A practical introduction to radio frequency electronics for NMR probe builders

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Abstract

In this tutorial paper, we describe some basic principles and practical considerations for designing probe circuits for NMR or MRI. The goal is building a bridge from material that is familiar from undergraduate physics courses to more specialized information needed to put together and tune a resonant circuit for magnetic resonance. After a brief overview of DC and AC circuits, we discuss the properties of circuit elements used in an NMR probe and how they can be assembled into building blocks for multi-channel circuits. We also discuss the use of transmission lines as circuit elements as well as practical considerations for improving circuit stability and power handling.

Keywords: NMR instrumentation, NMR probe, radiofrequency circuit, probe tutorial, transmission line

1. Introduction

The design of NMR probes is a specialized application of radiofrequency (rf) electronics. Many of the classic references on this topic are rich in information, but were intended for different audiences, e.g., post WWII radio engineers [1]. The purpose of this tutorial paper is to present an introduction to some general principles that are pertinent to probe design. The intended audience is students and others who are interested in building NMR probes and have some background in electricity and magnetism [2], but are not yet experts. Throughout, we note some of the classic references on radiofrequency circuit and transmission line design, and where available, suggest updated sources that may be more accessible to modern readers. A general discussion of direct current (DC) and alternating current (AC) circuits is included, followed by a description of the properties of common components used in probe circuits. There are many good references on this topic, including [3, 4], and works by Forrest Mims III, e.g. [5] and many others on specific practical topics. The use of transmission lines is also covered [6, 7]. The objective is to develop an understanding of the circuit components needed to assemble and tune a simple probe circuit as an entry point to NMR instrumentation development. Other useful pedagogical resources include a textbook on NMR probes [8] set of laboratory exercises for beginning probe builders [9], a more advanced tutorial on how to simulate complicated probe circuits for particular applications [10], a guide to calculating the circuit filling factor [11], which is a measure of performance for multiply-tuned probes [12]. Finally, a general overview of setting up and testing NMR instrumentation [13] provides a welcome update to some of the practical information provided in the classic textbook of Fukushima [14].

In its simplest form, an NMR probe is a tunable LC circuit that is impedance matched to the source it is used with, generally a radio-frequency (rf) amplifier operating at a resistance of 50 Ω. An inductor is used for excitation of nuclear spins in the sample and for detection of the signal. Capacitors are often used as the adjustable elements for tuning and matching the circuit. Transmission line segments are often used to deliver power to the probe circuit, and can also serve as circuit elements themselves. An overview of their behavior in both capacities is given. The general design and characterization of series and parallel resonant circuits is discussed, as well as efficiency and power handling capabilities.

2. DC circuits and Ohm's law

Although the treatment of DC circuits is not directly applicable to NMR probe design, some discussion of Ohm's law and the definition of resistance is useful as a starting point A DC circuit is an electric circuit in which electrons flow from a source to a sink in a single direction, and the applied potential and the current do not vary with respect to time. Here, current flow is impeded only by the resistance of the circuit elements. In practice, this is usually achieved using a constant voltage source such as a battery. The behavior of DC circuits can be understood by applying Kirchoff's rules. Briefly, these state that the sum of all currents entering a junction must equal the sum of currents leaving the junction, and the sum of all potential differences around a closed loop must equal zero [3]. These rules are based on Ohm's law,

$$V = IR \tag{1}$$

Resistance (R) is defined in terms of the current density passing through a conductor; the resistance of a material is its characteristic opposition to flow of electric current. This is analogous to friction in a mechanical system, and it is an extensive property: it depends on the size and shape of an object as well as the material. Resistivity (ρ [Ω-m]) is the resistance (R [Ω]) in a material having a cross section A [m^2] per length ℓ [m].

$$\rho = \frac{RA}{\ell} \tag{2}$$

This quantity is related to the conductivity (σ), which measures the ease of charge flow, with the expression $\sigma = 1/\rho$. The current density (J) is the rate of charge flow, or current (I), per a cross sectional area (A) of a conductor.

$$J = I/A \tag{3}$$

When a constant potential difference is applied across the conductor, proportionally generating a constant electric field (E) along the length, the current density becomes dependent on the conductor's *conductivity*,

$$J = \sigma E \tag{4}$$

The applied potential (V) at the ends of the conductor is

$$V = E\ell \tag{5}$$

where ℓ is the length. From current density, current (I) is given as

$$I = JA = \sigma EA = \frac{\sigma VA}{\ell} \tag{6}$$

Rewriting this expression in terms of resistivity (ρ) from conductivity (σ) and recalling resistance (R) from *Eq. 2* results in the derivation of Ohm's Law

$$I = \frac{VA}{\rho\ell} = \frac{V}{R} \tag{7}$$

DC circuits are often used in consumer electronics and small electronic devices that are used in the lab because of their relative simplicity and their efficiency for transmitting power over short distances.

3. AC circuits and the complex form of Ohm's law

In AC circuits, the applied current and voltage are not constant, but oscillate periodically in time, changing direction and magnitude with a characteristic frequency ω. Here there is a phase difference between I and V, described by the phase factor ϕ. Phase can be expressed either in terms of an angle or a displacement in time. A change in time by one period T corresponds to a change in phase by 360°. This change in phase in AC circuits is accounted for by the complex analogue of resistance, impedance (Z).

$$V(t) = V_0 \sin(\omega t + \phi) \tag{8}$$

$$I(t) = I_0 \sin(\omega t + \phi') \tag{9}$$

where ω is the frequency in radians and ϕ and ϕ' are the phases of V and I. The amplitude of the voltage can be measured as voltage peak to peak ($V_{PP} = 2V_0$) or root mean square voltage (V_{RMS}) [3]. V_{RMS} is defined as

$$V_{RMS} \equiv \left[\frac{1}{T}\int_0^T V^2(t)dt\right]^{\frac{1}{2}} = \left[\frac{1}{T}\int_0^T V_0^2 \sin^2(\omega t)dt\right]^{\frac{1}{2}} = \frac{V_0}{\sqrt{2}} = 0.707V_0 \tag{10}$$

In this way the value of V_{RMS} can be obtained from the value of V_{PP}, which can be measured easily with an oscilloscope.

Ohm's law is still valid for AC circuits as long as voltage and current are expressed as complex quantities with a phase term added to their real magnitudes. For simplicity, the following discussion assumes that the circuit is excited with only one frequency at a time. In the NMR context, this corresponds to a single-channel probe. The more complicated case with multiple frequencies is addressed in **Section 9**.

The complex form of Ohm's law relates the impedance Z to the current and voltage. In this expression, V and I each have a phase term and Z is a complex number.

$$V = IZ \tag{11}$$

The complex impedance (Z) contains a term for the resistance as well as a term for the reactance.

$$Z = R + jX \tag{12}$$

In Equation 9, R is the resistance, j is the square root of -1, following the convention used in electronics, and X is the reactance. Reactance accounts for energy that is stored in the electric field of a capacitor or the magnetic field of an inductor. At zero frequency (DC), an ideal capacitor has infinite reactance, and therefore infinite impedance (see **Section 4. Capacitors**). As a result, a capacitor behaves as an open circuit or an insulator with respect to DC. As the frequency increases, the capacitor has less time to build up charge in opposition to the current, and the reactance shrinks. On the other hand, an ideal inductor will have an impedance of 0, behaving as an ideal wire in a DC circuit (see **Section 5. Inductors**). As frequency increases, the impedance also increases, resulting in higher opposition to current flow. When a capacitor and inductor of equal and opposite reactance are used together, the impedance becomes purely resistive (R) and maximal current can flow through the circuit network (see **Section 6. Resonant circuits**). In the ideal case, inductors and capacitors are non-dissipative, but in practice, they are lossy. Loss can be modeled as a resistor, often with a value chosen to match empirical observations.

In series circuits, impedance adds in series, similar to resistance in DC circuits. For parallel AC circuits, the inverse of impedance, which is called admittance, is additive. The conductance, $G = \frac{1}{R}$, is sometimes used in referring to losses in dielectric materials. The conductance of a capacitor depends on the type and amount of dielectric used. Another useful quantity is susceptance (B), which is the imaginary part of admittance as reactance is the imaginary part of impedance;

$$B_L = \frac{-1}{2\pi f L} \tag{13}$$

$$B_C = 2\pi f C \tag{14}$$

In the ideal case where $R = 0$, B and X are inversely related. The susceptance of a capacitor is positive, while that of an inductor is negative. This is the opposite of the case for reactance. If impedance is known, it is possible to transform to admittance and vice versa. This is useful because it may be appropriate to use either impedance or admittance depending on whether the circuit to be analyzed is in series or parallel.

$$G_{L,C} = \frac{R_{L,C}}{R_{L,C}^2 + X_{L,C}^2} \tag{15}$$

$$B_{L,C} = \frac{-X_{L,C}}{R_{L,C}^2 + X_{L,C}^2} \tag{16}$$

$$R_{L,C} = \frac{G_{L,C}}{G_{L,C}^2 + B_{L,C}^2} \tag{17}$$

$$X_{L,C} = \frac{-B_{L,C}}{G_{L,C}^2 + B_{L,C}^2} \tag{18}$$

When adding capacitors or inductors in series, the complex impedance is written as in Eq. 6, where we add the resistive real component (R) to the complex component X. This transformation can also be accomplished using a Smith chart [15, 16, 17], which is a convenient graphical representation of the circuit's impedance behavior (**Figure 1A**). This graphical tool was developed before computers were widely available, and it was (and is) commonly used to plot how the addition of circuit elements changes the impedance. In the past, impedance values not directly labeled on the chart had to be calculated by interpolation, but now software is available (including free or inexpensive web-based options) that enables plotting of the exact values desired. Many network analyzers are also capable of displaying their output in this format. The Smith chart is particularly useful for impedance matching, because it is easy to keep track of the effect of multiple circuit elements on the chart. The most commonly used (impedance) version has circles representing constant resistance and circular arcs representing constant reactance. The horizontal line represents zero reactance, while values along this axis represent pure resistance. Infinite resistance and reactance are represented by the point intersecting this axis on the extreme right side of the chart. When components are placed in parallel, the impedance terms for those components are inversely added, also known as the admittance (Y). Recalling that admittance is the inverse of impedance,

$$Y = \frac{1}{Z} = \frac{1}{R + jX} \tag{19}$$

In this scenario, one might think it would be more convenient to use the admittance Smith chart, which contains the constant conductance circles and susceptance curves of a network. However, admittance values can be derived from the impedance values, and vice versa, on the Smith chart by inverting a data point 180° from the center of the chart, called *prime zero*. Therefore, many engineers simply use the impedance Smith chart, (usually just called a *Smith chart*), for most, if not all, of their circuit network's measurements. The impedance (Z) measured on the Smith chart is normalized, $Z = Z_{load}/Z_{source}$, where Z_{source} is the instrument's impedance, usually at 50 Ω, and Z_{load} is the circuit network's impedance. This means that when the circuit being measured is purely resistive, the impedance will be at the center of the graph at $Z = 1$. If the source impedance differs from $Z = 1$ (often, we want it to be 50 Ω), then the resulting impedance (Z) will be normalized to that value.

In the LC circuit networks used in NMR probes, components are placed strategically to achieve different resonant frequencies spanning a range of a few MHz to about 1 GHz, as well as isolation elements to minimize crosstalk between the channels in multiply-resonant circuits. This involves a combination of series and parallel components, introducing both complex impedance and admittance terms. However, in an optimal resonant circuit, the impedance should be purely resistive – ideally, the capacitive and inductive reactance terms are of equal magnitude and opposite sign so as to cancel each other out (See **Section 6** for more detail). In the Smith chart, this is represented by being on the resistive horizontal axis of the chart. If the impedance of a circuit is not purely resistive, then the impedance at the resonant frequency will have capacitive or inductive reactance, and the resulting complex impedance will either be below or above the horizontal line, respectively. An example is shown in (**Figure 1B**)

These aspects of the Smith chart show the residual complex impedance of a circuit, providing hints as to what changes could be made to obtain the desired resonant frequency (i.e., the Larmor frequency that will be excited in an NMR experiment) and purely resistive impedance (**Figure 1**). Impedance matching is one of the most important functions of the Smith chart, and a requirement in practical rf circuit design. Having an impedance-matched network means that any signal going from a source to your network will have zero wave reflectance. As the purely resistive impedance deviates from the ideal value, signal reflectance increases and results in poor signal propagation. In an NMR context, if the reflected voltage is very large, damage to

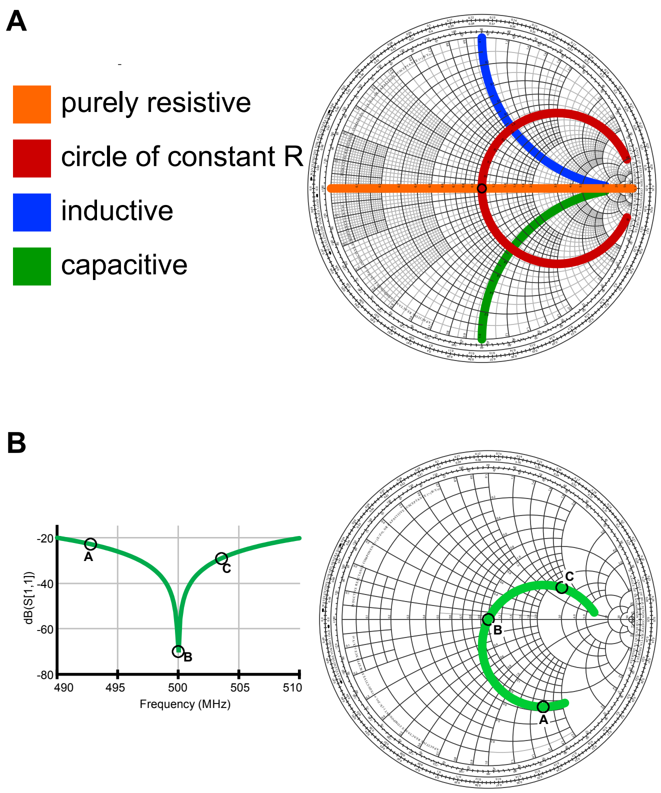

Figure 1: **The Smith chart A** A general Smith chart (template from https://commons.wikimedia.org/wiki/File:Smith_chart.svg). A data point lying on the horizontal line (*orange*) represents a purely resistive circuit ($Z = R$). As the point moves above or below the horizontal line, obtaining some inductive or capacitive reactance components, respectively, the resistance (R) can still be traced through the constant resistive circles (*red*). Similarly, the complex reactance component can be traced through the reactance curves, allowing users to find the inductive reactance (*blue*) or capacitive reactance (*green*). **B** An example is shown for a simple single-channel probe circuit. *Left*: Here, the reflection coefficient (S_{11}) is shown for a circuit tuned to resonance frequency of 500 MHz. Better transmission corresponds to a more negative S_{11} value, which occurs when the source and load impedances are matched. This is accomplished graphically using the Smith chart to match the impedance of the probe to the known impedance of the load (50 Ω). *Right* The impedance output of this simple resonating circuit is shown (*green*) as a curve plotted on the Smith chart. We can see that the circuit is well matched because the resonant frequency (point B) is purely resistive on the Smith chart. Points A and C on the Smith chart trace are useful when tuning the circuit to another frequency; they show how the reactance characteristics change as we tune.

equipment such as power amplifiers can result. By observing the Smith chart impedance, we can determine whether to add a series or shunt capacitor or inductor, or simply modify the magnitudes of existing components.

4. Capacitors

A capacitor consists of two conductive surfaces separated by a dielectric material. As discussed above, this arrangement behaves as a conductor for AC and an open circuit in the DC limit. The capacitive reactance X_C is given by the expression

$$X_C = \frac{1}{2\pi f C} \tag{20}$$

where f is the frequency in Hz and C is the capacitance. As a potential is applied, charge flows off one plate and onto the other at a rate determined by the applied voltage and the resistive components in the circuit. Initially, all the voltage is across the resistive elements and the current flow is maximized. As the charge flows onto the plates, the potential across the capacitor builds up until it reaches the applied potential and the flow stops. At this point, the potential is at a maximum and the charge flow is at a minimum.

If the voltage is alternated sinusoidally, the current lags 90° behind the voltage. DC cannot pass through a capacitor unless the dielectric breaks down. The voltage at which this occurs is referred to as the breakdown voltage. The capacitance of a circuit element is the property that makes it able to store a charge when there is a potential difference between the conductors. Capacitance is defined as the ratio of the charge on one of the conductive surfaces to the potential difference between them.

$$C = \frac{Q_S}{V} \tag{21}$$

C is the capacitance in farads (1 F is 1 amp second per volt), Q_S is the coulombic charge on one surface, and V is the potential difference. The simplest capacitor consists of two metal plates with a constant separation d filled with insulating material of uniform dielectric.

$$C = \frac{\epsilon_0 \epsilon_r A}{d} \tag{22}$$

where ϵ_0 is the permittivity of free space, $\frac{1}{36\pi \times 10^8}$ F/m, ϵ_r is the relative permittivity of the dielectric, and A is the area of the plate.

Transmission line segments can also be used as coaxial capacitors. The capacitance of a transmission line per unit length in picofarads per meter (pF/m) is given by

$$\frac{C}{\ell} = \frac{2\pi k \epsilon_r}{ln(b/a)} \tag{23}$$

where a is the diameter of the center conductor, and b is the inner diameter of the outer conductor and k is the dielectric constant of the insulating material.

Capacitors add directly in parallel and inversely in series.

$$C_p = C_1 + C_2 \tag{24}$$

$$C_s = \left[\frac{1}{C_1} + \frac{1}{C_2}\right]^{-1} \tag{25}$$

In addition to the capacitors that are intentionally part of the circuit, stray capacitance is also formed between circuit elements. Stray capacitances arise from two or more components forming an unwanted capacitor – two conductive elements separated by an insulating medium, including air. Thus, having components too close together, using high permittivity dielectrics, and poorly shielding conductors will result in stray capacitances. Even structural probe components can add stray reactances. This becomes increasingly important at high frequencies, because even small stray capacitances may be comparable to the values of the circuit elements needed. In transmission lines that carry signals, stray capacitances effectively introduce a new circuit element that affects the transmission line impedance, affecting wave propagation and signal reflection. In NMR probes, proper shielding and grounding will limit the effect of stray capacitances in the circuit, especially in preventing unwanted shorting, cross talk between different channels, and external electromagnetic interference (EMI). Having too many or too large stray capacitances can change the resonant frequency of the circuit, causing it to deviate from the Larmor frequency of the nucleus of interest. If the capacitance is altered outside of the tuning range of the circuit, then exciting a particular nucleus will be unachievable. In the more common case where the stray capacitance is small enough to permit retuning, it still creates problems because moving improperly grounded or secured circuit elements or even placing hands near them may cause unpredictable tuning changes.

The energy stored in the electric field of a capacitor is equivalent to the work required to charge the capacitor (W_C).

$$W_C = \int_0^V V dQ_S = \int_0^V V d(CV) = C \int_0^V V dV = \frac{1}{2} CV^2 \tag{26}$$

The voltage across a capacitor is $V = V_0 \cos(\omega t)$, so the current is

$$I \frac{dQ_S}{dt} = \frac{d}{dt} CV = \frac{d}{dt} CV_0 \cos(\omega t) = \omega CV_0 \sin(\omega t) \tag{27}$$

using the identity $-\sin(\omega t) = \cos(\omega t + \frac{\pi}{2})$,

$$I = \omega CV_0 \cos\left(\omega t + \frac{\pi}{2}\right) \tag{28}$$

The voltage and current can also be expressed as exponentials of the form

$$V(t) = Re\left[\omega(V_0 e^{j(\omega t)})\right] \tag{29}$$

$$I(t) = Re\left[\omega CV_0 e^{j\left(\omega t + \frac{\pi}{2}\right)}\right] \tag{30}$$

As we derived above, in ideal capacitors, V lags I by 90°. In real capacitors, this phase angle is smaller due to the added resistance produced by nonideality of the physical components, i.e., the small current leakage through the dielectric. For purposes of discussing dielectric effects in capacitors, it is necessary to define an additional phase angle, δ, which is the angle by which the current is displaced from exact quadrature with the applied voltage due to dielectric loss. In this case ϕ, the relative phase between V and I, is $90° - \delta$. The power factor is defined in terms of the phase angle ϕ; $PF = \cos\phi = \sin\delta$. This quantity depends on the dielectric, temperature, and frequency [18, 19].

The dissipation factor (DF $= \tan\delta$) is the ratio of the effective series resistance (ESR) to the reactance of the capacitor:

$$DF = \frac{ESR}{X_C} \tag{31}$$

For a given circuit or circuit element, the inverse of the dissipation factor is its quality factor, Q. The quality factor for capacitors is denoted here by Q_C to differentiate it from the overall Q of a circuit, which will be impacted by other circuit elements. This quantity is tabulated for commercial capacitors and ranges from approximately 50-50,000. In contrast to ideal capacitors, Q_C for practical chip capacitors degrades sharply with increasing frequency, even for small capacitance values. For example, a 1 pF chip capacitor has a Q_C of 3800 at 150 MHz, whereas at 800 MHz, the Q_C of the same capacitor is reduced to 260 [5]. Q is inevitably reduced at high frequencies because the effective resistance in the circuit increases. At low frequencies [20], dielectric loss in the capacitors dominates. At higher frequencies, AC resistance, which is proportional to the square root of the frequency, becomes more important.

In DC circuits, the current is distributed evenly over the cross-sectional area of the conductor. In AC circuits, the alternating magnetic fields induce eddy currents that are strongest at the center, forcing electrons towards the periphery of a cylindrical conductor, a phenomenon called the *skin effect* [21]. At very high frequency, the current effectively flows only on the surface of the conductor, meaning that a hollow tube is equivalent to a solid conductor of the same dimensions. Because the current distribution is exponential, the skin depth (δ_S) is defined as the depth where the current is $\frac{1}{e}$ of its surface magnitude [22]. The skin depth is a function of the material and the frequency.

$$\delta_S = \sqrt{\frac{2\rho}{\omega\mu}} \tag{32}$$

where ρ is the resistivity ($\rho = 6.787 \times 10^{-7}$ Ω/inch for copper) and μ_0 is the permeability of free space ($\mu_0 = 1.26 \times 10^{-6}$ Henry/m). Using this expression, the skin depth in copper is 0.023 mm at 800 MHz and 0.074 mm at 80 MHz. Since the skin depth is less than one mm even at moderate NMR frequencies, hollow tubes can be used instead of solid conductors without any difference in the behavior of the circuit. This strategy can be used to minimize mass of components and to save space inside the magnet bore by using concentric conductors where possible [23].

Assuming that the thickness and radius of curvature of the conductor are larger than the skin depth, the AC resistance (R_{AC}) in Ω per unit length of a conductor can be calculated as follows:

$$R_{AC} = \frac{\rho}{\delta_S}\left(\frac{1}{\pi d}\right) = \frac{\rho}{\sqrt{\frac{\rho}{\pi f \mu_0}}}\left(\frac{1}{\pi d}\right) = 2.61 \times 10^{-}7\sqrt{f}\frac{1}{\pi d} \tag{33}$$

where the values of ρ and μ_0 are the same as before and f is the frequency in Hz. Thus, AC resistance increases as the square root of frequency and decreases with increasing surface area of the conductor.

5. Inductors

An inductor, which is the magnetic equivalent of a capacitor, opposes a change in the current flowing through it. Any current-carrying wire has a self-inductance, which is normally low, but can be enhanced by winding it into a coil. For a given length and area, a flat strip has less inductance than a round wire. This is important in high-frequency applications where stray inductances in the leads connecting different parts of the circuit must be minimized. Even using flat ribbon leads, the distance between circuit elements that must be connected with a lead should be kept as small as possible. At high frequencies, stray reactances that could be tolerated in a lower frequency circuit could be on the order of the circuit elements required, causing interference with the performance of the circuit.

Inductance is given by

$$L = \frac{\phi_M}{I} \tag{34}$$

where ϕ_M is the magnetic flux, which is equal to the product of the magnetic field B and the area A. The time-dependent relationship of V to I is given by Faraday's law:

$$V = -\frac{d\phi_M}{dt} = \frac{d}{dt}(LI) = -L\frac{dI}{dt} \tag{35}$$

The expression for energy stored in an inductor is analogous to that for a capacitor.

$$W_L = \frac{1}{2}LI^2 \tag{36}$$

$$W_L = \int V dQ = \int \left(L\frac{dI}{dt}\right)(Idt) = L\int_0^I IdI = \frac{1}{2}LI^2 \tag{37}$$

The inductive reactance X_L is

$$X_L = 2\pi f L \tag{38}$$

The voltage across an inductor is $V = V_0 \cos(\omega t)$ and the current can be found by integrating the Faraday's law expression for voltage:

$$I(t) - I(0) = \int_0^t V dt = \frac{1}{L}\int_0^t V_0 \cos(\omega t) dt = \frac{V_0}{\omega L}\sin(\omega t) \tag{39}$$

If $I_0 = 0$, then

$$I(t) = \frac{V_0}{\omega L}\sin(\omega t) = \frac{V_0}{\omega_L}\cos\left(\omega t - \frac{\pi}{2}\right) \tag{40}$$

Thus, the current lags the voltage by 90°. This expression also shows that the higher the frequency, the smaller the current. End effects and loose winding have an effect on the coil's inductance. Most NMR sample coils are loosely wound and have a small number of turns. An approximate formula for inductance in nH of coils of length ℓ (cm) and n turns is:

$$L = \frac{10n^2D^2}{\ell + 0.45D} \tag{41}$$

where D is the coil diameter in cm.

Inductors also have losses represented by a series resistance, as well as parasitic capacitance. The parasitic capacitance arising from the total stray capacitances between the turns of the coil makes the coil's overall inductance appear less. The quality factor for an inductor (Q_L) is dependent on the coil's inductance and frequency (f), while being inversely proportional to its internal resistance (R). The Q_L-factor can be calculated by

$$Q_L = \frac{2\pi f L}{R} \tag{42}$$

Inductors also have a self-resonance at high frequency because of the distributed capacitance. At frequencies above the self-resonance frequency, the inductor behaves as a more complicated circuit element.

6. Resonant circuits

In a resonant circuit, energy is alternately stored in the magnetic fields of an inductor and the electric fields of a capacitor. The resonant frequency (f_0) is the frequency where the reactances of the capacitor (X_C) and the inductor (X_L) are of equal magnitude and opposite sign. The oscillation between the capacitor (C) and the inductor (L) defines the resonant frequency of the circuit. At time = 0 , the capacitor is fully charged and $I = 0$. Then the capacitor discharges through the inductor, and the current through L creates a magnetic field in which energy is stored, balancing the energy leaving the electric field in the capacitor. The field lines cutting the turns in L induce a voltage that opposes the current (Lenz's law), and this recharges the capacitor.

Resonant circuits can be series or parallel, as shown in Figure 2. The impedance of a series resonant circuit is at a minimum at the resonant frequency. For a series resonant circuit, the inductive reactance is $X_L = j\omega L$ and the capacitive reactance is $X_C = \frac{-j}{\omega C}$ [24].

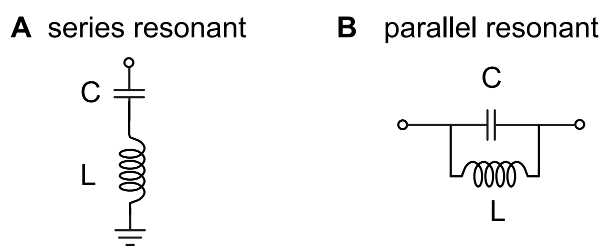

Figure 2: **Diagrams of resonant circuits A** A series resonant circuit has low impedance at resonance. **B** A parallel resonant circuit has high impedance at resonance.

Because the reactances have equal magnitudes and opposite signs, in an ideal resonant circuit, these cancel, and the impedance is zero. In a real circuit, these elements have resistance and therefore do not cancel each other perfectly. Real coils are often modeled as an ideal inductance L in series with a small resistance R, such that at resonance the impedance is:

$$Z = j\omega L + R - \frac{-1}{j\omega C} \tag{43}$$

The phase of the current relative to the voltage, (θ, where $\theta + \delta = 90°$) changes from lag to lead as ω passes through ω_0 [25], because

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{44}$$

The effect of a series resonant circuit is to attenuate all but a narrow band of frequencies centered about its resonant frequency. This type of circuit is often used as an isolation element in multichannel NMR probes, where it is desirable to give one frequency a path to ground before it reaches the electronics associated with another frequency. The quality factor Q of a resonant circuit is a measure of the relative amount of energy lost per cycle and is defined as:

$$Q \equiv \frac{W_S}{W_L} \tag{45}$$

The energy stored in L is

$$W_S = \frac{1}{2} L I^2 \tag{46}$$

where $I = I_0 \cos(\omega t)$. W is maximized when I and V are 90° out of phase, so when $I = I_{max}$, $V = 0$.

$$W_L = \int_0^T P dt \tag{47}$$

where P is the instantaneous power, and T is the period of the circuit, $T = \frac{2\pi}{\omega}$. Therefore,

$$Q = \frac{2\pi \frac{1}{2} L I^2}{\int_0^T i^2 t R dt} = \frac{\pi L I_0^2}{R \int_0^T I_0^2 \cos(\omega t) dt} = \frac{\pi L}{R} \frac{1}{\int_0^\pi \cos(\omega t) dt} = \frac{\omega L}{R} \tag{48}$$

at the resonant frequency,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{49}$$

Based on these expressions, it is evident that large inductances give larger Q values, but this effect is counteracted by the fact that in practice it is difficult to make large inductors without also increasing resistance.

A parallel resonant circuit has a sharp rise in impedance at the resonant frequency. For a parallel resonant circuit, the coil has a complex reactance of $j\omega L$, and the capacitor has a reactance of $\frac{-j}{\omega C}$. Therefore, the total reactance is

$$\frac{j\omega L}{1 - \omega^2 LC} \tag{50}$$

This is infinite when $\omega^2 LC = 1$, which is the resonance condition. In real circuits, this condition is not met because of non-ideal components. The inductor in the system has resistance R, making the impedance

$$\left(\frac{1}{j\omega L + R} + j\omega C\right)^{-1} = \left(\frac{R - j\omega L\left(1 - \omega^2 CL - R^2\frac{C}{L}\right)}{R + \omega^2 L^2}\right)^{-1} \tag{51}$$

Resonance then occurs when the impedance is real:

$$1 - \omega^2 CL - \frac{RC}{L} = 0 \tag{52}$$

and the value of the impedance at this point is:

$$Z = \frac{R^2 + \omega^2 L^2}{R} \tag{53}$$

Usually ωL is much larger than R, so the impedance approximately equals

$$Z = \frac{\omega^2 L^2}{R} \tag{54}$$

This value is large, but not infinite, and can be rewritten in terms of the quality factor Q.

$$Q = \frac{\omega L}{R} \tag{55}$$

$$Z = Q\omega L \tag{56}$$

In probe design, two-capacitor networks are often used (**Figure** 3). In one common type of design, the tune capacitor is placed in parallel with the coil such that the resonant frequency of this circuit is higher than the desired frequency. This value is chosen so that the real component of the parallel circuit's impedance matches the resistance of the load. The series capacitor is then chosen to cancel the remaining inductive reactance of the parallel resonant circuit. A useful heuristic is that the match capacitor will have about 1/10 the capacitance of the tune capacitor in this arrangement. Thus, the impedance looking into the network is 50 Ω at the resonant frequency. Impedance matching is critical in probe design in order to maintain probe efficiency and prevent damage to the amplifier due to excessive reflected power.

The quality factor Q for a resonant circuit is usually approximated as the Q_L of the coil, $\frac{L\omega}{R}$, where ω is the resonant frequency of the circuit

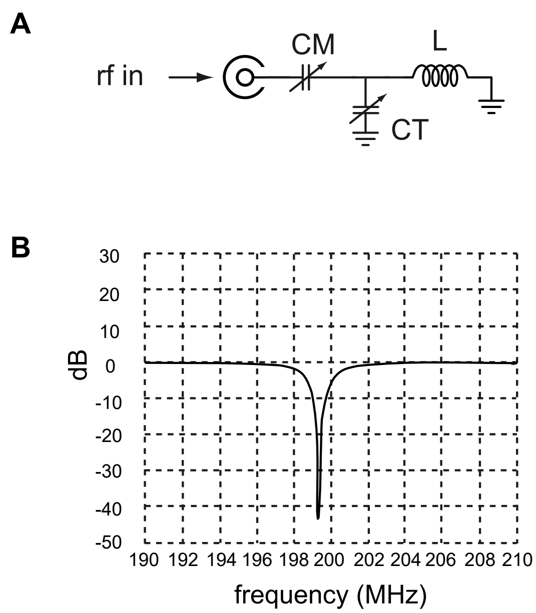

Figure 3: **Simple NMR probe circuit A** An example of a simple impedance-matched circuit of the type often used in NMR probes is shown. This single channel probe circuit consists of a parallel resonant circuit with a tunable capacitor (C_T) and fixed inductor (L). Impedance matching is achieved by adjusting the value of the match capacitor C_M. **B** The measured frequency response of a circuit of the type in **A**, with a resonant frequency around 200 MHz.

($\omega = \frac{1}{\sqrt{LC}}$). This approximation can be made because most of the loss in the circuit occurs in the coil. However, non-ideal capacitors can further degrade the Q of the circuit, as is discussed in **Section 4**.

The quality factor Q of a resonant circuit is related to R_L and R_C, the series resistances representing loss in the inductor and the capacitor, respectively.

$$Q = \frac{X_C}{R_C + R_L} \tag{57}$$

The impedance of a parallel resonant circuit has a maximum at:

$$Z_P = \left(\frac{1}{Q_C} + \frac{1}{Q_L}\right)^{-1} |X_{C,L}| \tag{58}$$

where Q_C is the quality factor of the capacitor, Q_L is the quality factor of the inductor, and $X_{C,L}$ is the reactance of the capacitor or inductor. The minimum impedance of a series resonant circuit is the total series resistance.

$$Z_P = \left(\frac{1}{Q_C} + \frac{1}{Q_L}\right) |X_{C,L}| \tag{59}$$

Q for a parallel resonant circuit can be discussed in the same terms as for the series case, except that the current is now given by

$$I(t) = I_0 \cos(\omega t - \phi) \tag{60}$$

$$Q = \frac{\pi L I_{max}^2}{\int_0^2 \cos^2(\omega t - \phi) R dt} = \frac{\pi L}{R} \frac{1}{(T/2)} = \frac{\omega L}{R} \tag{61}$$

I_0 is maximized at resonance, where its value is $\frac{V_0}{R}$ [6].

It is possible to measure the Q of a resonant circuit if the impedance magnitude or response magnitude is known over a range of frequencies. The Q is related to the point where the impedance magnitude is 3 decibels (dB) below the impedance at resonance for parallel circuits or above for series circuits. The impedance of these points is:

$$|Z_{3dB}| = |Z_{res}| \frac{1}{\sqrt{2}} \tag{62}$$

for the parallel case and

$$|Z_{3dB}| = |Z_{res}|\sqrt{2} \tag{63}$$

for the series case. In these circuits,

$$Q = \frac{f_0}{\Delta f} \tag{64}$$

where Δf is the difference between the upper and lower 3 dB frequencies and f_0 is the resonant frequency in Hz.

7. Transmission lines

In the above discussion of AC circuits, the implicit assumption has been made that all components are physically much smaller than the wavelength of the frequency of interest. When considering transmission line elements, this assumption is no longer valid. A transmission line is a structure that transfers energy and has its electrical properties such as capacitance and inductance distributed along its length. Several different geometries of transmission lines can be used, including striplines, twinlines, and coaxial lines. Transmission lines used in probe design are usually of the coaxial geometry. For traditional uses of transmission lines, the line is many times longer than the wavelength of interest. In circuit elements, it may be much shorter. Transmission lines have distributed capacitance and inductance and may therefore be used as AC circuit elements. By choosing line lengths between 0 and $\lambda/2$, it is possible to generate any value of reactance. The propagation mode of interest is the transverse electromagnetic (TEM) wave, also called the principal mode. This mode, which requires a transmission line structure rather that a single waveguide, is one where there is no component of E or H along the direction of propagation.

For propagation of the principal mode in a transmission line, the internal resistance of the line must be considered and is given by

$$R = \sqrt{\frac{f}{10^7}}\left(\frac{1}{a\sqrt{\sigma_a}} + \frac{1}{b\sqrt{\sigma_b}}\right)\Omega/m \tag{65}$$

where a and b are the diameters of the inner and outer conductors in meters, f is the frequency in Hz, and σ_a and σ_b are the conductivities of the inner and outer conductors in mhos/m^3. The value of σ for copper is $\sigma_{Cu} = 5.9 \times 10 - 7$ mhos/m^3.

A description of the propagation characteristics of a transmission line must include consideration of forward and reflected standing waves in the line. This description can be developed starting from the time dependence of voltage and current in the line. At point x, the instantaneous values of voltage and current are represented by V and I. These depend on position along the line:

$$\frac{\partial V}{\partial x} = -\left(RI + L\frac{\partial I}{\partial t}\right) \quad (66)$$

$$\frac{\partial I}{\partial x} = -\left(GV + LC\frac{\partial V}{\partial t}\right) \quad (67)$$

The solution to these equations that fits with the physical system of interest is one where the time dependent amplitudes of the voltage and current (V and I) are harmonic functions of time, i.e., $Ve^{j\omega t}$ and $Ie^{j\omega t}$. Assuming this form and taking the time derivatives of I and V gives

$$\frac{\partial V}{\partial x} = -e^{j\omega t}(R + j\omega L)I \quad (68)$$

$$\frac{\partial I}{\partial x} = e^{j\omega t}(G + j\omega C)V \quad (69)$$

These are the expressions for the complex time amplitudes of voltage and current as a function of position from the source [22]. Taking the second derivative with respect to voltage,

$$\frac{\partial^2 V}{\partial x^2} = (R + j\omega L)(G + j\omega C)V = P^2 V \quad (70)$$

where $P = \sqrt{(R = j\omega L)(G + j\omega C)}$. A general solution of this type of equation is

$$V_x = Ae^{-Px} + Be^{Px} \quad (71)$$

where A and B are constants that depend on the boundary conditions. From the expression for $\frac{dV}{dx}$, it is straightforward to get an expression for I:

$$I = \frac{1}{(R + j\omega L)}\frac{dV}{dx} = \frac{P}{(R + j\omega L)}\left[Ae^{-Px} + Be^{Px}\right] \quad (72)$$

upon writing out what P is and simplifying, this becomes

$$I = \sqrt{\frac{G + j\omega C}{R + j\omega L}} \left[Ae^{-Px} + Be^{Px}\right] \tag{73}$$

or, defining $Z_0 \equiv \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$I = \frac{1}{Z_0} \left[Ae^{-Px} + Be^{Px}\right] \tag{74}$$

Thus, for both V and I there is an expression that appears as a sum of forward and reflected traveling waves. For an infinite line, there are no reflections and hence no backward components. For lines of finite length, this effect is achieved by impedance matching the line with the source. This is important in probe design since backward reflections will reduce probe efficiency and possibly damage the amplifier or detector.

The quantity Z_0 defined in the description of current is called the characteristic impedance, because it arises from intrinsic properties of the transmission line. The characteristic impedance is a property that is specific to the construction of a particular line. For lines constructed with low-resistance conductors and low-conductivity dielectric material, the expression for Z_0 approximately reduces to $\sqrt{\frac{L_\ell}{C_\ell}}$. The characteristic impedance also depends on the geometry of the line and can be approximated as $Z_0 \approx 138\epsilon_r \log_{10} \frac{b}{a}$ for a coaxial transmission line [26]

When a line is terminated in a resistive load matched to Z_0, only the transmitted wave exists. If the line is terminated in any other impedance, there is also a reflected component. The impedance of a transmission line, as for any AC circuit element is $Z = R + jX$. The admittance is also defined the same as for any other AC circuit element $Y = G + jB$. Impedance and admittance are constant with respect to x in a transmission line of uniform geometry.

For a small section of line dx with a time dependent wave traveling through it, the distribution of the voltage is given by:

$$\frac{dV}{dx} = IZ \tag{75}$$

and the corresponding current distribution is:

$$\frac{dI}{dx} = VY \tag{76}$$

The second derivatives of these are

$$\frac{d^2V}{dx^2} = I\frac{dZ}{dx} + Z\frac{dI}{dx} = I\frac{dZ}{dx} + ZVY \tag{77}$$

$$\frac{d^2I}{dx^2} = V\frac{dY}{dx} + Y\frac{dV}{dx} = V\frac{dY}{dx} + YIZ \tag{78}$$

If the line is of constant geometry, these can be written so that:

$$\frac{d^2V}{dx^2} - ZVY = 0 \tag{79}$$

$$\frac{d^2I}{dx^2} - YIZ = 0 \tag{80}$$

In order to derive voltage and current distributions along the line, it is assumed that there is a solution of the form $V = e^{\gamma x}$, where $\gamma = \sqrt{ZY} = \alpha + j\beta$. In this expression, γ is the propagation constant, which is composed of the attenuation constant α and the phase constant β. The voltage and current can be described in terms of the attenuation constant and phase constant as a series of forward and reflected waves as follows.

$$V = V_1 e^{\alpha x} e^{j(\alpha x + \beta x)} + V_2 V_1 e^{-\alpha x} e^{j(\alpha x - \beta x)} \tag{81}$$

$$I = \frac{V_1}{\sqrt{(Z/Y)}} e^{\alpha x} e^{j(\alpha x + \beta x)} - \frac{V_2}{\sqrt{(Z/Y)}} e^{-\alpha x} e^{j(\alpha x - \beta x)} \tag{82}$$

These define the voltage and current at any point in the line. These have the components of two traveling waves propagating in opposite directions. For a single wave traveling in the $+x$ direction, the impedance is

$$Z = \frac{V}{I} - \sqrt{\frac{Z}{Y}} = Z_0 \tag{83}$$

The quantity G is a measure of the lossiness of the dielectric due to the fact that it is not a perfect insulator, as discussed in **Section 4**. The expression for G in a transmission line gives the conductance per unit length of continuous dielectric

$$G = 2\pi \frac{\sigma_d}{ln(b/a)} (mhos/m) \tag{84}$$

where σ_d is the conductivity of the dielectric material. This expression is not the most convenient to use when considering probe design because departures from ideality in dielectric materials are usually expressed in terms of the loss tangent, which is tangent of the angle δ by which the current is displaced from exact quadrature with the applied voltage.

$$\tan\delta = \frac{G}{C\omega} \tag{85}$$

The characteristic impedance can be expressed in terms of the loss tangent for a more convenient representation

$$Z_0 = \sqrt{\frac{R_\ell + j\omega L_\ell}{(\tan\delta)\omega C_\ell}}\Omega \tag{86}$$

where $j = -1$, R_ℓ is the resistance per unit length in the conductors, ω is the angular frequency, L_ℓ and C_ℓ are the inductance and capacitance per unit length of an infinitely long transmission line.

The conductance can be related to the loss tangent by using the expression for internal capacitance,

$$G = \frac{2\pi}{1.8\times10^{10}}\frac{\epsilon_r f\tan\delta}{\ln(b/a)}(mhos/m) \tag{87}$$

Factors such as resistance in the conductors and loss in the dielectric contribute to the overall loss of the transmission line. This quantity should be minimized in order to maximize probe efficiency. The loss per unit length is described by the attenuation constant α

$$\alpha = 4.34\left(\frac{R}{Z_0} + GZ_0\right)(dB/m) \tag{88}$$

which is related to the loss tangent by the expression

$$\alpha = 9.95\times10^{-6}\sqrt{f}\sqrt{\epsilon_r}\left[\frac{\frac{1}{a\sqrt{\sigma_a}} + \frac{1}{b\sqrt{\sigma_b}}}{\log(b/a)}\right] + (9.10\times10^{-8})f\sqrt{\epsilon_r}(\tan\delta) \tag{89}$$

$$\alpha = 4.34\left(\frac{R}{Z_0} + (C\omega\tan\delta)Z_0\right)(dB/m) \tag{90}$$

The minimum attenuation of a matched transmission line occurs for the condition [2]

$$\ln(b/a) = 1 + \frac{\sqrt{\frac{\sigma_a}{ab}}}{(b/a)} \tag{91}$$

It is important to consider this condition when choosing the relative dimensions of transmission line segments to be used as tunable probe circuit elements. For copper, the optimum value of b/a is 3.59. However, this condition is not very sensitive. In a transmission line with copper inner and outer conductors, this value is within 5% for ratios in the range 2.6-5.3 [27]. This flexibility makes it convenient to design transmission line components to fit in specific spaces within the probe. This expression applies to matched transmission lines, and is therefore appropriately considered in the design of matched lines carrying current to the probe elements as opposed to the mismatched transmission line segments used as capacitive and inductive elements. In addition to the attenuation per unit length, each line also produces a displacement in phase per unit length. This is called the phase constant (β). Together, α and β comprise the propagation constant for a particular line ($\gamma = \alpha + j\beta$)

The above expressions are useful for describing transmission lines used to propagate rf, but additional equations are needed to describe other transmission line circuit elements used in the probe design. The use of transmission line segments as coaxial capacitors has already been discussed in **Section 4**, but a more detailed description of the capacitive and inductive properties of transmission line elements is given below. The general expression for the input impedance of a transmission line given in terms of the propagation constant and the length of the line is:

$$Z_{in} = \frac{Z_0[Z_\ell + Z_0 \tanh(\gamma\ell)]}{[Z_0 + Z_\ell \tanh(\gamma\ell)]} \tag{92}$$

where ℓ is the length and Z_ℓ is the series impedance per unit length [2].

The expressions for input impedance in the special cases of lines terminated in a short circuit $\left(\frac{Z_\ell}{Z_0} = 0\right)$ and an open circuit $\left(\frac{Z_\ell}{Z_0} = \infty\right)$are

$$Z_{in}^{short} = Z_0 \tanh(\gamma\ell) \tag{93}$$

and

$$Z_{in}^{open} = \frac{Z_0}{\tanh(\gamma\ell)} \tag{94}$$

The imaginary part of the impedance, or the reactance, is given by the following expressions for shorted and open lines:

$$X_{in}^{short} = -j\frac{Z_0}{\tan\left(2\pi\frac{\ell}{\lambda}\right)} \tag{95}$$

and

$$X_{in}^{open} = jZ_0 2\pi\frac{\ell}{\lambda} \tag{96}$$

The reactances for transmission lines in open and shorted configurations are plotted as a function of the length of the transmission line in $\lambda/4$ increments in **Figures 4A** and **4B**, respectively.

As shown in **Figure 4A**, the input impedance behaves as an inductive reactance for shorted lines between 0 and $\lambda/4$ and capacitive reactance between $\lambda/4$ and $\lambda/2$. Thus, a small piece of grounded transmission line can be used as an inductor with approximate inductance:

$$L = \frac{9.55}{f}\ln\frac{b}{a}\tan\frac{2\pi\ell}{\lambda}(\mu H) \tag{97}$$

where f is the frequency in MHz, is the length of the shorted line, and λ is the wavelength.

Open quarter wave elements behave like high-Q series resonant circuits at short wavelengths. A transmission line of length $\lambda/4$ that is open on one end acts as a low impedance to ground, as can be seen from **Figure 4B**. Therefore, this type of structure can be used as a trap tuned to a particular frequency. Thus, transmission line elements may be used as capacitors, inductors, or tuned resonant circuits. These elements can be treated as simple circuit elements using the formulas given above. This means that every element in a probe circuit may be made of transmission line elements while maintaining the relative ease of analysis that is associated with discrete elements.

8. Power and impedance matching

Power is expressed in terms of current and voltage:

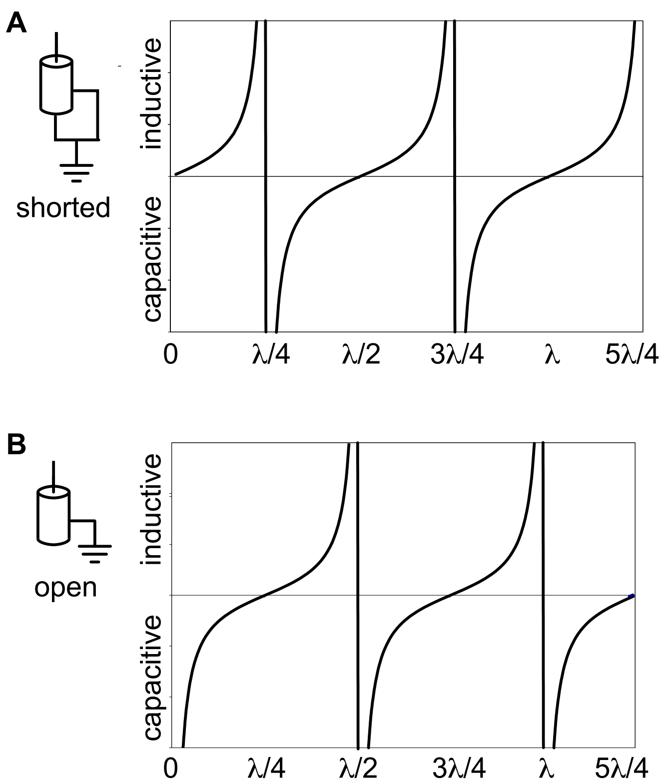

Figure 4: **Impedance of shorted and open transmission line segments as a function of length A** The impedance of a shorted transmission line is plotted as a function of length in units of $\lambda/4$. A high impedance condition occurs at odd multiples of $\lambda/4$ and a low impedance occurs at even multiples of $\lambda/4$. **B** An analogous plot is shown for an open-ended transmission line. In this case, odd multiples of $\lambda/4$ give a low impedance condition and even multiples of $\lambda/4$ result in a high impedance.

$$P = IV = \frac{V^2}{R} = I^2 R \tag{98}$$

$$P = \frac{V_{pp}^2}{8R} = \frac{V_{RMS}^2}{R} \tag{99}$$

In order to maximize the power delivered to a load from a source, it is necessary to match the resistance of the load to the internal resistance of the battery. This can be shown by considering a simple circuit consisting of a battery with voltage rating V_b and internal resistance R_b connected to a load R_L. The current in this circuit is

$$I = \frac{V_b}{(R_b + R_L)} \tag{100}$$

and the voltage at the load is

$$V_L = IR_L = \frac{V_b R_L}{R_b + R_L} \tag{101}$$

Therefore, the power P_L is given by

$$\left(\frac{V_b}{R_b + R_L}\right)^2 R_L \tag{102}$$

At the maximum power condition, $\frac{\partial P_L}{\partial R_L} = 0$. This reduces to $R_L = R_b$. Therefore, the power delivered to the load is maximized when the load resistance is matched to the internal resistance of the source. This result is important when discussing NMR probe circuits, because it is desirable to ensure that the ratio of forward to reflected power in the circuit consisting of the amplifier and the probe is high in order to maximize probe efficiency and prevent damage to the amplifier.

In AC circuits with reactive elements, the power has dissipated and non-dissipated terms.

$$P = PIV = (P_{dissipated} + jP_{stored}) \tag{103}$$

The instantaneous power in an AC circuit is given by

$$P(t) = I_0^2 R \sin^2(\omega t) \tag{104}$$

A more useful quantity is the average power (P_{av}) [3]. Because $V_{RMS} = \frac{V_0}{\sqrt{2}}$, it follows that $P_{av} = \frac{V_{RMS}^2}{R}$ and also $P_{av} = I_{RMS}^2 R$. If the voltage and current differ by phase ϕ, then

$$P(t) = V_0 \sin(\omega t + \phi) I_0 \sin(\omega t) \tag{105}$$

and

$$P_{av} = V_{RMS} I_{RMS} \cos\phi \tag{106}$$

The quantity $\cos\phi$ is called the power factor.

Another factor that affects probe performance is the power handling capabilities of the capacitors. At low frequencies, the maximum current rating for a capacitor is limited by the voltage and can be calculated by

$$I_V = V_{max} \times 2\pi f C \tag{107}$$

where I_V is the voltage limited maximum current rating of the capacitor, and V_{max} is its RMS voltage rating. At higher frequencies, the current rating is limited by power dissipation, described by

$$I_p = \sqrt{\frac{P_{max}}{ESR}} \approx \sqrt{\frac{P_{max}}{R_{AC}}} \tag{108}$$

I_V is the power dissipation limited maximum current rating and P_{max} is the maximum power dissipation defined relative to a mounting surface with well-defined thermal characteristics [28, 29, 30, 31].

At high frequencies, ESR can be approximated by R_{AC}. In the voltage limited (low frequency) case, the current rating is influenced mainly by the breakdown voltage of the dielectric material. In the power dissipation (high frequency) case, the resistance becomes important. Again, optimum performance is obtained from the capacitor that has more surface area and therefore less AC resistance. Another important factor is the thermal resistance (θ^R) of the capacitor, which influences P_{max}. P_{max} can be found by determining the temperature differential across the length of the capacitor and dividing by θ^R. The thermal resistance per unit length of a capacitor of cross-sectional area A is

$$\theta_\ell^R = \frac{1}{\lambda_T A} \left(\frac{^\circ C}{W}\right) \tag{109}$$

where λ_T is the thermal conductance of a particular material in $\frac{W}{(^\circ C)(cm^2)}$. This expression shows that thermal resistance is inversely proportional to the cross-sectional area [32]. This is another reason why physically larger capacitors have better power handling capabilities.

Although the power handling capability of capacitors at high frequency is usually limited by thermal rather than electric stress considerations, arcing can still occur at high voltage points in the coaxial capacitors and should be considered. The electric stress (in volts/m) for a length of coaxial line is given in terms of the voltage (V) across the plates by

$$E = \epsilon_r \frac{V}{a \ln(b/a)} \tag{110}$$

Corona discharge will occur at the center conductor when the electric stress exceeds the breakdown voltage of the dielectric. For air, corona discharge occurs at about 30 kV/cm. The breakdown voltage can be much higher in lines filled with a higher dielectric strength material. However, if there are air gaps at the interface between the conductor and the dielectric at high voltage points, arcing will occur in these areas, particularly in areas where there is no air movement. Air space can be tolerated better in areas where a continuous flow removes ions or particles that are prone to arcing. This factor should be considered when designing the geometry of the ends of coaxial capacitors. Hemispherical pin ends and cavities should be used whenever possible to minimize the possibility of corona discharge at sharp points [33]. The greater the radius of the sphere, the closer the two plates must be to permit arcing. For instance, at a voltage of 120 kV, the spark gap length in air for spherical electrodes of 25 cm diameter is 4.28 cm. For 10 cm diameter, the length is 4.78 cm, and this increases to 7.07 cm for spheres of diameter 5 cm. For needle points, the spark gap distance at 120 kV is 19.8 cm [24]. These data indicate that sharp edges should be avoided in the construction of electrodes and that larger diameter center conductors should be used for lower-frequency applications, which require more power handling capability.

Another factor that is important when considering arcing in probe design is the voltages in the sample coil and the tune capacitor required to resonate it. The voltages across the tuning capacitor $V_C = I_0(max)\left(\frac{1}{\omega C}\right)$ and the sample inductor $V_L = I_0(max)\,(\omega L)$ are large at the resonance condition. This can pose a problem with corona discharge occurring in a capacitor or between the turns of the coil in a resonant circuit. Another feature of real capacitors is the phenomenon of self-resonance. As the frequency increases, the inductance in the leads and plates of the capacitor becomes increasingly important. At the self-resonant frequency, this inductance becomes self-resonant with the capacitor, making its performance difficult to model. This becomes an important consideration when working at high frequencies [16].

Efficiency is a measure of the power delivered to the load as a fraction of the available power. In a real probe circuit, some means of impedance matching is needed to ensure acceptable efficiency of power traveling from the RF amplifier to the probe circuit. The maximum power is dissipated in a load when the load resistance equals the source resistance. In general, any load impedance can be matched to any source impedance, but in the case of RF amplifiers, the source impedance is usually 50 Ω. Many methods of impedance matching can be used, including transformers, transmission lines, and capacitive or inductive networks. Practical methods for calculating and measuring efficiencies in NMR probe circuits have been described by several instrument builders, e.g. [34, 35, 36, 11].

9. Considerations for building multi-channel circuits

The LC component analysis above is sufficient to build a simple resonant circuit at a single frequency. For multiply-resonant circuits, interaction among the channels must be considered, especially if more than two channels are connected. Let us consider a dual-resonant circuit where we will tune and match for a proton (^1H) Larmor frequency of 500 MHz and carbon (^{13}C) Larmor frequency of 125 MHz. In context of NMR, adequate impedance matching means efficient signal propagation at those frequencies, requiring minimal power for sample excitation. As the first step of the design process, the ^1H and ^{13}C resonant circuits can be tuned independently from each other, as shown in **Figure 5**. Although many different strategies for producing multiply-resonant circuits exist, an exhaustive discussion of them is beyond the scope of this tutorial: we provide an example of a double-tuned circuit with one coil.

The ^1H channel comprises the inductor, L_s along with a match (C_1) and tune (C_2) capacitor. The geometry, and hence the value of the inductance for L_s is partly determined by the desired sample dimensions. Taking this into account, the capacitors are selected to resonate it at ω_H, 500 MHz here and provide impedance matching to the load at 50 Ω as described in **Section 3**. Similarly, the ^{13}C channel, composed of another match (C_3) and tune (C_4)

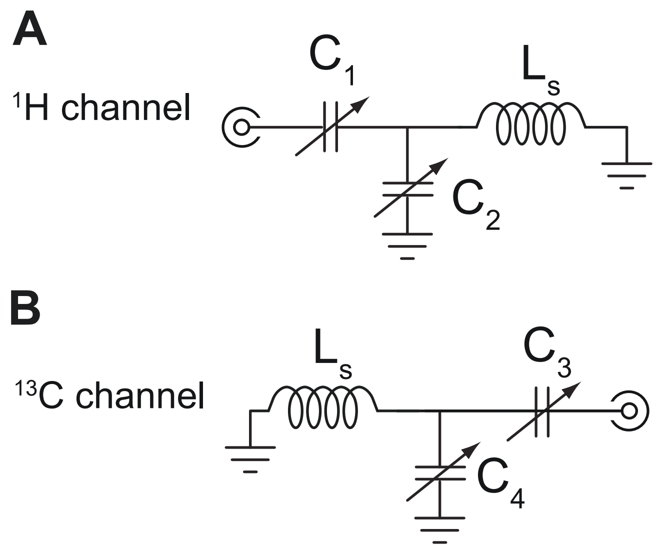

Figure 5: **Individual LC circuits** Single-frequency circuits for **A** the ^1H channel and **B** the ^{13}C channel. These circuits share the sample coil L_s and each is independently tuned and matched at its resonant frequency.

capacitors, is connected on the other side of the same inductor, L_s. Here, the (C_3) and (C_4) capacitor values are selected to resonate the same coil at ω_C, in this example approximately 125 MHz. Without additional circuit elements to isolate them, there will be cross-talk between the channels, which can result in unexpected resonant frequencies as well as more subtle issues, such as poor impedance matching and increased noise during data collection.

Once the two LC circuits have been tuned to their respective frequencies, isolation elements can be added. Although the design of a trap can vary depending on its purpose, its primary role in an isolation scheme is to behave as either an open or short circuit for a targeted frequency, while not impacting the performance at other frequencies of interest [37]. Since the resonant frequency of any particular channel must cross the sample coil L_s, traps are generally implemented after L_s while considering the other channel's tuning elements. **Figure 6** shows a doubly-resonant circuit with good isolation between the channels. The C_5L_5 parallel configuration resonates at ω_H and appears as an open circuit to this high frequency. This prevents ω_H from reaching the ^{13}C port by acting as a band-stop filter and reflecting the signal back to the ^1H channel. At the same time, it lets through the lower frequency range containing the resonant frequency of ω_C. On the other side, a series trap (C_6L_6) resonates at ω_C and serves as a low impedance (shunt) path to ground, preventing signal from reaching the ^1H port. Choosing the capacitance such that $C_1 << C_2$ is preferred, because this low capacitance in C_1 creates a high impedance path for the lower frequency, ω_C, and assists in the isolation of the ^1H channel [23].

Although isolation traps are represented as lumped elements with designated locations in the circuit design, in practice, they are made up of real capacitive and inductive elements that must be arranged within the probe and may interact with other components. They present additional impedance to the channel networks, influencing the resonant frequencies and potentially changing the values of tune and match capacitances to reach the desired resonant frequency. Depending on how they are placed relative to the rest of the circuit, they can also interact with other components to create stray reactances. During development, this can create a cycle of altering capacitances as the isolation traps and balancing capacitors (details below) are optimized, all while maintaining the resonant frequency of each channel. Simulating the rf circuit before beginning construction enables us to at least approximately predict the circuit's behavior, which becomes increasingly important as we add additional rf channels, traps, and balancing components [10]. Although the circuit simulation is imperfect, mostly due to properties of real components that are unaccounted for in the model, it saves a great deal of time during the probe engineering process and can be used as starting point for new designs.

After the target resonant frequency is achieved for each channel in the presence of the isolation elements, isolation performance between ports must be tested. With a network analyzer, we can perform an S-parameter measurement, specifically, the transmission coefficient (S_{21}). This is accomplished by performing a frequency sweep through port 1 and measuring the output signal through port 2. The ports can then be switched to measure isolation in the other direction. The S_{21} measurement gives us information about the input-output relationship through the circuit network in either the time or frequency domain. In our case, we are observing the frequency behavior as a measure of isolation, and if it is poor, the circuit cross-talk. If the isolation element for ω_H is improperly tuned, the measured signal coming out of the ^{13}C port will show a large peak at this frequency. In contrast, good isolation will result in an S_{21} isolation curve with a minimum, or at least a value close to zero, at the unwanted resonant frequency (**Figure 6C**).

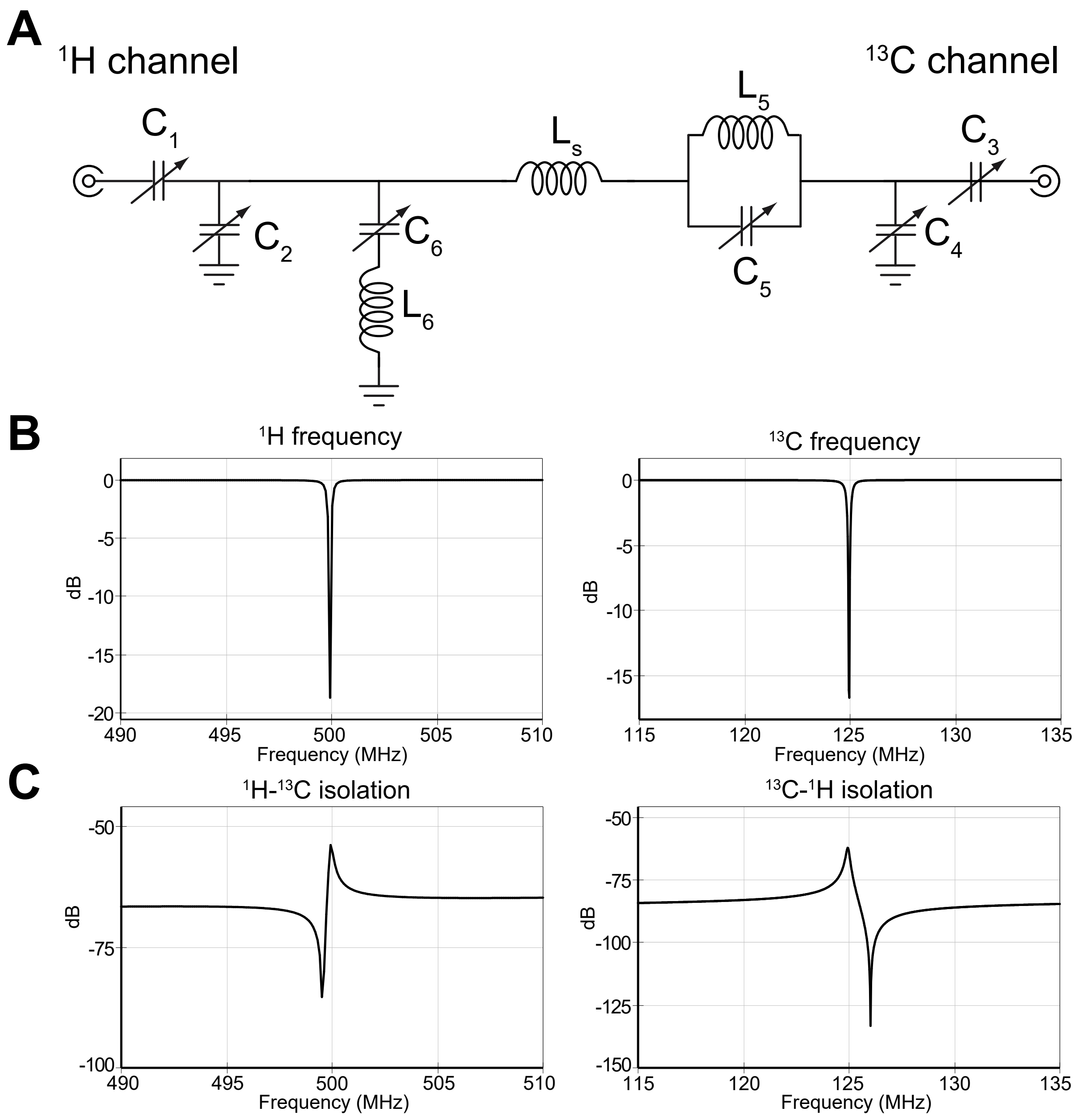

Figure 6: **Double-resonance LC circuit A** Double-resonance circuit with tuning and match elements to achieve ω_H and ω_C. **B** The reflection coefficient (S_{11} and S_{22}) in dB is shown for each channel. Each is centered on its resonant frequency. **C** Isolation between the two channels (S_{12} and (S_{21}) is simulated and plotted.

Another practical consideration that applies not only to multiply-resonant circuits, but also single-channel ones at high frequency is coil *balancing*. In an ideally balanced circuit, the reactances on either side of L_s are equal and opposite. Electrically, this causes the voltage have opposite phase and equal amplitude with respect to ground, creating a node at the coil's center [10]. In turn, this causes the average current to be distributed symmetrically across the coil as it alternates. If the coil is not balanced, the voltage will alternate between a maximum on one side of the coil and zero at the other. At high frequencies, where the length of the wire is comparable to $\lambda/4$ this can lead to an asymmetric B_1 distribution and degradation of performance [38]. In the context of multiply resonant circuits, lack of balancing can lead to the surprising result that the field profiles are very different for high and low frequencies, even when only one coil is used.

One way to balance the ^1H channel is by adding a balancing capacitor (C_7) on the opposite side of the coil (**Figure 7A**). When calculating the capacitance for C_7, it is recommended to double the tune capacitance of C_2 and give C_7 an equal value, splitting the impedance load across the inductor (L_s) [23] [8]. These theoretically re-add to the original total capacitance of C_2. In practice, the balancing capacitor is often a fixed value whereas the tuning capacitor is variable in order to tune to ω_H and adjust for small deviations caused by different samples. The effects of unbalanced circuits on B_1 have been previously explored with experimental [39] and computational analysis [40]. At lower frequencies, acceptable homogeneity and B_1 symmetry can be achieved despite the asymmetrical circuit load on the NMR sample coil. However, if balancing the ω_C network is desired, an additional capacitor C_8 can be added, using the same principles as discussed for C_7. This is labeled as optional in our design because in our experience the performance improvement gained often does not offset the added complexity and the required adjustment to the other capacitors.

As the circuit design becomes more complex, spurious (or secondary) resonances can arise. For example, in our balanced circuit, a spurious resonance with poor impedance matching ($\approx$ -2.8 dB) can be observed around 356 MHz (**Figure 8**). If these resonances are too close the primary resonance, there can be issues in determining the correct resonance frequency of the circuit as well as power loss during experiments. However, if all such resonances reside far from the primary resonances, one can safely ignore them. This is fortunate, because engineering the circuit to remove all spurious resonances can be difficult and may not be worth the effort for many applications.

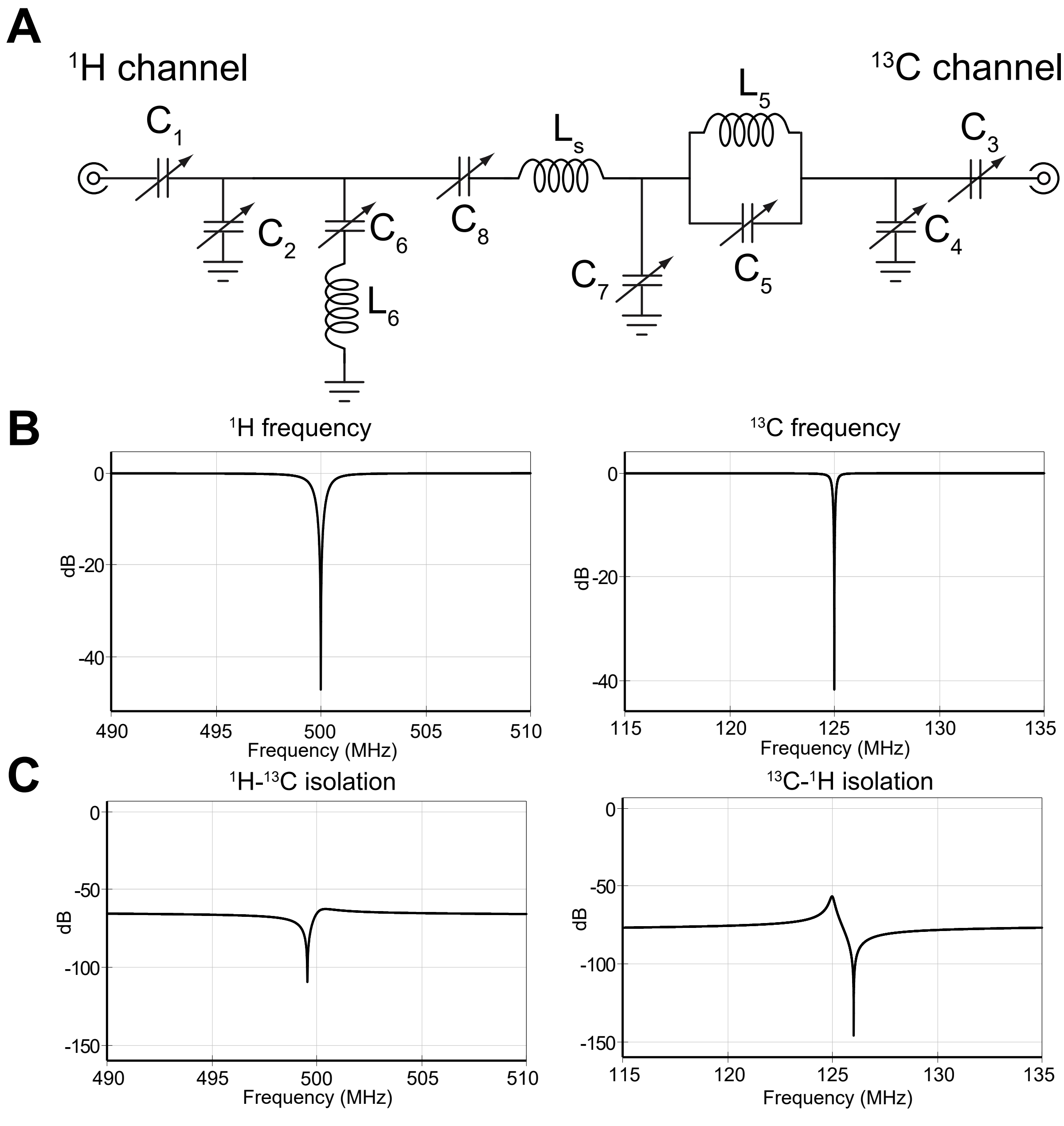

Figure 7: **Balanced double-resonance LC circuit A** A balanced double-resonant circuit network based on Figure 6 is presented. For the ^{1}H channel, the balancing capacitor (C_7) is utilized. If balancing of the ^{13}C channel is desired, an optional balancing capacitor (C_8) can be placed next to the sample coil across the ^{13}C circuit network. **B** S_{11} and S_{22} are shown for the 2-channel network, centered at their resonant frequencies. **C** In the now balanced network, the simulated isolation (S_{12} and (S_{21}) between the two channels is shown.

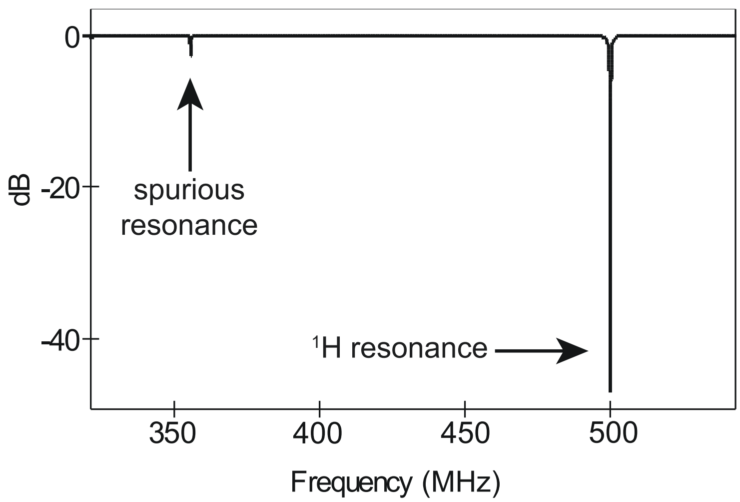

Figure 8: **Spurious resonance in dual-resonant circuit** A spurious resonance is found near 356 MHz in the 2-channel circuit tuned to 500 MHz (ω_H) and 125 MHz (ω_C). The resonant frequency for ^1H is shown for reference.

The example circuits shown here have been modeled and the code is available in the online **Supplementary Information**. The circuits were analyzed using the S-parameter simulation tool within the QucsStudio [41] circuit simulation software. Here, components can be fine-tuned while conveniently observing effects of the resonant frequencies via the reflection coefficient and impedance via the Smith chart, both described in **Section 3**.

10. Conclusion

In summary, it is possible to begin designing probe circuits for NMR or MRI with some knowledge of the behavior of capacitors and inductors and how they interact in a few simple configurations. In our experience, the most straightforward way to begin is to select a transceiver coil that fits the desired sample geometry and then select a tuning capacitor to resonate it at the Larmor frequency of a nucleus of interest. It is then possible to build on this simple circuit, adding a match capacitor, and if desired, additional channels and isolation elements. The Smith chart is a useful way to display the impedance of the circuit during the design process; this is an output available in many software packages and the displays of some experimental network analyzers. In some cases (i.e., when space is limited), it is more convenient to use of transmission lines elements rather than traditional lumped elements. We hope that this collection of information and references will be useful to students and others who are interested in building custom probes or fully understanding how existing ones work.

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