Force Feedback Model-Predictive Control via Online Estimation

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***Abstract*— Nonlinear model-predictive control has recently shown its practicability in robotics. However it remains limited in contact interaction tasks due to its inability to leverage sensed efforts. In this work, we propose a novel model-predictive control approach that incorporates direct feedback from force sensors while circumventing explicit modeling of the contact force evolution. Our approach is based on the online estimation of the discrepancy between the force predicted by the dynamics model and force measurements, combined with high-frequency nonlinear model-predictive control. We report an experimental validation on a torque-controlled manipulator in challenging tasks for which accurate force tracking is necessary. We show that a simple reformulation of the optimal control problem combined with standard estimation tools enables to achieve state-of-the-art performance in force control while preserving the benefits of model-predictive control, thereby outperforming traditional force control techniques. This work paves the way toward a more systematic integration of force sensors in model predictive control.**

I. Introduction

A. Motivation

Many tasks require accurate control of contact forces exerted on the environment: polishing, grinding, grasping, etc. This skill, trivial to humans, remains beyond most robot's abilities despite continuous progress in robotics research over the past decades. While Model Predictive Control (MPC) affords the online synthesis of complex motions, it remains fundamentally limited in its ability to control physical interaction. As a matter of fact, although force sensors have been used since the early days of robotics [1], they remain notably absent from modern control techniques relying on model-based optimization.

This is partly because predicting the evolution of contact forces is challenging in general and involves sophisticated models [2] that are too specific or impractical for real-time applications. Hence, the contact models used in practice for optimization-based control are kept simple for algorithmic convenience [3]. However, these simplifications hinder the ability to derive meaningful control policies in contact with explicit force feedback. To this day, the predictive feedback control of contact forces remains an open problem.

In this work, we address this issue and show that standard estimation tools [4] together with a reformulation of the optimal control problem can provide a simple yet effective framework to achieve force-output-feedback MPC.

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B. Related work

Force control techniques are classically divided into direct force control and indirect force control [5]. A full introduction is out of the scope, so we only provide here a brief overview and refer the reader to the concise introductory review on active compliant control proposed in [6].

Direct methods attempt to regulate the force explicitly using measurement feedback, typically in an integral controller - which is historically considered the best basic strategy for force tracking [7]. It can be combined with motion feedback in complementary task directions [8], or in parallel [9]. While the use of explicit force feedback enables high accuracy tracking, the artificial decoupling of force and motion tasks hides potential conflicts [10], [11] or phenomena such as contact friction [12] and exchange of mechanical work [13].

On the other hand, indirect methods, such as impedance control [14] or admittance control [15], [16], aim at regulating the dynamic relationship between force and motion. While this allows to generate stable and compliant contact interactions, such techniques are mainly limited by their force tracking capability: since the force is controlled indirectly through motion regulation, the tracking performance depends on a priori unknown environment parameters [17]–[20].

More recently, MPC has shown its ability to accommodate conflicting objectives through constrained nonlinear optimization [21]. Much research has focused on introducing MPC into direct [22], [23] and indirect [24]–[28] force control methods, mainly motivated by its ability to satisfy constraints. In contrast to [22], [23], [26], [28], [29], the proposed approach does not require a contact force *dynamics* model, which greatly simplifies the optimization. Unlike [24], [25], [27], we use a force sensor to achieve explicit force tracking rather than impedance/admittance regulation.

Estimation can also be used to improve performance in force tasks. In [30], external forces are estimated with a centroidal model. In [31], a state-dependent force correction model is adapted online. Closer to our work, [32] proposed an active Kalman observer in MPC to reject unmodeled disturbances at the input level, which can be viewed as a form of model-reference (direct) adaptive control. However, those lines of work do not consider the full dynamics model.

C. Contributions

In this paper, we propose a novel MPC formulation that allows to exploit direct feedback from force sensors. We show that simple contact models and standard estimation tools allow to incorporate force feedback in MPC and to achieve state-of-the-art performance. We claim that force feedback in MPC is not as challenging as it seems and that it solves many issues: it circumvents tedious modeling of complex phenomena (contact, friction, etc.), boosts performance of classical MPC in contact tasks, and does not conflict with optimization contrary to traditional force control methods.

We propose to use force measurements to estimate online the mismatch between the robot's dynamics model and measurements. This mismatch is used to correct directly the predictive model or the control objective. This idea resembles that of indirect adaptive control [33], where a model of the plant is identified online to adapt the controller's parameters. Our approach allows high-quality force tracking accuracy in challenging interaction tasks. Our main contributions are:

- a new framework affording direct force feedback control inside nonlinear MPC based on online estimation and feedback linearization
- a systematic comparative experimental study of our force feedback MPC against traditional techniques.

In particular, we demonstrate that the proposed approach outperforms integral control: it benefits from the same force tracking capability *without* impeding the benefits of MPC. In particular, in contrast to integral control, our approach maintains or improves the MPC running cost performance. It has also the advantage of being conceptually simple and cheap to implement with existing tools and software.

II. Background

In this section, we recall the classical MPC formulation for torque-controlled robots under rigid contacts, and point out its inherent inability to provide force-feedback policies.

A. Classical model-predictive control

MPC solves online the Optimal Control Problem (OCP)

$$\begin{aligned} \min_{x(.),u(.)} \quad & \int_0^T \ell\left(x(t), u(t), t\right) dt + \ell_T\left(x(T)\right) \\ \text{s.t. } & \dot{x}(t) = f(x(t), u(t)) \end{aligned} \tag{1}$$

where $x(0) = x^m$ is the initial (measured) state, f the dynamics model, and ℓ, ℓ_T the running and terminal costs. Note that hard constraints on the state and control can be added, as soft penalties or hard constraints - which may be more challenging for real-time applications. This OCP is transcripted into a non-linear program, i.e. the cost and dynamics are discretized using an Euler discretization scheme. This program is solved online at each control cycle. For the remainder, and without limitation, we assume that the robot is fully actuated with n joints, the state vector $x = (q, \dot{q}) \in \mathbb{R}^{2n}$ includes the joint positions and velocities and the control vector $u = \tau \in \mathbb{R}^n$ includes the joint torques.

B. Rigid contact model

In optimization-based control, it is convenient to assume that contacts between the robot and the environment are *rigid*, i.e., pure kinematic constraints that can be resolved at the dynamics level. The dynamics of a robot in contact is given by the following constrained dynamical system corresponding to the KKT conditions of Gauss' principle of least constraint [34]

$$\begin{bmatrix} M(q) & J^T(q) \\ J(q) & 0 \end{bmatrix} \begin{bmatrix} \ddot{q} \\ -F \end{bmatrix} = \begin{bmatrix} \tau - b(q,\dot{q}) \\ -\alpha_0(q,\dot{q}) \end{bmatrix} \tag{2}$$

where $M(q) \in \mathbb{R}^{n \times n}$ is the generalized inertia matrix, $J(q) \in \mathbb{R}^{n_c \times n}$ the contact Jacobian, $F \in \mathbb{R}^{n_c}$ the contact force, $b(q, \dot{q}) \in \mathbb{R}^n$ the nonlinear effects of Coriolis, centrifugal and gravity forces, and $\alpha_0(q, \dot{q}) \in \mathbb{R}^{n_c}$ the contact acceleration drift. For clarity, the dynamics f in (1), is in fact the solution map of system (2), i.e. $f : (q, \dot{q}, \tau) \mapsto (\ddot{q}, F)$. The dependencies in $q, \dot{q}$ will be dropped in the remainder.

C. The challenge of force feedback

While the rigid contact model conveniently fits the MPC framework, it inherently prevents force feedback. The contact force F corresponds to the Lagrange multiplier of the contact constraint, namely $J\ddot{q} + \alpha_0 = 0$ (second row of the system (2)) [35]. As such, it cannot be controlled in a feedback sense: once $x = (q, \dot{q})$ and F are measured, $u = \tau$ is already completely determined by (2). Hence, u cannot be optimized as a function of F without creating an algebraic loop. This issue is a typical pathology from control systems with non-zero input-output feedthrough and can be broken by introducing delay [36]. This point was discussed and addressed in our previous work [29], where actuation was modeled as a low-pass filter, and the joint torques were treated as part of an augmented state. In contrast, we propose in this paper to break this coupling thanks to the online estimation without augmenting the state of the MPC.

III. Method

This section presents a new approach using estimation to leverage force sensor feedback in MPC. It includes an estimator, a reformulation of the MPC problem to include force feedback in the MPC model, and a feedback-linearizing compensation term for unmodeled force directions.

A. Estimation

As explained previously, it is unclear how to achieve force feedback under the rigid contact assumption without introducing delays or more complex contact models. We show here that estimation is a simple way to circumvent this issue by keeping the rigid contact assumption and correcting the model. Indeed, due to numerous model inaccuracies, the force F predicted by (2) rarely matches the force measurement. Hence a natural idea is to keep track of this mismatch by estimating online the offset between the model and the measurement with standard Kalman filtering [4].

The idea of estimating an offset error to improve the closed-loop performance of the controller is standard in estimation (e.g., [30]). We show that a disturbance Δ in the dynamics can incorporate rich force sensor feedback information in the MPC. We consider a model of the form:

$$M\ddot{q} + b = \tau + J^T F + \mathcal{M}(\Delta), \tag{3a}$$

$$J\ddot{q} = -\alpha_0. \tag{3b}$$

Here, $\mathcal{M}$ models how Δ offsets the dynamics. While the mismatch can be modeled in many ways, we assume that $\mathcal{M}$ is linear. Specifically, we consider two different models:

- Torque offset (in joint space) : $\mathcal{M}(\Delta\tau) = \Delta\tau$
- Force offset (in task space) : $\mathcal{M}(\Delta F) = J^T \Delta F$

This offset is meant to correct the model mismatch due to inaccurate modeling of, e.g., the dynamics, contact model, external disturbance, etc. The idea is to estimate the offset online, given raw measurement. More precisely, given a prior on the offset $\widehat{\Delta}$, we use joint positions, velocities, accelerations, torque commands, and force measurements to update the force offset. We assume perfect joint position and velocity measurements, and Gaussian measurement noise:

$$\Delta = \widehat{\Delta} + w, \qquad w \sim \mathcal{N}(0, P), \tag{4a}$$
$$\ddot{q}^m = \ddot{q} + v, \qquad v \sim \mathcal{N}(0, Q), \tag{4b}$$
$$F^m = F + \eta, \qquad \eta \sim \mathcal{N}(0, R), \tag{4c}$$

where F^m is the force measurement and $\ddot{q}^m$ the acceleration measurement. P, Q and R are positive-definite covariance matrices. As it is traditionally done in Kalman filtering, each disturbance distribution is considered to be Gaussian, which allows to solve the Maximum Likelihood Estimation (MLE) problem [4]. Here, the MLE aims at finding the parameters $\Delta, \ddot{q}, F$ that maximize the probability density function given the observed measurement and prior force offset:

$$\max_{\Delta, \ddot{q}, F} \quad p(\Delta, \ddot{q}, F \,|\, \widehat{\Delta}, \ddot{q}^m, F^m) \tag{5}$$
$$\text{subject to constraint (3a)}$$

Applying the negative logarithm and leveraging the normal distribution assumption, the problem is equivalent to:

$$\min_{\Delta, \ddot{q}, F} \|\Delta - \widehat{\Delta}\|^2_{P^{-1}} + \|\ddot{q} - \ddot{q}^m\|^2_{Q^{-1}} + \|F - F^m\|^2_{R^{-1}}$$
$$\text{subject to constraint (3a)} \tag{6}$$

where $\|w\|^2_{P^{-1}} = w^T P^{-1} w$. If $\mathcal{M}(\Delta)$ is linear, Problem (6) becomes an equality QP and can be solved very efficiently with off-the-shelf solvers. This, in turn, allows high-frequency online estimation, e.g., 5 kHz for a 7 DoF robot. As in a Kalman filter, the obtained estimate Δ is used as a prior at the next time step.

Note that other constraints can be considered in the QP, such as inequalities on estimated quantities (e.g. force offset).

Remark 1. *If additional inequality constraints are unnecessary, one may solve the problem using a Kalman filter [4]. More specifically, one can use Recursive Least Squares (RLS) [37] with the transition equation, $\Delta = \widehat{\Delta} + w$ along with the observation equation*

$$\begin{bmatrix} \ddot{q}^m \\ F^m \end{bmatrix} = \begin{bmatrix} -M & J^T \\ J & 0 \end{bmatrix}^{-1} \begin{bmatrix} b - \tau - \mathcal{M}(\Delta) \\ -\alpha_0 \end{bmatrix} + \begin{bmatrix} v \\ \eta \end{bmatrix}, \tag{7}$$

in order to estimate Δ online. Note that if $\mathcal{M}$ is linear, this observation model is linear, and one can use the RLS equations to derive an update rule on Δ.

B. Force feedback in the MPC via estimation

Once estimated, the force offset must be considered by the controller. This will break the coupling between forces and torques discussed in Section II-C by adding a delay between the measurement and the corrective term ΔF.

1) Naive inclusion as a corrective control: A naive approach is to add a feedforward term to the optimal torque given by the MPC, τ_{MPC}, to compensate the estimated offset:

$$\tau = \tau_{\text{MPC}} - \mathcal{M}(\Delta). \tag{8}$$

Although this work focuses on MPC, this method is agnostic to the nature of the controller.

2) Inclusion in the predictive model: Alternatively, the offset can be considered directly in the model used by the MPC. More precisely, we can consider that the offset will be constant over the horizon of the MPC and solve the OCP using as dynamics Eq. (3a) (instead of Eq. (2)). The MPC model is then updated online at each offset estimate update.

Remark 2. *Interestingly, when $\mathcal{M}(\Delta F) = J^T \Delta F$, updating the predictive model is in fact equivalent to modifying the force reference in the cost function. More specifically, the modified dynamics can be written in the following way:*

$$\begin{bmatrix} \ddot{q} \\ F \end{bmatrix} = \begin{bmatrix} -M & J^T \\ J & 0 \end{bmatrix}^{-1} \begin{bmatrix} b - \tau \\ -\alpha_0 \end{bmatrix} - \begin{bmatrix} 0 \\ \Delta F \end{bmatrix}. \tag{9}$$

Therefore, the force offset only biases the predicted forces and does not affect the acceleration. This means that this force offset has no impact on the predicted trajectory. The offset will only impact terms of the cost function that include the predicted force. Given a cost of the form $\ell(x, u, F(x, u, \Delta F))$, we can simply consider $\ell(x, u, F(x, u) - \Delta F))$, and discard ΔF from the prediction model. This greatly simplifies the implementation and gives more interpretation to the method. Interestingly, if the cost function does not depend on the force, the force offset will not impact the solution of the OCP.

C. Direct compensation of unmodeled force directions

The above formulation assumes that force can only be exerted in the n_c constrained dimensions. However, in reality, forces can exist in the other $6 - n_c$ directions and may interfere with the task if not taken into account (e.g. friction during a polishing task if only the normal force is modeled).

Following [1], instead of using an explicit 6D force model to compute a feed-forward compensation term, we propose to use the force measurements directly. This is in fact a form of Feedback Linearization (FL) as emphasized in [38]. Concretely, we add to the optimal torque given by the MPC the following compensation FL term

$$\tau = \tau_{\text{MPC}} - J^T_{\text{6D}} S F^m_{\text{6D}}, \tag{10}$$

where $J_{\text{6D}} \in \mathbb{R}^{n \times 6}$ and $F^m_{\text{6D}} \in \mathbb{R}^6$ are the full 6D Jacobian and measured force, and the selection matrix $S : \mathbb{R}^6 \to \mathbb{R}^6$ nullifies the n_c constrained dimensions. In the experiment section, we will show that this simple FL term will lead to competitive performances with more established yet more complex friction models such as the Coulomb model.

From a control perspective, it could seem unsafe at first glance to use measured forces in the control torque because the robot would always maintain itself in a disturbed state, which would create divergence of the force (e.g., pushing harder). But this would happen only if unmodeled forces are unbounded (i.e. motion is actually constrained by the environment). If the unmodeled forces are bounded, the disturbance would simply generate motion in their directions. For instance, if the normal force on a plane is stably controlled, the lateral forces are bounded by it through the friction cone. In that case, a disturbance increasing the lateral forces would simply make the robot slip. So this FL term is a safe compensation term to use in practical situations.

Remark 3. *The FL compensation term in Eq.* (10)*, could instead be added directly inside the MPC model, assuming that it remains constant over the whole horizon.*

IV. Experimental study

In this section, we evaluate the performance of the proposed approach through a comparative experimental study on a torque-controlled manipulator. First, we show the major advantage in tracking performance of using explicit force feedback over classical MPC. This benefit is twofold: force feedback enables to effectively cancel friction, and it corrects the model mismatch thanks to online estimation. Second, we demonstrate the benefit of encoding the model mismatch in the task space (ΔF) rather than in the joint space ($\Delta \tau$). Finally, we show how the proposed approach outperforms the most established force control strategy (integral control) by demonstrating that its force tracking performance is identical, but that it additionally aligns with the MPC objectives.

A. Experimental setup

All experiments were performed on the torque-controlled KUKA LBR iiwa R82014. We used an ATI F/T Sensor Mini40 mounted at the tip of the arm on a custom end-effector mount piece. A short MPC horizon (4 nodes of $6ms$) allowed to run the MPC and the estimator synchronously at $1\,\mathrm{kHz}$. The estimation QP problem (6) is solved using ProxQP [39], the OCP (1) is transcripted using Crocoddyl [40], and rigid-body dynamics are computed using Pinocchio [41]. Our code is publicly available[1]. Moreover, the accompanying video illustrates the robustness of the proposed approach to external disturbances.

B. Tasks formulation

1) Polishing task: A constant normal force is exerted on a horizontal plane (e_x, e_y) while tracking a circular end-effector trajectory. The MPC includes a 1D rigid contact force model ($n_c = 1$) so that the constraint (3b) prevents motions in the normal direction e_z, and ignores tangential forces in the (e_x, e_y) directions. The cost function is

$$\begin{aligned}\ell(x,u,t) = \; & w_1\|x(t)-\bar{x}(t)\|^2_{Q_1} + w_2\|u(t)-\bar{u}(t)\|^2_{Q_2} \\ & + w_3\|p^{\text{ee}}(t)-\bar{p}^{\text{ee}}(t)\|^2_{Q_3} + w_4\|F(t)-\bar{F}(t)\|^2_{Q_4} \\ & + w_5\|v^{\text{ee}}(t)\|^2_{Q_5} + w_6\|\log_3\left(\bar{R}^{\text{ee}}(t)^T R^{\text{ee}}(t)\right)\|^2_{Q_6}\end{aligned}$$

where $(w_i, Q_i)_{i=1..6}$ are positive scalar weights and positive diagonal activation matrices, $\bar{x}(t) = (\bar{q}(t), 0)$ is a reference configuration, $p^{\text{ee}}(t), F(t), R^{\text{ee}}(t)$ are the position of the end-effector, contact force and end-effector orientation respectively, $, \bar{p}^{\text{ee}}(t), \bar{F}(t), \bar{R}^{\text{ee}}(t)$ are their respective references, $v^{\text{ee}}(t)$ is the end-effector velocity, $\bar{u}(t) = g(q(t)) - J^T F(t)$ is the gravity compensation torque under external forces, $\log_3 : SO(3) \mapsto so(3)$ is the logarithm map on rotations. The circular trajectory $\bar{p}^{\text{ee}}(t)$ has a diameter of $14\,\mathrm{cm}$ and a speed of $3\,\mathrm{rad\,s^{-1}}$, unless otherwise stated. The reference normal force is constant $\bar{F} = 50\,\mathrm{N}$.

2) Force step tracking task: A 3D contact force ($n_c = 3$) step signal is tracked. Hence the motion of the end-effector is constrained in normal and tangential directions. The cost function has the same form as the polishing cost function (11), with the only differences that $F(t), \bar{F}(t)$ are 3D, the reference end-effector position $p^{\text{ee}}(t)$ is now constant and the force reference is defined as $\bar{F}(t) = \left(\bar{F}_x(t), \bar{F}_y(t), \bar{F}_z(t)\right)$ where $\bar{F}_x(t)$ is a step signal from $-10\,\mathrm{N}$ to $10\,\mathrm{N}$, $\bar{F}_y(t) = 0\,\mathrm{N}$ and $\bar{F}_z(t) = 100\,\mathrm{N}$ are constant.

3) Energy minimization: A sinusoidal joint position trajectory is tracked while maintaining a fixed 3D contact with the horizontal plane and minimizing $\|\tau\|^2$. The cost function is similar to the polishing (11), except that the reference configuration $\bar{q}(t)$ is no longer constant, no end-effector cost is used ($w_3 = w_5 = 0$), the control regularization term is turned into an energy term ($\bar{u}(t) = 0$). The reference joint trajectory is a sine on the A3 joint with an amplitude of $0.2\,\mathrm{rad}$ and a frequency of $2\,\mathrm{Hz}$. Here the force objective acts as a regularization term to avoid slipping and large forces (i.e. $w_4 \ll w_1, w_2, w_6$) and the reference is $\bar{F}(t) = (0, 0, 50)$.

C. Friction model vs direct measurement feedback (FL)

We evaluate the effect of force feedback as a direct compensation of the contact friction (Section III-C). We compare its performance on the polishing task against the classical MPC (i.e., without compensation) and the well-known Coulomb's friction model

$$F_T = -\mu \frac{v}{\|v\|} F_N, \tag{11}$$

where $F_T \in \mathbb{R}^2$ is the tangential force, $F_N \triangleq F \in \mathbb{R}$ is the normal force, $v \in \mathbb{R}^2$ is the tangential velocity of the contact point and μ is the dynamic friction coefficient. This model is clearly discontinuous in v so in order to avoid chattering phenomena, we consider the following smooth relaxation

$$F_T = -\mu \frac{\tanh(\epsilon\|v\|)}{\sqrt{2}} \frac{v}{\|v\|} F_N, \tag{12}$$

where we used $\mu = 0.35$ and $\epsilon = 10$. Our results are reported in Table I for several polishing speeds. We can see that the Coulomb model is slightly better in fast

[1] https://github.com/machines-in-motion/force_observer

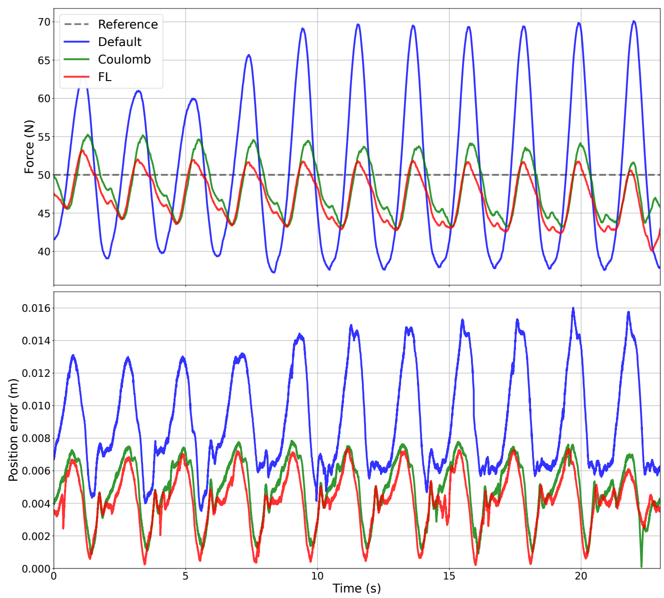

Fig. 1: Normal force trajectories of the medium-velocity polishing task. The blue curve is the classical MPC without friction compensation, the green curve is the classical MPC with the Coulomb model compensation, and the red curve is the classical MPC with FL compensation.

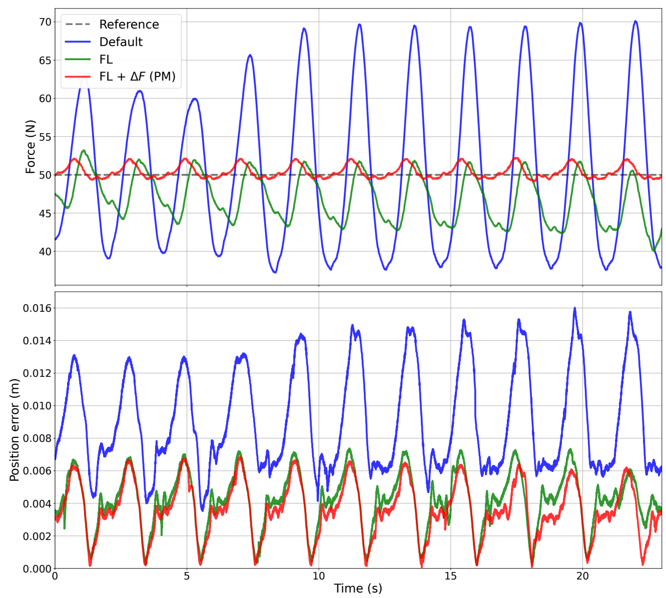

Fig. 2: Normal force (top) and end-effector position error (bottom) for the polishing task: in blue the classical MPC (II), in green the classical MPC with the FL compensation term (III-C), in red the proposed approach with FL compensation and the force offset in the predictive model (III-B.2).

		Default	FL	Coulomb
Slow	(1 rad/s)	7.67 ± 0.55	3.83 ± 0.17	4.72 ± 0.21
Medium	(3 rad/s)	9.66 ± 1.38	3.92 ± 0.56	3.99 ± 0.33
Fast	(6 rad/s)	16.42 ± 0.79	5.22 ± 0.32	4.82 ± 0.25

TABLE I: Mean-absolute error (MAE) of the normal force (in N) for the polishing task over 10 circles: classical MPC (Default), FL compensation (10) and Coulomb model (12).

	$\Delta\tau$	ΔF
Corrective control	2.01 ± 0.08	1.55 ± 0.03
Predictive model	1.95 ± 0.07	1.55 ± 0.04

TABLE II: MAE of the normal force (in N) for the polishing task: force offset ΔF vs. torque offset $\Delta\tau$, used in the control loop either in the "predictive model" way of III-B.2 or in the "corrective control" way of III-B.1.

motions but less performing in slow motions. Figure 1 shows the corresponding force trajectories for the medium-speed polishing task. Note that the FL compensation term only uses the $3D$ Jacobian as the contact torques a negligible in that task. These experiments confirm that considering the friction forces substantially increases performance w.r.t. classical MPC. Moreover, it shows that explicit force feedback from sensors can effectively be used as an FL term to directly to compensate friction effects and that it leads to a similar performance to well-established friction models.

As pointed out in Remark 3, it would be interesting to use the Coulomb model inside the MPC so that lateral forces are predicted using velocity and rigid normal force predictions, but this raises challenging issues (non-smoothness, insufficient software, breaks symmetry of KKT (2), etc.).

D. Comparison between force offset and torque offset

In this experiment, we compare the two mismatch models introduced in Section III-A, namely the torque offset $\Delta\tau$ and the force offset ΔF. Although capturing all disturbances in $\Delta\tau$ seems intuitive, experimental comparisons on the polishing task reveal a higher tracking accuracy for ΔF. For each model, we implemented the two ways of incorporating the correction into the MPC, namely

- The "corrective control" way of III-B.1: the correction is added to the optimal torque as a feedforward input
- The "predictive model" way of III-B.2: the correction is added directly to the model

Figure 2 illustrates how force feedback improves both the force tracking and the end-effector position tracking. Our results are summarized in Table II. There is a notable performance difference between ΔF and $\Delta\tau$ with a clear advantage for the force offset. Intuitively, the torque offset estimates perturbations unrelated to the contact (e.g. joint stiction) while the force offset only corrects what is necessary to improve the force tracking. There is, however, no clear difference in performance between using the estimate as a corrective control or in the predictive model. There seem to be a slight advantage for the predictive model, but the performance gap is too shallow to draw any conclusions.

E. Integral force control

Our approach is now compared to the most established direct force control approach - integral control. We were not able to find a difference in performance between using the integral term in the predictive model or as a corrective control. This question being out of the scope of this paper,

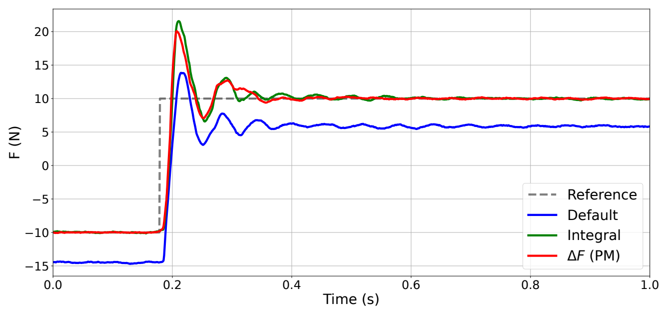

Fig. 3: Lateral force trajectories in the e_x direction for the force step tracking task: the blue curve is the classical MPC (Default), the green curve is the classical MPC with with integral control (Integral) and the red curve is the force offset estimation ΔF included in the predictive model (ΔF (PM)).

we propose to consider only the latter:

$$\tau = \tau_{\text{MPC}} - J(q)^T \left(-K_I \int_0^t \left(F(t') - \bar{F}(t') \right) dt' \right) \quad (13)$$

Note that we deliberately chose not to include a proportional and a derivative control term as Volpe et. al. [7] demonstrated both theoretically and experimentally that pure integral gain control was the best choice for accurate force tracking.

1) Polishing: We observed the same force tracking performance on the polishing task for the integral controller (1.69 ± 0.05 N) than for the proposed approach (cf. Table II, ΔF as corrective control).

2) Step experiment: We show in this experiment that the proposed approach and integral control have equivalent force tracking performances on a force step tracking task. The force trajectories are in Figure 3. We also report the average force tracking error of all the controllers in Table III.

	Avg. error
Default	1.99
ΔF (predictive model)	0.71
ΔF (corrective control)	0.60
$\Delta \tau$ (predictive model)	0.80
$\Delta \tau$ (corrective control)	0.87
Integral control	0.68

TABLE III: MAE of the normal force error for a step tracking task for different controllers: classical MPC (Default), force offset estimation (ΔF), torque offset estimation ($\Delta \tau$) and integral control. ΔF and $\Delta \tau$ are used as corrective control (III-B.1) or in the predictive model (III-B.2).

3) Energy minimization: In this experiment, we illustrate the ability of force feedback MPC to achieve contact tasks with conflicting objectives. Table IV shows how the proposed force estimation approach aligns with the MPC objectives by trading off force tracking against energy minimization: its overall cost is lower than the integral controller, which conflicts with the MPC and generates a high cost. These results also show interestingly that somehow, the torque offset estimation ($\Delta \tau$) uses less energy than the force offset estimation (ΔF), although it yields a slightly higher cost overall. This suggests that encoding the mismatch as a joint torque offset may have its own benefits, other than accurate force tracking. The accompanying video illustrates the relative importance of $w_2 \|\tau\|^2_{Q_2}$ w.r.t. the total cost.

	Avg. $\|\tau\|^2$	Total cost
Default	136 ± 21	0.44 ± 0.02
ΔF (predictive model)	139 ± 13	0.43 ± 0.01
ΔF (corrective control)	145 ± 18	0.43 ± 0.02
$\Delta \tau$ (predictive model)	131 ± 21	0.48 ± 0.01
$\Delta \tau$ (corrective control)	132 ± 22	0.51 ± 0.02
Integral control	1052 ± 29	0.82 ± 0.027

TABLE IV: Average squared torque and total cost for each controller for the energy task: classical MPC (Default), force offset estimation (ΔF), torque offset estimation ($\Delta \tau$) and integral control. ΔF and $\Delta \tau$ are used as corrective control (III-B.1) or in the predictive model (III-B.2).

V. Conclusion

In this work, we proposed a simple approach to achieve force feedback in MPC that relies on the online estimation of the mismatch between the predicted forces and the force measurements. Our experiments showed that force feedback effectively cancels friction and brings the force tracking performance to the level of the most established direct force control strategies. We also studied two variants of our approach: the estimation of a torque offset in the joint space, and the estimation of a force offset in the task space. Our experiments show that the force offset yields a more accurate force tracking while the torque offset is more generic and can enhance other criteria (e.g., energy minimization).

In conclusion, our experimental results show that current optimization-based control and estimation techniques are sufficient to incorporate force sensors in model-predictive controllers and suggest a more systematic exploitation of those modalities on real robots. In future work, it would be interesting to add the integral error as part of an augmented state in the MPC. Also, the estimation could be done over a horizon (although it has not led to any improvement so far in our trials), and the assumption of perfect joint position and velocity measurements could be relaxed, although this would turn the estimation problem into a nonlinear program. Finally, it would be interesting to extend the proposed methodology to floating base robots.

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