Entanglement Management Through Swapping Over Quantum Internets

Yiming Zeng, Jiarui Zhang, Ji Liu, Zhenhua Liu, and Yuanyuan Yang

Abstract

Quantum Internet has the potential to support a wide range of applications in quantum communication and quantum computing by generating, distributing, and processing quantum information. Generating a long-distance quantum entanglement is one of the most fundamental functions of a quantum Internet to facilitate these applications. However, entanglement is a probabilistic process, and its success rate drops significantly as distance increases. Entanglement-swapping is an efficient technique used to address this challenge. How to efficiently manage the entanglement through swapping is a fundamental yet challenging problem. This article considers two entanglement-swapping methods: (1) Bell state measurement (BSM) entanglement-swapping: a classic entanglement-swapping method that is able to fuse two successful quantum links, (2) Greenberger-Horne-Zeilinger (GHZ) measurement entanglement-swapping: a more general and efficient swapping method which is capable of fusing *n* successful quantum links. The goal is to maximize the entanglement rate for multiple quantum processor unit (QPU) pairs over the quantum Internet with a general topology. Two efficient entanglement management protocols are proposed which respectively make use of the unique properties of BSM and GHZ. Evaluation results highlight that the proposed protocols outperform the existing ones.

Introduction

Quantum computing is an emerging computing paradigm that holds great promise of harnessing quantum advantage to revolutionize information technology across various sectors, including finance [1], and cryptography [2]. Compared to classical computing, quantum computing applications have shown capabilities far beyond traditional computing ways. For instance, Shor's algorithm and the quantum linear system algorithm [3] significantly reduce time complexity.

In the broad context of quantum information science, the quantum Internet plays a crucial and foundational role, contributing significantly to both the theoretical analysis and practical realization of quantum computing and communication. A number of experimental quantum Internet have already been established in research laboratories. Examples include a long-distance (40 kilometers) teleportation link over fiber [4], and an integrated entanglement system facilitated by satellites, capable of supporting entanglement over distances exceeding 4600 kilometers [5].

Long-distance entanglement is essential for the quantum Internet, but the entanglement process is probabilistic and inherently unstable as quantum bits (qubits) created by photons are extremely fragile. The successful entanglement rate among qubits decreases exponentially with the transmission length. Meanwhile, quantum processor unit (QPU) pairs trying to be entangled may be too distant from each other to be directly connected through links. *Entanglement-swapping* is an important method that can establish an entanglement path between those pairs of QPUs that have not shared an entanglement. Some quantum repeaters are strategically placed within the Internet as relays, providing end-to-end entanglements for multiple users who require them [6], [7]. Quantum repeaters are quantum processors equipped with quantum memories (i.e., qubits) and have the ability to perform entanglement-swapping [8].

The *entanglement management* problem, which considers *how to efficiently manage qubits in quantum repeaters to build long-distance entanglements*, is crucial for the functionality of quantum Internet. Thoughtful design for the entanglement management on the quantum Internet can boost quantum Internet performance by efficiently utilizing resources, e.g., repeater memories. While large-scale quantum Internet has not been implemented outside of the research lab due to physical and experimental challenges, investigating the entanglement management problem from the network layer will be valuable to contribute to the successful implementation of quantum Internet in the future.

The entanglement management problem has drawn great attention in the research community recently, yielding some compelling outcomes. However, existing methods are still facing three major limitations: **(1)** A majority of studies have primarily introduced entanglement management algorithms and corresponding theoretical analyses for a limited set of specific system topologies; **(2)** Most research efforts concentrate on comparatively simplistic quantum Internet configurations, for instance, the processors are predominantly limited to performing Bell State Measurement (BSM); **(3)** Existing entanglement routing algorithms lack efficiency, which is mostly based on heuristics, and do not leverage the available system resources.

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Yiming Zeng, Jiarui Zhang, Ji Liu, and Yuanyuan Yang (corresponding author) are with the Department of Electrical and Computer Engineering, Stony Brook University, Stony Brook, NY 11794 USA; Zhenhua Liu is with the Department of Applied Mathematics and Statistics, Stony Brook University, Stony Brook, NY 11794 USA. Yiming Zeng and Jiarui Zhang contributed equally to this work.

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Motivated by the challenges in current studies, this article aims to improve the entanglement efficiency over the quantum Internet. In particular, we give:

- Protocol 1 for the entanglement management under BSM entanglement-swapping in a quantum Internet with a general topology.
- Protocol 2 for the entanglement management under Greenberger-Horne-Zeilinger (GHZ) projective measurement entanglement-swapping in a quantum Internet with a general topology.

Elementary Quantum Background

In this section, we introduce some basic terminologies and quantum backgrounds that we will use in this proposal.

Terminology

Qubit: A qubit is the basic unit of quantum information. Unlike an electronic bit that can only have either 0 or 1, a qubit can exist in a "superposition" of states where it can be 0 and 1 at the same time.

Quantum Link: A quantum link can transmit quantum states, such as superposition or entanglement, from one qubit to another. It can be established using a variety of physical systems, including optical fibers, free space, and satellites, among others.

Quantum Repeater: The loss in quantum information transmission increases exponentially with distance. A quantum repeater mitigates this problem by dividing the distance into shorter segments, thus reducing the loss exponentially. In this work, we consider a more generalized model of a quantum repeater, one that possesses multiple ports and can connect any port with any other port that is not currently in use. Such capability to connect between two arbitrary ports is similar to that of an Ethernet repeater. Its main function is to direct a photon from an input port to an output port.

Quantum Processor Unit (QPU): A QPU is a device designed to execute quantum algorithms and facilitate quantum communication. It consists of a collection of qubits that are the quantum analog of electronic bits, and quantum gates that are the basic building blocks for performing operations on qubits. In this work, QPUs try to entangle with others for quantum computing or communication.

Bell States: The Bell States or *Einstein-Podolsky-Rosen (EPR)* pairs are specific quantum states of two qubits that represent the simplest (and maximal) examples of quantum entanglement. The states are described by a wave function that is a superposition of possible states of individual qubits. In this article, we assume that all quantum links between repeaters share EPR pairs.

Entanglement: This is a quantum mechanical phenomenon in which the quantum states of two or more objects become intertwined so that one object can no longer be adequately described without the complete mention of the other(s), even though they may be spatially separated. This leads to very strong correlations between the observable physical properties of the systems. The entanglement is fundamental to supporting quantum communication and computing.

Entanglement-Swapping

Entanglement-swapping is a quantum process whereby qubits from two distinct QPUs, each entangled with a shared QPU, can become directly entangled through the intervention of this common QPU. This mechanism is fundamental to a quantum repeater and is conceptually analogous to an intermediate electronic node "connecting" two other nodes. In this sub-section, we will initially discuss the classic swapping method rooted in BSM, followed by an introduction to a more general entanglement method utilizing GHZ measurements.

BSM Entanglement-Swapping: As illustrated in Figure 1(a), a quantum repeater is entangled with two QPUs simultaneously through two distinct EPR pairs over quantum links. The quantum repeater then performs BSM. Following the measurement, the qubits in the repeater, which were previously entangled with the QPUs, are released, resulting in the two QPUs becoming entangled.

GHZ Entanglement-Swapping: A repeater entangled with $n (n \geq 2)$ QPUs concurrently. The repeater then executes GHZ projective measurements, which can concurrently fuse n quantum links, thereby releasing the qubits in the repeater that were previously entangled. Figure 1(b) illustrates an example of entanglement-swapping to concurrently fuse 3 quantum links.

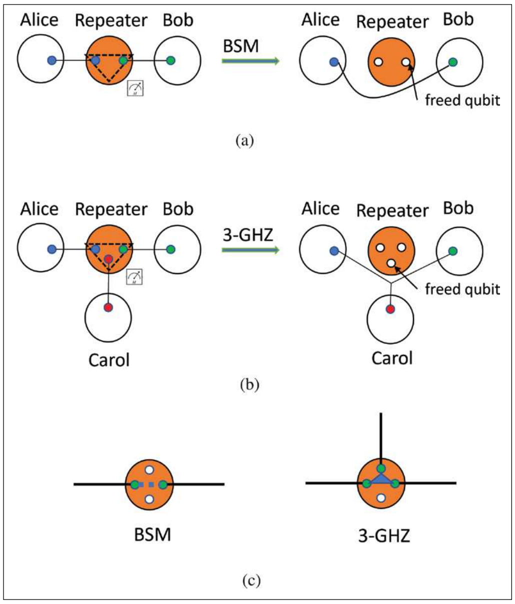

FIGURE 1. Example figures of entanglement-swapping under BSM and GHZ. a) A BSM measurement in the repeater that fuses two quantum links by connecting two qubits. b) A 3-GHZ measurement in a repeater that fuses three quantum links by connecting three qubits. In both figures, the small blank circle in the repeater denotes free qubits that are not entanglement, the small green circle in the repeater denotes entangled qubits, The orange line indicates the quantum links to be fused, and the blue line shows the connection between qubits to fuse quantum links. The triangle indicates the measurement of qubits inside the repeater. c) Examples of route building under BSM and 3-GHZ.

GHZ entanglement-swapping is a more general case compared with BSM, which is in principle not much harder than BSM in solid-state qubit memories [9]. BSM entanglement-swapping is a special condition when $n = 2$.

Entanglement Process

The entanglement process under BSM entanglement-swapping has been thoroughly explored in prior studies [10], [11]. This model, however, delves into a more generalized entanglement process where repeaters can execute GHZ entanglement-swapping. The classic swapping process can be regarded as a special case within this broader context.

The entanglement process under GHZ measurements encompasses two phases. Phase I involves the network preparing for the entanglement, while Phase II involves the network executing the entanglement over optical fibers and carrying out quantum link fusions through entanglement-swapping inside the repeaters. The details of these two phases are described as follows.

Phase I: the primary objective is to design routing paths for the entanglement management for QPUs in an offline manner and relay these designed paths to all repeaters involved in the entanglement process. The entanglement management is executed by classical computing devices in the cloud, as the computation time for tasks such as routing problems in the classical computing domain remains substantially shorter than in the quantum domain.

The cloud has access to the following information about the network: details about the QPUs, the network topology (including repeater placement and connections), and repeater information (such as the number of qubits in each repeater). With this comprehensive information, the cloud calculates routing paths for quantum states shared between QPU pairs, taking into account the capacity limits of the repeaters and optical fibers. The routing paths computed by the cloud are then transmitted via classical channels to QPUs for the entanglement process in Phase II.

Phase II: Phase II encompasses three steps.

- The first step entails synchronizing all repeaters' time before commencing the entanglement process.
- Subsequently, the network attempts to generate quantum entanglement over quantum links, using the fixed routing paths determined in Phase I. The second step involves repeaters implementing entanglement-swapping for successfully entangled links.
- It is important to note that the entanglement process is probabilistic, which may result in failures to generate or fuse certain entanglement links within repeaters. Furthermore, the duration of entanglement is quite short, making it impractical for the cloud to gather repeater information due to significant transmission delays and rescheduling paths for failed links during the entanglement duration. A repeater can only access information from other repeaters a few hops away. Given these constraints, the repeaters will endeavor to design and construct recovery routes for entanglement locally in an online manner.

Problem Statement

In this section, we first introduce the system model. Subsequently, we provide a definition and a formulation for the entanglement management problem prevalent in the quantum Internet.

In the present study, we focus on modeling the quantum Internet by considering only the basic and essential components required for long-distance entanglement. This foundational model paves the way for future applications with more intricate considerations such as coding, fidelity, purification, and error correction. Consequently, the protocol we design can be readily adapted to accommodate more complex conditions through tailoring protocols.

System Model

As shown in Figure 2, the structure of the quantum Internet is composed of QPUs and repeaters, interconnected through optical fibers.

QPUs: We define the set of QPUs as $\mathcal{M}$ that consists of M QPU pairs. We assume that QPUs have sufficient qubits for the entanglement.

Quantum Repeater: We define the set of quantum repeaters as $\mathcal{N}$ that consists of N repeaters. Each quantum repeater $n_i \in \mathcal{N}$ has Q_i qubits that can be assigned for the entanglement. Edge e_{ij} is an optical fiber link connecting v_i and v_j for transmitting qubits. We assume that repeaters have the same successful entanglement-swapping rate for both BSM and GHZ since GHZ is in principle not much harder than BSM in solid-state qubit memories [9]. The successful entanglement-swapping rate in each repeater for any pair of qubits is uniform and denoted as $q \in [0, 1]$.

Optical Fiber: QPUs are connected with quantum repeaters by optical fibers. In optical fiber cable e_{ij}, there are c_{ij} cores. Each core can be used as a quantum link for the entanglement of a pair of qubits. Therefore, multiple qubits can be assigned on an edge for the entanglement at the same time. The cable length of e_{ij} is denoted as L_{ij}. The success rate of each attempt to generate entanglement over e_{ij} is $p_{ij} = e^{-\alpha L_{ij}}$, where α is a positive constant depending on the physical material. Since p_{ij} only depends on the cable length and cable material, successful entanglement rates for different pairs of qubits over different cores on the same edge are the same.

Model Overview: We model a quantum Internet with N quantum repeaters and M QPU pairs as an undirected graph $G = (\mathcal{V}, \mathcal{E})$, where

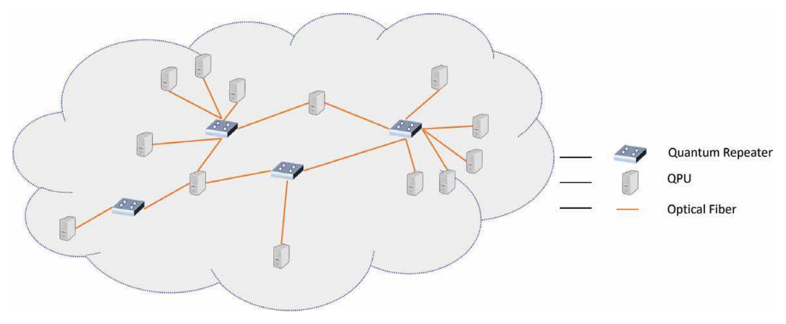

FIGURE 2. An example of network.

In summary, GHZ-based entanglement-swapping introduces more flexibility in entanglement management, as repeaters can establish connections with more than one other repeater.

$\mathcal{V} = \mathcal{M} \cup \mathcal{N}$ denotes the set of nodes, and $\varepsilon = \{e_{ij}\} \subset \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$ denotes the set of links.

Entanglement Management Problem Formulation

In this study, we explore an entanglement management problem within the framework of the quantum Internet model previously defined. Within the quantum Internet G, the objective of QPU pairs is to establish entanglement among themselves. A QPU could concurrently maintain distinct states with different QPUs, and a single QPU pair could share multiple quantum states. It is assumed that QPUs possess ample quantum memories (qubits) to facilitate entanglement.

We will mainly focus on two entanglement-swapping methods (i.e., under BSM and GHZ) and we will independently design entanglement management protocols for each method. There are two constraints we must contend with. The first is the capacity of the repeater, represented by the number of qubits, and the second is the capacity of the optical fibers. The total resources used for entanglement must not exceed the capacities of the optical fibers and the repeaters.

The objective of this study is to maximize the entanglement rate in the quantum Internet, which is defined as the expected number of shared quantum states between QPU pairs. Our proposed protocols are flexible and can be readily adapted to optimize other network metrics, such as the number of served QPU pairs, and the success rate of entanglement, among others.

Entanglement Management Protocol Design

In this section, we will present entanglement management protocols for BSM and GHZ entanglement-swapping methods, respectively.

Entanglement Management Impacts Under BSM and GHZ

As previously mentioned, the primary distinction between entanglement-swapping under BSM and GHZ lies in the number of quantum links that a repeater can simultaneously fuse. BSM facilitates the fusion of just two quantum links, whereas GHZ allows for the fusion of more than two quantum links at a time.

This difference between BSM and GHZ will have a significant impact on entanglement management. The Internet needs to manage qubits of repeaters to design routes for building entanglement.

As illustrated in Figure 1(c), under BSM, a pair of qubits within a repeater is dedicated to one virtual path (here, 'virtual path' merely denotes that these two qubits are exclusively assigned to a single QPU pair for entanglement through swapping). Consequently, under BSM, paths only exist between pairs of QPUs.

Figure 1(c) depicts an example of a repeater conducting 3-GHZ projective measurements. This results in three virtual paths linking three qubits within the repeater. As seen, 3-GHZ generates branches at the repeater rather than just linear paths. Thus, for the route connecting a pair of QPUs, GHZ entanglement-swapping will create a flow graph where repeaters along the route may exhibit branches that connect to more than one other repeater.

In summary, GHZ-based entanglement-swapping introduces more flexibility in entanglement management, as repeaters can establish connections with more than one other repeater. Additionally, GHZ allows for more efficient resource utilization since it can fuse a variable number of qubits, subject to capacity constraints, while BSM is limited to an even number of qubits. On the other hand, BSM-based entanglement-swapping is efficient for entanglement management as it only creates paths between QPU pairs. However, GHZ may introduce significant computational overhead in managing entanglement over the quantum Internet due to the increased complexity of entanglement management.

Overview of the Protocol Design

There are three main procedures involved in the design of the protocol for both BSM and GHZ entanglement-swapping methods.

The first procedure is to construct a subset of viable paths. This step reduces computational overhead and ensures sufficient paths for building routes for entanglement under both entanglement-swapping methods when designing entanglement protocols. A detailed analysis of the path selection process is provided in Subsection IV-C.

The second procedure is to develop the entanglement management protocol for *Phase I* of the entanglement process. This is an offline process where the protocol design can utilize all available network information.

The third procedure is to construct the entanglement management protocol for *Phase II* of the entanglement process. This is an online process where the repeaters have access only to local information within a few hops, due to the short decoherence time of the entanglement.

The subsequent sections will be structured into four parts. In Subsection IV-C, we will demonstrate the process of path selection to build a set of paths for the entanglement protocol design. In Subsections IV-D and IV-E, we will respectively design entanglement management protocols for *Phase I* under BSM and GHZ entanglement-swapping methods. In Subsection IV-F, we will develop an entanglement management protocol for *Phase II*, which is applicable to both BSM and GHZ entanglement-swapping methods.

Path Selection

Before introducing the protocols, it is crucial to establish a viable path set for the entanglements between QPU pairs. These paths will then serve as the foundation for constructing routes for the entanglement under both BSM and GHZ entanglement-swapping scenarios.

We cannot explore all potential paths for establishing entanglement routes due to the immense computational overhead it would cause. In a complete graph, there could be up to $|\varepsilon|!$ paths between a single QPU pair (given that quantum repeaters can be selected multiple times), where $|\varepsilon|$ represents the number of optical fibers in $\mathcal{G}$. This vast set of paths could significantly hamper problem-solving efficiency.

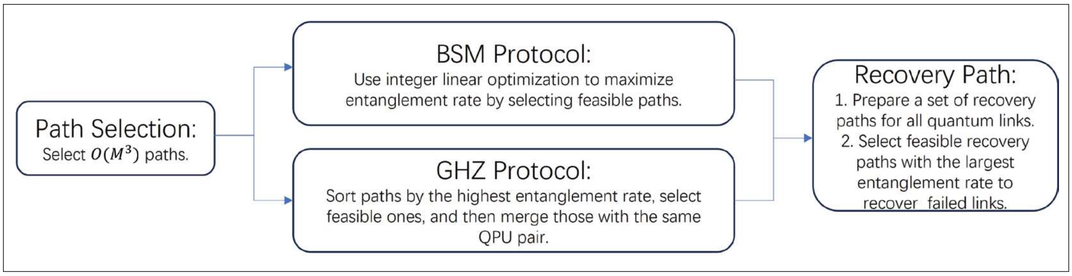

FIGURE 3. An overview of the protocol design is as follows: Initially, the protocol prepares a path set, selecting $O(M^3)$ paths with the highest entanglement rate. Subsequently, both the BSM and GHZ protocols leverage these paths to manage the entanglement of QPU pairs. Finally, the protocol attempts to recover any failed entanglement paths.

To address that, we select a subset of paths, forming a more manageable feasible path set, denoted as $\mathcal{A}$, for the entanglements of QPUs. The metric for path selection is *the successful entanglement probability of a path*. For each QPU pair, we prioritize paths with the top metrics to be included in the set. Specifically, we employ Yen's algorithm to concurrently identify $O(M^2)$ paths with the highest metrics for a given QPU pair. Yen's algorithm is to find k different shortest paths. Consequently, in set $\mathcal{A}$, we include $O(M^3)$ paths exhibiting the top metrics. This set size of $O(M^3)$ not only ensures an efficient algorithmic time complexity but also optimizes result performance. It's pertinent to highlight that during path selection, qubits can be reused, offering enhanced combinatorial flexibility for the routing protocol.

Entanglement Management Under BSM Entanglement-Swapping

Under BSM, we aim to maximize the entanglement rate of all QPU pairs. To solve this problem, we formulate an optimization problem. We construct an integer programming problem with the following four constraints.

- The paths are in the selected path set $\mathcal{A}$.
- Each path can be assigned an integer number of qubits.
- For any quantum repeater, the total number of qubits assigned for all paths through it cannot be larger than its capacity.
- For any optical fiber, the total number of quantum links over it cannot be larger than its capacity.

The first constraint limits the number of potential entanglement paths. The second constraint restricts that the number of quantum links should be a non-negative integer. The third and fourth constraints enforce that the quantum links used for entanglement cannot exceed the network capacity.

The problem is an integer optimization problem, and finding the optimal solution is NP-Hard [12]. To address this, we first relax integer variables to be continuous. After obtaining a continuous solution by standard linear programming methods, we round it to an integer version as a feasible approximate integer optimal solution.

We then describe the complete process of the protocol. First, the algorithm relaxes the integer constraint to allow the variables to be continuous non-negative real numbers, so that the algorithm can obtain an optimal continuous solution by the standard linear programming methods. Then, the algorithm retains the integer part of all variables and makes decisions through the fraction part. Specifically, for each variable Q^A which indicates the number of qubits assigned to the path A, assuming $Q^A = I + F$, where I is a non-negative integer, and $0 \leq F < 1$ a real number. The algorithm retains the integer part I of all Q^A as the integer solution and removes the occupied qubits in repeaters. For the remaining fractional part, we use a Branch-and-Bound algorithm to consume the remaining qubits in the network. The running time of the Branch-and-Bound algorithm is acceptable because the integer part of the solution has already consumed the most qubits in repeaters. Compared to Q-CAST, as described in [10], which is a heuristic algorithm that sequentially selects paths based on the highest entanglement rates, our method leverages optimization, allowing for more efficient utilization of network resources.

Entanglement Management Under GHZ Entanglement-Swapping

While GHZ entanglement-swapping introduces greater flexibility and options, it also presents significant challenges for entanglement management. Developing strategies to effectively utilize this increased complexity is crucial for optimizing the performance of quantum Internet.

We need to address two main challenges.

- First, determining routes between QPU pairs is challenging, as GHZ entanglement-swapping can generate flow graphs between QPU pairs, whereas BSM entanglement-swapping only produces paths. This added complexity makes route selection more difficult to optimize.
- Second, managing qubits within repeaters also presents a challenge, as minor variations in qubit management can lead to significant changes in routes, consequently impacting the overall performance.

To address these challenges, we adopt an alternative approach, rather than finding routes between QPU pairs directly.

We first select paths and subsequently merge them to form the final routes. This strategy allows us to better manage the complexities introduced by GHZ entanglement-swapping and optimize entanglement management in quantum Internet. More specifically, we begin by enumerating widths from high to low, and then sorting paths with the specific width in decreasing order of

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entanglement rate. Paths connecting the same quantum state will be merged. This merging process is inherently nonlinear; when qubits are commonly used for one QPU pair across different paths, they need to be deducted during the merging. The process will terminate when there are no feasible resources available in the network.

Entanglement Management for Recovery Routes

Recovery routes are designed for QPU pairs that failed to entangle through the designed management. The process of entanglement, as dictated by the algorithm running in the cloud, enables QPU pairs to attempt mutual entanglement. However, due to the probabilistic nature of quantum entanglement, some QPU pairs may fail to achieve successful entanglement. In such cases, recovery routes must be established to facilitate entanglement between these unsuccessful QPU pairs. Finding potential recovery routes in BSM-based and GHZ-based entanglement-swapping is largely similar. The distinction lies in the priority assigned to the routes based on their successful entanglement rate, as computed by either BSM-based or GHZ-based entanglement-swapping. Routes with a higher successful entanglement rate are prioritized for selection as formal recovery routes.

There are two challenges to building recovery routes.

- Each quantum repeater has repeaters' entanglement information within a limited range, i.e. K hops near the repeater. This is because the entanglement process does not last long enough for the quantum repeaters to spread the entanglement information over a large area.
- The number of qubits in repeaters for building recovery routes is limited. This is because the majority of qubits in quantum repeaters have been utilized for the previous entanglement processes.

For a path in $\mathcal{A}$, a potential recovery route can connect two quantum repeaters without involving other repeaters in the path, ensuring the hop distance between any two repeaters does not exceed K. To expedite the process, the cloud precomputes recovery routes for each path in $\mathcal{A}$ and sends the relevant set to repeaters.

During swapping decisions, repeaters share entanglement statuses with neighbors and aim for qubit entanglement from both ends. Repeaters organize recovery routes by expected successful entanglement rate and send setup requests sequentially. If all repeaters in a recovery route agree, they try to establish it. After all requests are made and qubits are not enough, no more routes can be set up. Due to time, communication, and qubit limitations, some QPU pairs might lack recovery routes.

Evaluation Results

We design controlled simulations under different parameters to demonstrate the performance of our proposed entanglement management protocols under both BSM and GHZ entanglement-swapping methods.

Settings

We generate the Internet through Waxman method [13], and Watts-Strogatz method [14]. The area of the quantum network is set as $10k \times 10k$ unit square, each unit may be considered as 1 kilometer.

We compare the network performance with the following algorithms.

- GHZ-P: We name our proposed entanglement management protocol under GHZ entanglement-swapping as GHZ-P. The protocol includes the recovery routes part.
- BSM-P: We name our proposed entanglement management protocol under BSM entanglement-swapping as BSM-P.
- Q-CAST: This is a benchmark from [10] under BSM entanglement-swapping.
- B1: This is a benchmark from [15] extended from single pair to multiple pairs, which uses GHZ entanglement-swapping.

Performance

GHZ versus BSM. From our simulations, it is observed that for a given network with identical resources, our proposed protocol GHZ-P outperforms protocols under BSM. To be specific, compared to Q-CAST, BSM-P, and B1, GHZ-P can boost the network entanglement rate by up to 277%, 2014%, 429%, respectively. This enhanced performance can be ascribed to the fact that n-fusion, being a more efficient swapping method, can utilize network resources better than BSM. Repeaters have the ability to fuse a larger number of quantum links, which can amplify the probability of successful entanglement of QPU pairs' qubits within the same network resources.

The results indicate that BSM-P outperforms most other algorithms, with the exception of GHZ-P. Many existing algorithms, such as Q-CAST and B1, employ a greedy approach, which involves repeatedly selecting a path based on the most optimal metric. Unlike these existing algorithms, BSM-P considers the network's overall performance and constructs an integer optimization problem to derive a solution.

Impact of the number of qubits in a repeater: Figure 6 demonstrates the impact of the repeater's

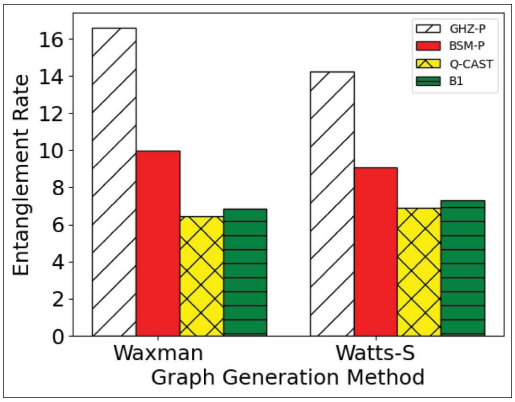

FIGURE 4. The network entanglement rate vs. different network generation methods.

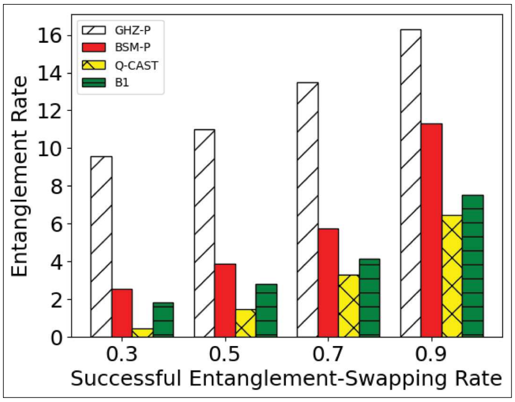

FIGURE 5. The network entanglement rate vs. different successful entanglement-swapping rate q.

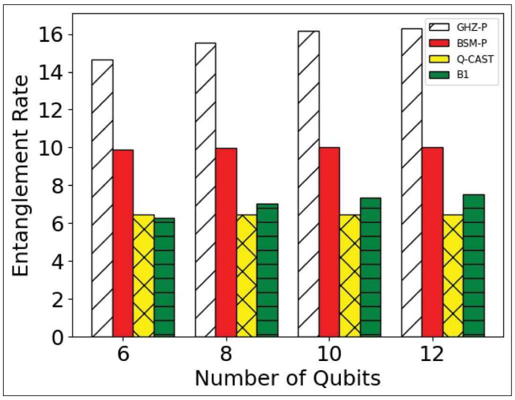

FIGURE 6. The network entanglement rate vs. the number of qubits in a repeater.

qubit number on network performance. As the number of qubits in the repeater increases from 6 to 12, a noticeable boost in the entanglement rate is observed. This can be attributed to the fact that, within the context of the problem, having more qubits equates to a larger network capacity for servicing QPU pairs.

Impact of the successful entanglement-swapping rate: Figure 5 illustrates the influence of the repeater's successful entanglement-swapping rate on network performance. A notable enhancement in the entanglement rate is observed as the entanglement-swapping rate increases. Therefore, constructing repeaters with high entanglement-swapping rates is crucial for the development of large-scale quantum Internet in the future.

Future Issue

The entanglement management raises a number of interesting research opportunities:

- **Multi-Partite Entanglement Management:** Existing papers only focuses on entanglement management between pairs of QPUs. However, many real-world communication scenarios involve more than two parties. Notably, classical multicast is not feasible in quantum networks due to the no-cloning theorem, but distributing multi-partite states to a group of users is possible. Concurrently, numerous quantum computing applications require millions of qubits, while current

Unlike these existing algorithms, BSM-P considers the network's overall performance and constructs an integer optimization problem to derive a solution.

QPUs only support hundreds of qubits. By entangling a set of QPUs, we can significantly enhance computing capabilities for applications that demand a large number of qubits. Designing multi-partite entanglement management and communication protocols is of paramount importance.
- **Quantum Internet Topology Design:** The topology of the Quantum Internet, which encompasses the placement of heterogeneous repeaters with varying capacities and their interconnection via optical fibers, will significantly impact the efficiency of entanglement management and its ability to support quantum applications.
- **Asynchronous Entanglement Management:** In this study, we have only considered the scenario where qubits are used once for the entanglement. However, after entanglement-swapping, the qubits at the two ends of the route become entangled, thereby freeing up the qubits in the repeater for new entanglement operations. These freed qubits can be reused much more quickly, circumventing potential delays in collecting input information.

Conclusion

In this article, we have provided a comprehensive introduction to the problem of entanglement management within the quantum Internet, focusing on both BSM and GHZ entanglement-swapping methods. We have proposed specific entanglement management protocols for both these methods and conducted simulations to evaluate the performance of these proposed protocols. Despite these efforts, many open questions remain. The field of entanglement management through entanglement-swapping offers a rich array of research opportunities.

Acknowledgment

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Biographies

Yiming Zeng (Member, IEEE) (yiming.zeng@stonybrook.edu) received the B.Eng. degree in information engineering from Shanghai Jiao Tong University, Shanghai, China. He is currently pursuing the Ph.D. degree in computer and electrical engineering with Stony Brook University, New York, NY. His research focuses on edge computing and quantum networking.

Jiarui Zhang (Member, IEEE) received the B.Eng. degree in computer science and technology from Shanghai Jiao Tong University in 2017. He is currently pursuing the Ph.D. degree in computer engineering with Stony Brook University. His research interests include blockchain and mobile edge computing.

Ji Liu (Member, IEEE) received the B.S. degree in information engineering from Shanghai Jiao Tong University, Shanghai, China, in 2006, and the Ph.D. degree in electrical engineering from Yale University, New Haven, CT, USA, in 2013. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering, Stony Brook University, Stony Brook, NY, USA. He is an Associate Editor of the IEEE Transactions on Signal and Information Processing Over Networks. His current research interests include distributed control and optimization, distributed reinforcement learning, epidemic, and social networks.

Zhenhua Liu (Member, IEEE) received the Ph.D. degree in computer science from the California Institute of Technology, under the supervision of Adam Wierman and Steven Low. He is an Associate Professor of Operations Research with the Department of Applied Mathematics and Statistics and affiliated with the Department of Computer Science, Stony Brook University (SUNY at Stony Brook). His research aims to develop analytical models, theoretical results, and deployable algorithms to manage complex distributed systems with limited information and network constraints. He has helped HP design and implement the industry's first Netzero Energy Data Center, which was named a 2013 Computer World Honors Laureate. He was recently awarded an IBM 2020 Global University Program Academic Award for his research on resource management of AI/ML systems. His research work is widely cited and recognized in academia, including the Best Paper or Best Student Paper Awards at IEEE INFOCOM, ACM GREENMETRICS, and IEEE Green Computing Conference, 2021 ACM SIGMETRICS Rising Star Research Award, 2021 ACM SIGMETRICS Test of Time Award, the Pick of the Month award by IEEE STC on Sustainable Computing, a SPEC Distinguished Dissertation Award (honorable mention), an NSF CAREER award, and several Excellence in Teaching awards.

Yuanyuan Yang (Fellow, IEEE) (yuanyuan.yang@stonybrook.edu) received the B.Eng. and M.S. degrees in computer science and engineering from Tsinghua University, Beijing, China, and the M.S.E. and Ph.D. degrees in computer science from Johns Hopkins University, Baltimore, MD. She is a SUNY Distinguished Professor of computer engineering and computer science at Stony Brook University, New York, and a National Academy of Inventors (NAI) Fellow. She worked as a Program Director at the National Science Foundation before. She has 30 + years of combined experience in parallel computing, cloud computing, optical networking, and quantum computing. He published 500+ papers in these areas, including three best paper awards and six best paper runner-ups. Her work has culminated in the development of fundamental theoretical advancements and empirical simulation tools, all important background and experiences for the proposed project. She is currently the Editor-in-Chief for IEEE Transactions on Cloud Computing and an Associate Editor for IEEE Transactions on Parallel and Distributed Systems and ACM Computing Surveys. She has served as an Associate Editor-in-Chief for IEEE Transactions on Cloud Computing, Associate Editor-in-Chief and Associated Editor for IEEE Transactions on Computers, and Associate Editor for IEEE Transactions on Parallel and Distributed Systems. She has also served as a General Chair, Program Chair, or Vice Chair for several major conferences and a program committee member for numerous conferences.