

AUTOMATED EMITTANCE AND ENERGY GAIN OPTIMIZATION FOR PLASMA WAKEFIELD ACCELERATION

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Abstract

At the Facility for Advanced Accelerator Experimental Tests (FACET-II) accelerator, a pair of 10 GeV high-current electron beams is used to investigate Plasma Wakefield Acceleration (PWFA) in plasmas of different lengths. While PWFA has achieved astonishingly high accelerating gradients of tens of GeV/m, matching the electron beam into the plasma wake is necessary to achieve a beam quality required for precise tuning of future high energy linear accelerators. The purpose of this study was to explore how start-to-end simulations could be used to optimize two important measures of beam quality, namely maximizing energy gain and minimizing transverse emittance growth in a 2 cm long plasma. These two beam parameters were investigated with an in-depth model of the FACET-II accelerator using numerical optimization. The results presented in the paper demonstrate the importance of utilizing beam-transport simulations in tandem with particle-in-cell simulations and provide insight into optimizing these two important beam parameters without the need to devote significant accelerator physics time tuning the FACET-II accelerator.

INTRODUCTION

A relatively new method of providing high accelerating gradients for charged particles in the Accelerator Physics community is known as Plasma Wakefield Acceleration (PWFA). This technique, which uses strong electromagnetic fields generated in plasma, has demonstrated accelerating gradients of over 10 GeV/m [1] which is orders of magnitude larger than traditional radiofrequency (RF) technology. The wake field, excited by a drive electron beam transfers energy to the witness electron beam trailing in the back of the wake. One of the challenges in the development of PWFA is that the plasma wake not only provides strong longitudinal fields which accelerate charged particles but it also makes strong transverse fields that can lead to deterioration of beam quality. The ability to sustain good beam quality and high accelerating gradients is a vital concern that we hope to address.

In the FACET-II portion of the SLAC linear accelerator, bunches of electrons are accelerated using RF cavity acceleration over the course of the one km long beamline before they reach an experimental chamber. This chamber contains a gas jet which can be used to produce a small length of plasma in which tests of PWFA can be performed [2].

It has been demonstrated that PWFA can be optimized with large datasets of accelerator measurements [3], which suggests that a search for an optimum could be automated [4]. At laser-driven plasma wakefield accelerators, Bayesian optimization was already applied successfully [5–7]. The objective of this work is to examine how simulations using a computational model of the FACET-II beamline could serve as a guide for experimental efforts at increasing the energy gain of PWFA while maintaining or reducing transverse emittance growth, a figure of merit for beam quality. A start-to-end model of the beam line was developed and used in a numerical optimization schema to determine the final focusing quadrupole magnet strengths which would best optimize these two beam characteristics. During the PWFA process, the energy gain is calculated by quantifying the difference between the mean energy of the particle beam at the beginning and end of the plasma process. The mean energy gain divided by the length of the plasma (L_{plasma}), gives the acceleration gradient, or the amount of energy the particles gained on average per meter of travel:

$$A = \frac{\langle E_f \rangle - \langle E_i \rangle}{L_{\text{plasma}}} \quad (1)$$

The normalized emittance is defined as the area of the beam in the corresponding position-momentum phase space, where x is the position of a particle in the beam and x' is the angle of its x momentum with respect to its momentum in the z -direction, multiplied by its Lorentz factor, (γ), to account for the beam's acceleration [8],

$$\epsilon_{nx} = \gamma \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}. \quad (2)$$

This quantity is often used as a figure-of-merit for the quality of a particle beam as it relates the distribution of the particles and how fast they are spreading out; therefore, a lower emittance is often desirable because it corresponds to a more focused and less divergent beam.

In PWFA, if the input parameters of the beam do not match the focusing fields of the plasma wake, phase-mixing of the transverse phase space or large oscillations can seed instabilities such as hosing [9] and lead to a deterioration of the transverse beam quality. This is known as emittance growth, and it was the aim of this study to reduce emittance as much as possible as the beam is accelerated in the FACET-II gas jet. We define the difference in normalized emittance at the start ϵ_{nxi} and end ϵ_{nxf} of the plasma as the normalized emittance growth,

$$\Delta \epsilon_{nx} = \|\epsilon_{nxf} - \epsilon_{nxi}\|. \quad (3)$$

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START-TO-END FACET-II BEAM LINE SIMULATION

The primary components of the FACET-II beam line are the photo-cathode, where the electron bunches are created via the photoelectric effect; the main acceleration section, containing RF cavities to accelerate the electron bunches in the beam to up to 10 GeV as well as bunch compressors to decrease the longitudinal spread of the beam and quadrupole magnets to focus the beam in the transverse direction; and the experimental chamber containing the gas jet where PWFA tests are performed.

The initial simulated beam was created using a program called GPT to simulate the photo-cathode [10]. The main acceleration section was simulated using LUCRETIA, a MATLAB-based physics toolbox for the simulation of electron beam transport systems [11]. LUCRETIA works predominantly by using matrix representations of the accelerator elements in the FACET-II beam line which transform the position and momenta of the electron bunches in the beam. This beam was stored as a six-dimensional phase space which kept track of the positions, momenta, and energies of each simulated macroparticle.

The next simulated portion of the beam line was the beam collimation process. This divides the beam into higher and lower energy components and can be performed in FACET-II by a notch collimator which physically blocks sections of the beam in a dispersive section. This was modeled by a MATLAB function which can remove a specified width, given in terms of energy, from the virtual beam, resulting in two beams: the drive beam at higher energies, and the witness beam at lower energies with a spatial separation on the order of millimeters.

The portion of the beam line directly before the particles reach the experimental chamber is known as the final focusing (FF) section which consists of six quadrupole magnets used to manipulate the transverse properties of the beam before it enters the plasma.

The quadrupole focusing strength for each of the six quadrupoles is given by, $k = \frac{qB_0}{\gamma mc}$, where B_0 is the integrated field strength of the magnets. The field strength of the quadrupoles in the FF section were the parameters varied in the optimization routine.

After passing through the FF section, the beam enters the experimental chamber and passes through the plasma. The plasma is simulated using a Particle-In-Cell (PIC) code written in Python called Fourier-Bessel Particle in Cell (FBPIC) [12]. Input parameters to the software can be both the plasma density and the plasma length, in this case that of a 2 cm long pre-ionized gas jet. The PWFA interaction is modeled using FBPIC: the drive beam enters the plasma first, displacing its electrons and creating an electromagnetic field. The witness beam enters the plasma afterwards, gaining energy from this wakefield. In order to calculate the emittance growth and energy gain, the beam is converted back into the beam structure used by LUCRETIA after this interaction.

OPTIMIZATION

The start-to-end simulation was used to implement an optimization loop aimed at iteratively reducing emittance growth while increasing energy gain. A schematic representation of the optimization process along the FACET-II beam line components is shown in Figure 1.

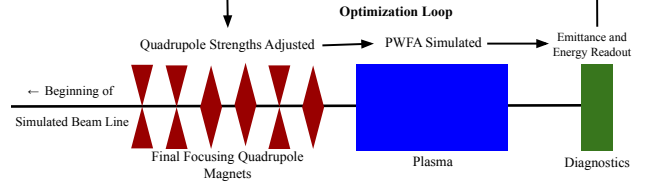


Figure 1: The performed optimization loop along the simulated sections of the FACET-II beam line.

Specifically, a minimization of the following function was performed,

$$f = \sqrt{\Delta\epsilon_{nx}^2 + \Delta\epsilon_{ny}^2} - k_E(\langle E_f \rangle - \langle E_i \rangle), \quad (4)$$

where the emittance growth in each transverse direction added in quadrature and then subtracted by the average energy gain. The energy gain term was multiplied by the factor k_E as it was a different order of magnitude than the emittance in order to prevent it from being favored in the optimization routine. As it was roughly two orders of magnitude greater, k_E was set to 0.01.

The algorithm used was the Nelder-Mead Algorithm [13]. This algorithm works by creating a geometrical structure known as a simplex in the function space of the problem by evaluating the function at the initial values of the input parameters to generate its vertices. At each iteration, the algorithm calculates the worst vertex of the simplex, corresponding to the largest function evaluation, and attempts to replace it using a set of defined transformations with the new calculated vertices corresponding to a function evaluation at different input parameters. This procedure of modifying the simplex continues until a stopping criterion is met [13]. The algorithm was run for 230 iterations, with each iteration taking approximately 90 minutes. The algorithm was also restarted twice during the process to prevent its convergence to a local minimum and simulate unpredictable experimental conditions.

The optimization resulted in the decrease in emittance growth in both transverse directions and the increase of energy gain; these results are shown in Table 1. The change in these characteristics through the optimization are shown in Figure 2.

With the exception of the spikes at initial simplex creation and the two algorithm restarts at around 80 and 120 iterations, the energy gain and the emittance reduction is steady with each iteration before approaching asymptotes. In Figure 3, the variation of the quadrupole strength with each iteration provides an indication of the manner in which these characteristics changed. The optimization is primarily

Table 1: Optimization Results

	$\Delta\epsilon_{nx}$ [m · Rad]	$\Delta\epsilon_{ny}$ [m · Rad]	A [GeV/m]
Initial	7.05×10^{-5}	1.06×10^{-4}	1.52
Final	1.18×10^{-6}	1.22×10^{-6}	1.87
Difference	-98.3%	-98.8%	23.0%

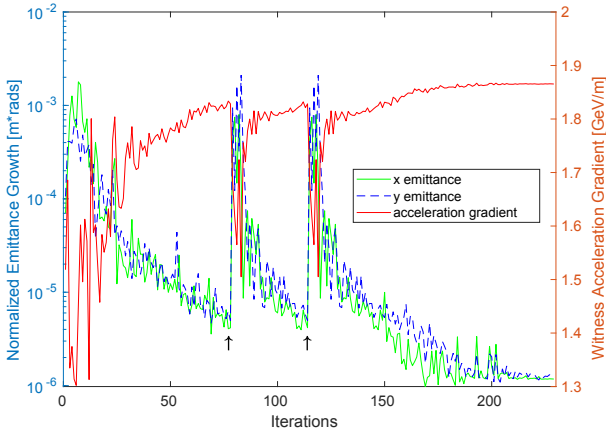


Figure 2: The normalized emittance growth in the x (green) and y (blue) directions and the acceleration gradient (red) of the witness beam at each iteration of the optimization algorithm. The arrows denote the two algorithm restarts.

driven by the increase in Q4's absolute strength at the outset. It appears, however, that the strength of Q5 also begins to increase significantly around the 150th iteration, indicating that there has been a change in how the function is being optimized. This can be better understood by examining the normalized emittance at the start and end of the plasma with successive iterations rather than only their growth in emittance, as shown in Figure 4.

Initially, the emittance changes were minimized by reducing the final emittance of the beam after the plasma, while the initial emittance remained relatively constant. At around the 150th iteration point, the initial emittance began to increase in order to minimize the overall increase in the plasma's emittance.

CONCLUSION

The described numerical optimization technique has demonstrated theoretical promise in improving beam figures of merit in a simulated model of the FACET-II accelerator. An important application of this work would be the physical implementation of the optimization process in some form in the FACET-II accelerator. There currently exists code in LUCRETIA which can be utilized to create images of the beam distribution similar to how a physical beam would appear on the diagnostic tools that are used in the experimental chamber at FACET-II. If good agreement can be established between these simulations and experimental data collected

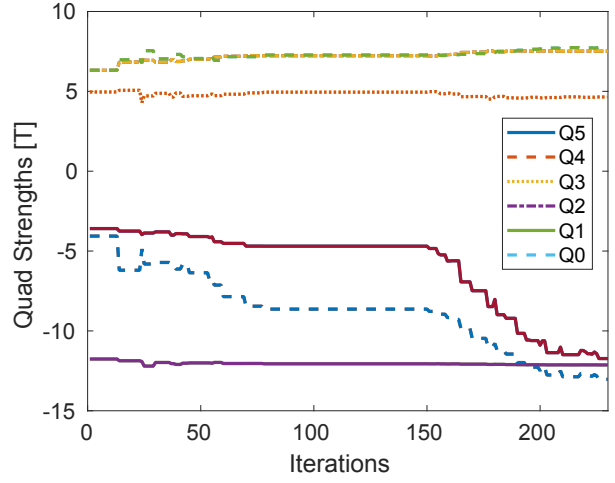


Figure 3: The integrated field strengths of the six final focusing quadrupole magnets at each iteration of the optimization algorithm.

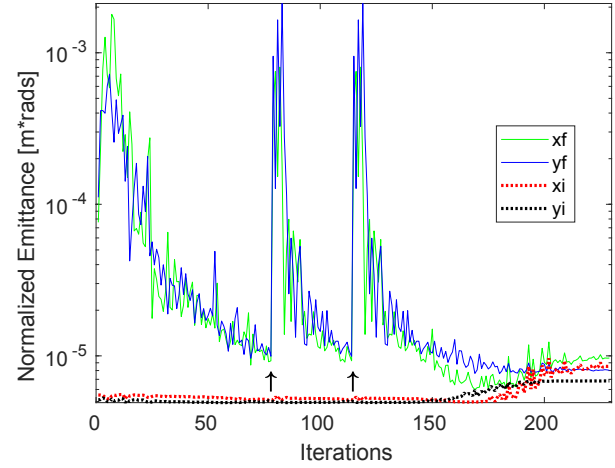


Figure 4: The normalized emittance of the witness beam in the x (green) and y (blue) directions at the entrance (dotted) and exit of the plasma at each iteration of the optimization algorithm. The arrows denote the two algorithm restarts.

at FACET-II for beams with similar parameters, future simulations may be used to guide future operating points to enhance the beam characteristics of PWFA.

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