



## Teachers' perceptions and uptake of professional development overtime

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### ARTICLE INFO

**Keywords:**  
Professional Development  
Mathematics education  
Teacher education  
Professional learning

### ABSTRACT

This study captured middle and high school teachers' perceptions of what they learned from professional development (PD) 3–4 years after participating in one of three National Science Foundation funded year-long PD projects. We surveyed 66 teachers from three different PD projects on the types of content, pedagogy, and resources that they remembered learning and continue to use when teaching mathematics. Results indicate that teachers remember and use many aspects from their PD experiences 3–4 years down the road. Most residual learnings from PD also appear to be highly aligned with the goals and intentions of the PD developers and researchers and may be related to the kind of PD design on the adaptive-specified continuum.

### Introduction

There is urgency to better understand the impact of professional learning opportunities in the field of mathematics to improve mathematics teaching and learning. One central challenge for the field is how to design and test interventions that target teacher knowledge, teachers' instruction, and student learning (Jacobs et al., 2020). Many researchers have worked to address both of these challenges and there is now a stronger research base to show for it. Significant progress has been made that focuses on delineating the critical features of effective PD to incorporate in PD design (e.g., Birman et al., 2000; Borko et al., 2010; Desimone & Garet, 2015). These features include a focus on mathematics content, student learning of content, active learning opportunities for teachers, coherence, duration, and collective participation (Sztajn et al., 2017). Yet surprisingly although many mathematics PD programs adhere to these design features results related to the impact of the PD intervention on teachers and students has been mixed (Koellner & Jacobs, 2015; Koellner et al., 2022). In some instances, incremental changes have been identified (e.g., Franke et al., 2001; Neumayer DePiper et al., 2021; Koellner et al., 2022) however others have proven less successful (e.g., Jacob et al., 2017; Santagata, 2014). Progress has been made but it is perplexing that research findings have not been conclusive enough to guide the field. This has caused researchers to take a more nuanced look to better understand the aspects that appear to be the impetus for teacher learning and those that have not supported learning and why.

Many funded PD projects are randomized control trial (RCT) studies aimed to better understand PD impact. The mixed results from recent impact studies of PD programs containing some or all these design elements brought on surprise and confusion (Arens et al., 2012; Bos et al., 2012; Jacob et al., 2017; Koellner et al., 2022). It is unclear why rigorous empirical studies and randomized trials produced differing results that contradict conventional wisdom among the field. There are many reasons that potentially could account for these varying results such as: the content of the specific programs evaluated may have been inefficacious, fidelity to the materials or pedagogical practices may have deviated from the identified goals and practices, difficulties may have resulted from scaling the program to multiple sites with different facilitators, or issues may have arisen with the research design and methodology. Our study is based on the hypothesis that teachers learn more than results have shown from these funded RCTs.

Understanding what teachers remember, take up and continue to use related to the intentions of the PD they attended may shed light on some of the mixed results from RCTs and more understanding about teacher learning and in turn how to account for teacher learning in PD research design. There are many facets of professional development interventions from the goals and objectives of the PD related to content and pedagogy, the nature of the facilitator, as well as the materials, resources and supports that can potentially provide more nuanced qualitative evidence of teacher learning. And more in-depth qualitative analyses could prove helpful to capture teacher learning and change overtime not necessarily to replace large scale studies but rather to use qualitative results to

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compliment quantitative findings. This study examined teachers' perceptions about what teachers learned in a PD experience 3–4 years after attending PD workshops and what they have taken up and continue to use related to content, pedagogy, resources and supports provided through the PD.

### Goals and purpose of the study

The goal of the present study, hereafter referred to as the Study, is to identify the residual impacts of three different PD programs on teacher learning. In other words, this Study examines teachers' perceptions about what they learned in their respective PD experiences 3–4 years after attending PD workshops, and what they have taken up and continue to use from the PD. PD models fall on a continuum from adaptive to specified (Borko et al., 2011; Koellner et al., 2015) depending on the extent to which the PD goals and content are pre-determined or not, and the three PDs examined in this Study fall on different parts of the continuum. The investigation uses surveys, videos, and think aloud protocols within case studies and cross-case analysis, to further inform (a) what teachers take up and use in different PDs in different contexts, (b) why some teachers appear to take up and use more than others, and (c) why some PDs have better results than others. While the Study includes multiple data sources, this paper shares findings from the survey that participants completed in May 2019 which was 3–4 years post their PD experience.

The research questions that guide the survey analysis in this paper are the following:

RQ1: *What do teachers take up and use from professional development (PD) workshops three to four years after participating in the PD?*

RQ2: *How do teachers' self-reported uptake differ across PDs located at different points on the adaptive-specified continuum?*

### Theoretical framework

#### Adaptive to specified continuum for PD models

PD models fall on a continuum from adaptive to specified (AUTHOR 2011; AUTHOR, 2015) and most typically embrace the agreed upon elements believed to be effective (i.e. (a) content taught in classrooms, (b) the mathematics practice and/or process standards, (c) collaborative participation, and (d) alignment of school curricula, missions and policies (see Fig. 1).

However, the structure of PDs can be quite different even when cast with the same elements. On one end of the continuum are adaptive models, in which the learning goals and resources are derived from the local context and shared artifacts (e.g., learning activities or tasks, student work) are generally from the classrooms of the participating teachers. In these models, the facilitator and/or the participating teachers selected and sequenced the artifact, and the related activities are based on general guidelines that consider the perceived needs and interests of the group. On the other end of the continuum, specified models of PD typically incorporate published materials that specify in advance teacher learning goals. In video-based specified PD, the video

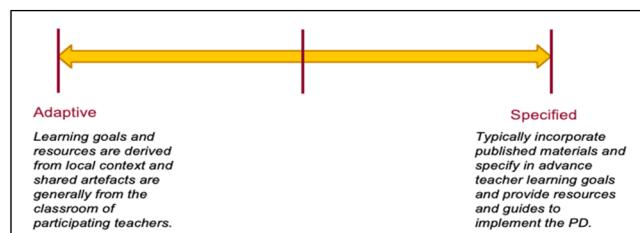


Fig. 1. Adaptive-specified continuum.

clips are typically pre-selected and come from other teachers' classrooms.

The nature of what teachers take up and use across the continuum has the potential to shed light on factors that are associated with teacher learning related to content and pedagogy. This study examines three PDs that fall on different parts of the continuum. The goal is not to determine which types of PD along the continuum are "best" because each has its affordances and challenges, but rather to better understand the variance of teacher uptake and use within and across these PD models and how they may or may not differ. Understanding, deeply analyzing and unpacking variance among and between types of PD offers the potential to identify the factors that impact uptake and use from PD. This paper examines how teachers' self-reported uptake differs across PDs located at different points on the adaptive-specified continuum. Specifically, one is highly adaptive, one is highly specified, and one lands between adaptive and the mid-point. We briefly describe the three different PD projects. We believe conducting a cross case comparison will aid in helping us understand the factors associated with uptake related to content, pedagogy, and resources.

### Methodology and methods

This three-year impact Study collects qualitative data from three large U.S. National Science Foundation (NSF) funded PD projects that are located at different points on the adaptive-specified continuum of PD models and are described in detail below.<sup>1</sup>

#### Lesson study (LS) PD

The first NSF project, *Collaborative research: TRUMath and Lesson Study: Supporting fundamental and sustainable improvement in high school mathematics teaching* (LS), aimed to engage in design research to develop and implement a replicable model for a coherent, department-wide approach to professional learning focused on creating classroom environments that produce students that can be powerful mathematical thinkers (Schoenfeld et al., 2020a, 2020b). In the PD, teachers worked to create lesson plans that were focused and coherent and allowed for a deeper and richer understanding of mathematics and the ability to make connections and implement curriculum more effectively. In this project, teachers were taught the Teaching for Robust Understanding (TRU) mathematics framework (Schoenfeld, 2017). This is an observation instrument that can be used to analyze mathematics classroom interaction across different dimensions. Teacher teams engaged in LS as a way to work on specific shifts in teaching practice that aligned with the TRU dimensions. LS is an adaptive form of PD that utilized the TRU framework but allowed for teachers' ideas to guide the workshops.

#### Learning and teacher geometry (LTG) PD<sup>2</sup>

The second NSF project, LTG, *An Efficacy Study of the Learning and Teaching Geometry Professional Development Materials: Examining impact and context-based adaptations*, sought to improve teacher's own knowledge and instructional strategies in transformations-based geometry (Jacobs et al., 2017). The goal of LTG was not only to improve teachers' conceptual content knowledge and increase their ability to engage students in mathematical practices but specifically to increase students' conceptual understanding of transformations-based geometry. This PD

<sup>1</sup> The four projects discussed in this article were funded by the National Science Foundation (NSF):, 1. *The Taking a Deep Dive* (TaDD) project; No. 1,812,438; [2018–2023], 2. *The Efficacy Study of the Learning and Teaching Geometry* (LTG) project; No. 1,503,399; [2015–2019], 3. *Visual Access to Mathematics* (VAM) project; No. 1,503,057; [2015–2019], 4. *TRUMath and Lesson Study* (LS) project; No. 1,503,342; [2015–2019]

<sup>2</sup> LTG original project, funded by NSF No. 0732757

consists of 54 h of highly specified video-based PD that is grounded in modules of dynamic transformations-based geometry which is aligned with the Common Core State Standards in mathematics (Akkus, 2016). Through video analysis, teachers work together to solve problems and further their knowledge in mathematics teaching in the domain of geometry. The PD allows teachers to better support students in their attempt to gain a deeper understanding of transformations-based geometry through activities like rate of change on a graph, scaling activities, and similarity tools. The material strongly connects to other critical domains including similarity, proportional reasoning, slope, and linear functions. LTG is a specified PD as the packaged materials for each workshop clearly articulated the content and pedagogical goals.

#### Visual access to mathematics (VAM) PD

The third NSF project, *Visual access to mathematics: Professional development for teachers of English learners* (VAM), aimed to build skills in mathematical problem solving and communication using visual representations (Driscoll et al., 2018). The PD sought to improve teachers' representational fluency, anticipate students' strategies, to interpret and construct various mathematical solutions, and to reason with and across representations with an aligned goal of interpreting students' unique solutions and representations. This PD consisted of face-to-face sessions as well as online workshops where teachers implemented problems from the PD and shared their student work to discuss access for English Learners (EL's) and all students. The project investigated the instructional strategies and supports that teachers of EL's need to provide students for access to mathematical learning and advancement of academic language development. The approach was grounded in the use of visual representations, such as diagrams and geometric drawings, for mathematical problem-solving with integrated language support strategies. The intended goals of VAM were to help teachers to properly select appropriate visual representations for the use of different rational number task types and communication tools to show and explain mathematical thinking. VAM fell in the middle of the adaptive-specified framework as the face-to-face workshops had specified and intentional goals and the online professional learning meetings were guided by the teachers and used artifacts of practice, mainly lesson plans, to guide their discussions.

#### Methodology

Participants from the above-mentioned PD projects were invited to be a part of the Study three-four years after the project and funding ended and completed a survey. This paper examines the survey responses and uses descriptive statistics, paired samples *t*-tests, and analyses of variance and covariance with pairwise comparisons to understand indicators, similarities, and differences of self-reported learning taken up and used from the three PD projects a few years on.

#### Sample

Sixty-six participants from the three NSF projects took a 25-question survey, 13 LS participants, 28 LTG participants, and 25 VAM participants. Teachers also provided educational and teaching background information, see Table 1. All teachers held an undergraduate degree and 88 % held a graduate degree, on average, but larger proportions of LTG (93 %) and VAM (96 %) teachers held graduate degrees compared to LS teachers (62 %;  $t = 3.29, p < .01$ ). In addition, VAM teachers reported over 16 years of experience teaching, significantly more than LS and LTG teachers who reported approximately 10 and 12 years, respectively ( $t = 2.81, p < .05$  and  $t = 2.57, p < .05$ , respectively). On average, 15 % of teachers were currently teaching Geometry with no differences between groups.

#### Measures

The survey was designed to understand uptake of teacher learning related to our first research question:

**Table 1**

Descriptive statistics of teacher background, by project ( $N = 66$ ).

Background measure	Lesson Study PD (LS, $n = 13$ )	LTG PD Efficacy Study (LTG, $n = 28$ )	Visual Access for ELLs in Math PD (VAM, $n = 25$ )	F	Pairwise comparisons
Undergraduate degree	100 %	100 %	100 %	ns	
Graduate degree	62 %	93 %	96 %	6.07**	LTG > LS* VAM > LS**
Currently teaching Geometry	15 %	25 %	4 %	ns	
Years teaching	10.08 (6.14)	11.79 (6.22)	16.52 (7.53)	5.11**	VAM > LS* VAM > LTG*

Note.

\*  $p < .05$ ;

\*\*  $p < .01$ .

RQ1: *What do teachers take up and use from professional development (PD) workshops three to four years after participating in the PD?*

Our review of research literature reporting impact of PD on teachers (Santagata & Yeh, 2014; van Es & Sherin, 2010) revealed that the most prominent goals and intentions of the PDs related to aspects of content, pedagogy, and resources, which research on teacher uptake revealed support as an additional contributing factor (Franke et al., 2001). We therefore designed the survey keeping in mind these four categories of content, pedagogy, resources, and support.

The survey included both closed- and open-ended questions that asked participants to reflect on their PD experience and characterize their past and/or current use of the PD content, pedagogy, resources, and support they received to implement new content and instructional practices. The survey included seven Likert scale questions, where participants responded to statements on a scale of 1–10, as well as 18 follow up questions that allowed the participants to explain and provide more details about their numeric response.

We created a coding manual (see Appendix) starting with *apriori* codes. The *apriori* codes were aspects of effective PD identified from the literature and organized around the four broad categories of content, pedagogy, resources, and support (examples include specific content, student thinking, representations used, resources supporting diverse learners, etc.). We then included emergent codes that came up frequently in the data and appeared relevant to the PD programs (example includes technology to support mathematics learning). We identified four content related codes: GCSL (general content student learning), GCTL (general content teacher learning), SCSL (specific content student learning), and SCTL (specific content teacher learning). More specifically, SCSL would refer to a comment on the survey that indicated specific content (e.g., dilations) and with a focus on student learning. We identified four codes related to teacher's pedagogy; these codes include GPS (general pedagogical strategies), MS (multiple solution strategies), SSDL (strategies to support diverse learners), and ST (student thinking). We identified six resource related codes; these codes included GR (general resources), RSDL (resource to support diverse learners), RTL (resource for teacher learning), SR (student resource), TSM (technology support math learning), and V (mention of video to support noticing). Lastly, we identified four support related codes; these codes include C (collaboration), FI (facilitator impact), CS (coach support), and PS (principal support). Examples from the data relating to each code are given in the coding manual in the appendix.

We coded the 18 open-ended questions on the survey from all 66 participants. We began with three researchers coding one survey from each project. We came together to discuss codes, add codes to the

manual, and reconcile differences. We then continued this process with seven surveys from each project to achieve inter rater agreement at 91 %.

Once all surveys were coded, we initially measured how many times a participant mentioned each code in their survey responses. For each of the four domains (content, pedagogy, resources, and support), we identified and averaged the specific codes included within each domain. Finally, percentages of comments resulted from the four domain averages as well as individual codes within domains were calculated for an in-depth understanding of teacher responses. A deeper analysis of the survey responses helped us answer our second research question: *How do teachers' self-reported uptake differ across PDs located at different points on the adaptive-specified continuum?* We will elaborate on this in the discussion section.

### Analyses

To analyze the data, we used descriptive statistics, paired samples *t*-tests, and analyses of variance and covariance with pairwise comparisons using the Bonferroni test to identify and understand the differences and similarities between uptake by project (LS, LTG, VAM). To control for pre-existing differences, graduate degree and years of experience teaching were included as covariates in the analyses of covariance. Measures of teacher undergraduate and graduate degrees and currently teaching geometry were included in preliminary analyses but found to be non-significant and dropped from subsequent analyses. The results, adjusted for teaching experience, are presented in the following section. Additionally in the discussion section we matched qualitative comments from the survey to highlight teachers' voices and help elucidate the nuances of uptake among and between the projects. This helps answer the second question about the difference between teacher uptake for each project vis-à-vis their location on the adaptive-specific continuum of PD design model.

### Results

We previously analyzed results from the Likert scale questions and found that the geographical context and the context of individual PD projects highly impacted the ways participants perceived their PD experiences (see Koellner et al., 2020). These analyses provided the impetus for the deeper analyses which resulted in this study.

#### Domains of teacher comments

Descriptive statistics of the four domain measures are presented both in Fig. 2 and Table 2. Fig. 2 colorfully illustrates the average percentages of the 1589 comments related to what teachers remembered from their

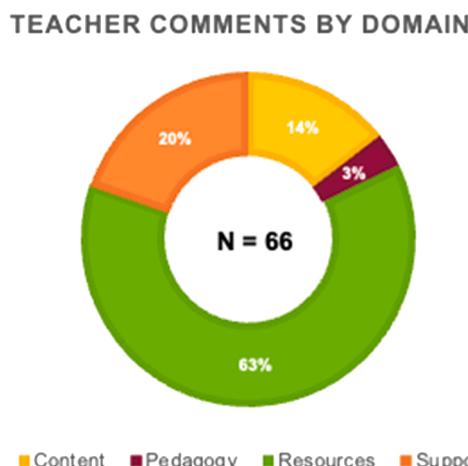


Fig. 2. Teacher comments by domain.

Table 2

Descriptive statistics of teacher comments about the content, resources, pedagogy, and support of their PD experiences, by project ( $N = 66$ ).

Domains of teacher comments	Number of comments		
	Total comments	Mean (sd)	Range
Content (4 codes)	229	3.47 (2.75)	0–10
Pedagogy (3 codes)	54	0.82 (1.28)	0–5
Resources (6 codes)	992	15.03 (10.50)	0–46
Support (4 codes)	314	4.76 (2.06)	1–10
Total comments (17 codes)	1589	24.08 (12.31)	3–61

PD and what they have continued to use in their classroom by domain. Table 2 provides the more detailed version that shows that over half of the 1589 total comments made by teachers focused on resources (992) followed by support and content (314 and 229, respectively). Only a very small number of comments mentioned pedagogy (54).

Table 3 presents the percentages of comments within domains and across projects and the results of the analyses of covariance adjusted for teacher years of experience teaching.

#### Types of comments within projects

Within projects, paired samples comparisons within the LS group identified a significantly larger percent of comments focused on support compared to content ( $t = 6.70, p < .001$ ), pedagogy ( $t = 4.76, p < .001$ ), and resources ( $t = 4.62, p < .01$ ). While this group also commented more on resources than on content ( $t = 3.38, p < .01$ ), both LTG and VAM emphasized resources more than all other domains: content ( $t = 2.86, p < .01$  and  $t = 14.21, p < .001$ , respectively), pedagogy ( $t = 10.70, p < .001$  and  $t = 17.89, p < .001$ , respectively), and support ( $t = 4.14, p < .001$  and  $t = 12.82, p < .001$ , respectively). LTG and VAM also focused more on content ( $t = 9.90, p < .001$  and  $t = 3.80, p < .01$ , respectively) and support ( $t = 8.29, p < .001$  and  $t = 9.48, p < .001$ , respectively) than on pedagogy.

To summarize, although the domain resources strands as the largest overall (63 % of total comments), it was only somewhat emphasized (23 %) in the TRU LS project which is an adaptive PD, while support was emphasized the most (54 %) and content and pedagogy were emphasized far less (10 % and 13 % respectively). On the other hand, the LTG PD project which was a specified PD had larger percentage of comments related to resources (43 %) and then percentages were fairly evenly distributed between content and support (29 % and 25 % respectively), but much less for pedagogy (3 %). The VAM project teachers most emphasized resources (65 %), followed by far fewer comments for support (21 %), content (10 %), and pedagogy (3 %).

Table 3

Results of ANCOVA on percent of teacher comments across the four domain averages, by project ( $N = 66$ ).

Domains of teacher comments	Lesson Study PD (LS, $n = 13$ )	LTG PD Efficacy Study (LTG, $n = 28$ )	Visual Access for ELLs in Math PD (VAM, $n = 25$ )	F	Pairwise comparisons
Content	10 %	29 %	10 %	25.76***	LTG > LS ** LTG > VAM ***
Pedagogy	13 %	3 %	4 %	7.34**	LS > LTG ** LS > VAM **
Resources	23 %	43 %	65 %	37.56***	LTG > LS *** VAM > LS ** VAM > LTG ***
Support	54 %	25 %	21 %	38.89***	LS > LTG *** LS > VAM ***
Total	100 %	100 %	100 %		

Note. Results from ANCOVA adjusted for years of experience teaching.

\*\*  $p < .01$ ;

\*\*\*  $p < .001$ .

### Types of comments across projects

Comparing teacher comments across projects, we look at two visual representations and analyses to gain perspective of the differences among project results related to uptake through the percent of comments by domain. Fig. 4 represents the percent of average number of teacher comments across the individual three projects. This figure compares how the teachers from three different PDs talked about uptake of resources, pedagogy, content, and support. Upon review of Fig. 3, VAM teachers clearly commented on resources from the PD most. LTG comments were the most distributed of the three PD projects. And LS mostly commented about the supports that were provided through the PD.

Table 3 represents the results of the analyses of covariance identified distinct patterns of comments about PD experiences for each group. LS participants were significantly more likely to mention support and pedagogy compared to both the LTG ( $t = 7.81, p < .001$  and  $t = 3.71, p < .01$ , respectively) and VAM participants ( $t = 8.28, p < .001$  and  $t = 3.17, p < .01$ , respectively). Their comments included principal and coach support as well as colleague support. Support was the domain qualitatively discussed most throughout the survey although how each project defined what support meant to them differed.

LTG participants emphasized content significantly more than both LS ( $t = 5.51, p < .001$ ) and VAM participants ( $t = 6.22, p < .001$ ) and resources more than LS participants ( $t = 4.35, p < .001$ ). On the other hand, VAM participants mostly emphasized resources and did so significantly more than both LS ( $t = 8.55, p < .001$ ) and LTG participants ( $t = 5.62, p < .001$ ).

Results indicate that the teachers' perceived uptake after 3–4 years was highly related to the goals and intentions of the PD projects. As the PD projects' goals and intentions were identified at different points on the adaptive – specified continuum, differences were highlighted based on comments related to content, pedagogy, resources, and support which will be discussed in more detail below. In some ways this is not surprising in that the different PD programs had different emphases, and these were revealed in the clusters of codes related to content, pedagogy, resources, and support yet it provides promising evidence that PD learning held residual value related to the PD.

Table 4 illustrates that LS participants were more likely to make comments related to the support category (53 % of comments were focused on support) compared to the other projects. These included principal and coach support as well as colleague support. Support was the item qualitatively discussed most throughout the survey. The LS

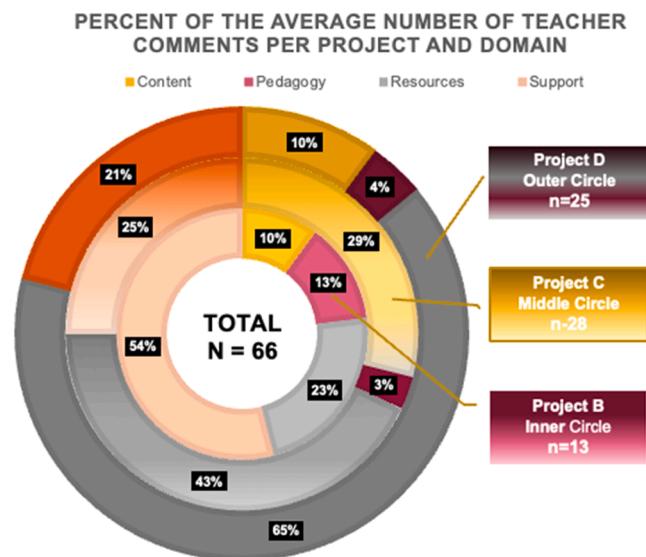


Fig. 3. Percent of average number of teacher comments across the three projects.

Table 4

Percent of the average number of teacher comments per domain (content, resources, pedagogy, and support) from PD experiences, by project ( $N = 66$ ).

Domains of teacher comments	Lesson Study PD (LS, n = 13)	LTG PD Efficacy Study (LTG, n = 28)	Visual Access for ELLs in Math PD (VAM, n = 25)	p-value	Pairwise comparisons
Content	8 %	29 %	12 %	0.00	LTG>LS*** LTG>VAM***
Pedagogy	13 %	3 %	5 %	.002	LS>LTG** LS>VAM*
Resources	26 %	44 %	62 %	.000	VAM>LS*** VAM>LTG***
Support	53 %	24 %	21 %	.000	LS>LTG*** LS>VAM***
Total	100 %	100 %	100 %		

project also commented somewhat frequently talking about resources (26 %). On the other hand, content and pedagogy were discussed far less in the LS project (8 % and 13 % respectively). The LTG project, a specified PD, had the most comments that were distributed among the categories. However, they had the most comments related to resources (44 %) and then comments were fairly evenly distributed between content (29 %) and support (24 %) and less so for pedagogy (3 %). The VAM teachers had the most comments related to resources (62 %) and the second most comments were related to support (21 %) and content and pedagogy trailed behind (12 % and 5 % respectively).

A close look at the four individual codes by project also illuminates some interesting uptake over time (see Table 2). For instance, in LS, teachers commented on content only 8 % of the time and when they did, they were discussing aspects of content that were general and related to teacher learning or student learning. When discussing pedagogy, most comments were related to working with diverse learners, and if they were discussing a resource, they were discussing a specific resource, most likely the TRU framework which was a centerpiece of their project. The LTG single project codes highlight content codes that were specific, meaning that LTG teachers referred to very specific content topics. In terms of pedagogy, the LTG teachers' most frequent comments were related to student thinking. In terms of resources, LTG teachers talked both about specific resources and general resources most often and when referring to support they discussed support from colleagues and the facilitator. The VAM teachers talked very specifically about the content for teacher learning and in terms of pedagogy, they discussed the strategies they learned to work with diverse learners most often. In terms of resources, the VAM teachers discussed specific resources and representations. Like LTG, VAM teachers discussed support from colleagues and the facilitator most often.

### Discussion

This study reveals that the teachers that participated in the three NSF funded PDs, 3–4 years before taking this survey, highlighted and wrote about the main goals and intentions of the PD that they attended. The teachers remember and use what the facilitator and PD developers intended; it shows promise that the PD's yielded high residue of intended teacher learning 3–4 years after the PD workshops.

#### Lesson study project

LS teachers generally tended to emphasize support (54 %) (see Fig. 4) when making comments on the survey which included support from facilitators, coaches, school administration or fellow colleagues. For example, a participant noted, "People from the district that have worked with us on TruMath and Lesson Study have been instrumental in building a culture around planning, observation, and reflection." These

### LESSON STUDY BY DOMAIN

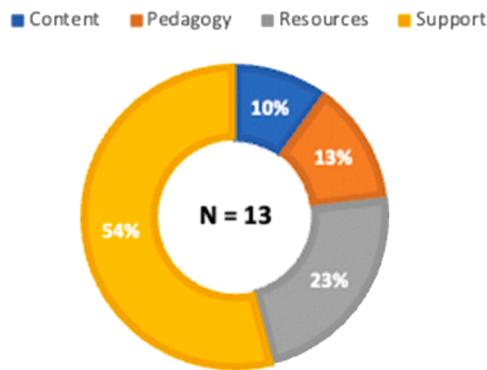


Fig. 4. Lesson study comments by domain.

teachers commented on resources 23 % of the time and pedagogy 13 % of the time. Resources for this project included the TRU framework as well as shared resources from the design of the lesson study lesson, such as the “talking cards and the activity pages that we prepped for the lesson study.” The LS PD workshops were school-based communities of teachers that planned lessons together identifying pedagogical strategies and appropriate content. These meetings were led by a coach and the teachers used the TRU framework, an observation tool, to not only have common language to use during debrief of observations but also as a tool to think about planning and assessment. The teachers observed one another teach lessons and provide feedback to one another with the help of coaches and facilitators.

When discussing pedagogy (13 %) most comments were related to working with diverse learners, this was a focus of the LS inquiry that they were planning for and attending to during observations. One participant noted, “I teach at a diverse high school with large class sizes and heterogeneous Algebra I classes. A lot of the strategies we discussed emphasized group work and low floor high ceiling tasks...both of which helped with my current teaching.” Therefore, the residual knowledge that stuck was highly relevant to the intentions of LS PD. Responses and comments tended not to emphasize content which is not surprising as the focus of the PD was not building content knowledge of mathematics in a specific domain. Rather when teachers discussed content it was in the context of teacher or student learning. As one participant noted, “I enjoyed watching videos [of other teachers in the group] and talking about them. I don’t really feel like we focused on learning specific content, rather on the process of teaching the content.” Teacher learning and student learning were the mediating processes used in the LS PD itself – when planning lessons, they focused on meeting the needs of their students and when observing teachers, they were actively discussing teacher thoughts and understanding.

These findings also are consonant with adaptive models of PD in many ways. Adaptive models are not pre-developed - rather the teachers and coach identify the goals, content of the lesson plan, instructional strategies and the focus of the inquiry (Borko et al., 2011; Koellner et al., 2015). In the case of this PD, since the coach and the teachers identified the goal to support diverse students, uptake appears to be connected with the strong collaboration between the coach and teachers as well as among teachers. This collaboration (or support) came in regard to developing plans and providing feedback to one another. One participant noted, “It was good that we had a lot of power in deciding what we wanted to pursue in our department team. It never felt like someone told us, ‘This is ‘great’; you have to do this or else.’ I think TRU helped my department work better together because it gave us a common goal where the outcome was increased student achievement for all of us.” Thus, teachers commented on collaboration more often compared to content or pedagogical strategies for that reason possibly.

### Learning teaching geometry project

The LTG teachers’ comments focused primarily on resources (43 %), then content (29 %), followed by support (25 %) (see Fig. 5). The LTG teachers referred to very specific content topics for teacher learning and student learning (e. g., transformations, dilations, within and between ratios). For example, one teacher mentioned, “The greatest thing that stood out, and something the students also found to be encouraging and useful was the use of the ‘transparent squares’ to perform geometric transformations instead of the basic (x, y) to (-x, y) functionality.” Another commented on how their learning changed, “A lot of the PD was just a learning process for the participants. We were learning new ways of engaging with and teaching content together. It was clear the goal was to help us understand new approaches to traditional concepts.” The comments from the LTG teachers were the most distributed across categories. In terms of resources, LTG teachers talked both about specific resources (e.g., geometric transformations field guide, geometric transformations workouts, patty paper, applets) and general resources (e.g., tasks, visuals, representations, materials) most. In terms of pedagogy, the LTG teachers’ most frequent comments were related to student thinking. They mentioned learning about “collaborative learning (lots of pairs and shares and turn and talks) to get students talking and discussing through discovery learning.” When referring to support, they emphasized support from colleagues and the facilitator: “Everyone was highly engaged and worked well together. We acted as students and teachers throughout the PD and supported each other in learning.”

These findings are aligned with the specified design of the LTG PD. The LTG PD was a funded development project originally.<sup>1</sup> Each aspect of the PD was carefully designed. Resources, pedagogical strategies, and facilitation (support) were carefully considered and sequenced for hopefully an ideal PD to teach the content of transformations-based geometry. The nature of the content was examined by a multiplicity of mathematicians, math educators and teachers to carefully engineer tasks and sequences for this particular content. Therefore, since each component of the specified PD was investigated in the design, one would anticipate that all categories could be taken up by participants given the consistent nature of the experience.

### Visual access to mathematics project

VAM teachers’ comments were overwhelmingly in the area of resources (65 %) (see Fig. 6) which is not surprising as the VAM project was geared to providing resources and strategies to support ELs with regard to access to learning. The VAM teachers most often made comments on specific resources and representations for teacher learning (65 %). One noted, “I really liked the activity that connected the rate of

### LTG PD BY DOMAIN

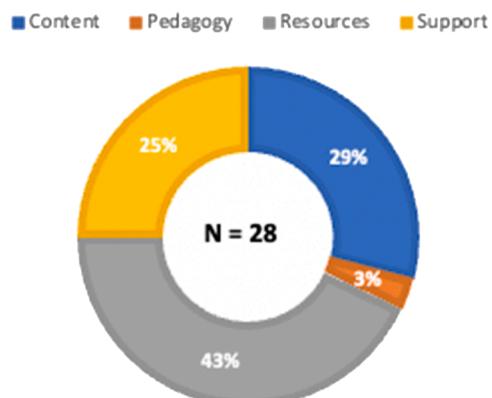


Fig. 5. LTG PD comments by domain.

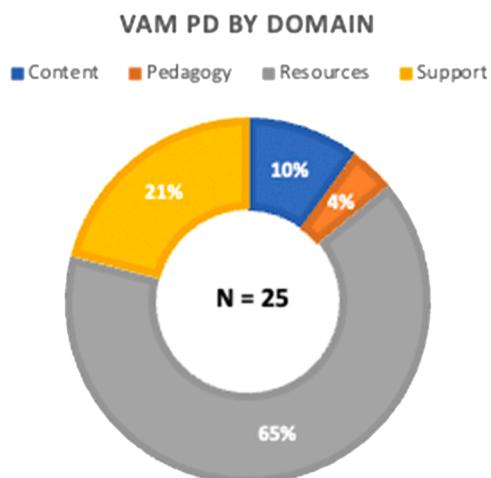


Fig. 6. VAM comments by domain.

change on the double number line and the rate of change on the graph. I think the applet that was shared made it really vivid for me. I have also shared this with students." The VAM PD introduced many different mathematical representations that could be used across mathematics disciplines to provide more access to ELs. Teachers remembered these and commented on them, "Double number lines, tape diagrams, and other visual models (number lines, pictures, proportions, etc.) to represent problems with proportional relationships. We also touched on EL strategies like pictures and careful wording of problems to help benefit the students who are ELs." When VAM teachers responded about content (10 %), they talked very specifically about teacher learning related to fractions, ratios, and proportions. In their comments about pedagogy (4 %), they most often discussed the strategies [often related to tools or resources] they learned to work with diverse learners. VAM teachers discussed support (21 %) from colleagues, the facilitator, and the principal. One participant expanded on how this support built community in the PD: "I think one of the reasons the community we built was so strong was because the PD was so relevant and the teachers were excited and engaged."

The VAM PD is both adaptive and specified in nature which is aligned with teacher comments especially how they were centered on resources. The specified aspects of VAM were pre identifiers resources and representations- specifically the double number line and the tape diagram- as well as the frameworks to support linguistically diverse students. Teacher comments were prevalent in this area, and they discussed the use of a representation as a resource that the PD provided (rather than using models as a pedagogical strategy). For example, one teacher commented, "I use or have adapted almost all the resources from the PD. For instance, there was an online applet for calculating percent change. I created a graphic based on the applet and a percent practice activity where students use a double number line to solve the problem, paired with their graphic, and then use the applet to check. I have also used and adapted the tape diagram tasks and double number line tasks that were given to us at the PD." The adaptive side of PD is related to an online component of the PD where they collaborated with colleagues about their experience using the representations and supporting linguistically diverse students in their classrooms throughout a whole school year. These meetings were organized by the teachers, and they could share artifacts of practice from their classroom like lesson plans or student work. This is clearly reflected in the second highest category of comments.

## Conclusion

This study highlights the importance of investigating and understanding how teacher learning is continuous, what teachers learn from

different PD projects (and structures) overtime and the potential reasons why. Theoretically these findings are aligned with the design principles of PDs along the adaptive-specified continuum. Our findings related to uptake show that teachers continue to use and hone their instructional practices in ways that are related to what they learned from PD.

The findings highlight the importance of continuing to investigate teacher learning over longer research cycles and certainly beyond the pre post design of a one to two-year RCT as we found that teacher learning continues and becomes more embedded in teachers' practice 3–4 years post PD. For instance, the use of a representation appears to become more intuitive for teacher application and time seems to allow teachers time to hone their craft using the representation and other knowledge learned from the PD as well as their own experiences of implementation with students in more sophisticated ways over time. This was clearly highlighted in both the LTG PD and the VAM PD. From the findings of this study, we hypothesize that representations play a critical role in teacher learning and the design of PD that has not been highlighted before (Placa et al., 2023).

The descriptive statistics and qualitative comments have provided more nuanced and stronger evidence of teacher learning overtime. Teachers remembered many aspects of the PD that they attended and were able to provide detailed examples of how these learnings were implemented, transformed, or adapted to their current classroom content. Thus, findings from this study are complementary to RCTs and illustrate that teachers do remember and continue to use what they learned in PD years after the study has ended and the descriptive statistics and qualitative comments have provided more nuanced and stronger evidence of teacher learning overtime. This study highlights the complementarity of survey analyses both descriptive statistics and qualitative aspects to RCT design studies and plays an important role in understanding teacher learning and professional development. Teachers who participated in PD highlighted and wrote about the PD that they attended and what they learned and continue to use in their classroom. Although this may not be surprising that the teachers remember aspects of the PD, it shows promise that the PDs yield higher residue of teacher learning years after participation than what may have been found initially in the RCT findings.

Neumayer DePiper et al. (2021) found the VAM PD had an impact on teachers' self-efficacy about teaching emergent bilinguals and using diagrams to support student learning. This study illustrates more closely how the teachers continue to use resources, or "diagrams" and how they use these diagrams in particular situations for different mathematical domains beyond proportional reasoning, the focus of the PD. Additionally they purport to use representations whenever they can as they believe this is a tool to provide access to linguistically diverse students. Teachers also mentioned resources, other than diagrams, that also impacted their learning including tasks, co-developed lesson plans, frameworks, and alternative representations to provide access to more children for complex mathematics. It is clear from our survey results that the resources contributed the most to VAM teachers' learning. They seem to have provided an anchor for their learning and teaching of proportional reasoning and this evidence adds to the findings of Neumayer DePiper et al. (2021).

Koellner et al. (2022) explain that studies related to teacher learning of content was limited to very specific objectives related to transformations-based geometry and that more pronounced incremental change was noted in teacher's instructional practice. Furthermore, they found that teachers continued to change instructionally in the post-post studies of observational change of classroom practice (Jacobs et al., 2022). This study adds to these results in more nuanced and important ways. LTG teachers learned new content and if they were teaching transformations-based geometry or if they made connections to other mathematical domains, they continued to use that knowledge based on survey comments. Additionally, the instructional strategies and apparent new uses of the applets and Geogebra had been taken up and used across mathematical domains.

As for LS, these teachers learned how to use a new tool, the TRU framework to differing degrees and their PD experience created a strong community of support, a main tenet of LS that appeared impactful at the time and possibly overtime (Schoenfeld et al., 2020a, 2020b). Community has been found to be an important contributor to teacher learning as previously reported (Horn, 2017).

Again, the survey results seem consistent with the large-scale studies from all three PDs. The findings reported here further illustrate that although learning and instructional change was only incremental in results published from RCTs, survey findings help elucidate more about what they learned and show that teacher change was sustained and that teachers attribute that change to the PD.

The three PDs studied were all designed with the effective design elements (Stjazin, 2017) but the PD structures of each were spread across the adaptive versus specified continuum. Teachers that were involved in adaptive PD or components of PD that were adaptive noted collaboration and support as pivotal in their learning process whereas teachers that participated in more specified models of PD noted content and resources as being critical in their learning and uptake. This is notable and helps administrators, coaches or teachers select different types of PD with an understanding that they yield different yet equally important impacts on teachers.

We acknowledge that there are confounds related to social desirability on surveys but on the other hand teachers remembered aspects of

the PD and attributed change in their own practice to the PD whether it is related to content, a pedagogical strategy, a resource or support provided. Furthermore, whether or not the PD was the sole contributor to a change in practice, is not the goal here, rather we see the PD experience as one niche in the larger ecosystem regardless of if it was the impetus for a new practice or supporting a burgeoning practice. We believe that this influence is causal and part of the larger ecology.

Findings that illustrate uptake in different areas related to the three different types of PD projects shed light on RCT findings that are incremental one-to-two-year post PD and the importance of longer research cycles to continue to study teacher learning. Additionally, understanding that teachers continue to shift and hone their practices years after PD provides a different perspective for how we might position teachers as learners across their career trajectories. Next steps for this work, is the development of case studies and cross case analyses which also will complement the results of RCT studies, and the research reported here. Case studies will use interviews, stimulated recall, and videotapes of classroom practice to illustrate teacher learning and threads to the connections between PD and teacher learning.

### Declaration of Competing Interest

None.

## Appendix. Survey codes

Category	Code	Description	Example
Content	GCSL	General content student learning	<i>We learned how students think about math concepts.</i>
	GCTL	General content teacher learning	<i>I learned more algebra.</i>
	SCSL	Specific content student learning	<i>I appreciated learning about how students use transformations to solve similarity problems.</i>
	SCTL	Specific content teacher learning	<i>I have never heard about dilation before to find similarity!</i>
	MS	Multiple strategies/representations	<i>I learned the importance of allowing my students to solve problems in different ways.</i>
	SSDL	Strategies to support diverse learners	<i>I gained new strategies to use with my EL students like pictures and careful wording.</i>
Pedagogy	ST	Student thinking	<i>I appreciated seeing the various ways that students thought about the problem.</i>
	GPS	General pedagogical strategies	<i>I learned new questioning strategies to use with my students.</i>
	GR	General resources	<i>I liked the LTG problems!</i>
	RSDL	Resource to support diverse learners	<i>I learned various strategies to use with my EL students.</i>
Resources	RTL	Resource for teacher learning	<i>The tracing paper was a great tool to help me visualize and understand transformations.</i>
	SR	Specific resource	<i>I really liked the activity that connected rate of change on double number line.</i>
	TSML	Technology to support math learning	<i>The applets that were shared made it really vivid for me.</i>
	V	Video to support noticing	<i>The videos of students explaining their methods helped me better understand different ways of solving the problems.</i>
Support	C	Collaboration	<i>We learned lots of pairs and shares and turn and talks to get students talking and discussing through discovery learning.</i>
	CS	Coach support	<i>My coach helped me in using the TRUMath framework.</i>
	FI	Facilitator impact	<i>I appreciated how the facilitator listened to all ideas without judgement.</i>
	PS	Principal support	<i>Our principal was very supportive of our participating in the VAM workshops.</i>

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