Nonlocal Mueller Polarimetry

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ABSTRACT

We present a method to determine the Mueller matrix of a sample using polarization-entangled photon pairs. One of the photons of a pair goes through a sample and is then subject to a polarization projection measurement. The other photon, which does not go through the sample, is also subject to a polarization projection. The measured quantum correlations are equivalent to polarimetry measurements, where the initial state of the photon going through the sample is determined by the polarization projection on the entangled partner that does not go through the sample. The correspondence with the classical system is acausal because quantum measurements apply to distinct Hilbert spaces. We tested this method with standard optical elements finding excellent agreement with the expectations. Thus it can be used as an alternative to classical Mueller polarimetry for conditions that would be challenging to do otherwise.

Keywords: Mueller polarimetry, Mueller matrix, Polarimetry, Entangled Photons, Nonlocality

1. INTRODUCTION

Non-locality is one of the most striking non-classical aspects of quantum mechanics. It is subtly manifested by the instantaneous correlation in the projection of the state of two particles in a non-separable or entangled state. This property of quantum mechanics was keenly recognized by Einstein in the early days of quantum mechanics, and challenged in the landmark paper by Einstein, Podolsky and Rosen in 1935[1] with the thought experiment involving the entanglement of the position and momentum of two particles. Since then, non-locality has been one of the mysteries of quantum mechanics. It became testable by Bell's proposals in 1964,[2] and tested, ever more stringently with polarization-entangled photons, initially by Aspect in 1982[3] and more recently by various collaborations in 2020.[4,5] The tests have always ruled out local realism, vindicating the predictions of quantum mechanics. Verifications of the violations of Bell inequalities are now integral parts of quantum mechanics textbooks and college laboratory exercises.[6]

With quantum principles now taking a firm entry into real-world applications, such as communications, security, and computation, it begs the question if there is a role for non-locality in metrology. There is, but perhaps not as one would think from a classical perspective. Because non-locality involves two projective measurements of the state of two particles, the outcome is obtained when the measurements on both particles are completed.

Non-locality involves the instantaneous correlation of projective measurements of particles prepared in an entangled state. It is often incorrectly assumed that non-locality involves a causal order. It does not. It involves separate measurements of the state of two entangled particles. The particles occupy separate Hilbert spaces, and consequently, the outcome is the same regardless of the order in which the measurements are made. Interpreting non-local measurements in a classical deterministic and causal way tends to produce paradoxes that are merely the result of making an incorrect parallel between two distinct physical situations. Post-selection is a frequent approach used within the context of quantum measurements. That is, a measurement decides what happened in the experiment, which is contrary to the deterministic and causal way of thinking in classical mechanics, where actions in a measurement follow a sequential order. Along the same lines, we can devise a type of metrology where the measurement decides what phenomenon is being tested and bypass classical causality.

The use of polarization correlations to do a non-local type of measurement was first reported on ellipsometric measurements,[7,8] a technique to obtain the properties of a reflecting surface based on the transformations

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it imparts on the polarization state of the input light. In this article, we report on another non-local type of polarization metrology: Mueller polarimetry. This is a mature classical technique that determines the polarization properties of an object.[9,10] With light's polarization fully described by Stokes parameters, represented by a 4-vector, the Mueller matrix is a 4×4 matrix that transforms an initial Stokes vector into another after the incident light interacts with the object, either by transmission or reflection. The classical method of determining the Mueller matrix is shown in Fig. 1(a), where the light incident with well-defined polarization is sent to interact with an object (a transmissive one in the figure), and the output polarization is measured. Determining the 16 elements of the matrix requires a minimum set of 16 measurements, with outcomes reduced by linear regression. Oversampling with a larger-than-minimum set of measurements allows one to determine the elements directly from measurements,[11] as shown in Table 1. There are numerous methods.[9,10]

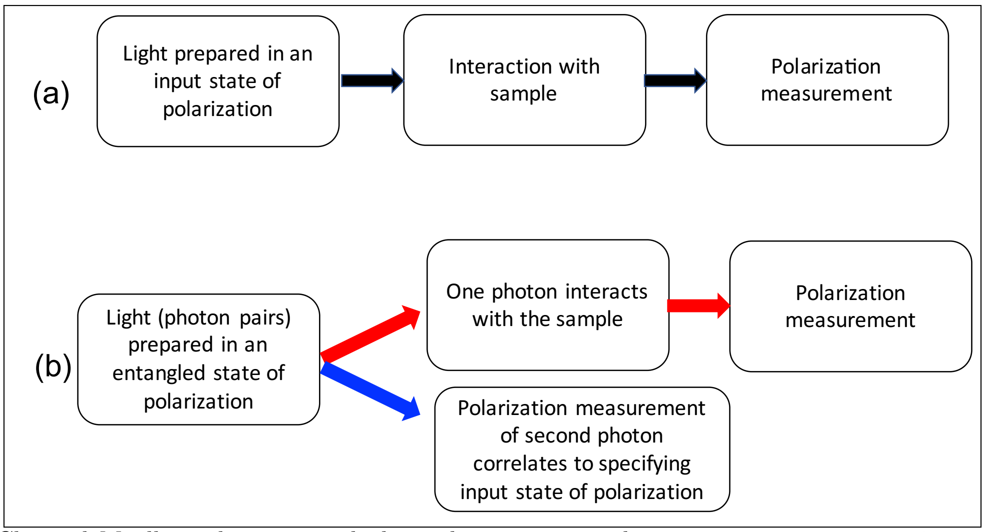

Figure 1. (a) Classical Mueller polarimetry, which involves 3 sequential steps: state preparation, interaction with the sample, and polarization-projective measurement. (b) Non-local Mueller polarimetry, where the correlation between the projective polarization measurements of the two photons works "as if" the measurement of one photon determines the initial state of the photon going through the sample.

By using polarization-entangled photons, the correlated measurement outcomes are equivalent to splitting the polarimetry into two parts, shown in Fig. 1(b): projective measurements on the photon not going through the sample correlated with measurements on the photon going through the sample. The correlated outcome works "as if" the measurement of the photon not going through the sample determines the initial state of the photon going through the sample. Because the correlation is independent of the order of the measurements, the measurement of the photon going through the sample can occur well before the other photon is measured. A causal interpretation would lead to a nonsensical action in the past. Because the correlation involves particles occupying distinct Hilbert spaces, the order of the measurements does not affect the outcome. It is a situation where acausal quantum measurements are identical to the outcomes of a causal classical measurement, yet they are physically distinct situations.

In this article, we apply this concept to Mueller polarimetry. In section 2, we present the basic method, and in Sec. 3, we present measurements of known and unknown objects.

2. METHODS

Polarization-entangled photons can be prepared in one of the maximally-entangled, "Bell," states:

$$\left|\Phi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle \pm |VV\rangle\right) \quad (1)$$

$$\left|\Psi^{\pm}\right\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle \pm |VH\rangle\right) \quad (2)$$

where the labels H and V denote horizontal and vertical polarization, respectively. They can be used in a form of non-local Mueller polarimetry. The various Bell states involve non-separable superpositions of states of polarization with equal probability, also known as maximally entangled states. Our apparatus is shown in Fig. 2. It uses two type-I beta-barium borate (BBO) crystals to prepare photons in state $|\Phi^+\rangle$. Because the other Bell states are prepared with additional optical elements, most of our data involves state $|\Phi^+\rangle$. The symmetry of this state is such that it leads to correlation in the photon's polarization in any linear basis, but anti-correlation in the circular basis. That is, the same state in the two other popular bases (D, A) and (R, L), is given by

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}\left(|DD\rangle + |AA\rangle\right) = \frac{1}{\sqrt{2}}\left(|RL\rangle + |LR\rangle\right), \tag{3}$$

where $|D\rangle = 2^{-1/2}(|H\rangle + |V\rangle)$ and $|A\rangle = 2^{-1/2}(|H\rangle - |V\rangle)$ stand for diagonal (+45° relative to the horizontal) and anti-diagonal (−45° relative to the horizontal) states, respectively; and $|R\rangle = 2^{-1/2}(|H\rangle - i\,|V\rangle)$ and $|L\rangle = 2^{-1/2}(|H\rangle + i\,|V\rangle)$ the right and left circular polarization states, respectively.

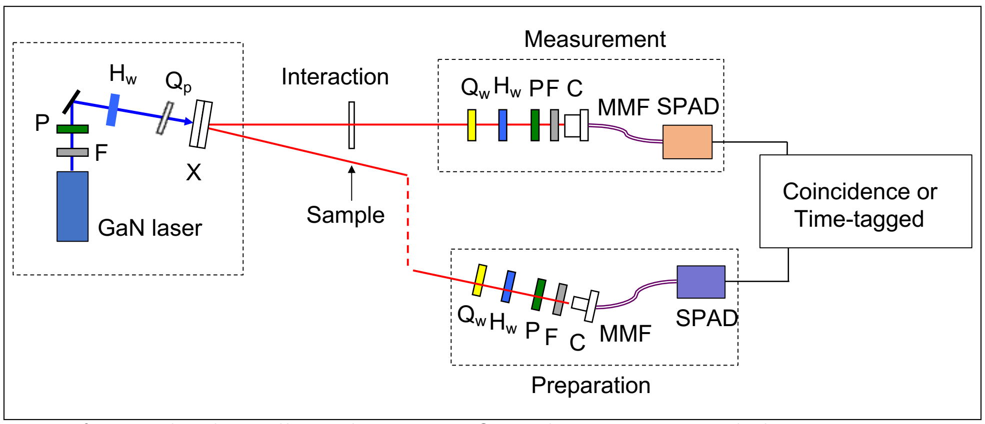

Figure 2. Apparatus for non-local Mueller polarimetry. Optical components include: spontaneous parametric down-conversion crystal (X), band-pass filters (F), polarizers (P), half-wave plates ($\mathrm{H_w}$), quarter-wave plates ($\mathrm{Q_w}$), fiber collimators (C), multimode fiber (MMF), single-photon detectors (SPAD).

Table 1. Table of measurements that yield the Mueller matrix elements directly using classical polarimetry. For simplicity of presentation we denote the Intensity I_{WU} by WU, where the polarization of the input light is W and the polarization of the transmitted light is U.

m_{11}	m_{12}	m_{13}	m_{14}
$HH+HV$ $+VH+VV$	$HH+HV$ $-VH-VV$	$DH+DV$ $-AH-AV$	$RH+RV$ $-LH-LV$
m_{21}	m_{22}	m_{23}	m_{24}
$HH-HV$ $+VH-VV$	$HH-HV$ $-VH+VV$	$DH-DV$ $-AH+AV$	$RH+LV$ $-LH-RV$
m_{31}	m_{32}	m_{33}	m_{34}
$HD-HA$ $+VA-VD$	$AA-AD$ $-DA+DD$	$AA-AD$ $-DA+DD$	$RD+LA$ $-RA-LD$
m_{41}	m_{42}	m_{43}	m_{44}
$HR+VR$ $-VL-HL$	$HR-VR$ $-HL+VL$	$AL-AR$ $+DR-DL$	$LL-LR$ $-RL+RR$

The Mueller matrix elements can be determined directly classically by preparing the light in one of the 6 states (H, V, D, A, R and L), and projecting the state of the transmitted light with each of them as well, as prescribed by Table 1.[11]

With polarization-entangled photons, the preparation and measurement sections are split between the two photons, as depicted in Fig. 1(b). We will refer to photon 1, the one that goes straight to the measurement section, and photon 2, the one that goes through the sample and then the measurement. The linear polarizations (H, V, D, A) are correlated (parallel). So the projection of a linear state, such as $|D\rangle$ on photon 1, correlates with the input polarization on the sample being state $|D\rangle$. If the projected state on photon 1 is one of the circular states, such as $|L\rangle$, then because the photons are anticorrelated in the circular basis (orthogonal), the results correlate with the input state on the sample being state $|R\rangle$. In table 2, we give the correlations for the four Bell states of Eqs. 1 and 2 in the three mutually unbiased bases: (H, V), (D, A) and (R, L). For example, if the initial state is $|\Psi^-\rangle$, the "singlet" state, then all the states are uncorrelated, and we would have to substitute in Table 1 H for V, V for H, D for A, A for D, R for L, and L for R for the projected state of photon 1 in all measurements, with the projections for photon 2 are left unchanged.

Table 2. Table of correlations of photon polarizations in the three mutually unbiased bases: (H, V), (D, A) and (R, L). Correlated means that the polarization states of the two photons are parallel, and uncorrelated means that the polarization states of the two photons are orthogonal.

Bell State	(H, V) Basis	(D, A) Basis	(R, L) Basis
Φ^+	correlated	correlated	anticorrelated
Φ^-	correlated	anticorrelated	correlated
Ψ^+	anticorrelated	correlated	correlated
Ψ^-	anticorrelated	anticorrelated	anticorrelated

The nonlocal measurements that directly give the unnormalized Mueller matrix with the correlations corresponding to state $|\Phi^+\rangle$, where linear states are correlated (parallel), and circular states are uncorrelated (orthogonal), are given in Table 3. The difference with Table 1 is that all the circular states for photon 1 have been switched to the orthogonal state.

Table 3. Table of measurements that yield the Mueller matrix elements directly using state $|\Phi^+\rangle$ as the entangled state. For simplicity of presentation we denote the photon counts N_{WU} by WU, where photon 1 is projected to state $|W\rangle$ and photon 2 to state $|U\rangle$.

m_{11}	m_{12}	m_{13}	m_{14}
$HH+HV$ $+VH+VV$	$HH+HV$ $-VH-VV$	$DH+DV$ $-AH-AV$	$LH+LV$ $-RH-RV$
m_{21}	m_{22}	m_{23}	m_{24}
$HH-HV$ $+VH-VV$	$HH-HV$ $-VH+VV$	$DH-DV$ $-AH+AV$	$LH-LV$ $-RH+RV$
m_{31}	m_{32}	m_{33}	m_{34}
$HD-HA$ $+VA-VD$	$AA-AD$ $-DA+DD$	$AA-AD$ $-DA+DD$	$LD-LA$ $+RA-RD$
m_{41}	m_{42}	m_{43}	m_{44}
$HR-HL$ $+VR-VL$	$HR-HL$ $-VR+VL$	$AL-AR$ $+DR-DL$	$RL-RR$ $-LL+LR$

A schematic of the apparatus is shown in Fig. 2. The quantum state was prepared in a standard way,[12] starting with an input pump laser beam at 405 nm, which was the output of a Toptica single-mode diode laser. The linear polarization of the pump beam was adjusted to be approximately 45 degrees to the horizontal using a half-wave plate so that the pair of thin BBO crystals produced photon pairs in states $|HH\rangle$ and $|VV\rangle$ with equal amplitudes via spontaneous parametric down-conversion. A tilted quartz crystal before the BBO crystals compensated the temporal and phase walkoffs of the light produced unsymmetrically by the BBO crystals to yield the pairs in state $|\Phi^+\rangle$. Using the quantum correlations just mentioned we recorded the results of 36 measurements. Because we need to detect photon pairs, we recorded coincidence counts N_{WU} when photon 1 was detected in state $|W\rangle$ and photon 2 was detected to be in state $|U\rangle$. The unnormalized Mueller matrix is shown in Table 3. Although we prepared the photons in this state using individual components, such entangled-photon source is available commercially as a unit.

After the production, photon 1 went straight to a polarization projection section that consisted of three optical components: a quarter-wave plate, a half-wave plate, and a fixed Thompson prism-polarizer. We selected to detect the photons with wavelength about their degenerate energy at 810 nm. A 40-nm bandpass filter centered at 810 nm allowed the passage of down-converted photons into a fiber collimator. The photons were channeled by multimode fibers to single-photon avalanche diode detectors (SPAD).

Photon 2 was transmitted by the sample, and its polarization state was projected with a setup similar to the one used for photon 1. The detector pulses were collected by an electronic unit recording the coincidence counts, or events where photon pulses arrived within 40 ns of each other. The mounts of the waveplates of the two photons were motorized so that all measurements were made in an automated fashion. We note that is not necessary to combine the electrical signals of the two photons into a coincidence circuit. It is only necessary to know when the electronic pulses were generated. This way, time-stamping pulses allow the selection of the signals from photon pairs by software rather than live electronic processing.

3. RESULTS

3.1 Verification

We verified this method by doing Mueller polarimetry of known optical elements. For a quarter-wave plate at 45° it is given by:

$$M_{\text{expected}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{-1} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{4}$$

The corresponding measured matrix is:

$$M_{\text{measured}} = \begin{pmatrix} 1.000(10) & 0.011(7) & 0.014(7) & -0.020(7) \\ 0.018(8) & 0.079(7) & -0.048(7) & -0.965(9) \\ 0.023(7) & 0.124(7) & 0.934(9) & -0.138(7) \\ 0.040(7) & 0.978(9) & -0.083(7) & 0.085(7) \end{pmatrix}. \tag{5}$$

Figure 3 gives a pictorial view of the matrix. It shows 3D bar graphs of the expected and measured Mueller matrices of the quarter wave plate, where the height of the bars correspond to the value of the matrix element.

Figure 4 shows a comparison of a half-wave plate with a fast axis forming 45° relative to the horizontal. The two waveplates were commercial zero-order for a design wavelength of 808 nm. Figure 5 gives the comparison for a Glan-Thompson polarizer. As can be seen pictorially, the agreement is excellent. We have tried several other cases, just to be sure, and in all cases, there is agreement. We have not made a direct comparison with polarimetry done by the classical method because it was difficult to have a classical source with the same center wavelength and bandwidth. Also, the article's point is not that non-local polarimetry is competing for a better result with the classical method but rather as an alternative for situations where classical Mueller polarimetry is challenging to set up.

3.2 Potential Uses

The present technique has two advantages over the classical method. One is the physical division of components. Since the initial state is provided by the projection of a photon that does not go through the sample. the associated hardware does not need to precede the sample, as in the classical case. This capability may be advantageous when geometry and limited space restrict the placement of optical elements. For example, using entangled photons channeled through optical fibers, one of the fibers could be embedded in the sample, so the initial state is obtained by the correlations with the partner photon.

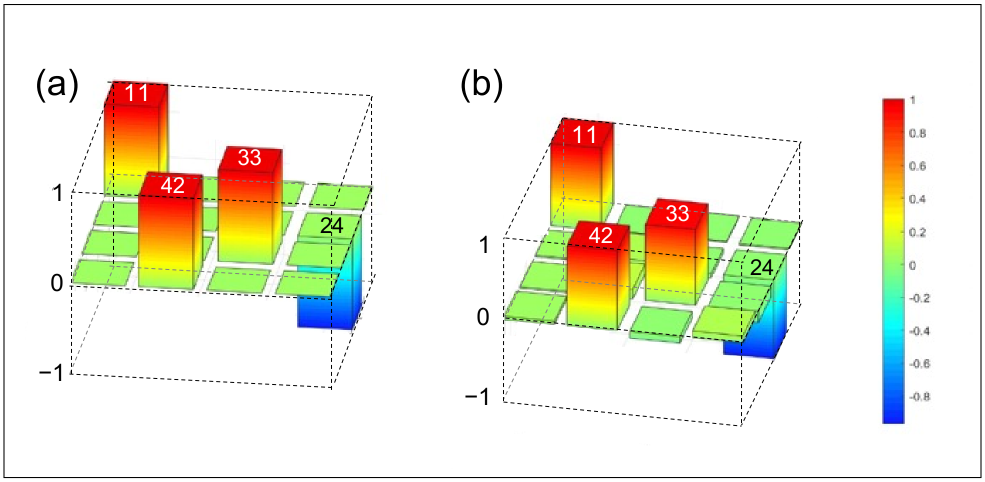

Figure 3. Bar graphs of the theoretical (a) and measured (b) Mueller matrices for a quarter-wave plate at 45°.

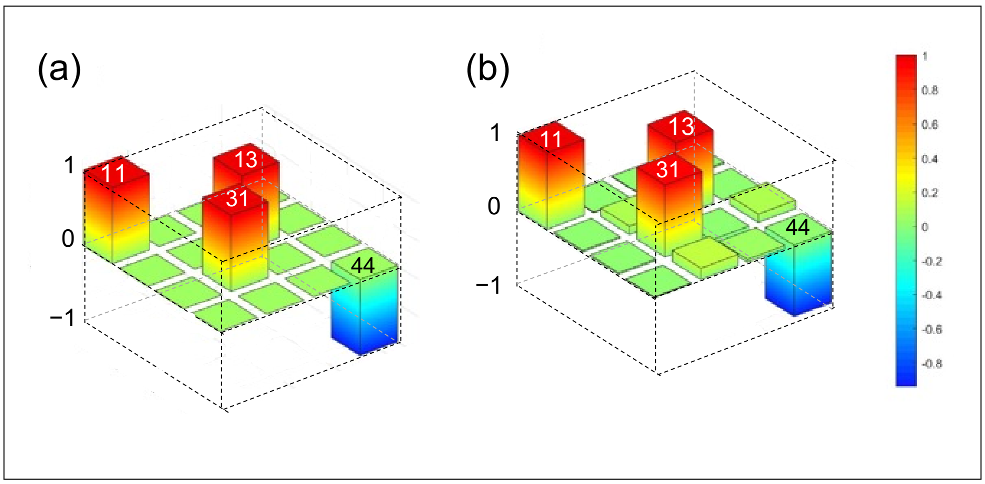

Figure 4. Bar graphs of the theoretical (a) and measured (b) Mueller matrices for a half-wave plate at 22.5°.

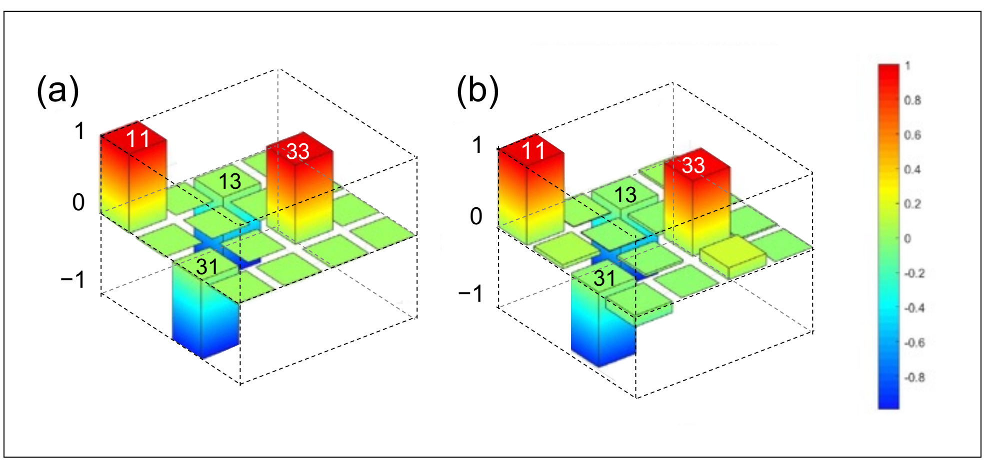

Figure 5. Bar graphs of the theoretical (a) and measured (b) Mueller matrices for a polarizer at −45°.

A second advantage is that we do this measurement with very low light levels of the order of femtowatts, which may be useful in diagnosing media that can easily be damaged by the light used to do the measurement. Just attenuating a classical beam to the single-photon level would make it difficult due to background noise reducing the signal-to-noise ratio. However, in the present method, coincident detection eliminates that noise.

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