

Non-Reciprocal Supercurrents in a Field-Free Graphene Josephson Triode

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Abstract. Superconducting diodes are proposed non-reciprocal circuit elements that should exhibit non-dissipative transport in one direction while being resistive in the opposite direction. Multiple examples of such devices have emerged in the past couple of years, however their efficiency is typically limited, and most of them require magnetic field to function. Here we present a device which achieves efficiencies approaching 100% while operating at zero field. Our samples consist of a network of three graphene Josephson junctions linked by a common superconducting island, to which we refer as a Josephson triode. The three terminal nature of the device inherently breaks the inversion symmetry, and the control current applied to one of the contacts breaks the time-reversal symmetry. The triode's utility is demonstrated by rectifying a small (nA scale amplitude) applied square wave. We speculate that devices of this type could be realistically employed in the modern quantum circuits.

Keywords: Superconductivity, Josephson Junctions, Superconducting Diode Effect, Quantum Devices

Diodes form one of the most important building blocks in electronic circuits, since they can be used in AC-DC conversion, signal rectification, and photodetection. The utility of diodes resides in their ability to offer non-reciprocity – a low resistance to current flowing in one direction and a high resistance to current flowing in the opposite direction. While traditional diodes exploit P-N interfaces in semiconducting materials, a flurry of theoretical interest has focused on developing their superconducting analogues [1–9].

These studies have followed a decade-long explorations of non-reciprocal supercurrents [10–23] which in turn stem from pioneering work in the late 1960s to 1980s [24–28]. Superconducting devices that use such non-reciprocity have shown promise in applications ranging from three-wave mixing [29] to single flux quantum circuits [30]. However, the recent interest is driven by the search of novel materials which break both the inversion and time-reversal symmetry, thereby intrinsically enabling the superconducting diode effect (SDE). Such materials have indeed been experimentally identified and investigated, ranging from metallic films and proximitized semiconductors to van der Waals heterostructures [31–42]. While this direction offers a probe into the symmetry properties of novel materials, the resulting devices typically have limited diode efficiency, which is defined as a ratio of supercurrent in the forward and backward directions.

In the meanwhile, it has been realized that higher superconducting diode efficiency can be achieved in properly designed nanostructures in which the time-reversal symmetry is broken via trapped fluxes or an external magnetic field [43–47]. However, magnetic field is often

undesired for integrating the devices in superconducting circuits. In this work, we rectify this problem by creating superconducting diodes which can operate at zero external magnetic field and achieve efficiency approaching 100%. Our devices are based on multiterminal Josephson junctions made in graphene. In the past few years, the multiterminal junctions have been realized in a variety of materials [48–54] and have even found a foothold as a solution to technological problems [55, 56].

We utilize the developments of multiterminal Josephson junction design to eliminate the necessity of an applied magnetic field to achieve the SDE. The structure is based on three graphene Josephson junctions tied at the central superconducting island (Fig. 1a). Without applying a DC offset bias, all junctions are superconducting and the $I - V$ curves are symmetric. By applying a dissipationless control current in one of the junctions, we break the time-reversal symmetry and tune the $I - V$ curves of the other two junctions, achieving the SDE efficiencies approaching 100%. Our devices are further tunable by electrostatic gating, which allows us to adjust the scale of the supercurrent that can be rectified.

Our devices feature a trapezoidal superconducting island and three superconducting contacts labeled left (L), right (R) and bottom (B), Fig. 1a. (An additional side-gate approaching the island from the top does not contact the device and is not used in these measurements.) The contacts are connected to the island through 500 nm graphene channels which are etched such that none of the contacts are connected through graphene alone – all transport must occur through the central island. We have measured two devices which yielded very similar results. For consistency, we present the results for one of

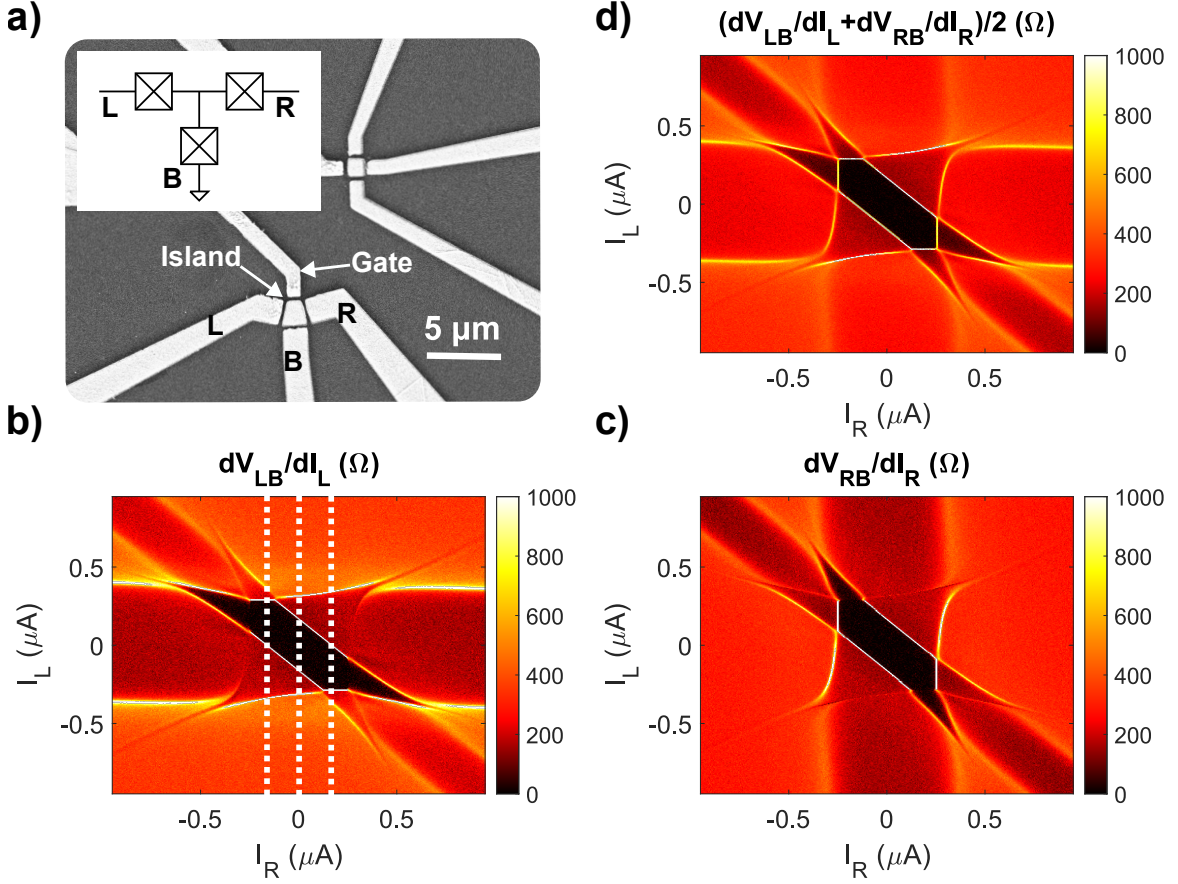


FIG. 1. a) An SEM image of the device and a schematic of the relevant Josephson network (inset). We measure the bottom one of the two similar devices, which is comprised of a superconducting island connected to three superconducting leads (L, R, B) via graphene junctions (not visible). The gate electrode approaching the island from the top is not used here. b) Map of $\frac{dV_{LB}}{dI_L}$ with a central diamond corresponding to junctions L and B both superconducting. The dashed lines mark the locations of the $I - V$ cuts further analyzed in Fig. 2. c) A similar map of $\frac{dV_{RB}}{dI_R}$ with a central diamond corresponding to junctions R and B both superconducting. d) An average of the maps in (b) and (c), which emphasizes all the boundaries. The black central region demonstrates the overlap of the central diamonds in (b) and (c) – an elongated hexagon where transport through the device is dissipationless.

them, shown at the bottom of Fig. 1a.

The contacts and the island are made of sputtered molybdenum-rhenium (MoRe), which offers high-transparency Ohmic contact to graphene [57, 58]. The devices are cooled in a Leiden Cryogenics dilution refrigerator. A significant hysteresis is observed between the switching and retrapping currents at the base temperature, indicating either underdamping, or more likely electron overheating [59]. To avoid this hysteresis, the measurements are conducted in the $T = 1.5 - 2$ K range, as indicated in the figure captions. We have verified that all the features measured at this temperature range exhibit negligible hysteresis and can be measured by sweeping the current in any direction. DC currents I_L and I_R are applied to the left and right contacts with respect to the grounded bottom contact, and a small AC excitation is used for extracting differential resistances. To maximize the critical current, a back gate voltage of $V_G = 20$ V is

applied in Figs. 1 and 2, tuning the carrier density to $\sim 1.1 - 1.3 \times 10^{12} \text{ cm}^{-2}$. (The Dirac point in this sample varied in the 2 – 5 V range between the cooldowns.) Importantly, all measurements take place at zero magnetic field.

Fig. 1b and Fig. 1c display differential resistant maps measured between the left and bottom (1b) and right and bottom (1c) contacts. As such, the prominent horizontal (1b) and vertical (1c) features correspond to the regimes where respectively the left and right junctions are superconducting. The diagonal feature common to both maps corresponds to the bottom junction being superconducting. The device's collective behavior can be gleaned from Fig. 1d where dV_{LB}/dI_L and dV_{RB}/dI_R are averaged. The distinct black region in the center of this map occurs where all three junctions are superconducting and the transport across the entire device is dissipationless.

The boundaries of the dissipationless region (an un-

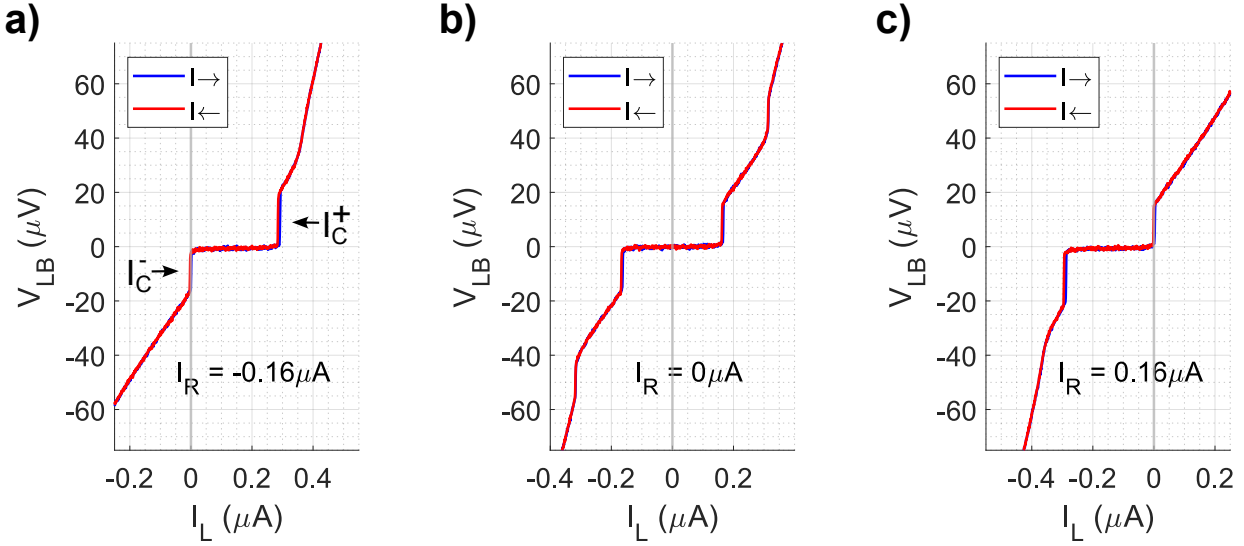


FIG. 2. $I - V$ curves of the left junction measured at 3 values of I_R : a) $-0.16 \mu\text{A}$, b) $0 \mu\text{A}$, and c) $0.16 \mu\text{A}$. In all cases, the control current I_R is well within the dissipationless range of the right junction. In all panels, I_L is swept back and forth, showing negligible hysteresis. In panel (b), the $I - V$ curves are reciprocal, as expected. In (a) and (c), either I_c^- or I_c^+ are approaching zero, resulting in a nearly ideal superconducting diode effect. Note that external magnetic field is equal to zero for all three panels.

even hexagon) correspond to points where at least one junction on the device switches to normal. The non-reciprocity we exploit here appears in the regions of the hexagon defined by the switching currents of two junctions. An example is the left dashed line in Fig. 1b, which cuts through the hexagonal region away from its center. As a result, the system stays dissipationless only for positive I_L and becomes dissipative at negative I_L (Fig. 2a). Note that all three junctions play a role in this process: the bottom junction switches at $I_L = I_c^- \approx 0$ while the left junction is responsible for the upper limit of the dissipationless range, I_c^+ ; finally, it is the biasing of the right junction which establishes the required asymmetry. For zero I_R , one recovers a symmetric cross-section along the I_L direction (central white line in Fig. 1b and Fig. 2b). Finally, at the opposite value of the control current I_R (right dash line in Fig. 1b and Fig. 2c) the system stays dissipationless only at negative $I_L \leq 0$ and the curves are reversed compared to Fig. 2a. Note that the right junction is biased below its critical current in all three cases.

Collectively, the boundaries of the hexagon enable tuning of the transport non-reciprocity between any pair of contacts by adjusting the current applied to the third contact. This leads to a diode efficiency,

$$\eta = \frac{I_c^+ + I_c^-}{I_c^+ - I_c^-}, \quad (1)$$

that in practice can be tuned to approach 100%, as will be seen in Fig. 2, where we plot the $I - V$ curves corresponding to the three cross-sections in Fig. 1b. Each set of curves is measured in both directions, showing negligible hysteresis. When $I_R = 0$ (Fig. 2b) the curves are expect-

edly symmetric, so $I_c^+ = -I_c^-$ and $\eta = 0$. However, as I_R is tuned away from zero, the non-reciprocity grows and $|\eta|$ increases until it reaches $\approx 100\%$ at $I_R = \pm 0.16 \mu\text{A}$.

Formally η can exceed unity when I_c^- becomes positive (same sign as I_c^+). However, this regime should be avoided if we are interested in rectifying small currents. Hence we stop increasing $|I_R|$ at the point where the high slope “knee” of the $I - V$ curve approaches the point $I_L = 0$. We then define I_c^- conservatively as a point at half the slope of the knee (see arrow in Fig. 2a), resulting in the $\eta \approx 100\%$. Finally, either positive (Fig. 2a) or negative (Fig. 2c) currents can be rectified depending on the desired operation.

To demonstrate the potential utility of the device, we apply a square wave of amplitude $\pm 25 \text{ nA}$ to the left contact (Fig. 3a). I_R can then be set to produce desirable device responses: namely, at $I_R = 0.16$ and $-0.16 \mu\text{A}$, the negative and positive portions of the square wave are respectively rectified. Further, when $I_R = 0$, the device is fully superconducting for the entire square wave, as its amplitude is smaller than the critical current of the device. As a result, the entire square wave passes through the device without dissipation. Interestingly, we can change the biasing scheme to further utilize the triode’s three-terminal nature. In Fig. 3b, we continue measuring V_{LB} but now use I_L as a control parameter, while applying a square wave to I_R . As a result, the voltage at the left contact, V_{LB} , switches depending on the sign of the square wave applied to the right contact.

Furthermore, the rectification demonstrated in Fig. 3(a,b) can be applied to square waves of varying amplitudes with a proper choice of control current. In Fig. 3c, I_R is kept at $0.16 \mu\text{A}$ while square waves of varying am-

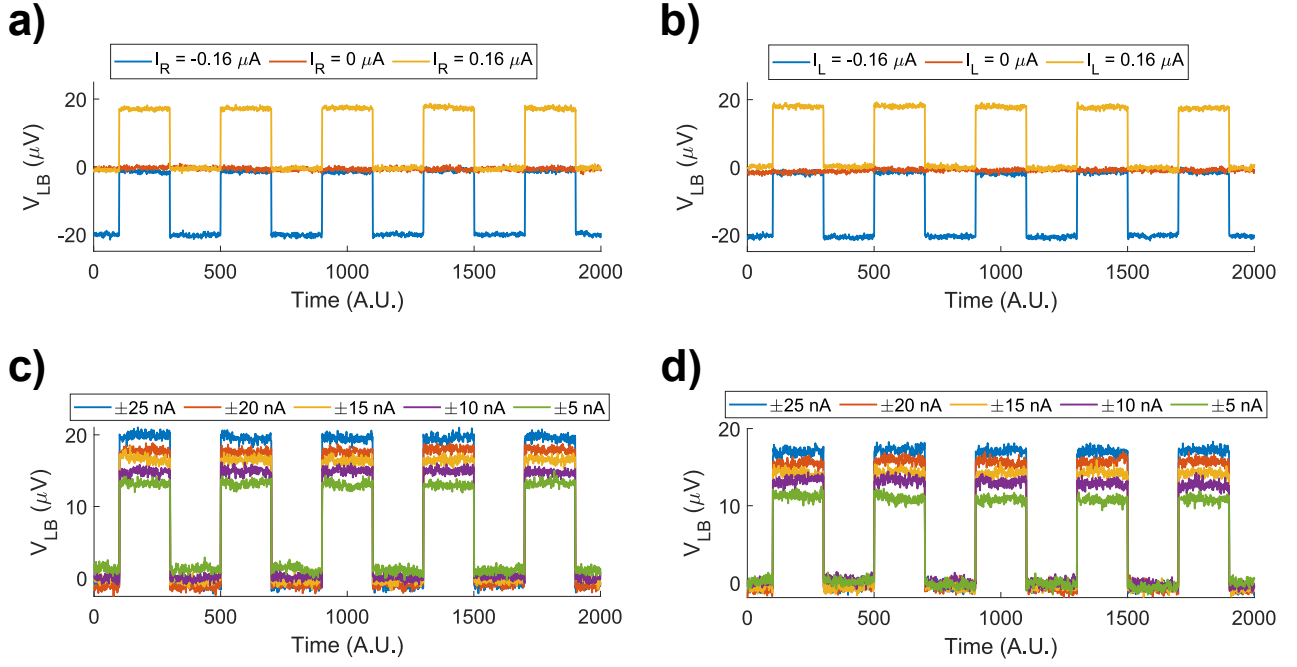


FIG. 3. a) V_{LB} measured as a square wave of a 25 nA amplitude is applied to I_L . Depending on the control current I_R , the negative and/or positive components of the square wave can be rectified (yellow and blue curve). For $I_R = 0$, all of the square wave lies within the dissipationless region. b) Now the square wave of amplitude 25 nA is applied to I_R instead of I_L , while still monitoring V_{LB} . Depending on the value of I_L , the negative and/or positive components of the square wave can be rectified. c) Rectification of square waves with varying amplitudes applied to I_L with I_R kept at $0.16 \mu\text{A}$. d) Rectification of square waves with varying amplitudes applied to I_R and centered around $I_R = 0.16 \mu\text{A}$ while $I_L = 0 \mu\text{A}$. Note that in panels (a,c) the square wave is applied to L, while in (b,d) it is applied to R.

plitudes are applied to I_L . In the measured V_{LB} , the negative portion of the square wave is rectified for waves with amplitudes as low as ± 5 nA. This is repeated in Fig. 3d, but here I_L is fixed at zero while square waves of varying amplitudes are applied to I_R centered at $0.16 \mu\text{A}$. Again measurements of V_{LB} demonstrate rectification of the negative half of the wave for amplitudes as low as ± 5 nA.

Finally, Fig. 4 explores the behavior of the triode as a function of the gate voltage, which changes the critical currents by a factor of ~ 4 . Pairs of maps and $I - V$ curves are measured at the same gate and temperature: (a,d) $V_G = 4$ V and $T = 1.6$ K, (b,e) $V_G = 10$ V and $T = 1.7$ K, and (c,f) $V_G = 20$ V and $T = 2.0$ K. For each value of the gate voltage, the measurement temperature is slightly adjusted to sharpen the transitions while avoiding hysteresis. Qualitatively similar behavior is observed in each column, including efficiencies approaching 100%. This feature enables a large range of working parameters in which the gate voltage and bias current could be chosen based on the experimental requirements, e.g. impedance matching and the magnitude of the rectified current.

Outside of the large non-reciprocity, the largest advantage of our design is that the time reversal symmetry is lifted without applying any sizable magnetic field. In-

deed, even small magnetic fields have been shown to induce vortices in superconducting electrodes, greatly reducing their quality factors. This would pose a problem for using magnetically controlled superconducting diodes in quantum electronic circuits. Namely, a global magnetic field on the required $> 1\text{mT}$ scale would harm all the superconducting circuitry on the chip; applying comparable local magnetic fields would require extra wires capable of carrying the appropriately large currents. Instead, by breaking the time reversal symmetry with a current bias, the required currents are greatly reduced, producing negligible magnetic fields $< 1 \mu\text{T}$ which are too small to produce vortices in the diode and surrounding circuitry.

While the near perfect diode efficiency of our device is compelling, integration into quantum circuits may be fundamentally limited by the frequency cutoff and the fluctuations in the diode efficiency. The frequency cutoff is limited by the RC time constant of the device. In the present case, this can be estimated by taking the normal resistance of the graphene junction ($\sim 500 \Omega$) and the capacitance of the bonding pads and leads to the silicon backgate (~ 2 pF). Therefore, our present cutoff frequency could be as high as ~ 1 GHz, indicating the promise for integrating our device into superconducting quantum circuits. Further improvements can be straight-

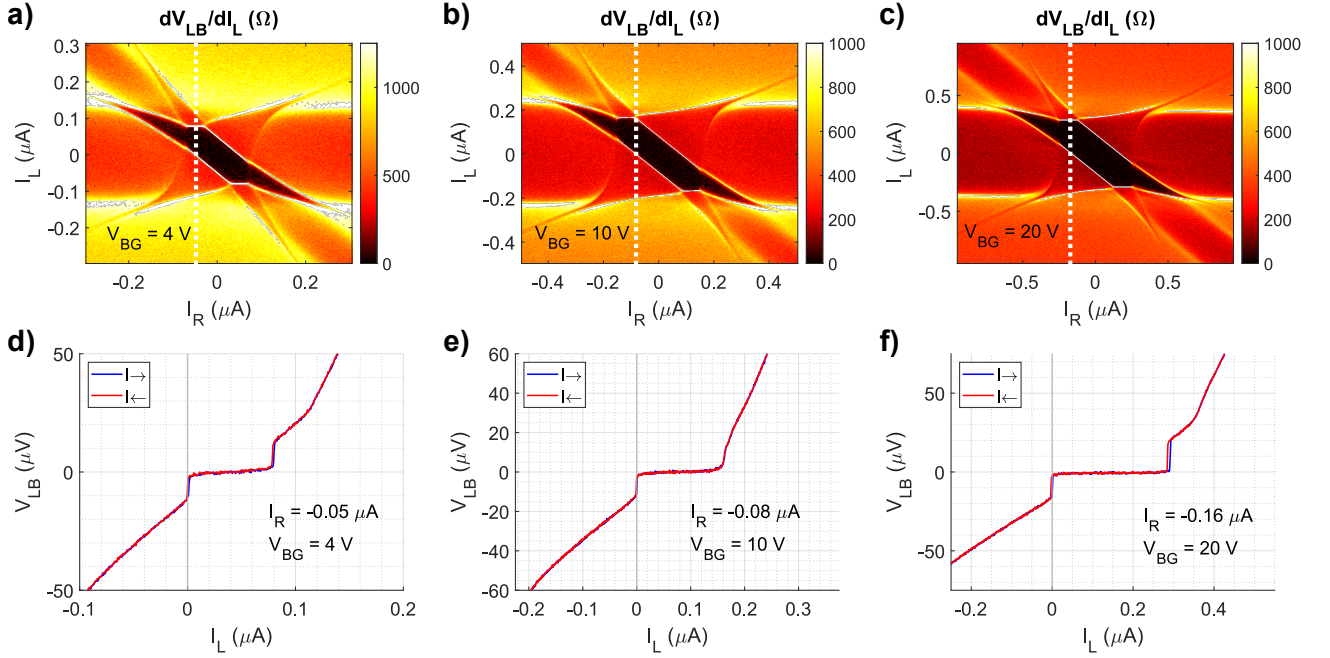


FIG. 4. (a-c) Maps of $R_{LB} = \frac{dV_{LB}}{dI_L}$ taken at (a) $V_G = 4$ V and $T = 1.6$ K, (b) $V_G = 10$ V and $T = 1.7$ K, and (c) $V_G = 20$ V and $T = 2.0$ K. Each map yields qualitatively similar results with expected changes of the normal-state resistance and critical currents. (d-f) $I - V$ curves measured under the same conditions as the maps in (a-c). In each case, tuning the control current I_R enables achieving an efficiency approaching 100%. Panels (c) and (f) correspond to the data in Figures 1, 2.

forwardly made by replacing the doped Si/SiO₂ substrate with undoped Si and moving to local gating schemes. This would greatly decrease the parasitic capacitance, especially when using the triode on-chip with the rest of the circuit.

Turning to the fluctuations of the diode efficiency, the noise of the DC control bias could be an important source. As this bias fluctuates, the diode will pick up slight changes of I_c^- , causing fluctuations of η . While in this measurement we have not optimized the current noise beyond reasonably acceptable levels, we have been able to limit it below ~ 1 nA in the past [60]. In principle the noise in the diode could be pushed down to the quantum limit, where it is dominated by critical current fluctuations [61]. The efficiency of the triode is additionally limited by rounding of the transitions due to the operation at elevated temperature. We chose to work at higher temperature to both demonstrate the robustness of the SDE and to suppress the hysteresis of the $I-V$ curves (see Supplementary Information). The hysteresis could be suppressed by designing the junction environment, which would in turn allow the device to operate at lower temperatures and suppress the knee.

Future implementations of our circuit geometry may also benefit by changing the junction material from graphene to tunneling junctions. Indeed, graphene allows one to tune the scale of rectified current via gate voltage (see Fig. 4), which in turn should enable better control of impedance matching and dynamic range. However, generating devices with Q-factors greater than

1000 in graphene Josephson junctions has been a challenge [62]. Meanwhile, a circuit of this device geometry implemented in tunneling junctions can maintain the high Q-factor and is more easily scalable, which can allow for rapid integration into existing quantum circuits.

Overall, our three-terminal device geometry provides field-free, highly tunable, near perfect non-reciprocities exceeding those reported in the current literature. Unlike the two-terminal devices that break time reversal symmetry (intrinsically, or via applied magnetic field), this circuit geometry should be easily scalable for rapid deployment into present-day microwave quantum systems. These benefits come at the cost of requiring the third terminal and an active biasing scheme, which may be susceptible to additional noise. However, even without optimization this platform has demonstrated rectification of very small currents.

In summary, we utilize a network of three Josephson junctions at zero magnetic field to produce a superconducting diode with efficiency approaching 100%. The device geometry – three graphene junctions connected by a common superconducting island – provides high degree of control through applied currents and gate voltage. While this device is made fairly simply in graphene, one can imagine copying the same Josephson circuit in other materials, including Al/AlO_x tunnel junctions. Although in that case the gate tunability would be lost, the advantage is that the fabrication techniques are well developed and easily scalable. Further effort in this direction could quickly present opportunities for including high-efficiency

superconducting diodes in quantum circuits.

METHODS

The sample is fabricated using standard techniques in exfoliation, stacking, and electron-beam lithography. The device is etched with CHF_3O_2 and SF_6 plasma and contacts are deposited from sputtered molybdenum-rhenium (MoRe).

Measurements were conducted in a Leiden Cryogenics dilution refrigerator at a base sample temperature of 60 mK. Two stage $1.5\text{ k}\Omega/1\text{ nF}$ RC filters are mounted at the mixing chamber. Current bias passed through $1\text{ M}\Omega$ room temperature resistors, and voltage is measured with $1\text{ nV}/\sqrt{\text{Hz}} \times 100$ homemade amplifiers. Interfacing with computer is performed with an NI USB-6363 DAQ board for both current biasing and voltage readouts.

To produce differential resistance maps as in Figure 1, the current bias on one contact is stepped in individual steps in an outer loop, while the bias on the other contact is swept quickly. From here, numerical derivatives can easily be computed to give resistance values (dV/dI). Currents are ramped in both directions (e.g. $-1.5\mu\text{A}$ to $1.5\mu\text{A}$ and back to $-1.5\mu\text{A}$) and all derivatives are computed (e.g. dV_{RB}/dI_R as well as dV_{LB}/dI_R) so several maps can be produced for each measurement.

DATA AVAILABILITY

Source data for figures (including the supplementary figures) will be made available in the public repository Zenodo prior to publication. All other data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

CODE AVAILABILITY

The codes used for the analysis will be made available in the public repository Zenodo prior to publication.

SUPPORTING INFORMATION

This material is available free of charge via the internet at <https://pubs.acs.org>.

- Additional data with varying temperature and magnetic field as well as mathematical modeling based on the RCSJ model

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