Optimal Entanglement Distribution Problem in Satellite-based Quantum Networks

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***Abstract*—Satellite-based quantum networks are emerging as a promising solution for the development of a global quantum internet in the near future. The ability to leverage the advantageous lower attenuation of optical signals from satellites to ground presents an exciting opportunity to establish a robust and secure quantum communication infrastructure on a global scale. By utilizing a constellation of satellites, it becomes feasible to continuously distribute high-fidelity quantum entanglements among ground stations over long distances, overcoming the limitations of traditional terrestrial-based quantum communication systems. In this article, we first provide a brief survey of existing solutions for satellite-based entanglement distribution, highlighting the various approaches and technologies that have been employed in this rapidly evolving field. We then delve into a formulated optimal entanglement distribution problem, aiming to optimize the distribution of quantum entanglement resources across the satellite network to maximize efficiency and reliability. This problem is addressed through a detailed exploration of several different methodologies and algorithms, each tailored to specific operational settings and constraints. Our experimental results confirm the efficiency of these approaches and provide valuable insights into their practical implementation and performance. Finally, we identify several key directions for further study and development in the realm of satellite-based quantum networks.**

***Index Terms*—Entanglement distribution, space-terrestrial networks, satellite networks, quantum networks.**

I. Introduction

Quantum networks [1], at the forefront of cutting-edge quantum technology, represent a new paradigm shift in secure communication and enable many emerging applications, such as quantum key distribution (QKD) [2], quantum sensing, and distributed quantum computing [3]. Quantum networks rely on the phenomenon of *quantum entanglement*, where particles become interconnected in a manner that the state of one particle instantaneously influences the state of another, irrespective of their separation distance. Quantum entanglement serves as the foundational cornerstone for all quantum communication protocols, enabling the establishment of secure and unforgeable connections between distant parties.

Although quantum entanglement plays a pivotal role in the functionality of quantum networks, distributing shared entanglement over long distances remains a very challenging task [4]. Typically, shared entanglement is distributed through single-photonic qubits transmitted via optical fibers. However, the success probability of transmitting such optical signals experiences an exponential decline as the distance increases. To mitigate this challenge, quantum repeater/memory and entanglement swapping have been introduced [5]–[7]. Unfortunately, there is still a significant gap between the demonstrated transmission distances and the requirements for realizing a global-scale quantum network, i.e., quantum internet.

One promising strategy to achieve efficient long-distance entanglement is through the usage of satellites. By taking advantage of the reduced signal attenuation in optical communication from satellites to ground stations and the growing availability of large constellations of orbiting satellites, satellite-based entanglement distribution emerges as a viable strategy for continuous and high-quality distribution of quantum entanglements over long distances to enable global-scale quantum networks. A notable example is the work of Yin *et al.* [8], where successful entanglement distribution to receiver ground stations separated by over 1,200 km was demonstrated by the Micius satellite in China. Mazzarella *et al.* [9] have also shown that a constellation of fifteen low Earth orbit (LEO) CubeSats can form a quantum backbone for ground-based quantum networks across the UK. Recently, Gonzalez-Raya *et al.* [10] have studied the effects of atmospheric turbulence on entanglement distribution and quantum teleportation in the optical regime between a satellite and a ground station.

While the satellite-based entanglement distribution leverages a large constellation of orbiting satellites to generate and distribute quantum entanglements, a central challenge lies in the optimal configuration of these satellites to meet the entanglement demands from multiple ground stations [11]. Such an optimal configuration problem can be formulated as different optimization problems within the complex space-terrestrial network consisting of multiple satellites and ground stations. In this article, we first review the most recent advancements in this field and then use the *optimal entanglement distribution problem* introduced by [12] to illustrate potential efficient solutions in satellite-based quantum networks.

The optimal entanglement distribution problem maximizes the aggregate entanglement distribution rate by considering various resource constraints at both satellites and ground stations. Although Panigraphy *et al.* [12] formulated their problem as an integer linear programming problem, they efficiently solved it for only two specific scenarios. In this article, we present a survey of existing satellite-based entangle-

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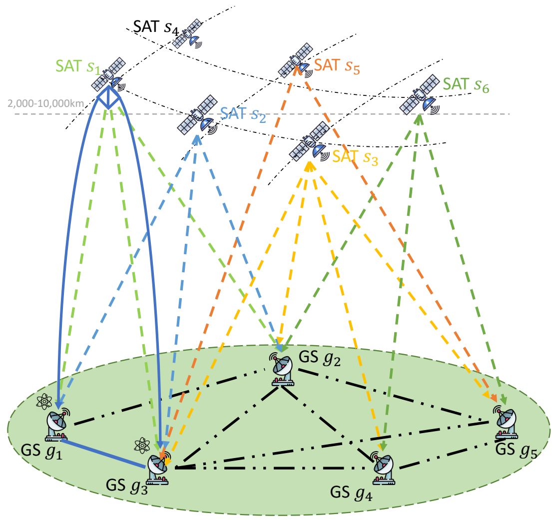


Fig. 1. The overall satellite-based entanglement distribution architecture.

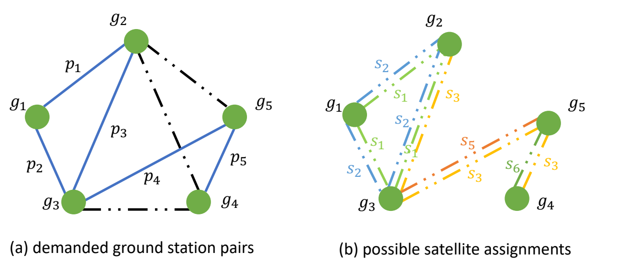


Fig. 2. Graph models of satellite-based entanglement distribution in the network of Figure 1. (a) five demanded GSPs, (b) all possible satellite distributions for each GSP.

ment distribution solutions, exploring various technologies and approaches. We then revisited the optimal entanglement distribution problem, aiming to maximize the total weighted utility across the satellite-terrestrial network. We explore different methodologies and algorithms tailored to specific operational settings and constraints, with experimental results confirming their efficacy. Additionally, we identify key directions for further study and development in the realm of satellite-based quantum networks.

II. Quantum Entanglement Distribution

The connection in quantum networks is different from that in classic networks. In classic networks, data packages are sent from the source to the destination by copying them at the intermediate routers. In quantum networks, a pair of entangled particles is distributed to the source and destination nodes so that quantum bits/status can be transferred between them. When two particles are perfectly entangled, they share the same state no matter the distance between them and thus can be used for many operations in quantum networks. Such shared entanglements are the key resource in quantum networks. However, unlike in classic networks, we cannot simply copy the state of one particle due to the no-cloning principle of quantum states.

Entanglement is typically generated and distributed through a process known as *entanglement swapping* or by directly creating entangled particles. Quantum sources, such as those utilizing spontaneous parametric down-conversion, can be employed to generate pairs of entangled photons. These photons can then be sent to different nodes in the network, establishing entanglement between the nodes. Such entanglements can be further extended towards a longer path by the quantum swapping operation. Due to the probabilistic success of entanglement generation and swappings, establishing long-distance entanglement can incur significant latency. Therefore, Ghaderibaneh *et al.* [13] and Pouryousef *et al.* [14] have proposed to pre-distribute entangled pairs over certain pairs of network nodes. By doing so, when needed, entangled pairs can be generated from these pre-distributed pairs quickly.

In this article, we consider the entanglement distribution of entangled pairs from satellites equipped with quantum sources to some pre-determined ground station pairs to support the global quantum network. There are existing works [5]–[7] on using quantum swapping and purification to generate and distribute entanglement along one or multiple paths in a multi-hop quantum network. Such works are orthogonal to the pre-distribution of entanglement we consider here, but complementary and used in conjunction with pre-distribution.

III. Satellite-based Quantum Networks

In satellite-based quantum networks [8]–[11], satellites equipped with entanglement-generating photon sources can create pairs of entangled photons and distribute entanglement between a pair of ground stations, as shown in Figure 1. The reduced signal attenuation in the near-vacuum of space enables the entangled photons to travel longer distances without significant loss of quantum coherence. We adopt a double downlink distribution architecture, where satellites and ground stations act as transmitters and receivers respectively. [11] has demonstrated that satellite-based entanglement distribution can achieve a better rate than the pure ground-based solution.

We consider a space-terrestrial network with certain satellites and ground stations, shown in Figure 1. The sets $S = \{s_i\}$ and $N = \{g_k\}$ represent the satellites (SATs) and ground stations (GSes). We define $P = \{p_j\}$ as the set of demanded ground station pairs (GSPs) requiring pre-distributed entangled pairs, as illustrated in Figure 2(a). For example, demanded pair p_2 needs entangled photons for delivery to GS g_1 and GS g_3.

In this network, an entangled photon can reach GS g_k if the elevation angle $\theta_{i,k}$ between satellite s_i and the horizon at g_k exceeds an elevation angle threshold θ_e. If both GSes of GSP p_j can satisfy the elevation threshold, the fidelity of entangled photon pair received at p_j from s_i can be approximated by $F_{i,j} = \frac{1}{4}\left(1 + \frac{4F_i^0 - 1}{(1+\frac{n_j}{\eta_{i,j}})^2}\right)$. Here, F_i^0 is the initial fidelity of the entangled photon pair at s_i, n_j is the number of background photons received by GSP p_j, and $\eta_{i,j}$ is the space-to-ground transmittance between SAT s_i and GSP p_j. Based on [11], [12], the space-to-ground transmittance $\eta_{i,k}^{sg}$ between s_i and g_k consists of two parts: the free-space transmittance $\eta_{i,k}^{fs}$ and the atmospheric transmittance $\eta_{i,k}^{atm}$, defined as below.

$$\eta_{i,k}^{sg} = \eta_{i,k}^{fs} \cdot \eta_{i,k}^{atm} = \frac{(\pi(d_i^T/2)^2)(\pi(d_k^R/2)^2)}{(\lambda l_{i,k})^2} \cdot e^{-\alpha h_{i,k}}. \quad (1)$$

Note that $\eta_{i,k}^{fs}$ depends on the orbital parameters, such as altitude and zenith angle, while $\eta_{i,k}^{atm}$ depends on the atmospheric conditions, e.g., turbulence and weather conditions. Here, $l_{i,k}$ is the distance length between s_i and g_k, $h_{i,k}$ is the distance height between g_k and atmospheric boundary when connected to s_i, and α is the atmospheric extinction coefficient. d_i^T and d_k^R are the diameters of the transmitter and receiver telescopes at s_i and g_k, respectively. These telescopes operate at a specific wavelength λ. We assume that $\eta_{i,j} = \eta_{i,k_1}^{sg} = \eta_{i,k_2}^{sg}$ if g_{k_1} and g_{k_2} are the GSes in GSP p_j.

IV. Optimal Entanglement Distribution Problem

Khatri *et al.* [11] have first considered a global-scale quantum network consisting of a constellation of orbiting satellites that provides a continuous, on-demand entanglement distribution service to ground stations. Particularly, they aim to determine optimal satellite configurations with continuous coverage that balances both the total number of satellites and entanglement-distribution rates. They propose a greedy approach to select the satellite assignments to maximize the achievable entanglement-distribution rates for multiple ground station pairs and show the advantage over the rates of ground-based quantum repeater schemes.

Most recently, Panigraphy *et al.* [12] then further formally defined the *optimal entanglement distribution problem* in satellite-based quantum networks, where the entanglement distribution from a constellation of orbiting satellites is assigned to a set of ground station pairs. The optimization goal of this problem is to maximize the aggregate entanglement distribution rate subject to various resource constraints at the satellites and ground stations. They formulated their problem as an integer linear programming problem but only solved it efficiently for two specific scenarios (i.e., when $L_j = 1$, $R_k = 1$ and $T_i = 1$ or when $L_j = 1$, $R_k = |S|$ and $T_i = 1$).

We now formally introduce the optimal entanglement distribution problem for a polar satellite constellation. Each satellite s_i has an entangled photon source and T_i transmitters to send entangled photon pairs to multiple GSPs. Each GS g_k has R_k receivers to receive photons and create entanglement for quantum applications. We assume that there is a set of GSPs P demanding entanglements. Figure 2(b) shows five pairs of GSPs and their potential coverage from all SATs. The optimization problem is to assign satellites to cover these GSPs. The binary decision variable $x_{i,j}$ indicates if SAT s_i provides entanglement for GSP p_j, with associated weight/utility $w_{i,j}$ representing the entanglement generation rate or arrival rate of requests. The objective is to maximize the weighted utility, leading to the formulation of the optimal satellite assignment problem as follows.

$$\max_{x_{i,j}} \quad \sum_{s_i \in S} \sum_{p_j \in P} w_{i,j} x_{i,j} \tag{2}$$

$$\text{s.t.} \quad \sum_{s_i \in S} x_{i,j} \le L_j, \quad \forall p_j \in P, \tag{2a}$$

$$\sum_{s_i \in S} \sum_{p_j \in P, g_k \in p_j} x_{i,j} \le R_k, \quad \forall g_k \in G, \tag{2b}$$

$$\sum_{p_j \in P} x_{i,j} \le T_i, \quad \forall s_i \in S, \tag{2c}$$

$$x_{i,j} = 0, \quad \forall F_{i,j} < F_j^{th}, s_i \in S, p_j \in P, \tag{2d}$$

$$x_{i,j} \in \{0, 1\}, \quad \forall s_i \in S, p_j \in P. \tag{2e}$$

Here, Constraint (2a) ensures that each GSP $p_j \in P$ can only connect to L_j satellites simultaneously. Constraint (2b) means that a GS g_k can be part of multiple GSPs and thus is not allowed to be allocated to more than R_k satellites due to its limited number of receivers. Constraint (2c) ensures that SAT s_i does not get allocated to more than T_i GSPs due to its limited number of transmitters on board. Constraint (2d) confirms that the fidelity of entanglement $F_{i,j}$ after transmitting is larger than the fidelity threshold F_j^{th} for those assigned. This also implies that the elevation angle is sufficient for achieving fidelity.

V. Solving The Optimization Problem

We now discuss several ways to solve the defined optimal entanglement distribution problem.

A. Greedy Heuristics

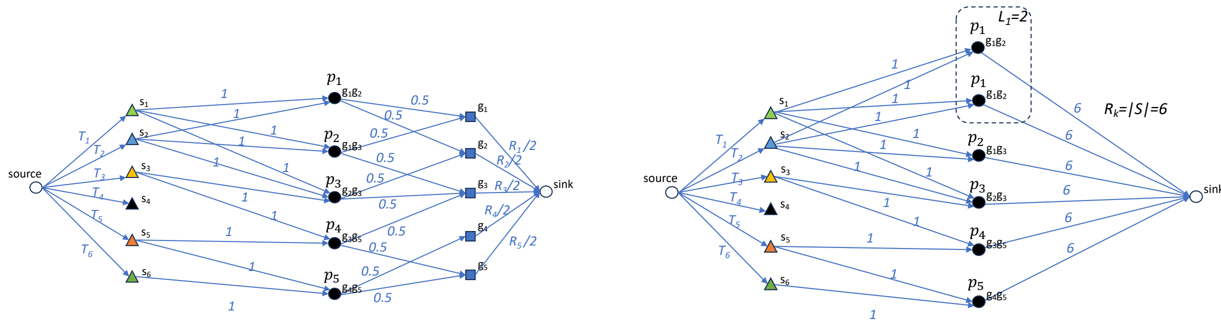
In [11], Khatri *et al.* consider the problem when $L_j = 1$ and $T_i = 1$, i.e., each GS can only distribute entanglement to one GSP, and each GSP can only be assigned one satellite. For this scenario, they propose a simple greedy algorithm: each satellite is assigned to the GSP that has the lowest loss among all GSPs within the visibility of the satellite. In addition, if a GSP has only one satellite in view, that satellite will be assigned to this lone GSP. In our formulation, the lowest loss between a SAT s_i and a GSP p_j corresponds to the highest fidelity ($F_{i,j}$). In other words, we can greedily select the maximal fidelity of satellite-GSP assignment among all remaining demanded GSPs and SATs in each step. We call this method ***Greedy-Loss***. This method selects assignments with good fidelity but may not maximize the overall utility. This has been confirmed by [12] in their simulations.

An alternative greedy strategy will be to greedily find an SAT-GSP assignment with the maximal utility $w_{i,j}$ among all remaining demanded GSPs and satellites at each step. We call this method ***Greedy-Utility***. Compared with *Greedy-Loss*, this method is more directly to optimize the optimization goal. However, it still cannot guarantee to find the optimal solution.

Both greedy methods are general enough to be applied in all possible cases (different values of T_i, L_j, R_k). The running time is also within polynomial time and is efficient for large-scale space-terrestrial networks.

B. Maximum Independent Set based Approach

In [12], Panigraphy *et al.* convert the original optimization problem when $L_j = 1$, $R_k = 1$, and $T_i = 1$ to a weighted version of the maximum independent set (MIS) problem, where each vertex is an assignment of s_i to p_j and an edge between two vertex exists if the two vertex share a satellite or a GS. However, MIS is NP-hard for general graphs. Therefore, they propose an approximation algorithm to solve the problem. We call such method ***MIS-Approx***, which is only useful for the special case of the problem with $L_j = 1$, $R_k = 1$, and $T_i = 1$. Obviously, the approximation algorithm cannot give the optimal solution even for such a special case.



(a) Case 1 (b) Case 2

Fig. 3. Examples of flow models for maximum flow-based method: Case 1, when $L_j = 1$, i.e., each GSP can only be served by one satellite; Case 2, when $R_k = |S|$, i.e., each GS has sufficient receivers to receive from all satellites.

C. Maximum Bipartite Matching based Approach

Panigraphy *et al.* [12] model the original optimization problem when $L_j = 1$, $R_k = |S|$, and $T_i = 1$ as the maximum weight bipartite matching problem where the SATs and GSPs are nodes in the bipartite graph. Such a maximum matching problem can be solved optimally by the Hungarian algorithm in polynomial time. We call such method ***Max-Match***. For this case, $R_k = |S|$, i.e., each GS has a sufficient number of receivers to receive from all satellites at the same time.

D. Maximum Flow based Approach

We now propose a maximum flow-based method to solve two more general cases of the optimization problem (beyond the special cases studied and solved by [12]), as shown in Figure 3.

Case 1 with $L_j = 1$: Each GSP can only be served by one satellite. Note that this case covers both special cases studied by [12] but is more general. We define a graph model, which includes all satellites, GSPs, GSs, a virtual source, and a virtual sink, as shown in Figure 3(a). A link between s_i and p_j with a unit capacity represents the assignment of that SAT to cover that GSP. For each GSP p_j, there are two outgoing links with 0.5 capacity towards g_{k_1} and g_{k_2}, respectively, who are the two GSs in p_j. There is a link from the source to each SAT s_i with the capability of T_i, and there is a link from each GS g_k to the sink with the capability of $\frac{R_k}{2}$. Weight $w_{i,j}$ is given to the link from s_i to p_j, when all other links have zero weights. The original optimal entanglement distribution problem now becomes a weighted maximum flow problem in such a weighted graph. We can use a weighted maximum flow algorithm to solve it. However, note that this problem is a little bit different from the traditional maximum flow problem in a general graph since we request the two outgoing links from a GSP to be selected or not be selected simultaneously. Therefore, we have to modify the traditional maximum flow algorithm to guarantee that during the argument path selection. A formal proof of such guarantee is left as future work.

Case 2 with $R_k = |S|$: Each GS has sufficient receivers to receive from all satellites. This will release the flow constraints from GSPs to the sink. Therefore, we can simply draw a direct link from GSP to the sink with a capacity of $|S|$. On the other hand, we can allow $L_i \geq 1$ and $R_k \geq 1$ compared with [12]. We duplicate the GSP node of p_j for L_j times. An example of $L_1 = 2$ is given in Figure 3(b). This case can be simply solved by the classical weighted maximum flow algorithm.

We call this method ***Max-Flow***. We also anticipate this approach may be extendable to more general cases (such as general L_j, T_i, and R_k), but we leave it as an open problem for future works.

E. Linear Programming based Approach

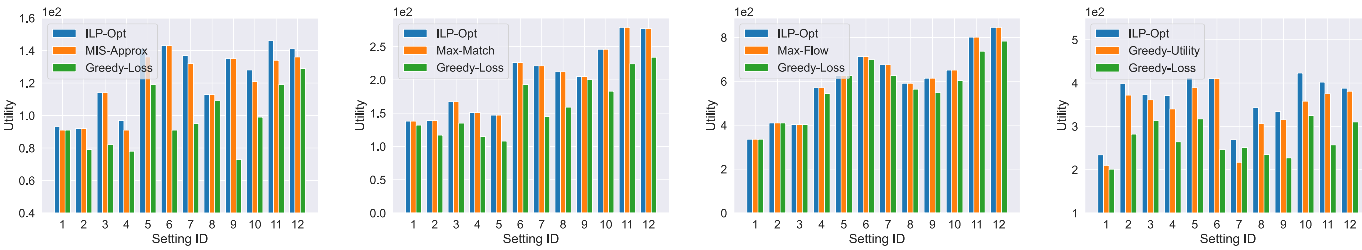
Last but not least, the formulated problem (Equation (2) is a basic integer linear programming (ILP) problem, thus a classical LP solver can be used. While linear programming (LP) is solvable in polynomial time, ILP is NP-hard in general, which makes it hard to solve efficiently. Fortunately, as experimental results are shown in Section VI, a classical LP solver (such as Gurobi) can perform nicely in solving our optimal entanglement distribution problem in most general cases. We call this method ***ILP-Opt***.

VI. PERFORMANCE EVALUATIONS

In this section, we evaluate the performance of all aforementioned entanglement distribution methods in Section V in terms of the achieved weighted utility.

Space-terrestrial Network Architecture: Our network architecture utilizes a polar satellite constellation, as detailed in [11], [12], with 10 rings of satellites in polar orbits, each containing 10 satellites at altitudes ranging from $2,000$ *km* to $10,000$ *km*. We focus on scenarios where only a few satellites are visible to specific ground stations within fixed time windows. Additionally, we designate several long-distance cities (e.g., New York, London, Rio de Janeiro, Mumbai, Cape Town, Beijing, Sydney, Singapore, and Vancouver) as GS, with a total of 36 GSPs, and randomly select a subset of GSPs for establishing entanglement links. The number of transmitters at each SAT and the number of receivers at each GS are randomly chosen from the ranges of 6 to 10 and 2 to 6, respectively.

Space-terrestrial Channel Parameters: We follow [11], [12] to set all parameters in our space-terrestrial quantum network. For the loss and noise parameters in photon transmission from satellites to ground stations, we set the atmospheric extinction coefficient α at 0.028125 and the wavelength λ at 737 *nm*. The transmitter telescope diameter at satellites and the receiver telescope diameter at ground stations are set to 0.2 *m* and 2 *m*, respectively. The elevation angle threshold θ_e for any satellite and GSP is set to $20°$ based on [8].



(a) Case A (b) Case B (c) Case C (d) Case D

Fig. 4. Simulation results for different cases: (a) Case A, $L_j = 1, R_k = 1, T_i = 1$; (b) Case B, $L_j = 1, R_k = |S|, T_i = 1$; (c) Case C, $L_j \geq 1, R_k = |S|, T_i \geq 1$; (d) Case D, $L_j \geq 1, R_k \geq 1, T_i \geq 1$.

We test all entanglement distribution methods under various cases (different ranges of L_j, R_k, T_i) in 12 different network settings. Figure 4 shows the results. Each of the plots arranges the network settings in ascending order based on the number of satellites and GS used. The smallest scale setting, for instance, has 13 satellites and 5 GSs. The largest scale setting involves 48 satellites and 9 GSs.

A. Case A: $L_j = 1, R_k = 1, T_i = 1$

This is the first case study in [12]. We compare the performances of *Greedy-Loss* [11], *MIS-Approx* [12] and *ILP-Opt*, and the results are reported in Figure 4(a). Obviously, *ILP-Opt* outperforms *MIS-Approx* and *Greedy-Loss* across all experimental settings, producing the highest total utility. Recall that *Greedy-Loss* more focuses on fidelity while *MIS-Approx* is only an approximation algorithm.

B. Case B: $L_j = 1, R_k = |S|, T_i = 1$

This is the second case studied in [12]. Figure 4(b) shows the performances of *Greedy-Loss* [11], *Max-Match* [12] and *ILP-Opt*. For this case, both *ILP-Opt* and *Max-Match* can solve the problem optimally, while *Greedy-Loss* still suffers.

C. Case C: $L_j \geq 1, R_k = |S|, T_i \geq 1$

This is a more general case than Case B, since it allows L_j and T_i to be larger than 1. Figure 4(c) shows both *ILP-Opt* and *Max-Flow* can solve the optimization problem optimally.

D. Case D: $L_j \geq 1, R_k \geq 1, T_i \geq 1$

Last is the most general case. Figure 4(d) confirms that (1) *ILP-Opt* can solve the optimization problem optimally in this general case; (2) *Greedy-Utility* outperforms *Greedy-Loss* since it more directly optimize the objective of utility.

E. Computation Time

Table I demonstrates the average computation time of different algorithms for 12 network settings in different cases. *Greedy-Loss* achieves the lowest computation time in all cases but with worse utility. *ILP-Opt* gains the best utility with acceptable computation time. In summary, *ILP-Opt* can solve the problem in all cases very efficiently, due to relevantly small scales. If the scale of the problem is too large to prevent the usage of *ILP-Opt*, *Max-Match*, *Max-Flow* and *Greedy-Utility* could be a good alternative solution for Case B, Case C and Case D, respectively. There is always a trade-off between performance and time complexity.

TABLE I
AVERAGE COMPUTATION TIME COMPARISON.

Algorithms	Avg. Computation Time (ms)			
	Case A	Case B	Case C	Case D
ILP-Opt	0.801	0.733	1.202	1.253
MIS-Approx	40.698	/	/	/
Max-Match	/	0.477	/	/
Max-Flow	/	/	1.435	/
Greedy-Utility	/	/	/	0.399
Greedy-Loss	0.133	0.198	0.466	0.367

VII. Opportunities and Challenges

While we have demonstrated *ILP-Opt* can solve the optimization problem relevantly easily, there are still some unanswered questions regarding the satellite-based entanglement distribution and many potential extensions to more challenging problems.

Polynomial Solution for All Cases or NP-Hardness: We need to explore the NP-hardness of these optimization problems in Cases A and D and investigate if polynomial algorithms exist for these problems.

Fidelity Consideration, Quantum Swapping, and Purification: In the mathematical formulation in this paper, the assumption is the entanglement can be generated and distributed as long as the elevation angle and fidelity satisfy the thresholds. However, if we integrate fidelity into the optimization objective function, the new optimization problem becomes more complex. In addition, if we allow the GSP to perform quantum swapping and/or purification, then we can satisfy or cover more GSPs. However, solving such optimization problems will become more challenging.

Quantum Memory and Entanglement Allocation: In the formulated problem, we only consider a unit of entangled pairs is needed for each GSP. However, if both SAT and GS are equipped with quantum memory and can hold more entanglement, then the optimization problem can include new constraints on quantum memory and also introduce new decision variables such as assigned entanglement number or rate for each assignment. Then, this problem will have more nonlinear constraints/objectives and become a mixed integer nonlinear programming model. It is more difficult to design algorithms for such a problem. Possible solutions include hybrid quantum-classical approaches [15].

Multi-Partite Entanglement Distribution: Here we only consider distributing bipartite entanglement to two GSes. If we take the most recent multi-partite entanglement into consideration, the entanglement distribution could be performed among multiple sets of GSes.

Dynamic Satellite-Ground Channels, Satellite Mobility, Satellite Constellation: In current studies, we treat the locations of satellites to be static during the entanglement distribution. However, it will be interesting to see how the dynamic channels and satellite mobility affect the distribution. In addition, we may investigate entanglement distribution within different types of satellite constellations or with the help of the intermediate stations. Note here we only distribute the entanglement via a direct downlink from SAT to GS.

VIII. Conclusion

This article provided a concise overview of recent studies concerning the optimal entanglement distribution problems in satellite-based quantum networks. We revisited the specific optimization problem introduced by [12] and presented various potential methods to address the problem under various scenarios. Notably, we introduced a new maximum flow-based approach and an ILP-based approach, demonstrating not only broader applicability but also superior performance compared to existing methods, as confirmed by experimental results. Furthermore, we discussed future opportunities and challenges in this area. We believe that while satellite-based entanglement distribution offers tremendous opportunities for realizing a global-scale quantum network, it also poses substantial research challenges in designing efficient optimal entanglement distribution algorithms for various scenarios. For example, the optimal entanglement distribution in a more complex and dynamic space-terrestrial integrated quantum network whose entities are from different organizations becomes much more challenging, and may require additional advanced techniques, such as deep reinforcement learning or game theoretical approaches, to tackle.

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