Editors' Suggestion

Physical Running of Couplings in Quadratic Gravity

Diego Buccio,[1,2,*] John F. Donoghue,[3,†] Gabriel Menezes,[4,‡] and Roberto Percacci[1,2,§]
[1] *International School for Advanced Studies, Via Bonomea 265, 34134 Trieste, Italy*
[2] *INFN, Sezione di Trieste, Trieste, Italy*
[3] *Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA*
[4] *Instituto de Física Teórica, Universidade Estadual Paulista, Rua Doutor Bento Teobaldo Ferraz, 271-Bloco II, 01140-070 São Paulo, São Paulo, Brazil*

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We argue that the well-known beta functions of quadratic gravity do not correspond to the physical dependence of scattering amplitudes on external momenta, and derive the correct physical beta functions. Asymptotic freedom turns out to be compatible with the absence of tachyons.

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Quadratic gravity is an extension of Einstein's theory whose action contains terms quadratic in curvature. In signature $-+++$ it reads

$$S = \int d^4x\sqrt{|g|}\left[\frac{m_P{}^2}{2}(R-2\Lambda) - \frac{1}{2\lambda}C^2 - \frac{1}{\xi}R^2\right], \qquad (1)$$

where $m_P = \sqrt{8\pi G}$ is the Planck mass, Λ is the cosmological constant, $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. We will not consider the Euler (Gauss-Bonnet) term here. This theory is renormalizable [1], and is a potential candidate for a full theory of quantum gravity. In addition to the massless graviton it propagates a massive spin-2 particle that is a ghost and if $\lambda < 0$ it is a tachyon [2]. It also has a massive spin-0 particle that is a tachyon for $\xi > 0$. In spite of these apparent pathologies, it has attracted renewed interest recently [3–9]. In these studies, it is suggested that it may be possible that the ghost state is acceptable, although tachyonic states are considered fatal.

The first attempt to compute beta functions for this theory was made by Julve and Tonin in Ref. [10], but that work missed the contribution of the Nakanishi-Lautrup ghosts. This was corrected in Ref. [11] and then, with some further corrections, in Ref. [12]. The final result is

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2, \qquad (2)$$

$$\beta_\xi = -\frac{1}{(4\pi)^2}\frac{5(72\lambda^2 - 36\lambda\xi + \xi^2)}{36}. \qquad (3)$$

Since then, these beta functions have been confirmed in several calculations using different techniques [13–17]. With these beta functions, full asymptotic freedom can only be obtained for the case of a tachyonic coupling $\xi > 0$. The goal of our Letter is to recompute these beta functions as appropriate for physical amplitudes and show that in fact full asymptotic freedom can be obtained without tachyons.

The beta functions (2) and (3) give the dependence of the renormalized λ and ξ on the renormalization scale μ. We call this the μ running. However, what one is really interested in is the dependence of the running couplings on external momenta, that we call physical running [18]. Physical scattering amplitudes are independent of μ after renormalization, but do depend on the momenta. In particular, the running of λ with momenta enters the spin-two component of the graviton propagator and that of ξ influences the spin-zero propagator. Note that there is no way to define a physical running for the coefficient of the Euler term, since it does not affect the scattering amplitudes in four dimensions.

In problems characterized by a single momentum scale p, e.g., the total center of mass energy $p = \sqrt{s}$, the p dependence is usually the same as the μ dependence, because for dimensional reasons they occur as $\log(p/\mu)$. In the presence of a non-negligible mass scale m, the amplitude generally contains, in addition to terms of the form $\log(p/\mu)$, also terms of the form $\log(m/\mu)$ and in this way the p dependence is no longer correctly reflected by the μ dependence. One clear source of such spurious μ dependence is tadpoles, Feynman diagrams that by construction do not depend on the external momenta. In such cases, the μ running is not the same as physical running.

In most familiar quantum field theories such as the standard model this is not a problem as one can use mass independent renormalization schemes. However, we claim that it is not always correct in higher derivative theories. Two of us have indeed found that in higher derivative sigma models the scale dependent beta functions calculated with a ultraviolet cutoff [23] or those obtained from the dependence on an infrared cutoff [24] are indeed contaminated by tadpoles, and hence not physical [25]. In the present Letter we claim that the same is true in quadratic gravity, and we compute the physical beta functions.

Calculations of the beta functions so far have been based on the background field method, expanding $g_{\mu\nu}=\bar{g}_{\mu\nu}+h_{\mu\nu}$ around a general background $\bar{g}$. In the following a bar always indicates a quantity calculated from the background metric. Almost all calculations used the heat kernel, which is very convenient because it preserves manifest covariance at all stages. These are standard techniques and there are many textbooks and reviews approaching the subject, see for instance [26–31].

The first step is always the linearization of the action and the choice of a suitable gauge-fixing term, leading to an action

$$S^{(2)} = \int d^4x\sqrt{|\bar{g}|}h_{\alpha\beta}\mathcal{H}^{\alpha\beta,\gamma\delta}h_{\gamma\delta}. \tag{4}$$

One can choose the gauge such that the operator governing the propagation of gravitons has the form (suppressing the indices)

$$\mathcal{H} = \bar{\Box}^2\mathbb{K} + \mathbb{J}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu + \mathbb{L}^\mu\bar{\nabla}_\mu + \mathbb{W} \tag{5}$$

and $\mathbb{K}$, $\mathbb{J}$, $\mathbb{L}$, $\mathbb{W}$ are matrices in the space of symmetric tensors, depending on $\bar{R}$ and its covariant derivatives. In particular

$$\mathbb{K} = \frac{1}{4\lambda}\mathbb{P}_{\rm tl} + \frac{9}{4(3\xi-2\lambda)}\mathbb{P}_{\rm tr} \tag{6}$$

where $\mathbb{P}_{\rm tr}^{\alpha\beta,\gamma\delta} = \frac{1}{4}\bar{g}^{\alpha\beta}\bar{g}^{\gamma\delta}$ is the projector on the trace part and $\mathbb{P}_{\rm tl} = \mathbf{I} - \mathbb{P}_{\rm tr}$ the projector on the traceless part. In flat space, $\mathbb{K}$ can be viewed as a tensorial wave function renormalization constant that gives different weights to the spin-2 and spin-0 components of h. As usual it is convenient to canonically normalize the fields by redefining $h \to \sqrt{\mathbb{K}^{-1}}h$, so that the action can be rewritten as

$$S^{(2)} = \int d^4x\sqrt{|\bar{g}|}h_{\alpha\beta}\mathcal{O}^{\alpha\beta,\gamma\delta}h_{\gamma\delta}, \tag{7}$$

where, suppressing again the indices,

$$\mathcal{O} = \bar{\Box}^2\mathbf{I} + \mathbb{V}^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu + \mathbb{N}^\mu\bar{\nabla}_\mu + \mathbb{U}, \tag{8}$$

and $\mathbb{V} = \sqrt{\mathbb{K}^{-1}}\mathbb{J}\sqrt{\mathbb{K}^{-1}}$, etc. Now $\mathbb{V}$ contains terms proportional to $\bar{R}$ and m_P^2, $\mathbb{N}$ contains terms proportional to $\bar{\nabla}\bar{R}$, whereas $\mathbb{U}$ contains terms proportional to $\bar{R}^2$, $\bar{\nabla}^2\bar{R}$, $m_P^2\bar{R}$, and $m_P^2\Lambda$. The logarithmic divergences, or equivalently the $1/\epsilon$ poles in dimensional regularization, are proportional to the heat kernel coefficient [27]

$$\frac{1}{32\pi^2}\int d^4x{\rm tr}\left[\frac{\mathbf{I}}{90}\left(\bar{R}^2_{\rho\lambda\sigma\tau} - \bar{R}^2_{\rho\lambda} + \frac{5}{2}\bar{R}^2\right) + \frac{1}{6}\mathbb{R}_{\rho\lambda}\mathbb{R}^{\rho\lambda} - \frac{\bar{R}_{\rho\lambda}\mathbb{V}^{\rho\lambda} - \frac{1}{2}\bar{R}\mathbb{V}^\rho_\rho}{6} + \frac{\mathbb{V}_{\rho\lambda}\mathbb{V}^{\rho\lambda} + \frac{1}{2}\mathbb{V}^\rho_\rho\mathbb{V}^\lambda_\lambda}{24} - \mathbb{U}\right], \tag{9}$$

where $\mathbb{R}_{\rho\lambda} = [\nabla_\rho, \nabla_\lambda]$ acting on symmetric tensors.

The operator (8) is of the same form as the one acting on the scalars in the higher derivative sigma models, and so are the divergences (9) (except for the terms ${\rm tr}\bar{R}\mathbb{V}$). We can then use the same arguments of Ref. [25]. The terms in the first line of (9) are the ones that we would get for $\mathcal{O} = \bar{\Box}^2$. Using the formula ${\rm Tr}\log\Box^2 = 2{\rm Tr}\log\Box$ one can conclude that none of those terms could be a tadpole, because with a standard p^2 propagator a diagram must have at least two propagators to be logarithmically divergent.

On the other hand consider the term in Eq. (9) which is linear in the $\mathbb{U}$ interaction, i.e., ${\rm tr}\mathbb{U}$. As this only involves one vertex, it is clear that the loop diagram involved must be a tadpole diagram as seen in the second diagram of Fig. 1. Some more detailed arguments lead to the conclusion that also some of the ${\rm tr}\bar{R}\mathbb{V}$ divergences are due to tadpoles. This is enough to conclude that the standard beta functions (2) and (3) cannot be the physical ones.

We thus wish to evaluate the physical beta functions. In order to use flat space Feynman diagrams, we go back to the original Julve-Tonin approach and assume that the background is itself a small deformation of flat space $\bar{g}_{\mu\nu} = \eta_{\mu\nu} + f_{\mu\nu}$. Expanding around flat space, the action (7) gives rise to an operator of the form

$$\mathcal{O} \equiv \boxdot^2\mathbf{I} + \mathcal{D}^{\mu\nu\rho\sigma}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma + \mathcal{C}^{\mu\nu\rho}\partial_\mu\partial_\nu\partial_\rho + \mathcal{V}^{\mu\nu}\partial_\mu\partial_\nu + \mathcal{N}^\mu\partial_\mu + \mathcal{U}, \tag{10}$$

where $\boxdot$ is the flat Laplacian, $\mathcal{D}$ and $\mathcal{C}$ come from the expansion of $\sqrt{\bar{g}}\bar{\Box}^2$, and $\mathcal{V}$, $\mathcal{N}$ and $\mathcal{U}$ are equal to $\mathbb{V}$, $\mathbb{N}$ and $\mathbb{U}$ plus terms coming from the expansion of $\sqrt{\bar{g}}\bar{\Box}^2$. Each of these terms is an infinite series in f. Recall that the

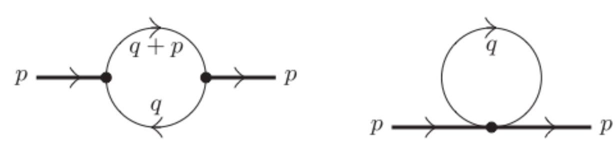

FIG. 1. Diagrams contributing to the two-point function: bubbles (left) and tadpoles (right). The thin line can be the h propagator or one of the ghosts, the thick line is the f propagator, with momentum p. The vertices can come from expanding any one among $\mathcal{D}$, $\mathcal{C}$, $\mathcal{V}$, $\mathcal{N}$, $\mathcal{U}$.

functional trace of the logarithm of an operator can be approximated by

$$\mathrm{tr}\log\mathcal{O} = \mathrm{tr}\log(\boxdot^2 + A) \approx \mathrm{tr}\left[2\log\boxdot + A\frac{1}{\boxdot^2} - \frac{1}{2}A\frac{1}{\boxdot^2}A\frac{1}{\boxdot^2} + \cdots\right]. \quad (11)$$

In the above expansion, A generically represents the remaining contributions to $\mathcal{O}$ appearing in Eq. (10), and again we are suppressing Lorentz indices for brevity. Furthermore, the first term in the perturbative expansion of Eq. (11) corresponds to tadpole integrals, while the second term can be evaluated as a bubble Feynman diagram. Our discussion here concerns how to compute terms proportional to $\log p^2$.

The physical running of λ and ξ comes from terms

$$b_\lambda \bar{C}^{\mu\nu\rho\sigma}\log\bar{\Box}\bar{C}_{\mu\nu\rho\sigma} + b_\xi \bar{R}\log\bar{\Box}\bar{R} \quad (12)$$

in the effective action, and the beta functions are

$$\beta_\lambda = -4b_\lambda\lambda^2, \qquad \beta_\xi = -2b_\xi\xi^2.$$

In flat space contributions to the coefficients b_λ and b_ξ can be read from the two point function of the background fluctuation f, which is represented graphically by the diagrams in Fig. 1.

The two h-h-f vertices in the bubble diagrams are obtained by expanding $\mathcal{D}$, $\mathcal{C}$, $\mathcal{V}$, $\mathcal{N}$, and $\mathcal{U}$ to first order, while for the tadpole one has to expand to second order. Being logarithmically divergent, the tadpole contributes to the μ running but not to the p dependence that we are interested in. Thus the bulk of the calculation consists of working out the Feynman integrals for each of the 15 possible pairs of vertices in the bubble and then evaluating the result for the specific form of the operator (10).

The calculation is simplified by neglecting the terms proportional to m_P. This is justified in the UV limit, as seen explicitly in the case of the simple shift-invariant scalar model [19]. The calculation of the relevant Feynman integrals becomes straightforward and the results are given in the Supplemental Material [32], where we also present all possible pairs of vertices appearing in the bubble integral.

In the calculation one sees in detail how it happens that the μ running differs from the physical running. In dimensional regularization the $\log\mu$ terms always appear together with the $1/\epsilon$ pole, so the μ running just traces the log divergences of the theory. We have checked that putting together all the bubble *and* tadpole diagrams one reconstructs the covariant expression (12) with the coefficients leading to the standard beta functions (2) and (3). If we just drop the tadpoles, the resulting function of f is not the linearization of a covariant expression. Thus, the physical running cannot be obtained from the μ running by just dropping the tadpole contribution. Instead, there are other contributions.

As we have explained earlier, in the presence of a mass, the μ dependence does not correctly describe the amplitude. In our theory the only mass is the Planck mass and one would expect that in the limit $p \gg m_P$, it becomes negligible. However, in this theory with four-derivative propagators the Planck mass also keeps the theory infrared finite. If we neglect m_P in the limit $p \gg m_P$ limit, one finds infrared divergences. This is a new phenomenon that does not occur in standard two-derivative theories. There are then two simple ways to deal with this situation. One is to continue to use dimensional regularization to regulate also the IR divergences which appear in the $m_P \to 0$ limit. In this case *all* the logs are again of the form $\log p^2/\mu^2$, but in addition to the UV logs there are now also IR logs, that change the beta function. As we explain more fully below, summing all the $\log p^2$ terms now gives again a covariant expression, but with a different coefficient. This is the physical beta function. One could alternatively reintroduce artificially a small mass m as an IR regulator by letting $q^4 \to q^4 + m^2 q^2$ [33]. The presence of the regulator mass leads to terms of the form $\log p^2/m^2$, and we are interested in the $\log p^2$ effects. We have checked that both procedures lead exactly to the same result. Notice that the small-time expansion of the heat kernel always gives only the UV divergences.

In our diagrams the IR divergences always appear with powers of the external momentum p in the denominator and therefore give rise to apparently nonlocal $1/\Box$ or $1/\Box^2$ terms. However, since the interactions always involve derivatives, they are offset by an equal number of powers of p in the numerator. Because of differential identities such as

$$\bar{\nabla}_\mu\bar{\nabla}_\nu\bar{R}^{\mu\nu\rho\sigma}\bar{\nabla}_\alpha\bar{\nabla}_\beta\bar{R}^{\alpha\beta}_{\rho\sigma} = \bar{R}_{\mu\nu}\bar{\Box}^2\bar{R}^{\mu\nu} - \frac{1}{4}\bar{R}\bar{\Box}^2\bar{R} + O(\bar{R}^3), \quad (13)$$

or their linearized versions, these momenta always appear in the combination p^2 and cancel the inverse powers of p. In this way also the logs of infrared origin appear as coefficients of local operators. However, these operators are not by themselves the linearization of a covariant expression. It is only when one adds them to the UV logs that they give rise to a covariant expression as in (12). Both types of logs are physical and both are needed to maintain general covariance.

These points can be seen easily by considering the vertex $\mathbb{U}$. It enters the heat kernel calculation linearly, see Eq. (9), corresponding to a tadpole. It is only the part of $\mathbb{U}$ quadratic in curvature that contributes to the beta functions (2) and (3) describing μ running. By contrast in our calculation we need two powers of $\mathbb{U}$ and hence only the part proportional to $\bar{\nabla}\bar{\nabla}\bar{R}$ contributes at order f^2.

These $\mathbb{U}-\mathbb{U}$ bubbles are UV finite but contain $\log p/m$ contributions coming from the IR region. These come with a factor p^4 in the denominator, from the propagators, but also p^4 in the numerator from the vertices. Thus they contribute to the terms (12).

Finally we observe that with our choice of gauge, the Faddeev-Popov ghost operators are of second order in derivatives and none of these exotic phenomena can happen. Thus their contribution can be taken from traditional heat kernel calculations.

Putting everything together, our final result is

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{(1617\lambda - 20\xi)\lambda}{90}, \tag{14}$$

$$\beta_\xi = -\frac{1}{(4\pi)^2}\frac{\xi^2 - 36\lambda\xi - 2520\lambda^2}{36}. \tag{15}$$

The flow lines around the free fixed point $\lambda = \xi = 0$ are shown in Fig. 2.

There are three separatrices, along which the motion is purely radial. The line $\lambda = 0$ is UV repulsive for $\xi > 0$ and UV attractive for $\xi < 0$; the line s_1 is defined by

$$\xi = \frac{569 + \sqrt{386761}}{15}\lambda \approx 79.4\lambda \tag{16}$$

and is attractive for $\lambda > 0$ and repulsive for $\lambda < 0$, and the line s_2 is defined by

$$\xi = \frac{569 - \sqrt{386761}}{15}\lambda \approx -3.53\lambda \tag{17}$$

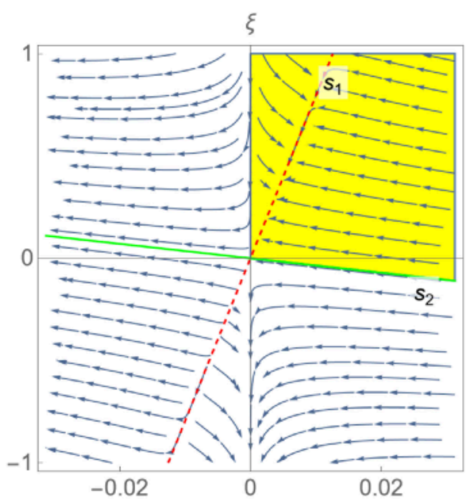

FIG. 2. Flow lines of the beta functions (14) and (15). The red dashed line corresponds to Eq. (16) and the green line to Eq. (17). Initial points in the shaded area are asymptotically free. In the two left quadrants the massive spin 2 state is a tachyon, in the two upper quadrants the massive spin 0 state is a tachyon.

and is repulsive for $\lambda > 0$ and attractive for $\lambda < 0$. Thus the region that is attracted towards the free fixed point is the upper right quadrant, plus a triangular slice of the lower right quadrant that lies above the separatrix s_2.

Recall that absence of tachyons requires $\lambda > 0$ and $\xi < 0$. There is a unique trajectory that is asymptotically free and lies entirely in the tachyon-free area, and that is the separatrix s_2. This behavior is to be contrasted with the flow related to μ running in Eqs. (2) and (3), for which the analog of the separatrix s_2 is the only asymptotically free trajectory, but with a positive slope, with the result that it lies entirely in the tachyonic region. The physical running couplings allow asymptotic freedom without tachyons. Moreover there may be an additional possibility. The pole in the spin-zero propagator appears at $p^2 = -\xi(p^2)m_P^2$, so that one can also have asymptotically free trajectories where the coupling $\xi(p^2)$ is negative at the pole in order to avoid a tachyonic state, but has a positive sign at higher momenta. One can see such trajectories above s_2 which have $\xi > 0$ in the far UV but eventually cross into $\xi < 0$ when one goes towards the IR.

In summary, a key result of our work is that asymptotic freedom can be obtained in quadratic gravity without tachyons. This is because the physical running with the momenta is the appropriate running coupling to be used in amplitudes. We have noted the difference between this running and that which just follows $\log\mu$ when applied to theories with higher derivatives, and have developed new methods to calculate the physical running in the gravitational theory. The possibility of tachyonless asymptotic freedom makes quadratic gravity more plausible as a complete renormalizable theory of quantum gravity.

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[*]dbuccio@sissa.it
[†]donoghue@physics.umass.edu
[‡]gabriel.menezes10@unesp.br
On leave of absence from Departamento de Física, Universidade Federal Rural do Rio de Janeiro.
[§]percacci@sissa.it

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[32] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.021604 for some intermediate results of the calculations.

[33] If we had kept the Hilbert term, m^2 would be either λm_P^2 or ξm_P^2 depending on the spin component of the propagator. However, the physical running with $\log p^2$ is independent of these masses so that it is most convenient to just use a common mass when using it as an IR regulator. No spurious poles are introduced by this procedure as long as the sign of m^2 is chosen to avoid tachyons.