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Statistical Analysis of Complex Shape Graphs

Aditi Basu Bal, Xiaoyang Guo, Tom Needham, and Anuj Srivastava, *Fellow, IEEE*

Abstract—This paper provides developments in statistical shape analysis of *shape graphs*, and demonstrates them using such complex objects as Retinal Blood Vessel (RBV) networks and neurons. The shape graphs are represented by sets of nodes and edges (articulated curves) connecting some nodes. The goals are to utilize nodes (locations, connectivity) and edges (edge weights and shapes) to: (1) characterize shapes, (2) quantify shape differences, and (3) model statistical variability. We develop a mathematical representation, elastic Riemannian metrics, and associated tools for shape graphs. Specifically, we derive tools for shape graph registration, geodesics, statistical summaries, shape modeling, and shape synthesis. Geodesics are convenient for visualizing optimal deformations, and PCA helps in dimension reduction and statistical modeling. One key challenge in comparing shape graphs with vastly different complexities (in number of nodes and edges). This paper introduces a novel multi-scale representation to handle this challenge. Using the notions of (1) "effective resistance" to cluster nodes and (2) elastic shape averaging of edge curves, it reduces graph complexity while retaining overall structures. This allows shape comparisons by bringing graphs to similar complexities. We demonstrate these ideas on 2D RBV networks from the STARE and DRIVE databases and 3D neurons from the NeuroMorpho database.

Index Terms—statistical shape analysis, shape graphs, blood networks, neuron morphology, geodesic deformations, shape registration, shape models.

1 Introduction

IMAGING provides a rich data source for studying *structures* found in diverse biological, ecological, bioinformatical, and anatomical systems. Due to rapid advances in imaging resolutions and processing techniques, this data increasingly involves objects with complex morphologies, resulting in growing challenges for data analysts. While