Analyzing the functions of multiple external representations of electric potential

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We present an excerpt from an in-class group activity where students generate equipotential curves for a quadrupole using: a whiteboard, a Mathematica notebook, and a 3D plastic graph. Applying Shaaron Ainsworth's framework for the functions of multiple external representations, we analyze how the students used the three representations in concert. We found that the students used different processes for generating each representation. The highly complementary nature of the representations facilitated the group's direct comparisons between representations, helping them to construct deeper understanding about the system and the representations. This case study also exemplifies a limitation of the Functions framework for multiple representations, namely that it does not consider the role of generating representations. We echo the calls to account for student generation in future analyses of the use of multiple representations, when relevant.

2023 PERC Proceedings edited by Jones, Ryan, and Pawl; Peer-reviewed, doi.org/10.1119/perc.2023.pr.Alfson

I. INTRODUCTION & METHODS

Physicists use a variety of representations for solving problems and communicating concepts. These representations are sometimes used separately, but are often used in concert, as a set of multiple external representations (MERs). Much research has already studied the uses of various representations in physics; a subset of which investigates environments with MERs. (See, e.g., Refs [1–3].) The research presented here contributes to the existing body of literature about the use of MERs by applying Ainsworth's Functions framework [4, 5] to a specific context: a single group of students working through an activity involving MERs (shown in Fig. 1). Although Ainsworth's framework is widely cited, more papers demonstrating its use, particularly within physics and in classroom environments, are needed. This paper is intended to fill that gap.

The case study presented here is *interpretive* [6], because it will both illustrate the use of the representations through the Functions framework for MERs, and challenge some of the assumptions of the framework. For example, we will discuss how the students had to generate certain aspects of each representation, which is not addressed by the Functions framework. Ainsworth *et al.* [7] have recognized the need for more focus on multi-representational construction when thinking about how students learn with representations. In the work presented here, we found agreement with Ainsworth, Prain, Tytler, and others that future research should consider how students *generate* multiple representations. We believe this particular case is also interesting because the representations used have significant overlap in information and include a novel representation (the plastic graph). Our research question is: *What facets of this group's use of the MERs are captured by the Functions framework?*

The instructional context of our research is an in-class activity that took place on the second day of an upper-division course on electrostatics in the *Paradigms in Physics* sequence, a set of reformed junior-level courses at Oregon State University [8]. For this activity, students were assigned to work in groups of three. Each group was prompted to draw on a tabletop whiteboard the equipotential curves due to four identical positive point charges at the corners of a square. After working together for some time to complete this part of the activity, the groups were directed to look at a pre-programmed Mathematica notebook displaying many different graphs of the electric potential for the system. Then, after some whole-class wrap-up discussion, the groups were given a second prompt: to sketch the equipotential curves for a new system consisting of a quadrupole square. During this time the groups continued to have access to the whiteboard and the Mathematica notebook and were invited to use a dry-eraseable 3D plastic graph, in which the height corresponds to the value of the electric potential in the plane of the quadrupole.

As part of a larger study of this activity, we examined video data of one three-student group (with pseudonyms Olive, Forest, and Sage, see Fig. 2). The larger study focuses on the

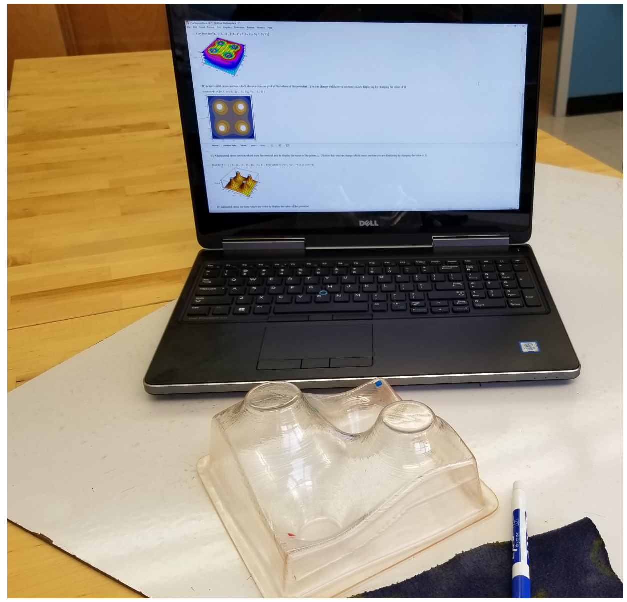

FIG. 1. The students produced equipotential curves on a tabletop whiteboard, a Mathematica notebook, and a dry-eraseable plastic graph.

relationship between each representation and the students' science practices. In the process of analyzing that data, we identified moments where students were using the representations together. The focus of this paper is on some of those moments, which took place primarily in the second part of the activity. Author JA selected a portion of the transcript to specifically code for possible instances of each Function, and then discussed together with the other authors what those lines of code demonstrated. Author JA originally transcribed the video and has also periodically reviewed the video and transcript to ensure accurate summaries throughout.

II. THEORETICAL PERSPECTIVE

We use Ainsworth's framework as the lens for studying the relationship between representations. This framework specifically addresses the use of *multiple* representations in terms of three different functions: *complement*, *constrain*, and *construct*.

Complement: MERs can *complement* each other by supporting different thinking processes or by containing different/partially redundant information about the same system. Processes contain strategies and tasks; a particular representation might be chosen to accomplish some task, or because it permits a particular problem-solving strategy.

Constrain: One (or more) representation(s) may also *constrain* the interpretation of some other representation(s), either by leveraging a user's greater familiarity with a representation or due to the inherent properties of the representation(s). For example, some representations may be ambiguous or unfamiliar to users and a more familiar representation can help with interpretation of the less familiar representation. Regarding inherent properties, we can consider the common practice of presenting a problem statement with both a diagram and a written description. The diagram constrains the written description to clarify a physical situation through the inherent properties of the diagram.

Construct: The third function of MERs is to help students *construct* deeper understanding via abstraction, extension, and relation. For this analysis, abstraction is how students use MERs to understand the essential elements of a concept itself, while extension and relation are the use of MERs to understand more about the representations themselves.

III. OVERVIEW AND DESCRIPTION OF THE ACTIVITY

In the first part of the activity, this group investigated the equipotential curves due to a collection of four positive charges on the corners of a square. The students drew curves on the whiteboard, considering the behavior close to single point charge as well as the spacing of the curves and behavior far away from the collection.

After some time working on this prompt, there was a whole-class wrap-up discussion where all the groups were directed to look at the pre-programmed Mathematica notebook and directly compare this with the whiteboard image. This notebook was available for the remainder of the activity.

The lead instructor then announced:

> *"Now I want you to make a quadrupole. So, the quadrupole is going to be two positive charges on opposite corners and two negative charges on opposite corners. Two positive charges and two negative charges. And I want you to draw the cross-sections of the equipotential surfaces for this plane."*

The group began drawing curves on the whiteboard and almost immediately modified the Mathematica notebook to represent the quadrupole. They then compared their whiteboard image and the Mathematica image. A few minutes later, the instructor announced that the plastic graphs were dry-erasable, and one of the students suggested that they should request a plastic graph and draw level curves on it. The group then drew level curves on the plastic graph and compared with their whiteboard image and the Mathematica image.

IV. RESULTS

We now describe the *complement* and *construct* functions of these three representations, and provide specific evidence for these two functions. Our data contains limited evidence of how any one representation *constrains* any other representation in this system, and we discuss this in Sec. VI.

A. Complementary Processes and Information

When this group studied the quadrupole, producing the three different representations of the equipotential curves each recruited different **processes** for reasoning about the shape. For the whiteboard image, the students first constructed some curves using their models for point charge electric potential. The students went through several brief reasoning steps for producing the equipotential curves; for example, they examined what happens very close to one of the point charges. They soon identified that, with the Mathematica notebook, they could change the signs of the charges to represent the new system:

> Olive: *"So then, we just need to change one of the negatives and one of the positives. So why don't I just make this positive and make the other one negative [pause] right [pause] here."*

Mathematica allows precise scalar superposition at all points, which is not feasible with the whiteboard but is done for the students in Mathematica. After configuring the Mathematica notebook to represent the quadrupole, the students compared with and corrected their whiteboard image. They also used the Mathematica image to help them draw the rest of the curves on their whiteboard.

The students' processes for generating curves on the whiteboard changed when the Mathematica notebook became available. Instead of using their physical reasoning to produce the curves on the whiteboard, the students simply referred to the Mathematica notebook. (Consider, for example, the first quote from Olive in Sec. IV B.) Forest recognized this at the very end of the activity, when he reflected:

> Forest: *"I also appreciate that we can successfully use technology to not have to think about stuff. I like that. [Olive and Sage nod]"*

These students thus acknowledged the shift in process that occurred during this portion of the activity.

The plastic graph introduced another new process: drawing level curves and viewing the projection. This different approach influenced the students to request a plastic graph. When the instructor interrupted the class to announce that the plastic graphs are dry-eraseable, Forest suggested to the group that they use one.

> Forest: *''Oh, shoot. Let's do that."*
> Olive: *"What?''*
> Forest: *''When we get the [plastic graph], let's draw some rings on them."*
> Olive: *"Oh! Cool."*
> Forest: *"We can look at the projection."*

The students then asked for a plastic graph and went through the steps of drawing level curves on it and comparing it to the whiteboard.

FIG. 2. Olive, Forest, and Sage (left to right) drawing equipotential surfaces. Olive points out the difference between the whiteboard image and the Mathematica image.

There is a great deal of overlap in the **information** each representation contains. The similarity between the contours on Mathematica and the whiteboard image facilitated direct comparisons by the students. Initially, the plastic graph represents potential using height, but once the level curves were drawn, the representations are scaled in such a way that the students could directly compare by overlaying the plastic graph on the whiteboard. This overlapping information is connected to the other function we observed, *constructing* deeper understanding, which we discuss next.

B. Construct Deeper Understanding

Here we describe examples of the students using the representations for **relation** and **abstraction**. **Extension** is about using a familiar representation to learn something about an unfamiliar representation, but we did not see particular evidence for extension is our data set. No one representation was more useful for understanding the others—once the group had all three representations, they **related** them bidirectionally for each pair.

At several instances, the students compared the same feature of each representation to determine the similarities/differences. Once the group modified the Mathematica notebook to represent a quadrupole, Sage exclaimed,

> Sage: *"Yeah, I was right! [Points to computer screen.] On the asymptotes it's zero because along those lines, there's equal push/pull."*

Which Olive followed up with,

> Olive: *"Right. And then, yeah, so it is actually spaced farther out that way and closer this way [Pointing to the computer screen. See Fig. 2]. So it's the opposite of what you [Forest] drew."*

We see in this interaction that the students immediately **related** what was on the computer screen with their whiteboard image and were particularly attentive to the lines of zero potential. They also discussed what the spacing of the equipotential curves looked like. This relation between the Mathematica notebook and the whiteboard continued through the rest of the activity.

When the group decided to get a plastic graph, they asked a learning assistant:

> Forest: *"Can we snag one? We're trying to decide whether or not we think it'll be fatter this way [toward the center of the collection] or fatter on the back end [on the outside of the collection]."*

We see in this quote a plan to **relate** the plastic graph with the other representations, and to continue investigating the spacing of the curves.

The learning assistant gave a plastic graph to Forest and, once the group finished drawing some level curves together, Forest pointed out that there are straight line equipotentials on the plastic graph and commented on the spacing of the other curves,

> Forest: *"I mean you can definitely see though that there's a line across this way [Draws straight line on plastic graph.], and you can see it, how they printed it, there's a line across that way. [Draws perpendicular straight line on plastic graph.]*
> Olive: *"Uh-huh."*
> Forest: *"So that's clear. And those get fatter out that way. [Draws curve on plastic graph around negative pole. Curves from this interaction can be seen in Fig. 3]*

This interaction demonstrates the group's efforts toward **abstraction** with regard to the lines of zero potential and the spacing of the oval-shaped curves. Moments after this, Forest sought to investigate outside the domain of the plastic graph,

> Forest: *"I'm just trying...I like our picture. I want to know what these do farther out...Is there a way?...Let's do this. [Pulls up the laptop. Fig. 3]*

The group then worked together to expand the range of the Mathematica notebook to view a more zoomed-out image. By comparing Mathematica and their whiteboard image, the group constructed deeper understanding about both spacing and the lines of zero potential. Not only did the students relate representations, but they also used this relating to reveal the essential aspects of the equipotential curves.

V. DISCUSSION & CONCLUSIONS

Our research question is: *What facets of this group's use of the MERs are captured by the Functions framework?* We

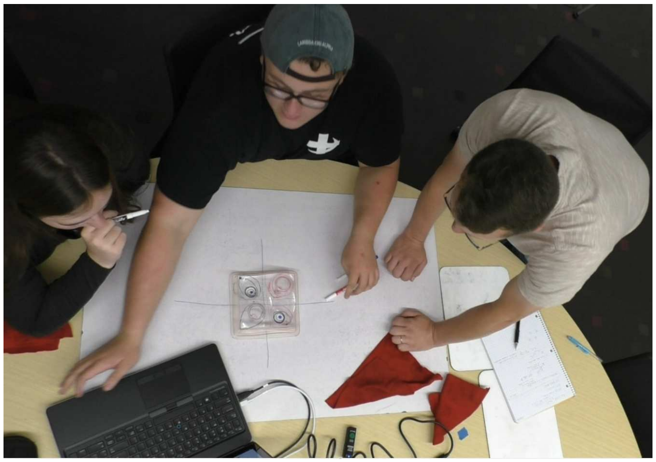

FIG. 3. The students drew level curves on the plastic graph, then overlayed it directly above the whiteboard image. We also see here that the students turned to the Mathematica notebook for investigating the curves outside the domain of the plastic graph.

identified *complementary* processes and information. The **processes** that the students used were different depending on the representation they chose. Producing the curves on the whiteboard resulted in the students making some qualitative arguments about the shape of the curves. Using the Mathematica notebook allowed the students to modify an equation and apply superposition. The plastic graph let the students draw level curves and see a projection of those curves. These latter two representations helped the students make decisions about how to generate the whiteboard image. The **information** that each representation contains is very similar once produced (they are all images of level curves) and this resulted in a significant amount of direct comparison between representations. While Ainsworth considers different or partially redundant information to be complementary, we have seen that the students found value in the highly redundant information contained by these representations.

The students used these MERs to *construct* deeper understanding. The availability of MERs provided opportunities for connecting between representations, letting the students explore the system to grasp underlying patterns (**abstraction**). The Mathematica notebook and the plastic graph were each used to explore something that was observed on the other representation. The students chose to draw the equipotential curves on the plastic graph and placed it on top of the whiteboard, despite having already viewed these curves on the Mathematica notebook. Conversely, the students explored Mathematica when it was necessary to expand the domain, since the other representations have limited domains. Although the plastic graph was likely a new representation for these students (they had not seen it in class before and these are custom graphs), they did not use the whiteboard and Mathematica notebook to **extend** understanding for the plastic graph. Rather, they **related** all three representations. This indicates to us that the (novel) plastic graph did not significantly increase the amount of interpretation these students had to exercise.

Using the Functions framework, we have identified a distinction in the students' processes due to complementing representations. Once the students had access to Mathematica for producing curves, they could use the software to generate the curves by choosing the appropriate signs of charges. With the plastic graph, the students could draw level curves and look at a projection. Our analysis also found that these students sought out the plastic graph precisely because of its potential for representing the level curves with high similarity to the whiteboard image. This implies that even representations with high degrees of similar information can result in constructing deeper understanding of a physical system, and students may even benefit from the highly similar nature. The implication for practitioners is that concerns about cognitive load from interpreting multiple representations may be minimal. Providing representations that support multiple processes engaged these students in forming deeper understanding of the physical system.

The representations are extremely similar—they are all 2-D curves—but each one is produced in a different medium using different reasoning and strategies. These considerations around generation are an interesting aspect of the complementary processes the students engage in, but it is difficult to describe with the Functions framework, because there is a tacit assumption that the representations have already been produced. Considering the affordances of representational construction [9, 10] appears to be a necessary part of understanding learning environments where students generate multiple representations.

VI. LIMITATIONS & FUTURE WORK

This study of in-class video has given us *in situ* information about the functions of these representations. Due to the in-class group setting, we did not gain much insight into each student's familiarity with the representations, and this limited our evidence about the *constraining* function of the MERs. The high degree of similarity between representations also meant that *constraint* by inherent properties was not apparent in the students' use of these representations. So we cannot make claims about the *constraint* function. A particularly valuable extension would be to conduct individual interviews to see more of how individual differences and inherent properties play a role in the *constraining* function of these representations.

VII. ACKNOWLEDGEMENTS

Aaron and Robyn Wangberg for their work on the Raising Physics to the Surface project, the OSUPER group, and the students who voluntarily participated in this research. Funded in part through the NSF DUE-1612480

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