresselhaus SOC, the presence of φ_0 in the anomalous Josephson effect can be expressed as^{57,58}

$$\varphi_0 \propto B_y(\alpha - \beta)(\alpha^2 - \beta^2). \quad (5)$$

With $\alpha(t)$ and B_y , considered in Fig. 3, we can then examine the influence of φ_0 on the bias current for JJ switching. While we again see the 2π -phase jump in Figs. 3(b) and 3(c), the sinusoidal $\alpha(t) \in [0, 116]$ meV $\text{\AA}$ requires a much higher bias current $i = 0.8I_c$ than $i = 0.65I_c$ from Figs. 2(b) and 2(d), at the same magnitude of B . This is surprising, as the JJ energy landscape in Fig. 3(b) suggests that the gray path with chosen evolution of α minimizes the local energy barriers.

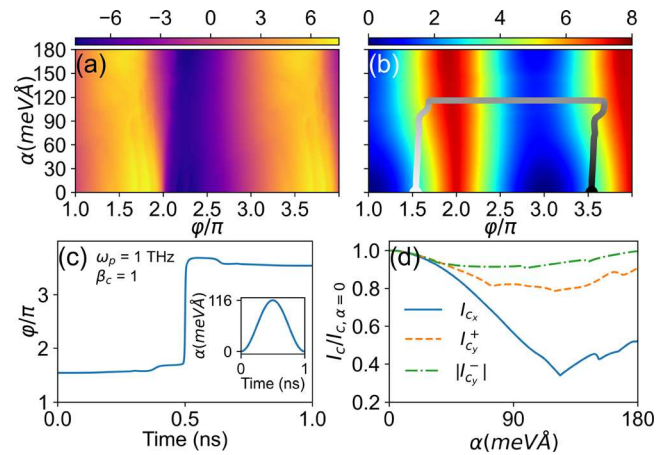


FIG. 3. The evolution of (a) CPR, normalized by I_0 , and (b) the JJ energy, normalized by Δ , as a function of φ and α , for $\mu = 2.5\Delta$ and $-B_y = 1$ T. The white to black path in (b) corresponds to the 2π -phase jump, shown also in (c). $\alpha(t)$ changes sinusoidally over 1 ns, as in Fig. 2(b), but with the amplitude of 116 meV $\text{\AA}$. (d) $I_c(\alpha)$ is nonmonotonic for a fixed $-B_y = 1$ T. For B_y , a Josephson diode effect shows an inequivalence in increasing or decreasing φ , $I_{c_y}^+ \neq |I_{c_y}^-|$.

To understand why such a higher bias current is needed for the anomalous Josephson effect and φ_0 , in Fig. 3(d), we compare the evolution of I_c with α for a fixed (i) $B = B_x\hat{x}$ and (ii) $B = B_y\hat{y}$. In the limit $\alpha = 0$, as expected, $|B| \neq 0$ leads to the suppression of the critical current, as compared to the zero-field limit. Unlike (i) with $I(\varphi) = -I(-\varphi)$ and the critical current I_{c_x} , independent of the current direction, (ii) is an example of the broken inversion symmetry and the Josephson diode effect,^{7,59} where the nonreciprocal transport implies that if $B_y \neq 0$, the positive and negative supercurrent has different maximum values, $I_{c_y}^+ \neq |I_{c_y}^-|$.⁶⁰ A superconducting diode effect is also possible with a single S region.^{7,61–63}

For a ballistic transport of Al/InAs-based JJs with an anharmonic CPR, as shown in Fig. 3(a), the Josephson diode effect was observed experimentally.^{7,29,64} The resulting difference between the critical supercurrent in the forward and reverse direction can be understood from the process of Andreev reflection at S/N interfaces^{46,65,66} and the formation of Andreev bound states,^{67,68} which have unequal contributions right and left moving subgap quasiparticles.⁶⁰ Such ballistic JJs allow for verification of this prediction for the nonmonotonic $I_{c_y}^{\pm}(\alpha)$ in Fig. 3(d). In contrast, a simple harmonic $I(\varphi) = I_c \sin(\varphi + \varphi_0)$, common to an anomalous Josephson effect in a diffusive regime, yields no diode effect.⁷

If we recall Fig. 1(b), a current-driven JJ switching with 2π -phase jumps is achieved for a current exceeding I_c (at $B = 0$).¹ We then expect that the evolution of the critical current in Fig. 3(d) is a guide for the minimum bias current needed in SOC-driven JJ switching. Indeed, for (i) and B_x , we get $I_{c_x}(\alpha = 80 \text{ meV}\text{\AA}) = 0.65I_c$, while for (ii) and B_y , we get $I_{c_y}^+(\alpha = 116 \text{ meV}\text{\AA}) = 0.8I_c$, just as we have independently obtained for the needed bias current by calculating the phase evolution from Eq. (1), which yields 2π jumps in Figs. 2(b) and 3(c).

Guided by the JJ parameters and the considered effective Zeeman field, our results from Figs. 2 and 3 allow us to establish some trends for the SOC-driven JJ switching, relying on the experimentally available range of gate-controlled α . If we seek to minimize the bias current for 2π -phase jumps, which could be both desirable for both low-

energy RSFQ and neuromorphic computing,^{3,12} Fig. 3(d) suggests that a large SOC and no diode effect are beneficial, while only in a range of α (near 0 and 124 meVÅ), there is a modest decrease in the needed minimum i , accompanied by the diode effect. The nonreciprocal transport in the diode effect, $I_{cy}^+ \neq |I_{cy}^-|$, implies that there is an inequivalence to increase or decrease φ . In the pendulum analogy from Fig. 1(c), the clockwise and counterclockwise 2π -phase rotations are energetically inequivalent, as in the ratchet effect.

From the above-mentioned discussion, it seems that our goal of a pure SOC-driven JJ switching at $i=0$ is not practical, since it would suggest the need to tune α to yield a vanishingly small I_c . Similarly, seeking to realize the Josephson diode effect only has limited advantages. However, there is a need to critically evaluate these assertions as in various III-V semiconductors Dresselhaus SOC strength, β , can play an important role,^{46,69} and as can be seen from Eq. (5), the presence of φ_0 and the implications of the diode effect could be more complex.

To examine this more general situation of the coexisting α and β , we consider results in Fig. 4, at $B_y \neq 0$, for a fixed $\beta = 130$ meVÅ, and $\theta_c = -\pi/4$ in Eq. (4). It is then impossible to combine spatial rotation and spin rotation around y axis to reverse I and φ .⁷ In Fig. 4(a), this leads to $I(-\varphi) \neq -I(\varphi)$ and the diode effect, as expected from Eq. (5) and $\varphi_0 \neq 0$.

The white to black path in Fig. 4(b) corresponds to choosing a harmonic $\alpha(t)$ changing over 1 ns as in Fig. 2(b), but with an amplitude of 183 meVÅ. Remarkably, the desired 2π -phase jump is now realized at $i=0$, for a pure SOC-driven JJ switching! The energy landscape reveals two phase jumps and plateaux, near $\varphi = 1.75\pi$ and 2π . At $\alpha=0$, the system starts from an energy minimum. As α increases, $\beta \neq 0$ is responsible for the formation of the $\varphi_0 \propto (\alpha - \beta)(\alpha^2 - \beta^2)$ state. The resulting diode effect favors the increase in φ (counterclockwise pendulum rotation) and the phase jump near $\varphi = 1.75\pi$. Subsequently, when α decreases, there is another phase jump near $\varphi = 2\pi$ or, equivalently, $\varphi = 0$ state, which completes the SOC-driven JJ switching.

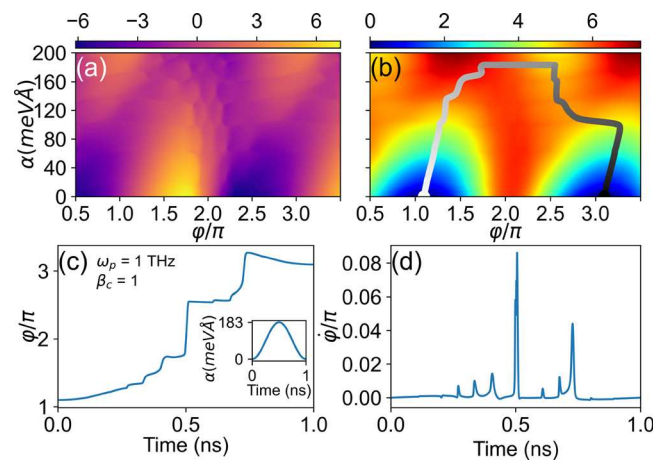


FIG. 4. The evolution of (a) CPR and (b) JJ energy with φ and α , at a fixed $\beta = 130$ meVÅ, $-B_y = 1$ T, and $\mu = 2.5\Delta$. An anharmonic CPR breaks the $I(-\varphi) = -I(\varphi)$ symmetry in (a), and the corresponding anomalous phase, φ_0 , increases with α in (b). (c) $\varphi(t)$, for $\omega_p = 1$ THz, $\beta_c = 1$, shows phase jumps near 1.75π and 2.5π with $\alpha(t)$ changing sinusoidally as in Fig. 2(b), but with the amplitude of 183 meVÅ. (d) The corresponding voltage pulses, each with a fractional SFQ.

Turning to the phase evolution calculated from Eq. (1) and using the microscopic CPR obtained in Fig. 4(a), we see the two main phase jumps and a plateau in Fig. 4(c). A manifestation of the resulting ac Josephson effect in Eq. (4) for such $\varphi(t)$ is the presence of two voltage pulses shown in Fig. 4(d). Each of them corresponds to the fractional SFQ, while their sum, along several much smaller peaks from additional small φ -jumps, yields SFQ. With the push for low-energy RSFQ, including using a half-SFQ pulses in more complicated JJ circuits,⁷⁰ it would be important to explore if using the fractional-SFQ pulses could be advantageous for the reduced power consumption in superconducting electronics.¹⁻³

With the current experiments limited to quasistatic SOC changes,^{29,35,36} we have focused on the parameters for the fabricated Al/InAs JJs. However, as the materials used in planar JJs expand to also include group IV Ge/SiGe heterostructures⁷¹ or Au-surface states,⁷² it would be important to explore other opportunities for SOC-controlled JJ dynamics, since both Rashba and Dresselhaus SOC have not only different strengths from III-V semiconductors, but also a different form, cubic in the wave vector.^{57,73}

The chosen GHz change in Rashba SOC is not the ultimate limit for the speed of the SOC-driven JJ switching, but simplifies our calculations.⁷⁴ In addition to considering a faster gate operation, available in JJ qubits,¹⁵ a more rapid phase jumps could be possible. For example, rather than being limited by the changes in the gate-controlled E -field and the resulting SOC, the curvature of the JJ energy could lead to an order-of-magnitude faster phase jumps,⁴² while Fig. 4(d) shows the feasibility of multiple phase jumps and voltage pulses, desirable for enhancing neuromorphic computing.^{1,12}

Beyond our focus on superconducting electronics and neuromorphic computing, in a future work it would be interesting to explore the role of time-dependent SOC in topological superconductivity and fault-tolerant quantum computing.⁷⁵⁻⁷⁷ The topological protection offers alternatives to the usual paths of decoupling qubits from their environment.^{15,78} Experimental support already reveals an approximately π -phase jump with an increase in B_y as a signature of the transition between trivial and topological superconductivity in SQUID based on two Al/InAs planar JJs.²⁹ With the gate-controlled SOC, such a phase jump should also be present at a fixed B_y , as an additional contribution to our studied phase dynamics, when extended to topological superconductivity. Adding multiple gates in this system is predicted to create and reposition Majorana states and therefore probe the non-Abelian statistics through their fusion,³³ which is crucial for implementing quantum gates and quantum measurements.^{77,79,80} Future studies could also explore the role of time-dependent gate voltage in other implementations of JJs.^{81,82}

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

David Monroe: Conceptualization (equal); Data curation (lead); Formal analysis (lead); Investigation (lead); Methodology (equal); Software (lead); Validation (lead); Visualization (lead); Writing – original draft (equal); Writing – review & editing (lead). **Chenghao Shen:** Data curation (equal); Formal analysis (equal); Investigation (equal); Methodology (equal); Resources (equal); Software (equal); Validation (equal). **Dario Tringali:** Data curation (supporting); Formal analysis (supporting); Investigation (supporting); Methodology (supporting); Validation (supporting); Visualization (supporting). **Mohammad Alidoust:** Conceptualization (supporting); Data curation (supporting); Formal analysis (supporting); Investigation (supporting). **Tong Zhou:** Data curation (supporting); Formal analysis (supporting); Investigation (supporting); Software (equal); Validation (supporting). **Igor Žutić:** Conceptualization (equal); Data curation (supporting); Formal analysis (equal); Funding acquisition (lead); Investigation (equal); Methodology (equal); Project administration (lead); Resources (lead); Software (supporting); Supervision (lead); Validation (lead); Visualization (supporting); Writing – original draft (lead); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

REFERENCES

- ¹*Fundamentals and Frontiers of the Josephson Effect*, edited by F. Tafuri (Springer Nature, Cham, 2019).
- ²M. Siegel and M. Hidaka, “Superconductor digital electronics,” in *Nanoelectronics and Information Technology*, 3rd ed., edited by R. Wasner (Wiley-VCH, Weinheim, Germany, 2012), pp. 421–430.
- ³I. I. Soloviev, N. V. Klenov, S. V. Bakurskiy, M. Y. Kupriyanov, A. L. Gudkov, and A. S. Sidorenko, “Beyond Moore’s technologies: Operation principles of a superconductor alternative,” *Beilstein J. Nanotechnol.* **8**, 2689 (2017).
- ⁴K. K. Likharev and V. K. Semenov, “RSFQ logic/memory Family: A new Josephson-junction technology for sub-terahertz-clock-frequency digital systems,” *IEEE Trans. Appl. Supercond.* **1**, 3–28 (1991).
- ⁵D. S. Holmes, A. L. Ripple, and M. A. Manheimer, “Energy-efficient superconducting computing-power budgets and requirements,” *IEEE Trans. Appl. Supercond.* **23**, 1701610 (2013).
- ⁶T. Yamashita, “Phase shift and control in superconducting hybrid structures,” *IEICE Trans. Electron.* **E101.C**, 378 (2018).
- ⁷M. Amundsen, J. Linder, J. W. A. Robinson, I. Žutić, and N. Banerjee, “Colloquium: Spin-orbit effects in superconducting hybrid structures,” *Rev. Mod. Phys.* **96**, 021003 (2024).
- ⁸F. Giazotto, J. T. Peltonen, M. Meschke, and J. P. Pekola, “Superconducting quantum interference proximity transistor,” *Nat. Phys.* **6**, 254 (2010).
- ⁹M. Eschrig, “Spin-polarized supercurrents for spintronics: A review of current progress,” *Rep. Prog. Phys.* **78**, 104501 (2015).
- ¹⁰J. Linder and J. W. A. Robinson, “Superconducting spintronics,” *Nat. Phys.* **11**, 307 (2015).
- ¹¹N. Banerjee, J. W. A. Robinson, and M. G. Blamire, “Reversible control of spin-polarized supercurrents in ferromagnetic Josephson junctions,” *Nat. Commun.* **5**, 4771 (2014).
- ¹²P. Crotty, D. Schult, and K. Segall, “Josephson junction simulation of neurons,” *Phys. Rev. E* **82**, 011914 (2010).
- ¹³T. Yamashita, K. Tanikawa, S. Takahashi, and S. Maekawa, “Superconducting qubit with a ferromagnetic Josephson junction,” *Phys. Rev. Lett.* **95**, 097001 (2005).
- ¹⁴K. Halterman, O. T. Valls, and C.-T. Wu, “Charge and spin currents in ferromagnetic Josephson junctions,” *Phys. Rev. B* **92**, 174516 (2015).
- ¹⁵P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, “A quantum engineer’s guide to superconducting qubits,” *Appl. Phys. Rev.* **6**, 021318 (2019).
- ¹⁶M. Bal, J. Long, R. Zhao, H. Wang, S. Park, C. R. Harrington McRae, T. Zhao, R. E. Lake, V. Monarkha, S. Simbierowicz, D. Frolov, R. Pilipenko, S. Sorzetti, A. Romanenko, C.-H. Liu, R. McDermott, and D. P. Pappas, “Overlap junctions for superconducting quantum electronics and amplifiers,” *Appl. Phys. Lett.* **118**, 112601 (2021).
- ¹⁷U. Gungördü and A. A. Kovalev, “Majorana bound states with chiral magnetic textures,” *J. Appl. Phys.* **132**, 041101 (2022).
- ¹⁸R. Cai, I. Žutić, and W. Han, “Superconductor/ferro-magnet heterostructures: A platform for superconducting spintronics and quantum computation,” *Adv. Quantum Technol.* **6**, 202200080 (2023).
- ¹⁹L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyenin, “Superconducting system with weak coupling with a current in the ground state,” *JETP Lett.* **25**, 290 (1977).
- ²⁰O. T. Valls, *Superconductor/Ferromagnet Nanostructures* (World Scientific, Hackensack, NJ, 2022).
- ²¹N. O. Birge and M. Houzet, “Spin-singlet and spin-triplet Josephson junctions for cryogenic memory,” *IEEE Magn. Lett.* **9**, 4509605 (2019).
- ²²*Spintronics Handbook Spin Transport and Magnetism*, 2nd ed., edited by E. Y. Tsymlal and I. Žutić (CRC Press, Boca Raton, FL, 2019).
- ²³S. S. P. Parkin, C. Kaiser, A. Panchula, P. M. Rice, B. Hughes, M. Samant, and S.-H. Yang, “Room temperature tunneling magnetoresistance in magnetic tunnel junctions with MgO(100) tunnel barriers,” *Nat. Mater.* **3**, 862 (2004).
- ²⁴S. Ikeda, J. Hayakawa, Y. Ashizawa, Y. M. Lee, K. Miura, H. Hasegawa, M. Tsunoda, F. Matsukura, and H. Ohno, “Tunnel magnetoresistance of 604% at 300 K by suppression of Ta diffusion in CoFeB/MgO/CoFeB pseudo-spin-valves annealed at high temperature,” *Appl. Phys. Lett.* **93**, 082508 (2008).
- ²⁵E. C. Gingrich, B. M. Niedzielski, J. A. Glick, Y. Wang, D. L. Miller, R. Loloee, W. P. Pratt, Jr., and N. O. Birge, “Controllable $0 - \pi$ Josephson junctions containing a ferromagnetic spin valve,” *Nat. Phys.* **12**, 564 (2016).
- ²⁶J. Shabani, M. Kjaergaard, H. J. Suominen, Y. Kim, F. Nichele, K. Pakrouski, T. Stankevicius, R. M. Lutchyn, P. Krogstrup, R. Feidenhans’l, S. Kraemer, C. Nayak, M. Troyer, C. M. Marcus, and C. J. Palmstrom, “Two-dimensional epitaxial superconductor-semiconductor heterostructures: A platform for topological superconducting networks,” *Phys. Rev. B* **93**, 155402 (2016).
- ²⁷A. Fornieri, A. M. Whicar, F. Setiawan, E. Portolés, A. C. C. Drachmann, A. Keselman, S. Gronin, C. Thomas, T. Wang, R. Kallaher, G. C. Gardner, E. Berg, M. J. Manfra, A. Stern, C. M. Marcus, and F. Nichele, “Evidence of topological superconductivity in planar Josephson junctions,” *Nature* **569**, 89 (2019).
- ²⁸W. Mayer, W. F. Schiela, J. Yuan, M. Hatefipour, W. L. Sarney, S. P. Svensson, A. C. Leff, T. Campos, K. S. Wickramasinghe, M. C. Dartailh, I. Žutić, and J. Shabani, “Superconducting proximity effect in InAsSb surface quantum wells with in situ Al contact,” *ACS Appl. Electron. Mater.* **2**, 2351 (2020).
- ²⁹M. Dartailh, W. Mayer, J. Yuan, K. Wickramasinghe, A. Matos-Abiague, I. Žutić, and J. Shabani, “Phase signature of topological transition in Josephson junctions,” *Phys. Rev. Lett.* **126**, 036802 (2021).
- ³⁰A. Banerjee, O. Lesser, M. A. Rahman, H.-R. Wang, M.-R. Li, A. Kringhøj, A. M. Whicar, A. C. C. Drachmann, C. Thomas, T. Wang, M. J. Manfra, E. Berg, Y. Oreg, A. Stern, and C. M. Marcus, “Signatures of a topological phase transition in a planar Josephson junction,” *Phys. Rev. B* **107**, 245304 (2023).
- ³¹M. Hell, M. Leijnse, and K. Flensberg, “Two-dimensional platform for networks of Majorana bound states,” *Phys. Rev. Lett.* **118**, 107701 (2017).
- ³²F. Pientka, A. Keselman, E. Berg, A. Yacoby, A. Stern, and B. I. Halperin, “Topological superconductivity in a planar Josephson junction,” *Phys. Rev. X* **7**, 021032 (2017).
- ³³T. Zhou, M. C. Dartailh, K. Sardashti, J. E. Han, A. Matos-Abiague, J. Shabani, and I. Žutić, “Fusion of Majorana bound states with mini-gate control in two-dimensional systems,” *Nat. Commun.* **13**, 1738 (2022).
- ³⁴T. Zhou, M. C. Dartailh, W. Mayer, J. E. Han, A. Matos-Abiague, J. Shabani, and I. Žutić, “Phase control of Majorana bound states in a topological X junction,” *Phys. Rev. Lett.* **124**, 137001 (2020).
- ³⁵W. Mayer, M. C. Dartailh, J. Yuan, K. S. Wickramasinghe, E. Rossi, and J. Shabani, “Gate controlled anomalous phase shift in Al/InAs Josephson junctions,” *Nat. Commun.* **11**, 212 (2020).

- ³⁶M. Gupta, G. V. Graziano, M. Pendharkar, J. T. Dong, C. P. Dempsey, C. Palmstrom, and V. S. Pribyl, "Gate-tunable superconducting diode effect in a three-terminal Josephson device," *Nat. Commun.* **14**, 3078 (2023).
- ³⁷A. A. Reynoso, G. Usaj, C. A. Balseiro, D. Feinberg, and M. Avignon, "Anomalous Josephson current in junctions with spin polarizing quantum point contacts," *Phys. Rev. Lett.* **101**, 107001 (2008).
- ³⁸A. Buzdin, "Direct coupling between magnetism and superconducting current in the Josephson ϕ_0 junction," *Phys. Rev. Lett.* **101**, 107005 (2008).
- ³⁹F. Konschelle and A. Buzdin, "Magnetic moment manipulation by a Josephson current," *Phys. Rev. Lett.* **102**, 017001 (2009).
- ⁴⁰H. Sickinger, A. Lipman, M. Weides, R. G. Mints, H. Kohlstedt, D. Koelle, R. Kleiner, and E. Goldobin, "Experimental evidence of a ϕ Josephson junction," *Phys. Rev. Lett.* **109**, 107002 (2012).
- ⁴¹E. Strambini, A. Iorio, O. Durante, R. Citro, C. Sanz-Fernández, C. Guarcello, I. V. Tokatly, A. Braggio, M. Rocci, N. Ligato, V. Zannier, L. Sorba, F. S. Bergeret, and F. Giazotto, "A Josephson phase battery," *Nat. Nanotechnol.* **15**, 656 (2020).
- ⁴²D. Monroe, M. Alidoust, and I. Žutić, "Tunable planar Josephson junctions driven by time-dependent spin-orbit coupling," *Phys. Rev. Appl.* **18**, L031001 (2022).
- ⁴³W. C. Stewart, "Current-voltage characteristics of Josephson junctions," *Appl. Phys. Lett.* **12**, 277 (1968).
- ⁴⁴D. E. McCumber, "Effect of ac impedance on dc voltage-current characteristics of superconductor weak-link junctions," *J. Appl. Phys.* **39**, 3113 (1968).
- ⁴⁵I. Žutić, A. Matos-Abiague, B. Scharf, H. Dery, and K. Belashchenko, "Proximitized materials," *Mater. Today* **22**, 85 (2019).
- ⁴⁶I. Žutić, J. Fabian, and S. Das Sarma, "Spintronics: Fundamentals and applications," *Rev. Mod. Phys.* **76**, 323 (2004).
- ⁴⁷B. Pekerten, J. D. Pakizer, B. Hawn, and A. Matos-Abiague, "Anisotropic topological superconductivity in Josephson junctions," *Phys. Rev. B* **105**, 054504 (2022).
- ⁴⁸J. D. Pakizer, B. Scharf, and A. Matos-Abiague, "Crystalline anisotropic topological superconductivity in planar Josephson junctions," *Phys. Rev. Res.* **3**, 013198 (2021).
- ⁴⁹B. Scharf, F. Pientka, H. Ren, A. Yacoby, and E. M. Hankiewicz, "Tuning topological superconductivity in phase-controlled Josephson junctions with Rashba and Dresselhaus spin-orbit coupling," *Phys. Rev. B* **99**, 214503 (2019).
- ⁵⁰A. Zagoskin, *Quantum Theory of Many-Body Systems*, 2nd ed. (Springer, New York, 2014).
- ⁵¹D. Van Tuan, B. Scharf, Z. Wang, J. Shan, K. F. Mak, I. Žutić, and H. Dery, "Probing many-body interactions in monolayer transition-metal dichalcogenides," *Phys. Rev. B* **99**, 085301 (2019).
- ⁵²S. J. Papadakis, E. P. De Poortere, H. C. Manoharan, M. Shayegan, and R. Winkler, "The effect of spin splitting of a two-dimensional system," *Science* **283**, 2056 (1999).
- ⁵³F. Dettwiler, J. Fu, S. Mack, P. J. Weigele, J. C. Egues, D. D. Awschalom, and D. M. Zumbühl, "Stretchable persistent spin helices in GaAs quantum wells," *Phys. Rev. X* **7**, 031010 (2017).
- ⁵⁴W. M. J. van Weerdenburg, A. Kamlapure, E. H. Fyhn, X. Huang, N. P. E. van Mullekom, M. Steinbrecher, P. Krogstrup, J. Linder, and A. A. Khajetoorians, "Extreme enhancement of superconductivity in epitaxial aluminum near the monolayer limit," *Sci. Adv.* **9**, eadf5500 (2023).
- ⁵⁵Z. Lin, Z. Li, H. Deng, T. Liu, G. Shi, N. Bonesteel, P. Schlottmann, Y. Li, and P. Xiong, "Giant enhancement of the in-plane critical field for thin Al films via proximity coupling to a topological insulator," *Phys. Rev. B* **102**, 144518 (2020).
- ⁵⁶G. P. Mazur, N. van Loo, J.-Y. Wang, T. Dvir, G. Wang, A. Khindanov, S. Korneychuk, F. Borsoi, R. C. Dekker, G. Badawy, P. Vinke, S. Gazibegovic, E. P. A. M. Bakkers, M. Quintero-Pérez, S. Heedt, and L. P. Kouwenhoven, "Spin-mixing enhanced proximity effect in aluminum-based superconductor-semiconductor hybrids," *Adv. Mater.* **33**, 2202034 (2022).
- ⁵⁷M. Alidoust, C. Shen, and I. Žutić, "Cubic spin-orbit coupling and anomalous Josephson effect in planar junctions," *Phys. Rev. B* **103**, L060503 (2021).
- ⁵⁸M. Alidoust, "Critical supercurrent and ϕ_0 state for probing a persistent spin helix," *Phys. Rev. B* **101**, 155123 (2020).
- ⁵⁹J. Hu, C. Wu, and X. Dai, "Proposed design of a Josephson diode," *Phys. Rev. Lett.* **99**, 067004 (2007).
- ⁶⁰T. Yokoyama, M. Eto, and Y. V. Nazarov, "Anomalous Josephson effect induced by spin-orbit interaction and Zeeman effect in semiconductor nanowires," *Phys. Rev. B* **89**, 195407 (2014).
- ⁶¹N. F. Q. Yuan and L. Fu, "Supercurrent diode effect and finite-momentum superconductors," *Proc. Natl. Acad. Sci. U. S. A.* **119**, e2119548119 (2022).
- ⁶²Y. Hou, F. Nichele, H. Chi, A. Lodesani, Y. Wu, M. F. Ritter, D. Z. Haxell, M. Davydova, S. Ilić, O. Glezakou-Elbert, A. Varambally, F. S. Bergeret, A. Kamra, L. Fu, P. A. Lee, and J. S. Moodera, "Ubiquitous superconducting diode effect in superconductor thin films," *Phys. Rev. Lett.* **131**, 027001 (2023).
- ⁶³D. Kochan, A. Costa, I. Zhumagulov, and I. Žutić, "Phenomenological theory of the supercurrent diode effect: The Lifshitz invariant," *arXiv:2303.11975* (2023).
- ⁶⁴C. Baumgartner, L. Fuchs, A. Costa, S. Reinhardt, S. Gronin, G. C. Gardner, T. Lindemann, M. J. Manfra, P. E. Faria Junior, D. Kochan, J. Fabian, N. Paradiso, and C. Strunk, "Supercurrent rectification and magnetochiral effects in symmetric Josephson junctions," *Nat. Nanotechnol.* **17**, 39 (2021).
- ⁶⁵M. Eschrig, "Phase-sensitive interface and proximity effects in superconducting spintronics," in *Spintronics Handbook Spin Transport and Magnetism*, 2nd ed., edited by E. Y. Tsymlal and I. Žutić (CRC Press, Boca Raton, FL, 2019), Vol. 1, pp. 635–682.
- ⁶⁶B. E. Nadgorny, "Point contact Andreev reflection spectroscopy," in *Spintronics Handbook Spin Transport and Magnetism*, 2nd ed., edited by E. Y. Tsymlal and I. Žutić (CRC Press, Boca Raton, FL, 2019), Vol. 3, pp. 35–97.
- ⁶⁷A. A. Golubov, M. Y. Kupriyanov, and E. Il'ichev, "The current-phase relation in Josephson junctions," *Rev. Mod. Phys.* **76**, 411 (2004).
- ⁶⁸S. Kashiwaya and Y. Tanaka, "Tunnelling effects on surface bound states in unconventional superconductors," *Rep. Prog. Phys.* **63**, 1641 (2000).
- ⁶⁹J. Fabian, A. Matos-Abiague, C. Ertler, P. Stano, and I. Žutić, "Semiconductor spintronics," *Acta Phys. Slovaca* **57**, 565 (2007).
- ⁷⁰F. Li, Y. Takeshita, D. Hasegawa, M. Tanaka, T. Yamashita, and A. Fujimaki, "Low-power high-speed half-flux-quantum circuits driven by low bias voltages," *Supercond. Sci. Technol.* **34**, 025013 (2021).
- ⁷¹A. A. Tosato, V. Levajac, J.-Y. Wang, C. J. Boor, F. Borsoi, M. B. C. N. Borja, S. Marti-Sanchez, J. Arbiol, A. Sammak, M. Veldhorst, and G. Scappucci, "Hard superconducting gap in a high-mobility semiconductor," *Commun. Mater.* **4**, 23 (2023).
- ⁷²P. Wei, S. Manna, M. Eich, P. Lee, and J. Moodera, "Superconductivity in the surface state of noble metal gold and its Fermi level tuning by EuS dielectric," *Phys. Rev. Lett.* **122**, 247002 (2019).
- ⁷³M. Luethi, K. Laubscher, S. Bosco, D. Loss, and J. Klinovaja, "Planar Josephson junctions in germanium: Effect of cubic spin-orbit interaction," *Phys. Rev. B* **107**, 035435 (2023).
- ⁷⁴For GHz switching, the uncertainty in energy is 3×10^{-4} meV $\ll \Delta_{\text{Al}} = 0.2$ meV.
- ⁷⁵S. Das Sarma, M. Freedman, and C. Nayak, "Majorana zero modes and topological quantum computation," *npj Quantum Inf.* **1**, 150001 (2015).
- ⁷⁶D. Aasen, M. Hell, R. V. Mishmash, A. Higginbotham, J. Danon, M. Leijnse, T. S. Jespersen, J. A. Folk, C. M. Marcus, K. Flensberg, and J. Alicea, "Milestones toward Majorana-based quantum computing," *Phys. Rev. X* **6**, 031016 (2016).
- ⁷⁷V. T. Lahtinen and J. K. A. Pachos, "A short introduction to topological quantum computation," *SciPost Phys.* **3**, 021 (2017).
- ⁷⁸Y. J. Rosen, M. A. Horsley, S. E. Harrison, E. T. Holland, A. S. Chang, T. Bond, and J. L. DuBois, "Protecting superconducting qubits from phonon mediated decay," *Appl. Phys. Lett.* **114**, 202601 (2019).
- ⁷⁹P. Bonderson, M. Freedman, and C. Nayak, "Measurement-only topological quantum computation via anyonic interferometry," *Ann. Phys.* **324**, 787 (2009).
- ⁸⁰C. W. J. Beenakker, "Search for non-Abelian Majorana braiding statistics in superconductors," *SciPost Phys. Lect. Notes* **15** (2020).
- ⁸¹P. Zellekens, R. S. Deacon, P. Perla, D. Grützmacher, M. I. Lepsa, T. Schäpers, and K. Ishibashi, "Microwave spectroscopy of Andreev states in InAs nanowire-based hybrid junctions using a flip-chip layout," *Commun. Phys.* **5**, 267 (2022).
- ⁸²P.-H. Fu, Y. Xu, S. A. Yang, C. H. Lee, Y. S. Ang, and J.-F. Liu, "Field-effect Josephson diode via asymmetric spin-momentum locking states," *Phys. Rev. Applied* **21**, 054057 (2024).