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Hierarchical Multi-Layered Sparse Identification for Prediction of Non-Linear Dynamics of Reconfigurable Microgrids

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***Abstract*—The increasing growth of the global energy demand has become one of the key factors that necessitate the development in the field of distributed energy resources (DERs). The interconnection of the DERs with the main grid makes the system extremely sensitive to disturbances, as they decrease the overall system's inertia. These disturbances are primarily caused by variations in the loads connected to the distribution system and environmental conditions affecting DER generation. The system dynamics exhibit strong non-linearity, requiring the use of various controllers to stabilize microgrids and maintain steady-state operation. Predicting future states is challenging yet crucial in power systems, as it enables the determination of control strategies to enhance transient stability following contingencies. This paper proposes a hierarchical multi-layered sparse identification technique to comprehend the system and forecast transient dynamics in different microgrid operation modes. The developed algorithm employs a multi-layered structure, which reduces the overall computational cost while ensuring accurate model dynamics. In the primary layer, the functions influencing the system dynamics are derived from measured data. These primary layer terms are then fitted into the secondary layer to determine the precise system dynamics under different disturbances. Numerical examples have been presented in this paper to validate the effectiveness of the proposed algorithm. The developed method proves particularly valuable in re-configurable and scalable networked microgrids where the system structure and the associated controls frequently change.**

***Index Terms*—Sparse identification, prediction, distributed energy resources, transient dynamics, microgrid**

I. Introduction

A microgrid is a localized energy system that operates independently or in conjunction with the main power grid [1], [2]. It consists of distributed energy resources, such as renewable energy sources (solar panels, wind turbines), energy storage devices (batteries), and local generators (diesel generators, fuel cells) [3]. Microgrids are increasingly being considered as a reliable and cost-effective option for power supply in remote or off-grid areas [4]. An islanded microgrid refers to a standalone power system that is disconnected from the main grid and operates independently. Various control strategies have been proposed to ensure the stability, reliability, and efficiency of islanded microgrids [5]. In an islanded microgrid, there is no direct connection to the main grid, which means there is no external support for voltage and frequency regulation. Control mechanisms are necessary to maintain stable voltage and frequency levels within acceptable limits [6]. A control system can also intelligently distribute the load to prevent overloading or under-utilization of distributed energy resources, thereby optimizing the microgrid efficiency and prolonging its lifespan. Model predictive control is a powerful tool to address the new control challenges that appear in microgrids [7].

Obtaining a prediction model for developing a reliable model predictive controller can present several challenges. Prediction of future states is a difficult, but an essential tool in power systems for determining different control strategies that can aide in maintaining the transient stability of the overall system following a contingency. Building an accurate prediction model that captures the dynamics of the system can be challenging, especially for complex systems nonlinearities, uncertainties, and interactions between multiple variables. Acquiring the data necessary for model identification can be challenging. Sufficient and representative data must be collected from the system under various operating conditions. Real-world systems often have uncertainties and measurement noise, which can affect the accuracy of the prediction model. Uncertainties may arise from unknown or immeasurable disturbances, model errors, or parameter variations [9]. Data-driven modeling is important for prediction because it allows us to build models that can capture the underlying patterns and relationships in the data without requiring a detailed understanding of the underlying physical processes or mechanisms [10]–[13], [19].

While it is advantageous to model complex systems whose underlying physics and mechanisms are not well understood or are difficult to model explicitly, data-driven models may not be able to capture the underlying physics accurately without abundance of data [14]. A physics-informed data-driven model is a combination of physics-based modeling and data- driven modeling. It incorporates prior knowledge and assumptions about the underlying physics or mechanisms into the model construction process, while also using data to refine and validate the model. This approach aims to improve the accuracy and physical consistency of the model by leveraging the strengths of both approaches.

The purpose of developing mixed approaches is to improve the reliability of the obtained relations through fundamental principles. A physics informed neural network with sparse regression has been discussed in [15] which possesses the salient features of interpretability and generalizability, to discover governing PDEs of nonlinear spatiotemporal systems from scarce and noisy data. Another advantage of physics informed data-driven modeling is its flexibility and adaptability. It can be easily updated and retrained with new data, allowing them to adapt to changing conditions and environments. They can also be easily integrated with other models or systems, such as control systems or optimization algorithms [16]. Developing a reliable data-driven model involves using statistical and machine learning techniques in combination with a physics based approach to extract patterns and relationships can be used to make future predictions. These models are often based on simple and interpretable functions, such as linear or nonlinear regression models, decision trees, or neural networks [17]. In the context of predictive models, computational complexity can be a significant factor to consider, as it can impact the feasibility and real-time applicability of the model [18].

Combining physics-informed model with sparse identification for non-linear dynamics is a topic that is currently being explored in multiple fields. The work in [21] shows that the learned relationships can be utilized as a surrogate model which, unlike typical data-driven surrogate models, relies on the learned underlying dynamics of the system rather than large number of fitting parameters. A structure-preserving neural SINDy algorithm has been presented is [22]. A physics-Guided Sparse Identification of Nonlinear Dynamics for Prediction has been developed specifically for application to Vehicle Cabin Occupant Thermal Comfort in [23]. The precise identification of bistable nonlinear stiffness force using the algorithm proposed in [24] is used to predict and enhance the system performance of the vibration energy absorption. The existing physics informed identification methods have been tailored to solve the prediction problem for different application domains using some basic knowledge of the system considered in the specific domains. Hence, it cannot be directly extended for identification of microgrid dynamics. Drawing inspiration from the aforementioned works, a physics informed sparse identification has been proposed in this paper for application in the power systems domain.

The major contributions of this paper can be listed as:

1. A hierarchical multi-layered sparse identification algorithm is developed for power systems application to capture the non-linearities in the microgrid transient dynamics. This method is successful in predicting the future operating states of the microgrid model under various disturbances.
2. While the offline training and identification of the model has a high computational cost, it has been verified that the validation and prediction of the model is much more cost effective and can be considered as an improved step towards developing a reliable model predictive controller.

The remainder of the paper is organized as follows. Section II outlines the different mathematical concepts that drive the design and development of the prediction model. It includes the physics behind the selection of a multi-layered hierarchical sparse identification method. Section III provides some numerical examples to validate the efficiency of the proposed algorithm. Conclusions are drawn in Section IV.

II. Hierarchical Multi-Layered Sparse Identification

A. *Essential idea of hierarchical SINDy*

In the hierarchical multi-layered Sparse Identification of Non-linear Dynamics (SINDy)-based identification method, two layers of sparse identifications are used to develop the data-driven identification model. The first layer is used to create different standard knowledge power system terms which are primarily responsible for the non-linearities in the overall system. The second layer uses the knowledge of the terms identified in the primary layer to fit the dynamics and obtain the set of all the differential equations that describe the transients in the system.

The transient dynamics of the microgrid model can be mathematically described by a set of differential-algebraic equations (DAEs),

$$\dot{\mathbf{x}}(t) = \mathbf{f}\big(\mathbf{x}(t), \mathbf{y}(t), \mathbf{u}(t)\big), \tag{1a}$$

$$\mathbf{0} = \mathbf{g}\big(\mathbf{x}(t), \mathbf{y}(t), \mathbf{u}(t)\big), \tag{1b}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state variable vector, e.g., state variables in the controller of DER power-electronic interfaces, $\mathbf{y} \in \mathbb{R}^m$ is the algebraic variable vector, e.g., bus voltage amplitude and angle, and $\mathbf{u} \in \mathbb{R}^p$ represents the input variations/ disturbances, e.g., power output fluctuation of PV and power load changes. Before moving on to detailed explanation of the hierarchical SINDy model, the preliminary knowledge of some of the non-linear terms in a microgrid are given below:

B. *Trigonometric non-linearities*

The dq transformation is used to convert a three-phase signal into two orthogonal components, namely the d-axis and the q-axis, which simplifies the analysis and control of three-phase systems. The d-axis component is proportional to the average value of the three-phase signal, while the q-axis component is proportional to the quadrature component of the three-phase signal. This consists of cosine and sine terms - *trigonometric non-linearities*.

The dq transformation is a mathematical tool used in power systems to convert a set of voltage or current phasors in a rotating reference frame into a two-dimensional space in the dq reference frame.

$$\begin{bmatrix} d \\ q \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_{\mathrm{R}} \\ y_{\mathrm{I}} \end{bmatrix} \tag{2}$$

Here, x_{R} and y_{I} denote the real and imaginary components of the phasors respectively. The dq transformation allows for easier control of the power electronic converters, as it separates the control of the active and reactive power flows. The d-axis component is responsible for controlling the active power flow, while the q-axis component controls the reactive power flow. This allows for more efficient and effective control of the power flow, leading to better performance and stability of the microgrid.

This transformation can be used to obtain the d- and q-axis components of the voltage and current phasors.

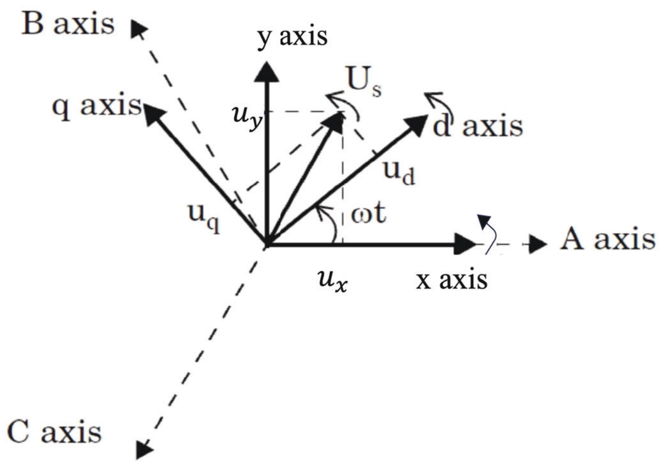

Fig. 1. Reference frame for voltage and current phasors transformation

Here, x_{R} represents the real part of the phasor and y_{I} represents the imaginary part of the phasor. θ represents the phasor angle from the phase locked loop.

C. Quadratic non-linearities

The voltage magnitude is another important parameter that is used to describe the state of the system. The non-linearity of this term arises from the following equation formulation:

$$V_m = \sqrt{V_x^2 + V_y^2} \tag{3}$$

The *quadratic term (polyorder = 2)* contributes to the non-linearity.

The input data required for the primary layer of the hierarchical SINDy model are the system state variables data ($\mathbf{x}(t)$) and the output bus voltage ($\mathbf{v}(t)$) and line current data ($\mathbf{i}(t)$). This is given by Eq. 4.

$$\mathbf{X_{L1}} = \begin{bmatrix} \mathrm{x}_1(t_1)\cdots\mathrm{x}_n(t_1) & \mathrm{v}_1(t_1)\cdots\mathrm{v}_p(t_1) & \mathrm{i}_1(t_1)\cdots\mathrm{i}_p(t_1) \\ \mathrm{x}_1(t_2)\cdots\mathrm{x}_n(t_2) & \mathrm{v}_1(t_2)\cdots\mathrm{v}_p(t_2) & \mathrm{i}_1(t_2)\cdots\mathrm{i}_p(t_2) \\ \vdots & \vdots & \vdots \\ \mathrm{x}_1(t_m)\cdots\mathrm{x}_n(t_m) & \mathrm{v}_1(t_m)\cdots\mathrm{v}_p(t_m) & \mathrm{i}_1(t_m)\cdots\mathrm{i}_p(t_m) \end{bmatrix} \tag{4}$$

Candidate functions are the set of different possible non-linear functions that are typically used to represent the dynamics of the desired application domain. The library of candidate functions for the first layer of the heirarchical SINDy can be established as:

$$\Theta_{L1}(\mathbf{X},\mathbf{V},\mathbf{I}) = [\begin{matrix} \mathbf{1} & \mathbf{X} & \mathbf{X}^{P^2} & \cdots \\ \sin(\mathbf{X}) & \cos(\mathbf{X}) & \mathbf{V} & \mathbf{XV} \\ \sin(\mathbf{V}) & \cos(\mathbf{V}) & \cdots & \\ \mathbf{I} & \mathbf{XI} & \cdots & \\ \sin(\mathbf{I}) & \cos(\mathbf{I}) & \cdots & \\ \mathbf{X}\sin(\mathbf{X}) & \cdots & & \\ \mathbf{I}\sin(\mathbf{X}) & \mathbf{V}\sin(\mathbf{X}) & \cdots & \end{matrix}] \tag{5}$$

Let the non-linear functional space of terms be defined by $\mathbf{\Phi}$.

$$\mathbf{\Phi} = \Xi_{L1}\Theta_{L1}^{\mathsf{T}}(\mathbf{X},\mathbf{V},\mathbf{I}) \tag{6}$$

Sparse Regression is performed to identify these special non-linear terms using input data sent to the first layer of the SINDy algorithm. Eq. (7). represents the set of sparse coefficients that can fit the overall dynamics of the system with the proposed candidate functional space. Eq. (8). provides the details of the regression method used in this paper.

$$\Xi_{L1} = [\xi_{l11} \quad \xi_{l12} \quad \cdots \quad \xi_{l1n}] \tag{7}$$

$$\xi_k = \arg\min_{\xi_k'} ||\mathbf{\Theta}\xi_k' - \mathbf{\Phi_k}||_2 + \lambda||\xi_k'||_1, \tag{8}$$

In addition to the already gathered system state variables data, the identified non-linear functional data, represented by $\mathbf{\Phi}$ are added to develop the input data for the second layer of the heirarchical SINDy model. Since, the non-linearities are already identified in the previous layer, the computational effort of the second layer can be significantly reduced. This is because, the simple case of polyorder '1' can be chosen to build the candidate functions.

$$\Theta_{L2}(\mathbf{X},\mathbf{\Phi},\mathbf{U}) = [1 \quad \mathbf{X} \quad \mathbf{\Phi} \quad \mathbf{U}] \tag{9}$$

Here, $\mathbf{X}$ is the set of all the state variables. $\mathbf{\Phi}$ is the output of the first layer of the developed model which comprises of the algebraic terms contributing to the non-linearity in the model. $\mathbf{U}$ is the external disturbances that drives the system dynamics.

$$\dot{\mathbf{X}} = \Xi_{L2}\Theta_{L2}^{\mathsf{T}}(\mathbf{X},\mathbf{\Phi},\mathbf{U}) \tag{10}$$

Ξ_{L1} and Ξ_{L2} are the sparse vector coefficients identified by regression. Multiple regression methods such as Ordinary Least Squares (OLS) regression, Ridge regression, Least Operator Shrinkage and Selection Operator (LASSO) can be used. The LASSO-type optimization problems can be solved by using various proximal Newton methods which are also used for solving convex composite optimization problems [20].

TABLE I
DER GENERATIONS AT EACH BUS

Bus	$P_n(kW)$	$Q_n(kVAR)$
1	62.9	77.8
3	80.2	30.5
6	40.5	0

TABLE II
POWER LOADS AT EACH BUS

Bus	$P_n(kW)$	$Q_n(kVAR)$
2	12.7	7.9
5	42.5	26.3
7	61.1	37.9
8	40.0	24.8
9	12.7	7.9

This paper utilizes the LASSO regression technique which is outlined by Eq. (7) in which Λ gives the sparsity constraint. The differential equations of the microgrid DAE model are identified using the heirarchical multi-layered SINDy algorithm. In this work, the network topology details are assumed to be known to the system operator and the algebraic part can be modeled based on the prior knowledge.

III. NUMERICAL EXAMPLES

The microgrid system used for verifying the identification and prediction of the transient dynamics using hierarchical multi-layered sparse identification is given in Fig. 2. The circuit breaker 1 in the test system is open making the microgrid operate in islanded mode. The three generation units in this system are the micro-turbine, battery storage and PV system connected as buses 1,3 and 6 respectively. Constant impedance load is considered at buses 2,5,7,8 and 9. The system topology, the generation details and the load details are given in Table I, II and III. Multiple disturbances are introduced in the system to understand the transient dynamics of the model which is used for preparing the training data. The details of the disturbances used for training the model are provided in Table IV. The input disturbances used in the prediction model is given by Table V.

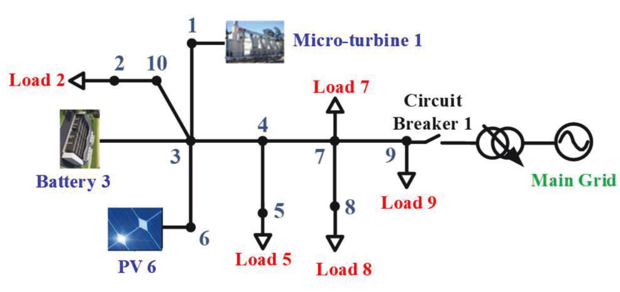

Fig. 2. 10 bus test system with 3 generation units

The circuit breaker connecting the microgrid with the main grid is open. Thus, the simulated microgrid is being operated

TABLE III
LINE IMPEDANCE BETWEEN BUSES

From	To	R(Ω/km)	L(H/km)	Length(m)
10	2	0.0153	1.039×10^{-6}	45
10	3	0.0020	1.606×10^{-6}	30
3	1	0.0086	1.360×10^{-6}	30
3	6	0.0096	3.761×10^{-6}	50
3	4	0.0024	1.927×10^{-6}	50
4	5	0.0032	8.061×10^{-6}	50
4	7	0.0041	3.212×10^{-6}	45
7	8	0.0345	2.338×10^{-6}	20
7	9	0.016	4.449×10^{-6}	20

in the islanded mode. Fig. 3 and Fig. 4 provides the plot to show the effectiveness of the training and testing of the proposed algorithm. The observable output variables of interest are mainly the bus voltage magnitude and phase angle. The grid forming generator's bus voltage magnitude is depicted in these figures. The identified data closely follows the training data-set with minimal error.

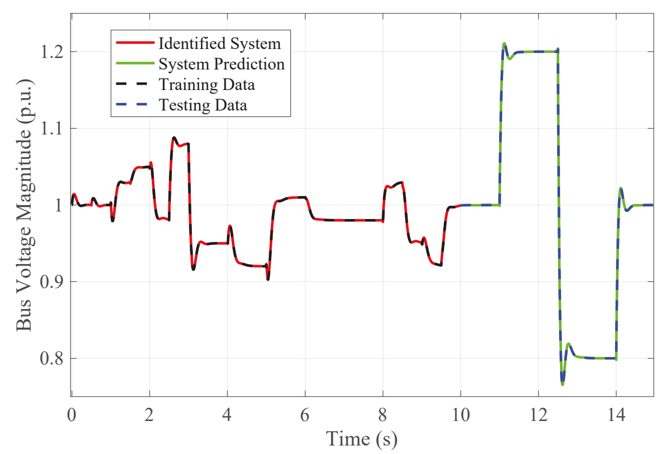

Fig. 3. Voltage Magnitude Training and Prediction - Grid forming generator bus

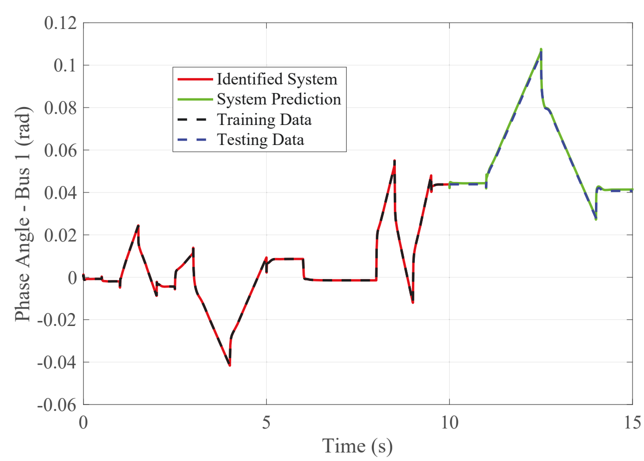

Fig. 4. Voltage Angle Training and Prediction - Grid forming generator bus

TABLE IV
INPUT DISTURBANCES - TRAINING

Time (s)	V_{ref}	$\Delta\omega_{\text{ref}}$	P_{ref}	Q_{ref}
0.0 - 1.0 s	1.0	0.0	-10%	$+20\%$
1.0 - 1.5 s	1.03	0.05	-30%	$+30\%$
1.5 - 2.0 s	1.05	-0.05	$+1\%$	-80%
2.0 - 2.5 s	0.98	0.0	-15%	$+10\%$
2.5 - 3.0 s	1.08	0.02	-60%	$+10\%$
3.0 - 3.5 s	0.95	-0.04	-40%	$+10\%$
3.0 - 4.0 s	0.95	-0.04	-40%	$+10\%$
4.0 - 5.0 s	0.92	0.04	$+10\%$	$+80\%$
5.0 - 6.0 s	1.01	0.0	-60%	-40%
6.0 - 8.0 s	0.98	0.0	-90%	-80%
8.0 - 8.5 s	1.03	0.07	$+30\%$	$+40\%$
8.5 - 9.0 s	0.95	-0.08	-50%	-70%
9.0 - 9.5 s	0.92	0.08	$+40\%$	$+50\%$
9.5 - 10.0 s	1.0	0.0	-20%	0%

TABLE V
INPUT DISTURBANCES - PREDICTION

Time (s)	V_{ref}	$\Delta\omega_{\text{ref}}$	P_{ref}	Q_{ref}
10.0 - 11.0 s	1.0	0.0	-60%	$+5\%$
11.0 - 12.5 s	1.2	0.04	$+10\%$	$+20\%$
12.5 - 14.0 s	0.8	-0.04	$+5\%$	-15%
14.0 - 15.0 s	1.0	0.0	-40%	$+85\%$

The trigonometric and quadratic transformations of the voltage and line current data are obtained from the primary layer of the proposed method. The secondary layer fits the non-linear functions to determine the transient dynamics. It can be observed that the effect of the dynamics propagate to the algebraic network constraints, i.e. the bus voltage magnitudes and phase angles obtained through power flow.

Based on the identified model, new input disturbances are provided after $t = 10s$ to verify the prediction capabilities of the proposed data-driven technique. The same input disturbances are used to develop a test data set using the actual microgrid model. Satisfactory performance is observed through the prediction results.

It can be seen that the loads are connected to buses 2, 5, 7, 8 and 9. The input disturbances are usually provided in the form of load changes and generation changes. Thus, it is important to observe the tracking performance of the bus voltages at the load buses as well. Fig. 5 provides the plot consisting of the voltage magnitude at the load bus 2. It can be seen that the predicted data is very close to the testing data under constant load and power generation variations. This work utilizes a constant impedance load model. As per the authors' knowledge, constant impedance, constant current and constant power loads can also be leveraged in this identification algorithm to develop a satisfactory prediction that can be used for controlling the microgrid's future operations.

The bus voltage magnitude and phase angles can be obtained through the measurement devices. While, it is not common to measure all the bus voltages, this work presumes the localized measurement of all the bus voltages and line currents. The state variables are not easily obtainable in real-time applications. In this work, we assume the availability of all the state variables data to further validate the proposed algorithm. In an inverter

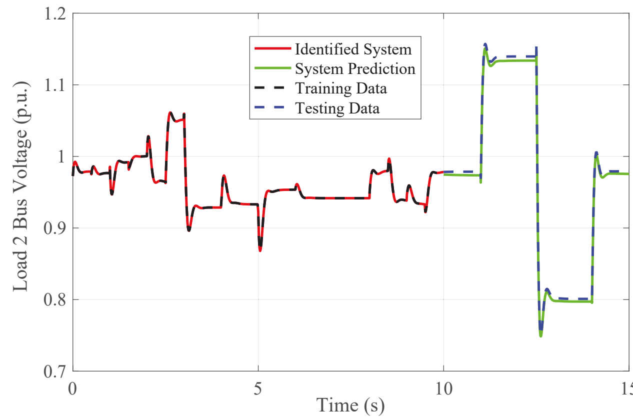

Fig. 5. Training and prediction results of the load bus voltage

based integration, the PLL output and the dq-axis modulation indices are some of the most dominant state variables and the training and prediction results of these states are provided to elucidate the effectiveness of the developed prediction model.

The effective prediction of the dominant state variable can be seen in Fig. 6. The average root mean square error between the training data and the identified data was found to be around $\pm 1.79e^{-4}\%$ across all the state variables. Similarly, the average root mean square error between the testing data and the predicted data was also calculated and was found to be around $\pm 3.46e^{-2}\%$ across all the state variables. The comparison of the proposed algorithm with SINDy is shown in Fig. 7. It can be seen from the plot, that the identification and the prediction of the proposed method performs significantly better than the SINDy algorithm with the given set of data.

Another advantage of the proposed method is based on the computational resource requirement. To successfully train $10s$ of the model in MATLAB, $137.593s$ of simulation time was required. To predict the model dynamics for $5s$, the simulation time required was $7.469s$. Thus, it can be concluded, that the model can be trained offline and the developed data-driven model has can operate in a range close to real-time operation which is advantageous while controlling the system in real-time.

While the proposed algorithm is advantageous to determine the dynamics of a microgrid subject to changes, the authors observed a significant increase in the amount of data required as the system complexity increases and can possibly pose some challenges in the data collection process.

IV. CONCLUSION

A hierarchical multi-layered sparse identification algorithm has been developed in this paper to capture and predict the non-linear transient dynamics of the microgrid. Numerical results have been provided to elucidate the successful training and prediction of the operating states and output variables of the microgrid model under varying input disturbances. An accurate prediction model is advantageous while developing model predictive controller as it enables the anticipation of

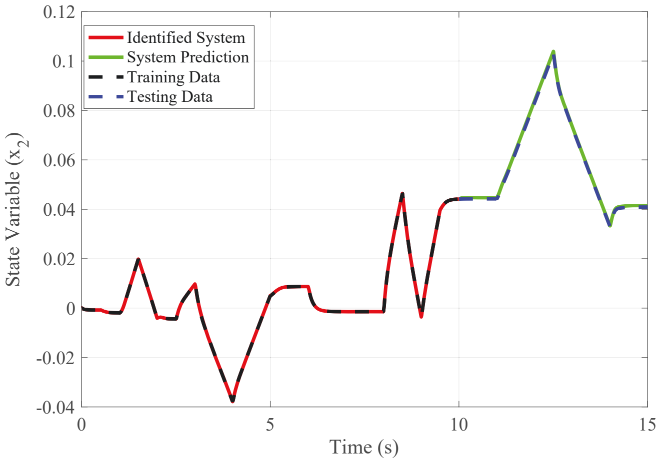

Fig. 6. PLL phase angle training and prediction data

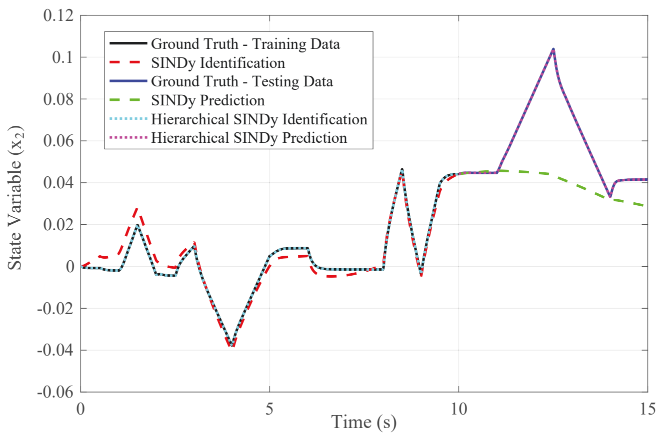

Fig. 7. Training and prediction of the inverter side modulation index

the future behavior of the microgrid system. To further elucidate the advantages of the proposed method, the authors' future work comprises of comparison with various network topologies to verify the statistical significance of this method.

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