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ORIGINAL PAPER

Worlds and words: entangling mathematics, language, and context in newcomer classrooms

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Abstract
This work studies mathematics word problems' use in a classroom of recent immigrants, or newcomers, to a United States public elementary school. I study how word problems foster the recontextualization of mathematical concepts in a lived reality experienced by newcomer students in their new cultural and educational setting. In this study's setting language plays a significant role in the process of meaning-making. I describe how language use in word problems remains intertwined with mathematics instruction. This opens a space for questioning word problems' purpose and role in multilingual classrooms, and I highlight how the creative process of co-constructing problems' meaning in this context can expand notions of genre applied to word problems. Throughout I adopt a theorization of translanguaging as a language practice and apply it in problem discussion. This helps probe how language use impacts students' ways of understanding and utilizing mathematical concepts.

Keywords Newcomers · Word problems · Elementary school · Translanguaging · Multilingual

1 Introduction

This work emerges in the context of recent immigrant students in the US educational system often termed *newcomers*[1] (Culbertson et al., 2021). Many such students are only just learning English, inhabiting a liminal space often urging them to shed previous experiences and rapidly find a place in a new setting foreign in both language and practice. Newcomers' dramatic dislocation and relocation suggest rethinking practices of language use and content instruction to support their finding new understandings and voices. This work focuses on one particular aspect of this context: mathematics word problems (WPs).

I aim to answer the question: what interaction structures support newcomer students' sense making when working with word problems? With this I explore how word problems foster the recontextualization of mathematical concepts in a lived reality experienced by newcomer students in a new cultural and educational setting. I use Li's (2018) theorization of translanguaging to resignify WP language use and interrogate the role WPs play in mathematics education research and instruction, specifically in multilingual[2] classrooms. Here I employ translanguaging as a practice of language and apply it in problem discussion. I probe how language opens spaces to explore mathematical meaning-making.

I first situate WPs in mathematics teaching and research, then outline Li's (2018) description of translanguaging. I provide and discuss evidence from a classroom in light of the focus posed above. I conclude by highlighting this work's contribution to furthering mathematics education research.

2 Framing and context

2.1 WPs and their use in multilingual classrooms

Research has long investigated WPs in mathematics teaching and learning in multilingual contexts (Adetula, 1990; Barwell, 2009; Krause, 2022). Barwell (2009) places WPs at

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1 The term Students with Limited or Interrupted Formal Education (SLIFE) also has currency in this context (Center for Applied Linguistics, 2022). Here I have used the term *newcomer* in accordance with the school system where this study took place.

2 Throughout this paper *multilingual* refers to places where, and people whose relationships and experiences with, two or more languages and ways of conveying ideas interact, encompassing complex "historicities, relationships, practices, and identities" (Lee, 2023, p.84).

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the fulcrum balancing mathematics and language. Teaching mathematics through WPs has gained value in mathematics instruction (Boaler & Staples, 2008; Carpenter et al., 2015; Lampert, 2001), because WPs allow students to connect familiar mathematical ideas by solving problems in a variety of ways (Krause et al., 2021). However, the complexity involved in parsing the language, while making connections with mathematics, remains insufficiently addressed in the theory supporting problem-based instruction (Krause, 2022).

Instructional practices centered on eliciting student understanding, like children's mathematical thinking, focus on teaching moves probing students' grasp of mathematical concepts (Jacobs & Empson, 2016). Such practices deemphasize standard algorithms, which express students' procedural knowledge, rather than conceptual understanding. But the practices can be difficult to apply in linguistically diverse classrooms. Maldonado Rodríguez et al. (2022) note how these practices overlook "the intentionality needed to question how race, power, status, and identity are interwoven with what it means to teach and learn in mathematics classrooms" (p. 193). Their and other scholars' work (Bartell & Aguirre, 2019; Battey & Chan, 2010) suggests that focusing on children's mathematical ideas alone is insufficient for supporting children from historically marginalized groups and, as argued here, more so newcomers. I argue that this derives in part from the need to allow newcomers to latch on to cultural, sociological, linguistic, and educational concepts that they recognize from their own experience, and this view has not yet permeated the majority of mathematics classrooms. When students come from an educational context imbued with methods or practices foreign to their new context, these can provide a sense of stability in the new context. For some students, a particular method itself might be what they know and understand at a particular moment. As they adapt to a new language and culture, and perhaps to new notation, they can at least rest assured that what they learned still works.

Some mathematics education research posits a general definition for WPs "as verbal descriptions of problem situations" (Verschaffel et al., 2020, p. 1). The descriptions arise in an academic setting and pose questions to be solved using data in the situation (Pongsakdi et al., 2020; Verschaffel et al., 2020). This aligns with operationalizations of mathematics WPs in efforts at automatic parsing and solution (Zhang et al., 2020), though Verschaffel et al.'s (2020) discussion emphasizes numeric values in a way that seemingly excludes geometric problems studied elsewhere (Zhang et al., 2020). Its framework also leaves little room, e.g., for problems setting students to discern *if* they have been given sufficient information to answer the questions posed.

Several researchers have studied WPs' structure and use in mathematics teaching. Land (2017) gives four WP components: (a) the mathematics addressed, (b) the type of problem, (c) a story context, and (d) number choice. Each component influences students' access to a problem (Krause, 2022). While "number choice" applies to problems focused on numeric data, interpreting this as "the data given" allows these criteria to serve as a general characterization of WPs. Hiebert et al. (1997) center students' perceptions, characterizing a WP as a task or activity where students do not carry an expectation of a single "correct" method, nor seek to apply learned rules or methods. Hiebert et al. (1997) identified three components facilitating WPs' productive use in teaching: the problem (1) must be accessible to a range of students, (2) can be solved in different ways, and (3) addresses worthwhile mathematics. Gerofsky (2004) posits WPs as forming their own literary genre, in part characterized by a tripartite structure: (1) the set-up, (2) the information, and (3) the question. Gerofsky (2004), Land (2017), and Hiebert et al. (1997) provide a basis for a conversation on WPs' use and impact in teaching and learning mathematics.

While these characterizations yield formal criteria for a WP, they do not specify how WPs' stories contribute to students' engagement with mathematical concepts. Instruction often presents WPs as "real-world" applications of mathematical ideas. While the entities in WPs often recall real-world objects, they need not: a problem can refer to the number of *pencils*, or of *zapbots*, with equal facility. A WP goes on to posit relationships among these entities. These relationships likewise need not represent the real world. But during instruction teachers invite students to inhabit a universe where such entities and relationships are *assumed* real, and then to explore how this universe works. Language use, especially as applied to capture experience and convey ideas, plays an important role in navigating the students' and teacher's co-construction of new universes.

In structural terms, the WP genre's boundaries seem permeable. The linguistic or narrative presentation of the WP's universe can draw from different genres: poetic verse in mathematical problems dates at least from ancient India and the Middle East (Katz, 2018) to James Clerk Maxwell's "A Problem in Dynamics" (Campbell & Garnett, 1882). Gerofsky's (2004) study of WPs as genre provides a particular window on how WPs fluidly cross other genre boundaries. A problem set-up might construct a universe using an adjacent genre to engage students, presenting the information and question with narrative novelty.

Not only do WPs' narrative structures interplay with students' mathematical understandings, but also WPs' linguistic structures. For example, a semantic frame, or situation, like Giving (cf. FrameNet, as described in Fillmore and Baker (2010)) might take several lexical forms across WPs: *give*, *donate*, *receive*, etc. But each time the frame entails certain roles: a *donor* (doing the giving), a *recipient* (being given something), and a *theme* (the thing given).

In crossing language boundaries the frames can shift. With WPs students practice linking semantic content, such as frames and their roles, with mathematical understanding: Giving can correspond to *adding*, Taking to *subtracting*, etc. Applying this process to the WP "set-up" and "information" (Gerofsky, 2004)—"situation" (Pongsakdi et al., 2020) or "model" (Verschaffel et al., 2020)—amounts to exploration or creation of a universe. Here *universe* denotes, in a logical or semantic sense, the encompassing set of logical discourse in a given context, or the collection of referents for a given lexicon. But it also carries a narrative connotation: e.g. the semantics of *talk* might shift because, in the universe described in the set-up, the word denotes something not only a human can do, but also a rabbit (Kroeger, 2019, p. 22). Students engage in this exploration with others. They co-construct universes to create and test relationships between meaning structures and mathematical structures, so that WPs provide a venue for exploratory learning in multilingual contexts.

As students and teachers negotiate a WP's solution, the solution method might strike them as applicable to other problems, implicitly defining a problem solution type. A problem's *solution type* is the collection of all problems soluble by the same method. These do not form mathematical equivalence classes, since a given problem can be soluble by more than one method and belong to various solution types. But in the co-constructive process students gradually build up a recognition that problems fall into types, and these types correspond to possible mathematical methods of solution. This type recognition intertwines with parsing the linguistic structures on which their meaning is built.

2.2 Translanguaging

Li (2018) details the development of the term *translanguaging* in research: Baker (2001) attempted to translate Welsh *trawsieithu* into English when studying Williams's (1994) work in Welsh revitalization programs. Li (2018) describes how Williams (1994) observed that the teacher would try to teach in one language, and the students would respond in another. Li (2018) suggests this practice "helped to maximize the learner's, and the teacher's, linguistic resources in the process of problem-solving and knowledge construction" (p. 15). Recently researchers have shown translanguaging's pedagogical effectiveness when the learners' language differs from the language of instruction (Li, 2018). It purposefully breaks "the artificial and ideological divides" (Li, 2018, p. 15) used to assert difference and uniqueness (e.g. named languages, cultures, race, etc.) (Creese & Blackledge, 2015; García, 2009). As Li (2018) notes, "Translanguaging empowers both the learner and the teacher, transforms the power relations, and focuses the process of teaching and learning on making meaning, enhancing experience, and developing identity" (p. 15).

Translanguaging in this article arises as a practical theory of language (Li, 2018): a *pedagogical practice* that facilitates a focus on, and expression of, concepts being learned in the classroom (Krause et al., 2022). In a multilingual mathematics classroom translanguaging permits communication and understanding by the speakers' fluid use of language as they make sense of their worlds, identities, and mathematical ideas (Planas & Chronaki, 2021).

Planas and Chronaki (2021) describe how immigrant students adapt language, "not *a* language or *a* single set of linguistic features" (p. 153), to their new circumstances in a mathematics classroom. However, these translanguaging practices remain hidden to the class as the learners "seem to adapt their speaking utterances to meet the norms of classroom engagement and please the teacher, even if this may imply the omission of mathematically significant reasoning developed during small group discussions" (p. 153). This suggests the complex intertwining of language and mathematics, and the complexity of teaching mathematics in a multilingual context. Work on translanguaging has extended more widely into mathematics education research. And the literature on translanguaging as a theory of practice in the mathematics classroom continues to develop rapidly (Krause et al., 2022; Tai & Li, 2021a, 2021b). Any conjectures as to what translanguaging as a theory of practice might explain in the mathematics classroom must therefore remain tentative. However, this work hints at the need for investigations of the detailed dynamics of multilingual classrooms. Translanguaging as a pedagogical practice supports students' creativity and idea expression. I follow Li's (2018) definition of creativity as multilinguals' ability "to push and break boundaries between named language and between language varieties, and to flout norms of behaviour including linguistic behaviour" (p. 23). He also characterizes criticality as multilinguals' "ability to use evidence to question, problematize, and articulate views" (Li, 2018, p. 23). In the classroom space described, creativity and criticality are crucial (Tai & Li, 2021b).

In this work, translanguaging empowered the learners in the mathematics classroom. Here translanguaging practice facilitates meaning-making and enhances the students' and researcher's learning experience. Translanguaging is not "a linguistic structural phenomenon" (Li, 2018, p. 15), but rather a "practice and a process" (p. 15), which "involves dynamic and functionally integrated use of different languages and language varieties, but more importantly a process of knowledge construction that goes beyond language(s)" (p. 15). These views inform the understanding I propose of how WPs can be used in teaching mathematics in multilingual classrooms with newcomer students.

3 Method

This work constitutes a case study (Bogdan & Biklen, 1992; Hatch, 2002) of WPs' use in a specific newcomer classroom, given meaning through an interpretive analysis (Hatch, 2002). With the interpretative model I highlight the active participatory role I played in the study as researcher (Hatch, 2002). The interpretations presented result from a process of identifying patterns for salient working hypotheses, then refining and grouping the data according to patterns from successive rereadings to focus on those best supported by the data. I identified data clusters where students used particular strategies to solve the problem, e.g. modeling or an algorithm. I also marked clusters where specific linguistic structures were salient, e.g. student use of only one language or mixed languages. I noted how students first approached a given problem. Within that, I identified how I delivered the problem, any modifications to the initial problem, and in-the-moment decisions. I categorized linguistic structures used in delivering the problem and subsequently as we interacted. I noted what spurred my decisions for any modifications. I marked instances where my own notes combined languages, or when I identified interactions with salient use of non-verbal cues (e.g. signaling with hands or fingers). I clustered the data by student to help construct narrative descriptions of our process of engaging and solving WPs in each interaction. Section 4 presents some of these narratives.

This research also draws on *weaving* as a research tool (Tachine & Nicolazzo, 2022). Within this methodology learning involves creation of a narrative of interconnectedness among not only the persons coming together (e.g. students and researcher), but also the external elements and contexts we often other, or separate ourselves from (e.g. the classroom, playground, languages), when instead we might acknowledge their connection to and importance for interpersonal interactions. The methodology fosters a recognition of the tapestry woven through subjects, researchers, participants, and contexts, and how at different moments any individual can inhabit any number of roles.

Weaving extends the case study's bounded system (Hatch, 2002)—the class—to include the boundaries themselves—the class*room*. Within the framework, the students and I not only interacted with*in* the classroom's four walls, but *with* those boundaries, yet another study participant. As mentioned below, characteristics of the room itself occasionally acted upon the classroom inhabitants, impacting our interactions.

This methodology's use aims to aid the understanding of the complexities and interrelatedness of three strands participating in this study: (1) the learners (students and the researcher), (2) the subject (mathematics), and (3) the spoken languages (Spanish, English, Pashto).

3.1 Participants

This study occurred in a public elementary school in the southeast United States. Seven students participated. All attended one English as a Second Language elementary class, taught by Ms. Ulysses. Many students entered and left this classroom during the study. Only seven remained constant throughout the year. Five hailed from Central and South America, two from Afghanistan.

I established a relationship with the teacher and students for the 2021–2022 school year. For the first 3 months I only observed and recorded field notes. In January I started working with students one on one, once a week. Sometimes I worked with them individually, sometimes in groups. Only one student, Juno, was present in all sessions, even when other students joined. I share here work from the interactions with all seven students present for the entire school year. Juno's work appears more often given her participation in all interactions.

A 5th-grader, Juno comes from El Salvador; she was 11 when our work began, 12 by the time we finished. The other students were Gloria, Martín, Saafi, Saba, Manuel, and Eddy. Gloria and Martín are siblings, 10 and 9 years old, respectively. Originally from Venezuela, their family moved to Colombia at the end of 2018, where Gloria and Martín attended school for 1 year. Then the family moved to the US. In that process Gloria and Martín stopped attending school for several months. They joined Ms. Ulysses' classroom in late October of 2021. They both speak Spanish as their first language. Saafi and Saba, siblings, came from Afghanistan with their parents. Saafi was 10 years old, Saba 8. Their first language is Pashto. Manuel, from Guatemala, was 12 years old. His first language is Spanish, as with Eddy, 11, who comes from Costa Rica. Eddy's mother is fluent in English, and on a few occasions I chatted with Manuel's father about Manuel's love for and facility with mathematics.

3.2 Data source

The study's data consisted of one-on-one and small group interactions to solve WPs and equations. I recorded my written reflections as contemporaneous notes after each of those interactions, also collecting photographs and scans of their written work. I met with the students in these one-on-one and small group settings 64 times and recorded 64 reflection entries. These instantiate my process of noting what I learned and experienced interacting with the students. The reflections helped me to verbalize some of my decisions in these interactions. Through weaving, I expanded my conceptualization of the participants, noting interactions with the inanimate and with external factors. These reflections are a fundamental piece in constructing the narrative of the different roles I inhabited as I interacted with the students.

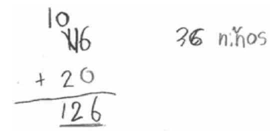
4 Data and analysis

The data below outlines problems solved by the students, my interactions with them during problem solving, and their solutions and responses. The examples are not intended as exemplars for generalizations on how newcomers solve WPs. Rather they illustrate elements of how linguistic practice, problem-solving strategies, and WPs can interact in working with a group of newcomers advancing their mathematical understanding in a new environment. Examples include written and oral interactions. Each mode could lie anywhere along the translanguaging spectrum: from employing any one of English, Spanish, or Pashto, to involving elements from various. I focused on interactions that fit Li's (2018) definitions of creativity and criticality: e.g., interactions where we crossed the boundaries of individual languages (Li, 2018), and where students articulated views on solving a problem or proposed ideas of where to learn. I sought examples where I delivered problems that showed our experiences. I marked examples where the classroom itself, or its abandonment, seemed to play a role in the situations' dynamics. From among these I chose examples exhibiting one or more of the major patterns I found in my data coding.

4.1 WP 1: playground

The initial session with Juno started with a WP: There are 16 children on the playground. 20 more children run out and join them. How many children are on the playground?[3] [*Hay 16 niños en el parque. 20 niños más corrieron a jugar con ellos. ¿Cuántos niños hay ahora en el parque?*][4] (See Fig. 1.) I intended to gain an understanding of how she was

Fig. 1 Juno's solution for WP 1



thinking about problems and how she would attempt to solve them, without prompting her to use methods she had been taught explicitly.

When I read the problem in English, Juno seemed not to understand. I asked, in Spanish, if she would like to discuss it in Spanish, and she agreed. I did not read the problem in Spanish as written. Instead I told her a story about us on the playground, aligning the story with the problem I had written. Based on my notes, the conversation went as follows:

> **Researcher:** *Imagínate que estamos en el parque y vemos que hay 16 niños jugando. Imaginemos que están jugando algo. ¿Qué crees que están jugando?* [Imagine we are at the playground and we see that 16 children are playing. Let's imagine what they are playing. What do you think they are playing?]
> **Juno:** *Mmmm … ¿la lleva?* [Hmmm … tag?].
> **Researcher:** *OK. Imaginémonos que están jugando a la lleva. Ahora imaginémonos que llegan 20 niños más a jugar con los niños que están en el parque. ¿Cuántos niños hay ahora jugando a la lleva?* [OK. Let's imagine that the children are playing tag. Now, let's imagine that 20 more children arrive to play with the children who are at the playground. How many children are there now playing tag?]
> **Juno:** *¿Nosotras no estamos jugando a la lleva?* [Are we not playing tag?]
> **Researcher:** *No, nosotras sólo estamos mirando.* [No, we are just watching.]

She then attempted a solution. She immediately placed the numbers in the format of a standard algorithm. (See Fig. 1.) She added 6 plus 0 and wrote 6 under the line. Then she wrote a 1 between the 1 and the 6 of 16, crossed out the two 1 s, and wrote 10 above them. I followed up on this and she was not sure why she "had" to write the 1 in front of the 6; but she was "certain" that that number was now a 10, not 11. To this she added 2 (from the 26) and wrote 12 next to the 6 in the answer.[5]

I suggested that she also approach the problem using base-10 blocks. She solved the problem, wrote *36 niños*, and solved the following problems in similar fashion. She

[3] Here and in the sequel, numerals (e.g. *16*) rather than written words (e.g. *sixteen*) appear in problems given to the students in written form. In all reporting of dialog, unless otherwise specified, numerals appear as shorthand for the actual words pronounced: students actually *said* words like *sixteen* or *dieciséis*, but I write *16* to help readers follow the mathematical argument.

[4] The English uses a simple present *run* (which does not connote a true present-time action, but rather a general truth or gnomic statement) flanked by instances of *are* (which can be present-time or gnomic). Given this was the initial session with Juno, it seemed appropriate to provide a more colloquial rendition in Spanish. In the moment, I chose a paratactic style, where each successive sentence envisages a new *now* as if scenes in a movie, even though the morphological tenses do not align strictly with those of the original English.

[5] I interpreted the result as due to a mechanical, rather than conceptual, error. From context, it appeared clear to me in the moment that Juno would not suppose adding 20 to 16 would give an answer over 100. She showed good number sense, and the missteps in the calculation appeared notational.

found counting by ones straightforward with manipulatives, but counting by larger intervals led at times to miscounts. This marked the beginning of working with the students on solving problems.

Here the problem statement, whether in English or in Spanish, was insufficient to grant Juno access to the problem. In practice, what granted access was the conversation as a whole. From my position, Juno needed more time to inhabit the situation—to understand it as a reality, understand the processes involved, and to see how those happenings corresponded to mathematical notions. The initial problem statement appeared too terse for this to happen all at once. But as we conversed, she began to ask questions, and I recognized she was beginning to see the event and wonder about its details.

The interaction also suggests that, when considering a WP, difficulty can arise in talking about *the* problem as confined to the problem statement. This was an initial clue to the importance of students inhabiting the problem, often through an open-ended discussion rather than through a narrow problem statement. The entire context bears on the problem. This suggests, on the one hand, the relation of the context can potentially cross genre boundaries, including different styles of narrative and world representation. On the other hand, seeing two problems as the same based on formal characteristics might pose obstacles. This hints at the utility of considering WPs as equivalent if they admit solution by the same method, i.e. belong to the same solution type.

4.2 WP 2: Elsa's boxes

In a different interaction, with all seven students, I gave the following problem: *Elsa has 36 coins. She puts 9 coins into each box. How many boxes can she fill with 9 coins each?* Because all seven students were present, I decided not to render the problem in Spanish. Anticipating this, I brought coins and, as I read the problem in English, I showed them the coins and moved my hands to represent what was happening. Juno and Gloria asked for the Spanish translation. As I spoke in Spanish to them, I also showed the other students what I was saying and paraphrased in English. The discussion of this particular problem moved seamlessly among three languages (Spanish, English, and Pashto), the coins, and hand gestures.

Another component of this interaction stood out, which I had not planned for ahead of time. By this point in our interactions, the process of co-constructing our universes and trying to inhabit them had led us frequently to try to identify each of us as a character in the story. In this instance I had planned ahead enough to bring coins to provide added reality. But the problem was about Elsa, yet there was no Elsa in our classroom. In delivering the problem to the students,

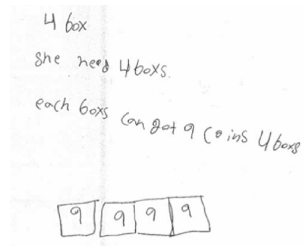
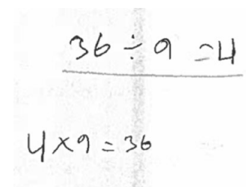


Fig. 2 Saafi's solution for WP 2

Fig. 3 Saafi's numerical representation of his solution for WP 2



I found it difficult to identify that character. It occurred to me that, in our previous interactions, I needed to find ways for students to inhabit the problem. But introducing Elsa added a new element to the problem: in semantic terms, this extended the universe of discourse unnecessarily. After reading the problem the first time, I decided to change the character. I said something like

> **Researcher:** … Elsa? Who is Elsa? [I pointed to everyone in the classroom and shrugged my shoulders.] No Elsa here, right?

I crossed out the name and said, "How about Gladys?" I pointed to myself, and from then on the problem was about Gladys putting coins in boxes. Figure 2 shows how Saafi solved the problem. Juno and the other students also represented their solutions with drawings.

After drawing the boxes and placing a 9 inside each box, Saafi also wrote a numerical representation of his strategy (Fig. 3):

I followed up: "Where are the boxes in this equation? Where are the coins in this equation?" Saafi pointed out the numbers 4, 36. I asked what the 4 represents in this equation and he said, "The boxes." Saafi also said, "I noticed that 36 divided by 9 is 4. And I know that 4 times 9 is 36."

I interpreted what he shared as him noticing that dividing 36 coins into equal groups of 9 coins would give him 4 groups. I also suspected that he noticed this by doing what the WP said Elsa—or Gladys—was doing. I interpreted what he wrote as him noticing that dividing these two numbers gives 4 and also knowing this multiplication fact that seemed to match what he was doing. In essence, Saafi solved the particular problem given as a unique exercise. But in his subsequent statements, he seems to begin to access the

Fig. 4 First fraction equations solved with the students

Fig. 5 Juno's work on the fraction problem 5–3/4

underlying mathematical concepts that point toward identifying the solution type to which this problem belongs.

Since the dynamics of this interaction were mediated largely through English, each student worked in a language space that yielded some unsurety. In this and similar instances, I found students more able to latch onto the situation when we imagined a member of the group as a participant in the problem.

4.3 WP 3: speedy's cucumbers

At times it seemed that no amount of preparation, or of trying to inhabit the constructed reality of the problems, yielded engagement. The students appeared claustrophobic, cramped into Ms. Ulysses' tiny, windowless classroom. The space itself seemed to play an ever greater role in our interactions and their focus on the problems. So as the weather warmed up, we decided to work outside, and this change of boundaries seemed to renew their energy and engagement.

In later interactions we worked on fraction WPs. For this particular interaction rather than starting with WPs, we began with fraction equations, such as __1/6 = 4/6, or 1/2 + 1/2 = __ (Fig. 4.). I chose to start with equations because the students had been solving fraction equations with their mathematics teacher that week. All seven students, including Juno, showed facility with these problems, exhibiting comfort in manipulating fractions with equal denominators. On several occasions, referring to such denominators, Juno said, *Porque estos dos son el mismo, no hago nada con ellos* [Since these two are the same, I don't do anything with them].

Continuing this fraction work, I started a session with Juno with the problem 5–3/4 = __. She appeared unsure of how to approach this problem. She began by drawing and shading a circle to represent the fraction in the expression. (See Fig. 5.) But she did not progress further, so I said we would come back to it, and we moved on to other problems.

As we continued the session, it seemed that a more contextualized approach might help reduce some of the confusion surrounding the manipulation of fractional quantities. So we shifted to WPs. I had selected a problem that I thought could help add context to our work with fractions. Because the problem was about a turtle eating cucumbers, I started by asking questions about pets: what they eat, how much care we give them, etc. Some parts of our conversation are described below:

> **Researcher:** *Si pudieras escoger tener una mascota ¿qué mascota tendrías?* [If you could pick a pet to have, which pet would you have?]
> **Juno:** *Un perro*. [A dog.]
> **Researcher:** *¿Y qué nombre le pondrías?* [And what name would you give it?].
> **Juno:** *Mmmm* … Skippy.
> **Researcher:** *Qué lindo nombre. El perro de mi hijo se llama Rocco. Es muy pequeño, pero ¡come mucho para ser tan pequeño!* [What a cute name. My son's dog is named Rocco. It's very small, but it eats a lot for being so small!]

In that conversation I introduced the following problem: *La tortuga Relámpago come 1/2 pepino cada día. ¿Cuánto pepino come en 5 días?* [Speedy the turtle eats 1/2 a cucumber every day. How much cucumber does she eat in 5 days?] (Empson et al., 2018). When Juno approached the solution to the problem, she drew 5 cucumbers and split them in halves. (See Fig. 6.) She said the turtle would eat 2 whole cucumbers and ½.

We then repeated the scenario, changing it to 1/5 of a cucumber per day over 7 days. (See Fig. 7.) She drew 7 cucumbers and split them into 1/5s. Her answer was that the turtle would eat 1 cucumber and 2/5. I asked how many 1/5s made a whole cucumber, and she said 5.

Juno's solution to this problem highlights the interconnectedness among WPs, how the information in the problem is presented, and how children think about their solutions.

Fig. 6 Juno's strategy for solving WP 3

Fig. 7 Juno's strategy for WP 3 with 1/5 of a cucumber over 7 days

For instance, she drew 5 cucumbers perhaps influenced by the number 5 in the information of the problem. However as she worked on the solution, she identified that only half of a cucumber was eaten in a day, so she dealt out cucumbers one half at a time.

I interpreted this interaction as an illustration of Juno's sense-making. In contrast to Saafi's example above, she apparently did not initially think of the problem as I intended. Saafi modeled the problem directly, putting 9 coins at a time in boxes. Here I expected Juno to model as well, drawing a 1/2-cucumber for each day the turtle ate: $1/2 \times 5$ or 1/2 added 5 times. Instead she created a different problem starting with 5 cucumbers. The numbers seemed to have given her something to latch on to in the problem, and she seemed to have made sense of them as she worked through the story in the WP itself. One possible origin of her impetus to latch on to the 5 might have been in the notion of a 1/2-cucumber itself: the 1/2 does not make much sense without reference to a whole. The search for a whole could have been one factor that drew her to the 5. Even if not her thinking, it is important to foreground how she gave the 5 a reality of its own beyond the statement of the problem, and worked from there to a solution by inhabiting the problem and modeling its action. This exemplifies the intricate interplay between different WP components.

Fig. 8 Gloria's strategy for WP 4

4.4 WP 4: clay bars

In continued work with fractions, 1 day all seven students solved a problem about children in an art class sharing bars of clay. Again I presented the problem in English and did not prepare a Spanish translation. To help myself present the problem, I displayed on the computer images of bars of clay and of a clay turtle. I pointed to the images, read the problem in English, and asked students to solve the problem. They solved the problem in different ways. The Spanish speaking students, even when given the problem in English, provided written and oral explanations in Spanish. Also, during this interaction the Pashto speaking students counted the pieces of clay in Spanish and Pashto during their sharing process. Figure 8 shows how Gloria solved the following problem: *In art class, the children are making clay sculptures. If 4 children share 13 bars of clay equally, how much clay does each child get?* (Empson et al., 2018).

In a subsequent interaction, I worked again with all the students. I decided to use the same problem to save time; I could simply talk about what we did last class and then move on to working with the fractions. I would emphasize that we were solving the same problem, just with 6 children and 10 bars of clay. I did not read the problem, but we talked about the bars of clay and what we did last class.

Saafi solved the problem by explaining that each child gets one whole "cookie", and then there are 4 "cookies" left. He split each of the 4 cookies into 4 equal parts, counted each part in Pashto, then in English, then paused. He wrote 1/4 inside each piece in the first cookie. Then he wrote 1/4 inside two of the pieces on the second cookie and paused again. He said he was missing pieces and created more pieces in each cookie. I counted in Spanish, and he said, "There are 6 pieces". I asked about the size of each piece, now that he had more pieces, and he said 1/6. He said each person got 1 whole cookie and 4/6 s (Fig. 9).

This interaction highlights how naturally the students began to expand their linguistic horizons, sliding into translanguaging along with me. We unselfconsciously began to infuse our conversation with elements of English, Spanish,

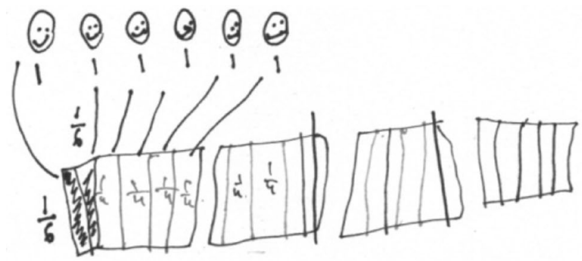


Fig. 9 Saafi's strategy for WP 4, with 6 children and 10 bars of clay

and Pashto, mindful of the limits of others' understanding while inviting them to step beyond those limits.

Moreover, Saafi's switch from clay bars to cookies was revealing. It could initially have been a linguistic slip (even though he used "clay bars" in our prior discussion) or creative invention. It suggested, however, that he was inhabiting the problem and conceiving of it in his own way. The clay bars were now cookies, marking either a simple linguistic substitution (using the *word* "cookies" where "clay bars" was intended) or a conceptual shift (making the problem *about* cookies instead of clay bars). But as he proceeded, it appeared to denote more likely a conceptual shift: it hints at his process of stepping beyond the specific problem to subconsciously identify the problem's solution type. The particular thing being divided could be clay or cookie: the difference was immaterial, and the method of solution became the central focus. Through action he seemed to begin constructing a relationship between a semantic frame of equal sharing and mathematical operations on fractions.

In what follows, I try to highlight ways in which the episodes described above contribute to an understanding of how WPs aid in recontextualizing concepts in these students' new setting.

5 Findings

This study focuses on how WPs foster newcomers' recontextualization of mathematical concepts within a new cultural and educational setting. I found that, in the specific classroom in this study, WPs act as locus for aligning linguistic meaning structures with mathematical concepts. A WP served as a sandbox, a delimited arena for exploratory play or discovery, where students tested the boundaries of their linguistic and mathematical understanding, and how these two relate to one another. The element of play—the interaction with others and interrogation of the problem set-up and information—was important. These same WP elements reduced the (linguistic or physical) objects in focus, and the applicable mathematical concepts, to a specific few, so students could explore a handful of concepts without being overwhelmed. At least three aspects of WPs facilitated this recontextualizing play, outlined below.

5.1 Discerning linguistic and mathematical structures through languaging and translanguaging

The interaction surrounding WP 1, about the playground, suggested that Juno found an algorithm for addition familiar. I noted that, where the problem represented a situation clear to her, she knew what to do, and that knowledge could be encapsulated in an algorithm. When the problem situation remained opaque to her, Juno appeared at a loss; the WP did not seem to make sense to her as a mathematical problem. From one perspective, Juno seemed caught in the process of knowing a solution method, but not recognizing in the problem a corresponding semantic frame—an expression of a scenario whose meaning coincided with her conception of applying the method.

From another perspective, Juno appeared to look at a problem and try to decide to what solution type—i.e. the collection of problems amenable to the same solution method—it belonged. Asked to find a solution in a different way, she initially found little sense in the question: if she implicitly looked for a problem to be an instantiation of a particular solution method she was familiar with, then solving the same problem a different way became at best irrelevant, at worst nonsensical. She seemed to expect solution types to be disjoint, not yet recognizing how one problem could yield to several solution methods. The method and the solution type are inextricably intertwined: a solution method defines a type (the collection of problems soluble in a particular way), and any WP is just a particular instance of its type. Likewise for students who solved the problem by modeling: they seemed to find the solution type and the particular problem intertwined in the same way. When Saafi solved WP 4, about the clay bars, and inadvertently converted the clay bars into cookies, he implicitly recognized the solution type, and the particular details of the context fell into the background.

With WP 3, on cucumbers, it is hard to tell what the initial representation of the 5 meant for Juno. In the problem information, the 5 represents days; but in Juno's representation the 5 became cucumbers that she then split in halves as Speedy the turtle ate them. This strategy provides an interpretation slightly different from descriptions in previous research on how children think about solving problems. In research on children's mathematical thinking, when students first attempt to solve problems new to them, they tend to directly model the action in the problem (Carpenter et al., 2015). However, Juno's strategy might present a different possibility. Students might alternatively represent the numbers in the problems first, as they make sense of how to use them in the solution. This is a subtle, but potentially important, difference. The numbers marked a point of departure for solving the WP, and what they represented seemed, initially, irrelevant: as with Saafi above, whether art class students are sharing clay or cookies is immaterial; rather the relationship between the quantities and the solution method remained salient. What the numbers represented started playing a role as the students began to inhabit the universe depicted in the WP. With Juno, how the story was narrated seemed critical for this sense-making.

This has implications for work with newcomers. As students learn new languages and contexts, the numbers in a WP might at times be more salient than the information conveyed through words. The numbers might give students an anchor as they make sense of the problem. In a way, newcomers can start accessing the problem and its mathematical content (the algorithm or solution type) before they even understand its linguistic and contextual meaning. This suggests subtle divergences from other WP models (Gerofsky, 2004; Hiebert et al., 1997; Land, 2017). Firstly, the numbers can play a more central role than noted elsewhere. Secondly, the situation assumes a central position, helping students explore the quantities' import as they model the situation. Thirdly, the background situation frequently comes to light more through discussion than through the problem statement, and translanguaging can facilitate student inventiveness. This last point can impact classroom settings, as discussion implies a collaborative setting for instruction.

5.2 Co-constructing universes and storytelling

Spanish use during the interaction on WP 1, about the playground, seemed critical for letting Juno access and relate to the situation. Throughout, if I gave a problem in English, she asked me to render it in Spanish. Importantly, in that and following interactions, although English and Spanish were present in the WPs as translations of each other, Spanish provided the opportunity to create stories to add context to the WPs, to imagine ourselves as part of the stories, as in the WP on Elsa's boxes.

At first this opportunity arose in the switch to Spanish. Later the storytelling in the context of problem delivery became more than storytelling in Spanish. As a group, all seven students and I narrated stories in ways that merged different languages and that transcended a written problem on a piece of paper. Gestures and elements of our surroundings contributed to our communication. As a group we communicated creatively (Li, 2018; Tai & Li, 2021a, 2021b) by explaining our mathematical understandings in ways we all understood, but that at the same time differed from how we initially expected teachers and students to share problems and ideas.

Learning outside Ms. Ulysses' classroom became commonplace. On walks to the playground we "prepped" the stories we were going to discuss. We selected the characters, and these helped us decide where they would take place. Our individual experiences and interactions with the confines of the classroom—the way it drove us to this new arena—impelled us toward new communicative modes and constantly shifting roles. As a group we found communication that allowed us new ways of expressing our understandings (Tai & Li, 2021a, 2021b).

5.3 Overlapping with other genres

The situations described here hint that some students in multilingual settings might benefit from more expansive notions of WP form and equivalence. The translanguaging context and creativity inherent in sense-making might require that WPs not be defined by any particular linguistic criterion. This would serve a purpose: in a setting with newcomers, where a strict problem form might be linguistically too terse for newcomers to parse initially, an entire discussion might function as the "problem statement". This discussion, moreover, provides a locus for translanguaging, transcending any particular language. Such multilingual contexts might view the discussion itself as the WP—so long as the discussion requires the same mathematical content for its solution—because the initial problem statement might be linguistically opaque to newcomers.

A problem set-up, moreover, might draw on other genres to elicit student engagement. Inasmuch as WPs invite the co-creation of novel universes, other genres might enter the discourse. And as teaching and learning practices start to involve translanguaging contexts, genre identities themselves might break down: a genre in one language or culture might be absent in another, or the boundaries of a shared genre might not coincide. As these languages and cultures mix, the genres themselves could break down. And the same could happen to the WP genre itself.

6 Discussion

The problems I posed to students contained all the components (in English and Spanish) outlined by research (Gerofsky, 2004; Hiebert et al., 1997; Land, 2017). Yet having all these formal components and rendering the problems in a specific language that students nominally understand might be insufficient to grant access to the problem for students new to a particular learning context. In overcoming this lack of access, a factor that appeared relevant—to the students and to myself rendering the problems—was the enactment of the WP, the elaboration of their contexts and details, through an unencumbered fluidity of linguistic expression. Moving, enacting, situating ourselves within the contexts, and sharing our own experiences filled in the missing pieces to grant students access and allow them to imagine a solution.

WPs as commonly defined (Gerofsky, 2004; Hiebert et al., 1997; Land, 2017) appear as static tools of instruction. This facilitates listing components that define the tool. But in multilingual classrooms—at least with this group of students new to the US educational system—effective use of this tool requires it to be dynamic (Li, 2018; Tai & Li, 2021a, 2021b). This process necessitates creativity and fluency in and between languages, modes of expression, and meaning-making. The meaning of such a tool becoming dynamic was unveiled as I worked with students. Juno showed discomfort with her understanding of problems presented in English, and only when presented with problems in Spanish did she gain the confidence to attempt a solution. However, she showed little inclination simply to solve a WP once presented in Spanish. Rather the WP served as a springboard for broader discussion, and through this she encountered the ideas to work her way to a solution.

In the discussion of WP 2, about Elsa's boxes, the solution shared by Saafi allowed me to identify his understanding of the context, as he solved the problem correctly by modeling the situation presented. And as he continued to solve the problem, I could identify how he started to link the situation to a more concrete mathematical expression. Unlike the situation with Juno, who first resorted to an algorithm, for Saafi the situation allowed him to explore a process that started with a simple solution and moved to more general mathematical relations. In this interaction I did not merely read the problem to Saafi and the others; I enacted the problem and shared the story occurring in the problem moving flexibly between English and Spanish. This supports findings described by Barwell (2009) in work conducted in multilingual classrooms: the interpretation of WPs in multilingual contexts might impose linguistic challenges comparable to the challenges posed by the mathematical content. In this example we can see that, once the interpretative demand has been surmounted, students can not only solve the problem, but also describe and make mathematical connections. In the examples shared here, this interpretative demand was facilitated by translanguaging.

When discussing WP 3, about Speedy's cucumbers, Juno seems to begin solving the problem by latching on to the numbers, rather than to the story in the problem, in contrast to Saafi. The numbers themselves seem to play a key role in developing WPs as described by Land (2017) and others. In the example shared here the numbers themselves might play the initial role of providing access, rather than the context. The numbers seemed to open the door to making sense of the story.

The situations described here highlight the profound complexity of WPs and language in a specific setting involving newcomers. Elements of my interaction with students hint at how opening a space for creatively inhabiting co-constructed universes, within interactions mediated by practices of translanguaging, can break down barriers to sense-making as they transcend boundaries between languages and shift fluidly among narrative genres. At the same time, the very boundaries of the classroom can interact in unanticipated ways with these dynamics and facilitate the cross-pollinating of ideas. In this unique context, the researcher and students alike discovered ways of understanding that they might not have guessed before entering.

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