Learning Problem Decomposition-Recomposition with Data-driven Chunky Parsons Problems within an Intelligent Logic Tutor[*]

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ABSTRACT

Problem decomposition into sub-problems or subgoals and recomposition of the solutions to the subgoals into one complete solution is a common strategy to reduce difficulties in structured problem solving. In this study, we use a data-driven graph-mining-based method to decompose historical student solutions of logic-proof problems into *Chunks*. We design a new problem type where we present these chunks in a *Parsons Problem* fashion and asked students to reconstruct the complete solution from the chunks. We incorporated these problems within an intelligent logic tutor and called them *Chunky Parsons Problems* (CPP). These problems demonstrate the process of problem decomposition to students and require them to pay attention to the decomposed solution while they reconstruct the complete solution. The aim of introducing CPP was to improve students' problem-solving skills and performance by improving their decomposition-recomposition skills without significantly increasing training difficulty. Our analysis showed that CPPs could be as easy as Worked Examples (WE). And, students who received CPP with simple explanations attached to the chunks had marginally higher scores than those who received CPPs without explanation or did not receive them. Also, the normalized learning gain of these students shifted more towards the positive side than other students. Finally, as we looked into their proof-construction traces in posttest problems, we observed them to form identifiable chunks aligned with those found in historical solutions with higher efficiency.

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Keywords

Parsons Problem, Intelligent Tutors, Data-driven Subgoal, Problem Decomposition

1. INTRODUCTION

Computational thinking, a set of skills and practices for complex problem solving, provides a foundation for learning 21st-century skills, particularly computer science (CS). Educational researchers and teaching professionals acknowledge problem decomposition-recomposition skill as a key component of computational thinking and complex problem solving [11, 23, 13, 22]. Efficient problem solving using the problem decomposition skill or strategy involves several steps: 1) identifying sub-problems (i.e. subgoals) to reduce the difficulty associated with the problem, 2) constructing a solution for each of those sub-problems, and 3) recomposing the sub-problem solutions to form the larger solution [5]. Research showed that experts carry out problem decomposition and recomposition (PDR) steps more than novices [41]. However, several studies also showed that novices often attempt to decompose problems [26, 40]. But while they may demonstrate correct decomposition in easier problems, novices fail to decompose sophisticated problems [26].

Despite problem decomposition-recomposition (PDR) being vital to complex problem-solving, it is rarely mentioned explicitly in instructional materials for computer science (a discipline focused on complex problem solving using computers) [30]. Also, existing research lacks guidance on how to motivate students to adopt this PDR process or how to improve their skills associated with PDR. A few studies analyzed the differences between experts and novices in adopting this PDR process, indicating that experts use PDR more than novices [27, 20]. And, a few studies aimed at introducing this PDR skill to students, mostly during programming problem solving, using varying methods [for example, using pattern-oriented instruction [32], programming problem decomposition exercise [42], guided inquiry-based instruction [36], etc.]. However, PDR remains under-explored in the instruction of other structured problem-solving domains.

In this study, we design, implement, and evaluate a problem-based training intervention, named *Chunky Parsons Problem* (CPP), that introduces to students the concept of problem decomposition-recomposition (PDR) while engaging them in these processes during problem solving within an intelligent logic tutor, DT (Deep Thought). To generate CPP, we decomposed students' historical solutions to each logic-proof construction problem stored in DT's problem bank into sub-proofs (referred to as ***chunks***) using a data-driven method. These chunks are presented in a *Parsons Problem* fashion. In a traditional Parsons Problem, all steps contributing to the complete solution of a problem are presented in a jumbled order. On the contrary, within CPP, the solution to a logic-proof problem is presented as jumbled-up chunks (groups of connected statements) instead of individual statements. By design, CPP is a partially worked example where all the required statements are shown in chunks. However, the missing connections among the chunks give the students an opportunity to recompose the solution while having them pay attention to the decomposition to understand the composition of each chunk, how each chunk contributes to other chunks, and the overall solution. Thus, CPPs can be thought of as problems that are partially worked examples and partially problem-solving (PS) problems.
We deployed DT with CPP implemented within its training session in an undergraduate classroom of CS majors and conducted a controlled experiment. In the controlled experiment, we implemented three training conditions: 1) Control(C): received only worked example (WE) and problem-solving (PS) logic-proof construction problems, 2) Treatment 1(T_1): received CPP (**without** explicit explanation of the chunks) along with PS/WE, and 3) group who receive CPP (**with** explicit explanation attached to the chunks) along with PS/WE. Since prior research showed that explicit instruction on what to learn or take away from an intervention may help to improve students' decomposition ability [36], we introduced the last training condition to identify the more effective representation (between with/without explanation) of CPP. Finally, we evaluated the efficacy of CPP by answering the following research questions:

- ***RQ1:*** How do *Chunky Parsons Problems* impact students' performance and learning?
- ***RQ2:*** What are the difficulties associated with solving a *Chunky Parsons Problem*?
- ***RQ3:*** How do *Chunky Parsons Problems* impact students' Chunking (problem decomposition-recomposition) behavior and skills while solving a new problem?

2. BACKGROUND AND MOTIVATION

Existing research has identified problem decomposition-recomposition (PDR) as difficult for novices as problems get more complex [26]. However, we found only a few studies investigating methods to improve this skill. For example, Pearce et al. [36] explored explicit instruction (openly instructing students to learn problem decomposition and describing how to go about that learning) to improve students' problem decomposition skills and concluded that explicit instruction can lead to significant gains in mastering this skill. Muller et al. [32] found that pattern-oriented instruction can have a positive impact on problem decomposition skills. We found some studies where researchers in the domain of mathematics and programming, using problem-based methods, aimed at improving students' subgoal learning which is equivalent to the skill of identifying sub-tasks required to solve a problem (i.e. problem decomposition). The most common method explored by researchers in this regard is subgoal-labeled worked examples or instructional materials [29, 8, 7]. Studies showed that worked examples with abstract labels that give away structural information help improve students' problem-solving skills measured by test scores. However. these studies do not evaluate or measure students' problem decomposition or subgoaling skills after training. Also, we did not find any established guidance on how problem-based interventions can be generated automatically and how they should be designed to be used within tutors to improve students' problem decomposition-recomposition (or chunking) skills.
From our literature review, we concluded that problem-based interventions specifically designed for tutors to improve students' chunking skills are under-explored. Thus, in this paper, we set our aim to design and implement CPPs to be used within DT to improve students' problem-solving and chunking skills. While extracting chunks to present within CPP and designing its representation within DT, we considered three goals: 1) Automating the solution-decomposition process to extract chunks so that expert effort is not required; 2) Designing the problem to demonstrate chunking and engaging students in the process to improve their skills, and 3) keeping the difficulty-level low so that students can persist and learn. To set the difficulty level of our problem, we explored problem types that are of low difficulty as established by literature: Worked Examples and Parsons Problems.
Worked Examples: Worked examples (WE) reduce learners' intrinsic load (i.e. working memory load which is caused by the complexity of the problem) and help them to learn better [35]. This improvement in learning due to worked examples is referred to as the Worked Example Effect [44] in literature. However, several studies argued the applicability of worked examples in certain situations. For example, worked examples may not be useful for students with high prior knowledge [34], when problems are structured [34], or if the problem is strategic but involves only a few interactive elements [10]. In such cases, problem-solving (PS) supports the learning process better [10]. Also, for goal or product-directed problems, a worked example only shows the construction of the solution and does not help students to grow an understanding of the rationale behind the selection of certain steps [49]. In this scenario, students fail to acquire a schema of the problem-solving approach which leads to the failure to transfer problem-solving skills. Renkl et al. [39] suggested that worked examples help students to learn better only when the examples give away structural information of the solution and isolate meaningful building blocks.
Parsons Problems: Parsons problems ask students to construct a solution from a given set of jumbled solution steps [14]. Poulsen et al. showed the application of the Parsons problem in a mathematical proof construction tool, Proof Blocks [38].

They found that Parsons Problems within Proof Blocks significantly reduced the difficulty associated with proof construction. Parsons problem is heavily explored in programming education. Studies found that Parsons problems can improve students' code writing capability [48, 24, 14, 17] or can help in completing programming tasks efficiently without impacting performance on subsequent programming tasks [51]. Studies also showed that attached explanations [18] and subgoal labels [31] can help students solve Parsons problem and improve the learning process.
From the overview of the impact of WEs and Parsons Problem, we designed CPPs as partially WE and partially PS, which represent meaningful building blocks of a proof (i.e. Chunks) in a Parsons Problem fashion. Additionally, we explored attaching explanations to the chunks to further reduce difficulties and support students' learning process.
Data-driven Solution Decomposition Techniques: Data-driven solution decomposition refers to the process of automatically decomposing a problem or its solution into subgoals or sub-solutions based on the properties found in historical solutions. These historical solutions often come from tutors or learning platforms that collect students' solution traces and thus, are often redundant. While performing data-driven decomposition, researchers mainly focused on identifying independent or dependent components of a solution. To do so, they often presented student solutions as graphs depicting how they moved from state to state to reach the final solution [37, 50, 45, 33, 4]. Prior research showed the application of clustering [37] or connected component detection [50, 45, 16] to extract independent sub-solutions or chunks in these graphical models. On the other hand, some researchers [12, 19] proposed constraint-based decomposition techniques for linear problem solutions such that decomposed sub-solutions can be replaced with alternate solutions without causing any problem. To decompose computer programs, researchers have used methods where they looked at the usage of different program components (for ex., variables) to identify independent parts of the program [46, 47]. In this paper, we demonstrate a solution decomposition method that extracts chunks by applying rules/constraints on graphical representations of historical student solutions similar to Eagle et al.'s work [16].
Evaluation of Students' Chunking Skills We observed that in prior research, researchers have only used test scores to evaluate methods that were set to teach students chunking/PDR. Only a few recent studies explored methods to measure students' chunking skills from data. Kwon and Cheon [26] mapped predefined sub-tasks and program segments in Scratch programs to observe how students decompose and develop programs. In a recent study, Charitsis et al. [9] used NLP to identify key components and students' approaches to develop programs and then relate those to performance metrics using linear regression to quantify their decomposition skills. Kinnebrew et al. [25] also mined frequent patterns in students' action sequences and relate that to performance to explain their learning behavior. Overall, to evaluate decomposition skills, these studies each sought a baseline to compare students' solutions against and explained performance using solution characteristics. In this paper, to measure students' chunking/PDR skills after being trained with CPPs, we analyzed students' step sequences during proof construction to identify potential chunking/PDR instances and tried to explain their performance through the chunking/PDR characteristics.

3. METHOD

In this study, we explored Chunky Parsons Problems (with or without explicit explanations attached to them) to improve students' problem-solving skills (with an emphasis on *Chunking/PDR* skills) and learning gain in the context of logic-proof problems. We derived CPP using a data-driven method and incorporated them into the training session within DT [28], an intelligent logic tutor. In the subsequent sections, we first provide a brief introduction to DT. Then, we discuss how we derived CPP from data, designed explanations explaining the chunks, and presented them within DT. Finally, we present the design of our experimental training conditions and data collection method to facilitate analyses to answer our research questions

3.1 Deep Thought (DT), the Intelligent Logic Tutor

DT is an intelligent logic tutor that teaches students logic-proof construction. Each logic-proof problem within DT contains a set of given premises and a conclusion presented as visual nodes [Figure 1a]. To solve a problem, new propositions (or nodes) are needed to be derived by applying valid logic rules on the given premises and subsequently on derived premises to reach the conclusion. Usually, each problem in DT is either of type Worked Example (WE) or problem-solving (PS). WEs are solved by the tutor step-by-step as the students click on a next step (>) button [Figure 1b]. On the other hand, PSs are required to be solved by the students where they have to derive all the steps of a proof [Figure 1a]. Here, a step refers to the process of deriving a single node or proposition.
DT is organized into 7 levels. In the first level, the tutor starts by showing two sample logic-proof problems (one WE and one PS) to help students understand how to use different features of the tutor. Then, the students solve two pretest PS problems. After the pretest level (i.e. level 1), the students go through 5 training levels with 4 problems in each level. Each of the first three problems in the training levels is either a WE or PS. For these training-level PS problems, on-demand step-level hints are available. The last problem in each training level is always of type PS and is called the training-level test problem. After the 5 training levels, students enter into a posttest level containing 6 PS problems. During the pretest, training-level test, and posttest problems, the tutor does not offer any hints or help and the students have to solve them independently. For each of these problems, students receive a score between 0 and 100 (efficient proof construction [less time, fewer step counts, and incorrect rule applications] receives higher scores) [3, 1]. The pretest scores represent students' mastery level before training. On the other hand, the training-level posttest and posttest scores track how much students learned after each level of training and after all 5 training levels. More Details on DT interface and features can be found in Appendix A.

3.2 Deriving Chunky Parsons Problem (CPP) using a data-driven Graph-Mining Approach

Data for Deriving CPP: DT has been being deployed in an undergraduate logic course offered at a public research uni-

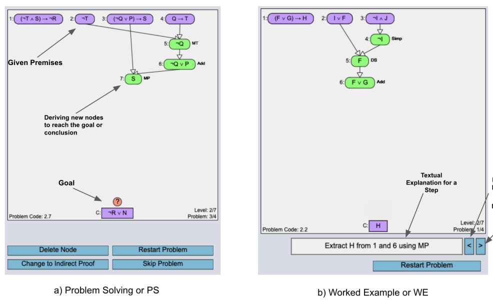


Figure 1: (a) PS and (b) WE Interface in DT

versity in the Fall and Spring semesters since 2012. To derive CPP representation for logic proofs, we used the most recent log data collected by DT in the Fall and Spring of the years 2018-2021. These log data detail students' historical step-by-step proof-construction attempts for each problem they solved within DT. Using these data, we generated high-level graphical representations (Interaction Networks [16] and Approach Maps [15]) of students' solution approaches for these problems to derive CPP from them. For each problem, data of approximately 170-200 students containing altogether 2000-5500 solution steps were used. To select the students, we performed equal random sampling from the semesters mentioned before so that the data is representative of different student groups who took the course over the years. The reason for not using all data is to mainly reduce the computational complexity of the adopted graph mining approach. Also, the data used is assumed sufficient enough to capture common student approaches to solve each logic-proof problem in the problem bank of DT [43].

Interaction Network and Approach Map: For each of the problems in the DT problem bank, we generated a graphical representation of how students moved from one state to another during the construction of a proof for the problem. Here, a state refers to all nodes (or propositions) a student had at a particular moment during their proof construction attempt. Students move from state to state by deriving or deleting nodes, i.e. via a step. To limit the number of states in the graph, the propositions at a particular state are lexicographically ordered, which means that the order of derivation of the nodes is not considered in the graphical representation. This graphical representation of students' proof-construction attempts for a problem is called an interaction network since it represents the interaction among the states [16]. Since interaction networks are often very large and visually uninterpretable, we applied Girvan-Newman community clustering [21] on the interaction networks to identify regions or clusters of closely connected states. Each cluster contains a set of states containing effective propositions that contributed to the final proof submitted by the students and also unnecessary propositions (i.e. propositions that did not contribute to the final proof) that they derived along the way. We represented each cluster with one single graphical node containing only the effective propositions. Thus, we obtained a graph where the start state containing the given premises is connected to the conclusion through clusters of effective propositions. Each path from start to conclusion represents one student approach (or solution) to the logic-proof problem [sample approach maps are visualized in Figure 2]. Thus, this representation is called an approach map [15]. Later, we used a rule-based approach to extract chunks from the approach maps.

Extracting Chunks from Approach Maps: As discussed in Section 2, researchers [12, 19] have decomposed problem solutions using constraints such that the decomposed sub-solutions can be replaced with alternate solutions without causing any problem. Based on this idea, we defined two rules to extract pivot[1] or subgoal propositions that are present in multiple approaches and/or have multiple replaceable derivations within an approach map (for example, $\neg K \vee N$ in Figure 2a has two possible derivations from the start state.). The rules to identify such pivots are:

Rule 1: First proposition derived within a cluster where multiple clusters merge is a pivot [$\neg K \vee N$ in Figure 2a].

Rule 2: Last proposition derived within a cluster that generates a fork is a pivot [$\neg K \vee N$ and $\neg(K \wedge \neg N)$ in Figure 2a].

Recall that an approach is a path from start to goal in an approach map. And, being present in multiple paths or approaches means that a proposition is possibly vital to the proof and a subgoal in student approaches. Finally, we defined a third rule to identify pivots in approaches that do not have a common proposition with other approaches, i.e. they are simply a linear chain of clusters of propositions[Figure 2b]. The third rule is described below:

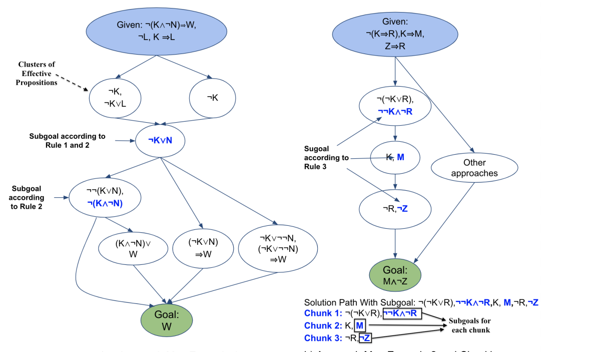
Rule 3: In a chain of clusters, the last derived node in each cluster is a pivot [M or $\neg Z$ in Figure 2b]. Note that in this rule, we simply exploit the clusters identified by Girvan-Newman algorithm to dismantle a complete solution into sub-solutions or subgoals.

Finally, Using the three rules, we extracted the subgoals within the most common student-solution approach for each DT logic-proof problem while traversing its approach map from top to bottom. We validated our pivot/subgoal- extraction process by comparing our rule-based subgoals from approach maps against expert-identified[2] subgoals for 15 problems. And, our method was successful in identifying all expert subgoals for those problems. After validation, we used the subgoals to decompose the solution to derive *Chunks* from them. An example of deriving chunks can be found in Figure 2b. In the example, pivots/subgoals are colored blue, and using the subgoals three chunks are extracted from the complete solution. Note here that each chunk is associated with a subgoal.

Explanations for Chunks: To accompany each of the chunks, we generated automated explanations using a script that explains the composition and purpose of the chunks. Before writing the script, a format for the chunk explanations was decided through discussion with an expert. Each explanation is written in natural language and tells what a chunk derives (i.e. the associated subgoal), how the subgoal is derived within the chunk, and why it is derived [Figure 3b]. The why part simply tells that each subgoal is necessary for the derivation of another subgoal or the final goal. Note that we paid close attention while crafting the explanation format so that it does not give away any information about the final solution beyond the visual representation of the chunks. Overall, the purpose of the explanations is just to

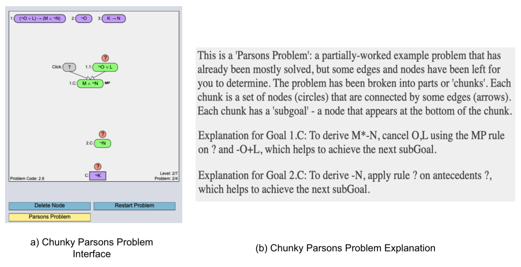
[1]major propositions within a proof that can be used to decompose the proof, also referred to as *Subgoals*.

[2]The experts are two academic professionals with 10+ years of experience with logical reasoning



a) Approach Map Example 1 b) Approach Map Example 2 and Chunking

Figure 2: Demonstration of a) Rule 1 and 2; and b) Rule 3 and Chunk Extraction



This is a 'Parsons Problem': a partially-worked example problem that has already been mostly solved, but some edges and nodes have been left for you to determine. The problem has been broken into parts or 'chunks'. Each chunk is a set of nodes (circles) that are connected by some edges (arrows). Each chunk has a 'subgoal' - a node that appears at the bottom of the chunk.

Explanation for Goal 1.C: To derive M*-N, cancel O,L using the MP rule on ? and -O+L, which helps to achieve the next subGoal.

Explanation for Goal 2.C: To derive -N, apply rule ? on antecedents ?, which helps to achieve the next subGoal.

a) Chunky Parsons Problem Interface (b) Chunky Parsons Problem Explanation

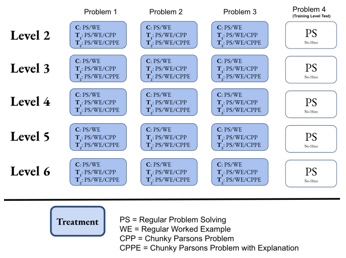
Figure 3: a) Parsons Problem Interface in DT; b) Explanation Given to Specific Student Groups for Chunks Presented in a Parsons Problem.

highlight what the chunks represent (i.e. they are building blocks of a complete solution each deriving a subgoal).

3.2.1 *Chunky Parsons Problem Interface*

The Chunky Parsons Problem representation is shown in Figure 3a. In the presentation, the given premises and conclusion are presented as usual. And the chunks are presented as groups of connected propositions (or nodes) in a Parsons Problem fashion. The problem shown in Figure 3a has two chunks. All nodes within the chunks are connected to each other. However, the givens, the chunks, and the conclusion need to be connected by students to complete the proof. Each node within a chunk can be either justified (both antecedents present), partially justified (one of the antecedents missing as for $M \wedge \neg N$), or unjustified (all of the antecedents missing as for $\neg O \vee L$ or $\neg N$). For justified and partially justified nodes, the associated logic rule is also shown. For example, in Figure 3a, $M \wedge \neg N$ is labeled by MP, i.e. Modus Ponens is required for its derivation from the antecedents. In addition to the visual components, textual instructions containing chunk explanations [Figure 3b] were also provided to students who were assigned to a specific training condition (more details about training conditions in the next subsection). Note that the chunk or subgoal IDs (for example, 1.C, 2.C, etc. in the figure) are used to associate an explanation to a chunk and the IDs do not confirm the order of how the chunks should be connected to each other.

3.3 Experiment Design



	Problem 1	Problem 2	Problem 3	Problem 4 (Training Level Test)
Level 2	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	PS No Hint
Level 3	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	PS No Hint
Level 4	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	PS No Hint
Level 5	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	PS No Hint
Level 6	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	**C**: PS/WE **T$_1$**: PS/WE/CPP **T$_2$**: PS/WE/CPPE	PS No Hint

Treatment

PS = Regular Problem Solving
WE = Regular Worked Example
CPP = Chunky Parsons Problem
CPPE = Chunky Parsons Problem with Explanation

Figure 4: Problem Organization in the Training Levels for the Three Training Conditions. Note: '/' indicates a random selection. For example, 'PS/WE/CPP' indicates that the problem will be randomly presented as either a PS, or a WE, or a CPP.

Using existing problem types within DT (PS and WE) and the new problem types (CPP), we designed three training conditions. The three training conditions are described below:
Control (C): Students assigned to the Control (C) condition received only PS or WE (selected randomly) during training.
Treatment 1 (T_1): Students assigned to this condition, may receive CPP without explanation in addition to PS/WE (selected randomly) during training.
Treatment 2 (T_2): Students assigned to this condition may receive CPP with an explanation (i.e. CPPE) in addition to PS/WE (selected randomly) during training.
Problem organization for each condition in the 5 DT training levels is demonstrated in Figure 4. Note that the Control (C) condition gives us a baseline for comparison between students who received CPP or CPPE [i.e. T_1/T_2 students]) and those who did not receive CPP at all (i.e. C students). On the other hand, a comparison between T_1 and T_2 helps to understand the impact of the explicit explanation attached to each chunk in a CPP.
System Deployment and Data Collection: We deployed DT with the three training conditions in an undergraduate logic course offered at a public research university in the Spring of 2022. Each participating student in that course was assigned to one of the three training conditions after they completed the pretest problems. Our training condition assignment algorithm ensures that the pretest scores of students in each of the training groups have a similar distribution. Finally, we had 50 students assigned to C, 50 students assigned to T_1, and 45 students assigned to T_2 who completed all 7 levels (pretest, all training levels, and the posttest level) of the tutor. We collected their pretest, training-level test, and posttest scores to compare performance/learning across the training groups. Additionally, we collected their solution traces to analyze differences in their proof construction approaches. Note that access to these data is restricted to IRB-authorized researchers. To answer our research questions, we carried out statistical and data-driven graph-mining-based analyses on the collected data that we report in the subsequent sections.

4. RESULTS

4.1 RQ1: Students' Performance and Learning Gain

To understand the impact of each of our training conditions on students' performance and learning, we analyzed students' test score-based performance and normalized learning gain (NLG) after training. For these analyses, we focused on the training-level test problems (2.4-6.4) and the posttest problems (7.1-7.6) that students solved independently without any tutor help. We adopted a combination of regression and statistical analysis (Kruskal-Walis test with posthoc pairwise Mann-Whitney test with Bonferroni corrected $\alpha = 0.016^3$) to compare the performance and learning gain across the three training conditions. These tests do not make an assumption about the data being perfectly normal. Since most of our collected data were skewed, these tests were considered suitable in this case. Note that there were no significant differences found in performance across the three groups in the pretest problems.

4.1.1 Test Score-based Performance

To identify the association between the training conditions and performance, we performed two mixed-effect regression analyses: one for the training-level test problems and one for the posttest problems. In each of these two analyses, problem IDs were defined as the random-effect variable (to eliminate the impact of differences across problems), training conditions were defined as the fixed-effect variable, and problem score was the dependent variable. The analysis for the training-level test problems [avg. training-level test scores(C, T_1, T_2) = 65.1, 61.7, and 65.5] gave a p-value of 0.8 [$p < 0.05$ indicates significance] indicating that there was no significant association between training-level test performance and the training conditions. However, the analysis for the posttest problems [avg. posttest scores(C, T_1, T_2) = 69.3, 68.2, and 73.5] gave a p-value of 0.06 demonstrating a marginally significant association between the training conditions and posttest performance. Also, the average posttest scores showed that T_2 (who received CPPE) marginally outperformed the other two groups after 5 levels of training. To further investigate each training group's posttest performance, we statistically compared scores across the three training groups in each of the independent posttest problem-solving instances (7.1-7.6). The trend in scores for these problems across the three training groups is shown in Figure 5. While analyzing the scores in the posttest problems, we observed that T_2 had significantly higher scores than T_1 and C in problems 7.1-7.3 [for 7.1, $P_{MW}(T_2 > T_1)^4 = 0.003$ and $P_{MW}(T_2 > C) = 0.01$, for 7.2, $P_{MW}(T_2 > T_1) = 0.02$ and $P_{MW}(T_2 > C) = 0.01$, and for 7.3, $P_{MW}(T_2 > T_1)$ = 0.03 (marginal) and $P_{MW}(T_2 > C) = 0.012$] and higher average scores in problems 7.4-7.6. Note that posttest problems in DT are organized in increasing order of difficulty. Our analyses indicate that even though T_2 could not significantly outperform the other two groups in the harder posttest problems(7.3-7.6), they performed comparatively better. This trend can be observed in the 'Posttest' fragment in Figure 5.

As shown in the figure, although T_2 did not show a significantly higher average than the other two conditions in all problems, starting from problem 6.4, T_2 students always had higher scores (shown by the solid green line) than the other two groups (shown by the dotted blue line and dashed orange line). Overall, from our regression and statistical analysis, we concluded that students' posttest performance was associated with the training conditions, and the T_2 training condition that involved CPPE was more helpful in improving students' performance after training. However, T_2 students showed evidence of improved performance around the end of training and in the posttest rather than showing gradual improvement over the period of training. A consistent pattern that indicates improved performance could not be identified for T_1 students who received CPP without an explanation attached.

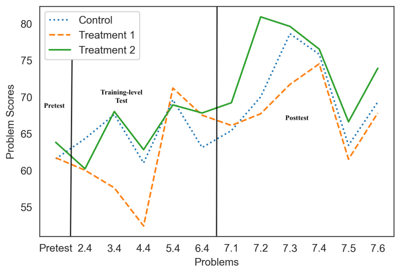


Figure 5: Training-level Test and Posttest Scores across the Three Training Groups

4.1.2 Normalized Learning Gain

To identify the training condition that was most effective in promoting learning, we analyzed students' normalized learning gain (NLG) across the three training conditions. NLG is defined as the ratio between how much the students learned and the maximum they could have learned between the period of pretest and posttest and is represented by the following equation:

$$NLG = (post - pre)/\sqrt{(100 - pre)} \quad (1)$$

Note that NLG is normalized between -1 and 1. A negative NLG value represents that the posttest scores are lower than the pretest scores. Negative NLGs could occur if the students did not learn enough from training or if the posttest problems are significantly harder than the pretest problems. NLG for the three groups is shown in Table 1. We compared the NLGs across the three training groups using statistical tests. A Kruskal-Walis test demonstrated significant differences in the NLGs across the three training groups (statistic=5.8, p-val=0.05). As we carried out posthoc pairwise Mann-Whitney U tests with Bonferroni corrected α=0.016, we observed T_2 students had significantly higher NLGs than Control (C) (statistic=1283.0, p-val=0.01) and T_1 (statistic=1235.0, pvalue=0.02) students. As we plotted the distribution of NLGs for the training groups in Figure 6, we observed that the distribution of NLGs for T_2 is centered around positive (+) values, whereas the other two groups had tails on the negative (-) side. Also, as reported in Table 1, 80% of T_2 students had a positive NLG, whereas the percentage for the other two groups are only 70% and 72% respectively. The results of this analysis on NLG indicate that training condition T_2 (combination of CPPE with PS/WE)

[3]In the pairwise tests each datapoint was used in at most three tests: (C, T_1), (C, T_2), and (T_1, T_2). Thus, corrected $\alpha = 0.05/3$

[4]$P_{MW}(T_2 > T_1)$ refers to the p-value obtained from the Mann-Whitney U test for the hypothesis "T_2 had significantly higher values than T_1 for the metric under consideration." $p < 0.016$ indicates significance.

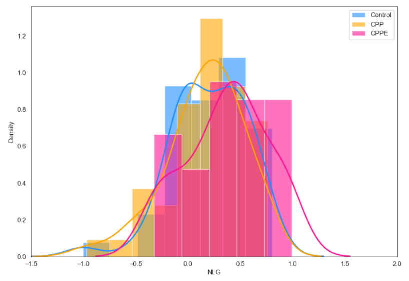


Figure 6: NLG across the Three Training Groups

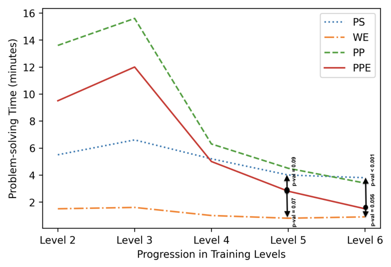


Figure 7: Problem-solving Times for Different Problem Times over the Period of Training.

Table 1: Normalized Learning Gain (NLG) across the Three Training Groups

Group (n)	Pre	Post	NLG	%Student with(+) NLG
C (50)	61.6(19.7)	70.4(14.4)	0.20(0.35)	70%
T1(50)	61.7(18.3)	68.2(15.1)	0.16(0.37)	72%
T2(45)	60.8(18.9)	73.2(14.8)	0.31(0.33)	80%

helped the students to learn better which moved their NLG above 0. Possibly, CPPE helped the students to perform comparatively well even in the harder problems (7.3 to 7.6) that could have caused negative NLG otherwise.

4.2 RQ2: Difficulties Associated with Solving Chunky Parsons Problems

The results from students' performance analysis showed that CPP with explanations attached to chunks (i.e. CPPE) has the potential to improve students' performance and learning gains. However, since it is a new type of problem-based training intervention, we acknowledged the necessity of analyzing its difficulty level in comparison to traditional training interventions like PS or WE. Since tutors like DT are often used by learners in the absence of a human tutor, our aim was to avoid increasing the training difficulty so that the students can persist and learn. Thus, we carried out a comparative analysis between the difficulty level of training CPP/CPPE and PS/WE problems. The difficulties associated with each problem type were measured by the average time that the students needed to solve them (i.e. the problem-solving time). Additionally, to guide future improvements so that the students are better supported during training with CPP/CPPE, we carried out an analysis to identify difficulties that could be associated with specific problem structures where students may need additional help to succeed. In the subsequent sections, we report the findings from the two analyses.

4.2.1 Comparative Difficulty Level of CPP/CPPE

To understand the comparative difficulty level of CPP/CPPE, we compared the problem-solving times of CPP/CPPE against the problem-solving times of PS/WE using Mann-Whitney U tests. The plot representing problem-solving times for each of these problem types over the period of training is shown in Figure 7. Notice that in the first two training levels, students' problem-solving time for CPP/CPPE was almost twice the problem-solving time of PS. This higher problem-solving time in the early training levels could be potentially associated with the additional time that the students needed to figure out how different components in the CPP/CPPE interface within DT work. However, as training progressed problem-solving time for CPP became more aligned with that of PS [notice the problem-solving times and comparative p-values at training levels 5 and 6 in Figure 7]. On the other hand, CPPE problem-solving times were marginally or significantly lower than that of PS at levels 5 and 6 respectively. Additionally, CPPE problem-solving times were only marginally higher than that of WEs in these two levels. These statistics indicate that the difficulty level of Chunky Parsons Problems (with/without explanation) lies in between the difficulty levels of PS/WE. However, with explanation, it can be a low-difficulty training task (difficulty level similar to WEs and lower than PS in terms of problem-solving time) that can help improve students' learning gain.

4.2.2 Difficulties Associated with Specific Problem Structure

To identify difficulties associated with specific problem structures, we calculated the average time students spent to complete the proof of each chunk presented in a CPP/CPPE (by connecting all nodes within a chunk to their correct predecessor). We call this chunk-solving time. We identified 10 problems that contained chunks with chunk-solving time above the 75th percentile ($>$ 2.5 minutes) for at least 10% ($>=$ 10 students) of all T_1 (CPP) and T_2 (CPPE) students. To identify the difficulty patterns in these problems, we carried out an exploratory analysis of the structures of these problems and how the students approached to solve the problem. For simplicity, while explaining the problems associated with student difficulties, we present only the abstract structure of the problems [Figure 8a, b, and c]. In the abstract structure, we show how the chunks need to be connected to solve the problem and rule categories instead of the specific rules required to connect the chunks. We grouped the available logic rules in DT into 3 categories: 1) Transformation rules: transform the logic operator in between variables or reorganize the variables in a proposition (Comm, Assoc, DN, De Morgan, Impl, CP, Equiv, Dist), 2) Elimination: remove one or more variables from proposition(s) (MP, MT, DS, Simp, HS), 3) Combination: combines variables from two propositions in one proposition (Add, Conj, CD). For the 10 problems, we identified three abstract problem structures that are shown in Figure 8. Structure 1 was associated with 6 problems. Structures 2 and 3 were asso-

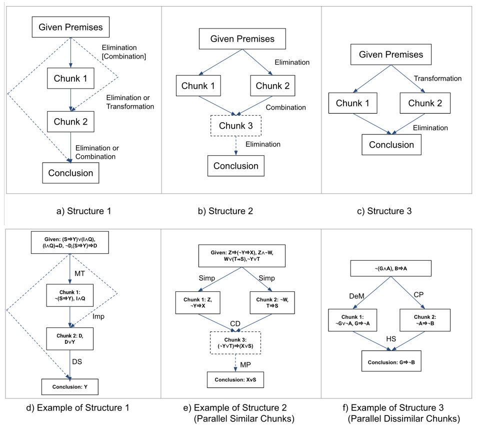


Figure 8: Abstract Structure of Problems where Students Spent Higher Times when Presented as CPP or CPPE. Note: Dashed components are missing in some problems.

ciated with 2 problems each. Below we present our observations on student difficulties (i.e. when and where in these structures students spent more time) associated with each problem structure:

Structure 1: In structure 1 [Figure 8a], the chunks are sequentially connected with different categories of rules. We observed that within each of the six difficult problems with this structure, there are almost no visual commonalities across the chunks. An example problem with this structure is shown in Figure 8d. In the figure, notice that each chunk contains propositions composed of variables from almost exclusive sets (Chunk 1 variables=S, I, Y, Q), Chunk 2 variables=D, Y), and also each chunk requires a different rule.

In these 6 problems, we found 71 students (T_1 = 39, T_2 = 32) who spent time above the 75th percentile to derive a chunk within at least one of these problems. A total of 247 difficult instances were found for these students solving problems with structure 1. 132 of those instances were associated with forward-directed sequential derivation (i.e., the students completed the problem in the following sequence, chunk 1 $\rightarrow$ chunk 2 $\rightarrow$ conclusion), 11 were associated with backward-directed sequential derivation (i.e., the students completed the problem in the following sequence, conclusion $\rightarrow$ chunk 2 $\rightarrow$ chunk 1), and 104 instances were associated with random derivation where students moved from chunk to chunk without demonstrating a strategical pattern.

Overall, after the analysis of the structure of the 6 problems and student approaches to solving the problems (forward, backward, or random), we could not associate a specific approach with the chunk-solving difficulty. Rather, we concluded that the difficulties could be associated with the diversity in rules/variables across chunks within the problems that possibly increased cognitive load[5] introducing difficulties for students.

Structure 2: In structure 2 [Figure 8b], two parallel chunks (chunk 1 and chunk 2) with very similar derivations are combined to derive the conclusion or a third chunk (chunk 3) that later helps to derive the conclusion. We found 2 difficult problems associated with structure 2. An example problem for this structure is shown in Figure 8e.

We identified 40 students (18 T_1 students, 22 T_2 students) who at least had one difficult instance (i.e. spent above 75th percentile of time) while solving one of the problems associated with this structure. 50 difficult instances [16 associated with forward-directed sequential derivation, 1 associated with backward-directed sequential derivation, and 33 with random derivation] were found for these students while solving one of these 2 problems. We observed that the students spent more time on either chunk 1 (31 instances) or chunk 2 (19 instances) depending on whichever they attempted to complete first. We also observed that they spent average or below-average time while deriving the rest of the chunks.

These observations indicate that the students were able to identify similarities across the chunks within a problem. Thus, although they spent more time on the first chunk, after figuring out the derivation of the first chunk, they needed less time to derive the rest.

Structure 3: Structure 2 and structure 3 [Figure 8c] are visually very similar. However, the main difference is that the derivations of chunk 1 and chunk 2 within structure 3 have no similarities (an example problem is shown in Figure 8f). We found 2 difficult problems associated with structure 3.

33 students were identified (19 T_1 students, 14 T_2 students) who had at least one difficult derivation (i.e. spent above 75th percentile of time) while solving one of the problems associated with this structure. 45 difficult instances [28 associated with FW-directed sequential derivation, 13 associated with BW-directed derivation, and 4 with random derivation] were found for these students while solving one of these 2 problems. And, we observed that in most of the cases, students spent higher time on both chunk 1 and chunk 2 (total 35 instances).

Overall, our observations indicate that difficulties mostly occurred when chunks within a problem were very dissimilar (in Structure 1 and Structure 3). On the other hand, if there are similar chunks within a problem, after deriving one chunk, the students figured out the derivation of other similar chunks very quickly.

4.2.3 *Learning Efficiency and Correlation Test between NLG and Training Time*

Overall, the training time for T_1 (this group received CPP without any explanation) and T_2 (this group received CPP with explanations) was higher than the control group [Control (C): 66.4(35.3) minutes, T_1 (CPP): 89.7 (60.7) minutes, T_2 (CPPE): 81.8 (46.0) minutes]. The skewed distribution of training times across the three training conditions is visualized in Appendix B, Figure 12.

Since the training times were higher for the treatment groups, we calculated the learning efficiency (NLG/Training Time) for each group. However, we did not find any difference in learning efficiency across the groups [Control (C): 0.007(0.011), T_1 (CPP): 0.003(0.005), T_2 (CPPE): 0.004(0.009). Kruskel-Wallis Test: (statistic=2.84, p-value=0.24); Pairwise post-hoc Mann Whitney U Tests: (C, T_1)=(statistic= 986.0, p-value=0.30), (C, T_2)=(statistic=1020.0, p-value = 0.11), (T_1, T_2)=(statistic=1231.0, p-value=0.43)]. We also did not find any significant correlation between NLG and training times [Control (C): coefficient = -0.09, p-value =

[5]The amount of working memory being used

0.32; T_1 (CPP): coefficient = -0.21, p-value = 0.96; T_2 (CPPE): coefficient = 0.01, p-value = 0.43]. Therefore, it is unclear whether or not the differences in NLG across the training conditions occurred due to differences in training times.

5. RQ3: STUDENTS' CHUNKING BEHAVIOR

CPP and CPPEs were incorporated within DT training levels to demonstrate chunking (i.e. decomposed problem solutions), engage students in the process (by having them recompose complete solutions from chunks), and motivate them to adopt chunking (i.e. decomposition-recomposition (PDR)) to reduce difficulties while solving new problems. To investigate if students successfully captured the notion of chunking while solving CPP/CPPE and if they tried to form chunks when solving problems independently (which we refer to as chunking behavior), we adopted a data-driven approach. From log data collected within DT, we tried to infer if the students showed chunking/PDR behavior and how the behavior was associated with their performance.

Method to Identify Chunking Behavior: We analyzed students' chunking behavior in the posttest problems that they solved independently [7.1-7.6]. To do so, first, we derived baseline chunks from historical student solutions to these problems. For problems 7.1-7.6, using the method described in Section 3.2, we generated approach maps using historical data collected in DT to capture previous students' solutions and identified baseline chunks in those solutions. The baseline chunks answer 'What to look for in the solutions of the students participating in this study'. Next, to confirm the presence of the baseline chunks or chunking/PDR behavior in a student's proof construction attempt, we sequentially scanned through the student's steps while constructing a proof and identified consecutive steps as chunking when the step sequence has the following characteristics:

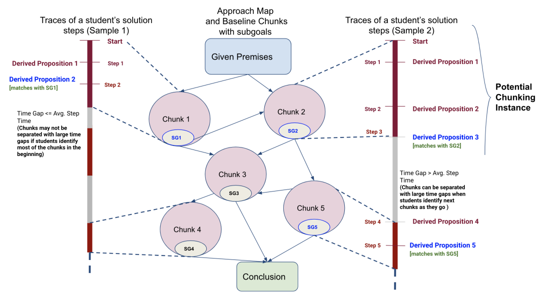
1. The propositions derived in the step sequence overlaps with propositions and subgoal associated with only one of the baseline chunks [we applied the 'Intersection' set operation to find an overlap].

2. The step sequence may or may not be separated from the rest of the steps by a time gap above the average step time (the time spent on a single step) of 1.6 minutes. The two cases of separation by time gaps are shown in Sample 1 and Sample 2 in Figure 9.

The process of identifying chunking in a student solution attempt is further illustrated in Figure 9.

Learning and Chunking Behavior: Following prior research [9, 25], to validate our method to identify chunking behavior, we sought to explain students' learning gain that reflects their problem-solving skills through derivation efficiency in the chunking instances detected using our method.

We hypothesized that a higher number of treatment group students (who received CPP/CPPE) will have chunking instances in their solutions of posttest problems than the control(C) group students. However, we identified that most of the students (121 out of the 145 students) regardless of their training conditions had baseline chunks present in their solutions. Only 24 students (C=8,T_1=9,T_2=6) never showed any identifiable chunking behavior. This indicates that students might have a natural tendency to identify sub-problems and construct logic proofs in chunks. Whereas the presence of some chunking instances is desirable, too many chunks in a problem solution do not represent better performance and



Traces of a student's solution steps (Sample 1)
Approach Map and Baseline Chunks with subgoals
Traces of a student's solution steps (Sample 2)
Start
Derived Proposition 1
Step 1
Derived Proposition 2
[matches with SG1]
Step 2
Time Gap <= Avg. Step Time
(Chunks may not be separated with large time gaps if students identify most of the chunks in the beginning)
Given Premises
Chunk 1
SG1
Chunk 2
SG2
Chunk 3
SG3
Chunk 4
SG4
Chunk 5
SG5
Conclusion
Start
Step 1
Derived Proposition 1
Step 2
Derived Proposition 2
Step 3
Derived Proposition 3
[matches with SG2]
Potential Chunking Instance
Time Gap > Avg. Step Time
(Chunks can be separated with large time gaps when students identify next chunks as they go)
Step 4
Derived Proposition 4
Step 5
Derived Proposition 5
[matches with SG5]

Figure 9: Method to Identify Chunking using Approach Map and Students' Solution Traces [showing the mapping between step sequence and chunks].

more learning. Deep Thought proofs are usually 7-15 steps long and an ideal solution for each problem mostly contains 2-3 chunks. Since the Deep Thought problem score which impacts NLG is designed as a function of time, step counts, and rule application accuracy, to achieve higher scores and NLG, students need to demonstrate only correct baseline chunks within their solutions and each of those chunks needs to be derived efficiently with less time and fewer steps. This fact was validated by a mediation analysis. In the analysis, we used training condition as the independent variable (IV), NLG as the dependent variable (DV), and average # chunks/problem as the mediator (MD). The analysis gave an insignificant p-value [Appendix B, Figure 14] indicating that the impact of the training treatments on learning or NLG is not mediated by the amount of chunking present in students' logic-proof solutions.

Thus, in subsequent analyses, instead of focusing on the number of chunks present in student solutions, we focused only on students' efficiency in deriving the baseline chunks. We calculated efficiency in terms of time spent on deriving different chunks and # of steps within the chunks. We also analyzed NLGs across different pretest score groups to understand the impact of CPP/CPPEs on students with different levels of prior knowledge.

Moderation Analysis on Different Pretest Score Groups: Prior studies showed that both worked examples and Parsons problems may have a different impact on students based on their prior knowledge or skill level [34, 17]. We carried out a moderation analysis to understand how NLG and chunking behavior and efficiency varied across different pretest score groups and training conditions. The pretest score distribution is visualized in Appendix B, Figure 13. We classified students based on their pretest scores (low, medium, and high) and used this classification as the moderator in our analysis. Within this classification, we considered training condition as the independent variable. For each pretest score group and training group, we analyzed 4 dependent variables: average # chunks/problem, average chunk time, avg. chunk step count, and the NLG. Table 2 shows the groupings and values for the dependent variables. To compare the dependent variables across pretest score groups and training conditions, we carried out Kruskal Wallis test and pairwise posthoc Mann Whitney U tests with Bonferroni correction (corrected $\alpha = 0.05/3$ or 0.016). Note that a total of 36 tests(3 pretest score groups, 4 dependent variables, and 3 pairwise tests for each group and dependent variable) were carried out to compare the metrics presented in Table

2. Thus, a more conservative Bonferroni correction could be carried out to eliminate false positives. However, to not introduce many false negatives while eliminating false positives, we decided the level of correction based on the number of unique pairwise tests each datapoint participated in [6] rather than on the number of related tests (for example, the 9 tests on NLG across the pretest scores groups though independent could be considered related and a more conservative correction could be carried out). We explain the results for each pretest score group below:

Low Pretest Scorers: We considered students with pretest scores below the 25th percentile as low scorers. In this group, we observed that T_2 who received CPPE had significantly higher and less negative NLGs than the other two training conditions [$p_{KW} < 0.001$, $p_{MW}(T_2 > C) = 0.011$, $p_{MW}(T_2 > T_1) < 0.001$]. We also observed that T_2 students with low pretest scores had lower average chunk time and significantly lower step counts per chunk [$p_{KW} < 0.03$, $p_{MW}(T_2 < C) = 0.004$, $p_{MW}(T_2 < T_1) < 0.014$]. Overall, T_2 students with low pretest scores demonstrated comparatively more efficient chunking (in terms of less time and step counts) and higher NLG. However, a difference in the number of chunks per problem was not found across the training conditions as expected.

Medium Scorers: Students with pretest scores between 25th-75th percentile were identified as the medium scorers. Within this group, we observed that the T_2 condition again showed significantly or marginally higher NLG than the other two training conditions [$p_{KW} < 0.001$, $p_{MW}(T_2 > C) = 0.002$, $p_{MW}(T_2 > T_1) < 0.021$]. We observed differences in the averages of chunk time across the three training conditions, however, a significant difference was not found in chunk count, time, or step counts.

High Scorers: We did not observe any significant differences in NLG across the three training conditions for students with high pretest scores (above 75th percentile). However, we observed that T_1 and T_2 students in the high pretest score group demonstrated comparatively more chunking (in the range of 3-4 chunks per problem) in posttest than the control (C) group (in the range of 2-3 chunks per problem).

Overall, the results of the moderation analysis indicate that T_2 students with low and medium pretest scores achieved significantly higher NLGs than students from the other two training conditions with similar levels of prior knowledge. There were no significant differences in the amount of chunking per problem across the training conditions. However, we observed differences in the chunking efficiency where T_2 had lower chunk derivation times and fewer steps within chunks in some cases. Thus, next, we analyze and present the chunking efficiency across the three training conditions on different posttest problems in further detail.

Chunk Derivation Efficiency: We compared the chunk derivation efficiency of the students across the three training conditions who had identifiable chunks in their solutions. Toward that, we analyzed two metrics for the baseline chunks identified in student solutions to the posttest problems: 1) Time to derive a chunk (shorter chunk derivation time [CTime] indicates students figured out '*how to derive the chunk*' quickly), 2) unnecessary proposition count [UProp][6] (fewer unnecessary propositions indicate students correctly identified '*what to derive within the chunk*'). Lower values for these two metrics indicate higher chunk derivation efficiency.

In Figure 10, we show the chunks commonly found in students' proof for each of the posttest problems. For simplicity, for each of the chunks, we only show what subgoal the chunk derives. To identify significant differences in the derivation efficiency of these chunks (in terms of CTime or UProp) across the three training conditions, we carried out Kruskal-Wallis tests. The chunks for which there is a significant difference in derivation efficiency across the training conditions in terms of at least one of UProp or CTime are marked with thicker edges and green nodes in the figure. The results of the statistical tests are shown along the thicker edges. We observed that the significant differences were found mostly for non-trivial chunks, i.e. chunks that involve several proposition derivations. For example, there are two chunks in the solution of 7.1: the first chunk derives $\neg R$ which requires multiple steps (i.e. non-trivial), and the second chunk derives $R \vee \neg T$ which can be derived after a *Simplification* rule application on the given premise $(R \vee \neg T) \wedge X$ (trivial derivation). We found significant differences only in the derivation efficiency of chunk 1 (the non-trivial chunk). To identify the training condition that was the most efficient in deriving the green chunks in Figure 10, we carried out posthoc pairwise Mann-Whitney U tests [for the pairs (C, T_1), (C, T_2), and (T_1, T_2)] with Bonferroni correction (corrected α=0.016) comparing UProb and CTime. The results of the tests are shown in Table 3. As shown in the table, in most of the cases, T_2 is the most efficient group in deriving the chunks, i.e. the tests for the hypotheses '$T_2 < C$' and '$T_2 < T_1$' in terms of UProp/CTime gave p-value < 0.016. Overall, these results indicate that although most students naturally derived chunks, T_2 students achieved higher efficiency in deriving non-trivial chunks.

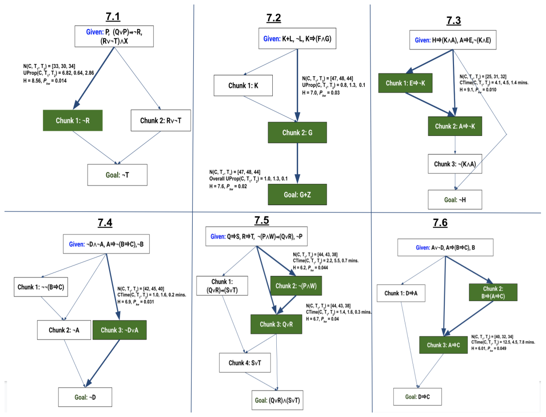

7.1
Given: P, (Q∨P)⇒¬R, (R∨¬T)∧X
N(C, T1, T2) = [33, 30, 34]
UProp(C, T1, T2) = 6.82, 0.64, 2.86
H = 8.56, Pkw = 0.014
Chunk 1: ¬R
Chunk 2: R∨¬T
Goal: ¬T
7.2
Given: K+L, ¬L, K⇒(F∧G)
Chunk 1: K
N(C, T1, T2) = [47, 48, 44]
UProp(C, T1, T2) = 0.8, 1.3, 0.1
H = 7.0, Pkw = 0.03
Chunk 2: G
N(C, T1, T2) = [47, 48, 44]
Overall UProp(C, T1, T2) = 1.0, 1.3, 0.1
H = 7.6, Pkw = 0.02
Goal: G+Z
7.3
Given: H⇒(K∧A), A⇒E, ¬(K∧E)
Chunk 1: E⇒¬K
N(C, T1, T2) = [25, 31, 32]
CTime(C, T1, T2) = 4.1, 4.5, 1.4 mins.
H = 9.1, Pkw = 0.010
Chunk 2: A⇒¬K
Chunk 3: ¬(K∧A)
Goal: ¬H
7.4
Given: ¬D∧¬A, A⇒¬(B⇒C), ¬B
Chunk 1: ¬¬(B⇒C)
N(C, T1, T2) = [42, 45, 40]
CTime(C, T1, T2) = 1.0, 1.6, 0.2 mins.
H = 6.9, Pkw = 0.031
Chunk 2: ¬A
Chunk 3: ¬D∨A
Goal: ¬D
7.5
Given: Q⇒S, R⇒T, ¬(P∧W)⇒(Q∨R), ¬P
N(C, T1, T2) = [44, 43, 38]
CTime(C, T1, T2) = 2.2, 5.5, 0.7 mins.
H = 6.2, Pkw = 0.044
Chunk 1: (Q∨R)⇒(S∨T)
Chunk 2: ¬(P∧W)
N(C, T1, T2) = [44, 43, 38]
CTime(C, T1, T2) = 1.4, 1.6, 0.3 mins.
H = 6.7, Pkw = 0.04
Chunk 3: Q∨R
Chunk 4: S∨T
Goal: (Q∨R)∧(S∨T)
7.6
Given: A∨¬D, A⇒(B⇒C), B
Chunk 1: D⇒A
Chunk 2: B⇒(A⇒C)
Chunk 3: A⇒C
N(C, T1, T2) = [40, 32, 34]
CTime(C, T1, T2) = 12.5, 4.5, 7.8 mins.
H = 6.01, Pkw = 0.049
Goal: D⇒C

Figure 10: Chunk Derivation Efficiency in Posttest Problems.

6. DISCUSSION

Overall, our analysis showed that *Chunky Parsons problem* could be a low-difficulty training intervention, specifically when presented with an explanation hinting at what the chunks mean and how they contribute to the complete solution. However, while being a low-difficulty training intervention, it has the potential to improve students' learning gain and problem-solving skills, specifically chunking skills. We observed that most students formed some chunks dur-

[6]Unnecessary propositions are propositions that students derived during proof construction but later deleted and those were not part of the final proof.

Table 2: **Moderation Analysis across the Three Training Conditions Categorized on Pretest Scores. [Note: Blue* indicates a significant difference. Boldface indicates comparatively better averages (e.g. higher for NLG/lower for extra steps).]**

Moderator	Independent Variable (IV)	Dependent Variables (DV)			
Pretest Quantile	Training Condition	Avg. # chunks/ prob.	NLG	Avg. Chunk Time (minutes)	Avg. Chunk Step Count
Low Scorers (< 25th percentile) N = 35	Control(n=12)	2.02(3.25)	-0.20(0.35)	3.42(1.45)	2.01(0.81)
	T_1-CPP(n=12)	2.47(3.62)	-0.40(0.39)	2.78(3.80)	2.06(1.03)
	T_2-CPPE(n=11)	2.19(3.54)	**-0.07(0.22)***	**2.21(3.26)**	**1.94(0.90)***
Medium Scorers (25th-75th percentile) N = 75	Control(n=25)	2.51(3.98)	0.34(0.23)	2.88(4.51)	2.25(1.38)
	T_1-CPP(n=26)	2.58(3.96)	0.35(0.25)	3.03(5.15)	2.05(1.08)
	T_2-CPPE(n=23)	2.26(3.60)	**0.48(0.22)***	**2.45(3.90)**	2.14(1.14)
High Scorers (> 75th percentile) N = 35	Control(n=13)	2.92(3.72)	0.30(0.17)	3.09(4.22)	2.45(1.34)
	T_1-CPP(n=11)	**3.50(4.5)**	0.32(0.13)	4.03(5.64)	2.16(0.89)
	T_2-CPPE(n=11)	**3.13(3.96)**	0.33(0.17)	**2.94(3.47)**	2.31(1.26)

Table 3: Chunk Derivation Efficiency across the Three Training Groups (only significant p-values are shown).

Problem	Chunk	Metric	Pairwise Mann-Whitney U Test
7.1	Chunk 1	UProp	p(T1<C)=0.005 p(T2<C)=0.012
7.2	Chunk 2	UProp	p(T2<C)=0.016 p(T2<T1)=0.006
7.3	Chunk 1 + Chunk 2	CTime	p(T2<C)=0.015 p(T2<T1)=0.002
7.4	Chunk 3	CTime	p(T2<C)=0.013 p(T2<T1)=0.014
7.5	Chunk 2 + Chunk 3	CTime	p(T2<C)=0.010 p(T2<T1)=0.030
7.6	Chunk 2 + Chunk 3	CTime	p(T1<C)=0.010 p(T2<C)=0.020

ing proof construction. However, students from all training conditions were not equally efficient in chunking. Our statistical tests showed that T_2 (who received CPP with an explanation attached to chunks) derived non-trivial chunks with higher efficiency. However, this efficiency often was not observed for all chunks within a problem. Another limitation of CPP/CPPEs is the difficulties associated with it when students first encounter CPP/CPPE in early training levels or when the chunks within a CPP/CPPE are very diverse. Thus, we recommend providing additional guidance or tutor help in these scenarios to ensure a better student experience while solving and learning through CPP/CPPE. Nevertheless, our analyses have established that Chunky Parsons Problems with explanations can be an effective problem-based training intervention to improve students' Chunking skills (i.e. problem decomposition into chunks and recomposing them to construct a complete solution).
Also, our data-driven method to derive subgoals can be adopted for any structured problem-solving domain as long as each step during problem solving can be presented as a state transition. For example, in a math-expression evaluation problem, a state can be the set of all evaluated parts of the equation at a particular moment. A step or an action (for example, applying a math operator) changes the problem state. Once the state transitions or interaction is defined within a domain, generating the approach maps and extracting subgoals from them can be carried out generically (graph construction, applying clustering, and simplifying the graph in approach maps). Similarly, the chunking efficiency evaluation method can be adopted in other domains, as long as each point in the students' sequential problem solution traces can be presented as a state from a finite state space.

7. CONCLUSION AND FUTURE WORK

The contributions of this paper are 1) the demonstration of a data-driven graph-mining-based method to decompose problem solutions into expert-level chunks, 2) the design of a problem-based training intervention called *Chunky Parsons Problem* to be used within an intelligent tutor to teach students the concept of structural decomposition-recomposition (or Chunking) of problems, 3) an evaluation of the impact of *Chunky Parsons Problem* on learning and students' chunking skills, and 4) a mechanism to identify *Chunking* in students' solution traces using historical baseline chunks. As discussed earlier, our data-driven methods to derive *Chunky Parsons Problem* and to identify *Chunking* in student solution traces can be adapted for any domain where problem solving is structured and the states and transitions of students during problem solving can be defined definitely. Likewise, *Chunky Parsons Problem* can be adapted for any problem-based tutor within such domains.
However, this study has several limitations. First, the design decisions for Chunky Parsons Problems (CPPs) and their explanations were made based on prior literature, without any user studies to validate them. Second, while we validated chunks found in participants' solutions, our data-driven evaluation method may not be able to detect new chunks that were not previously seen in prior student data. Also, the outcomes of this study are dependent on how we defined different data-driven metrics (for example, difficulty or efficiency). Third, although our evaluation can identify the impact of interventions, it cannot validate the source of the impact. Thus, future user studies involving interviews or talk-aloud protocols could help address these three issues and validate the findings on the usability and impact of *Chunky Parsons Problem*. Finally, our study focused only on logic-proof problems and should be replicated in other domains to understand the generalizability of the findings.

8. ACKNOWLEDGMENTS

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APPENDIX

A. DEEP THOUGHT INTERFACE AND FUNCTIONALITIES

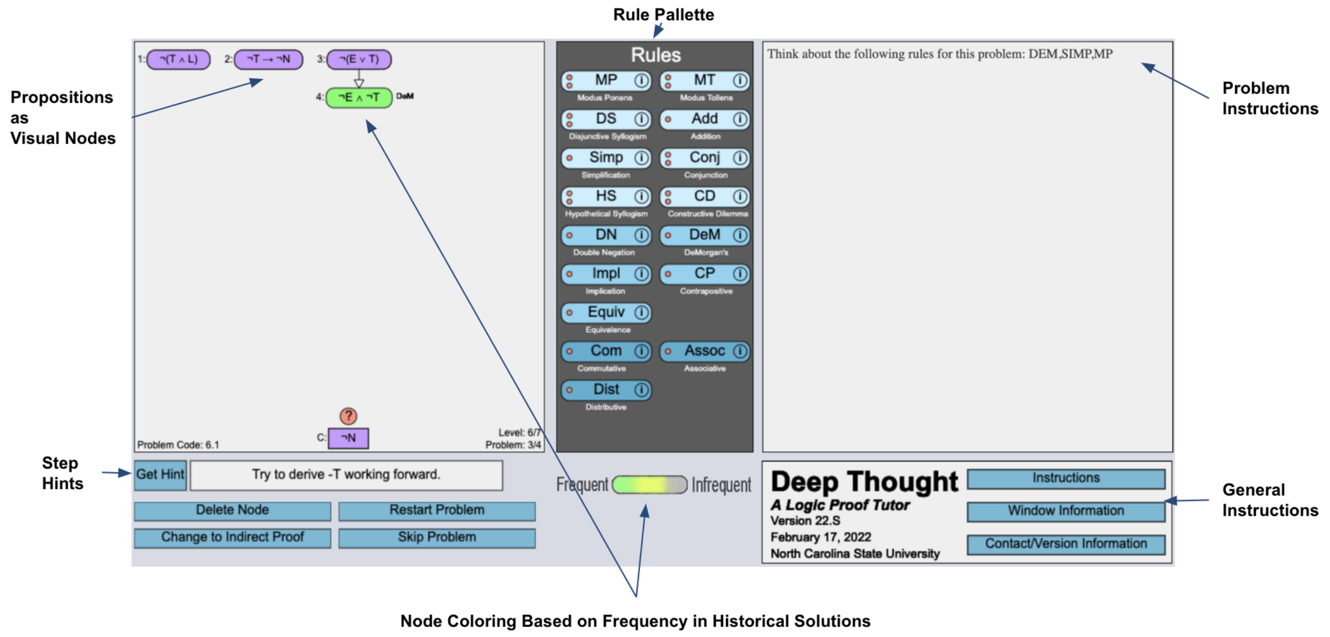
Figure 11 shows the different components and architecture of Deep Thought or DT, including one property worth mentioning: during training problems, the tutor colors student-derived propositions based on their frequency in prior student solutions. This coloring is designed to help the students to understand if they are on the right track or not.

A.1 Problem-Solving Strategies within Deep Thought

Logic Proof Construction problems in Deep Thought can be constructed using one of the three following strategies: 1) Forward Problem Solving; 2) Backward Problem Solving; and 3) Indirect Problem Solving. The three strategies are briefly described below:

Forward Problem Solving: In this strategy (Figure 15a), proof construction progresses from the given premises toward the conclusion. At each step, a new node is derived by applying rules on the given premises or derived justified nodes. To derive a new node in the forward direction, students first need to select the correct number of premise(s) or already justified node(s) and then select the rule to apply to the selected node(s).

Backward Problem Solving: In this strategy (Figure 15b), proof construction progresses from the conclusion toward the given premises. At each step, the conclusion is refined to a new goal. In this strategy, students can add unjustified nodes in the proof that they wish to derive from the given premises. To derive a node backward, students need to select the '?' button above a node, then select the rule, and then input the proposition(s) which are the antecedents of the selected node as per the selected rule. For example, in Figure 15b, the conclusion $\neg N$ is first refined into antecedents $\neg T \rightarrow \neg N$ and $\neg T$ using the Modus Ponens (MP) rule. $\neg T \rightarrow \neg N$ (given) is already justified. So, $\neg T$ becomes the new goal since it is still unjustified. Then, $\neg T$ is refined



Think about the following rules for this problem: DEM,SIMP,MP

Problem Code: 6.1

Level: 6/7
Problem: 3/4

Try to derive -T working forward.

Deep Thought
A Logic Proof Tutor
Version 22.S
February 17, 2022
North Carolina State University

Figure 11: Deep Thought Interface

to a new goal $\neg E \vee \neg T$ using the Simplification (Simp) rule. In this way, the unjustified goal(s) are refined to the given premises to complete the proof.

Indirect Problem Solving: Indirect problem solving [2] refers to the 'Proof by contradiction' approach. To construct a proof using this strategy, students first need to click on the 'Change to Indirect Proof' button (Figure 11) which adds the negation of the original conclusion to the list of givens (as in $\neg\neg N$ in Figure 15c). From there, students need to derive two contradictory statements (for example, $\neg\neg N$ and $\neg N$) to prove the contradiction ($\emptyset$).

Note that usually within Deep Thought, students are not required to follow any particular strategy. They can use any strategy at any point in a proof construction attempt.

B. SUPPLEMENTARY FIGURES

Figure 12, 13, and 14 supplement the analyses presented in Section 4.2.3 and 5.

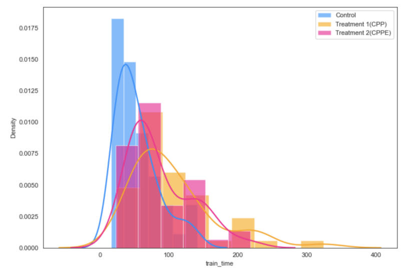


Figure 12: Training Time across the Three Training Conditions.

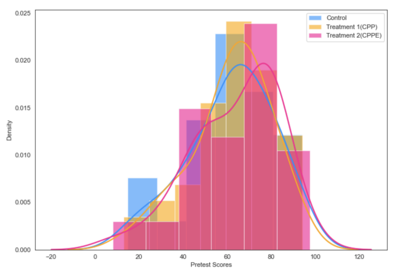


Figure 13: Distribution of Pretest Scores across the Three Training Conditions.

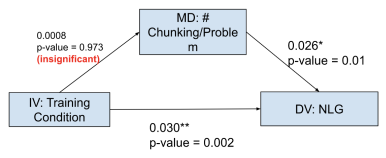
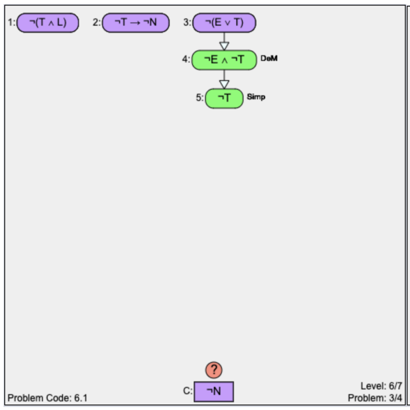
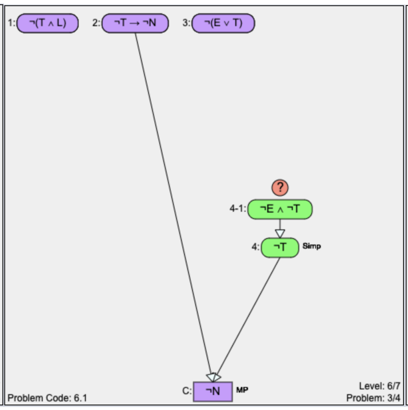


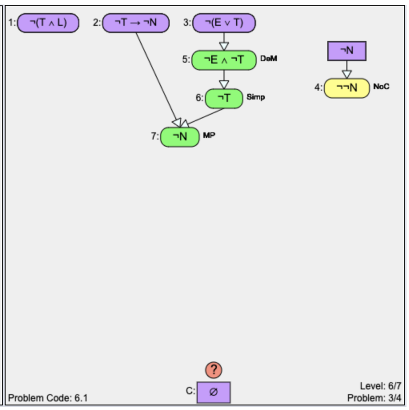
Figure 14: Mediation Analysis to Analyze the Impact of Amount of Chunking on Learning.



(a) Forward Strategy



(b) Backward Strategy



(c) Indirect Strategy

Figure 15: Problem-Solving Strategies Implementable in Deep Thought