Magnonic Purcell Effect in a Cavity System

Guogan Zhao[1] and Xiaofeng Qian[1,*]

[1] *Department of Physics, and Center for Quantum Science and Engineering, Stevens Institute of Technology, Hoboken, NJ 07030, USA*

[*] *xqian6@stevens.edu*

Abstract: We analyze the spontaneous decay of magnon with arbitrary excitations when coupled to a cavity. A quantitative enhancement of the decay, i.e., Purcell effect, is achieved analytically. Our results agree well with existing experimental data.

The spontaneous emission rate of an excited emitter can be substantially enhanced when coupled with a resonator. This phenomenon, called Purcell effect, was first predicted by Purcell in 1946 [1] for a two-level atom. It has then been demonstrated in various quantum emitter systems including atoms and molecules, quantum dots, solid-state electronic system, photonic and plasmonic crystal, etc [2]. Recently it has been experimentally demonstrated in cavity magnonic systems [3]. Different from the conventional two-level emitters, the enhancement of magnon decay has its unique feature based on the magnon's intrinsic multi-level nature. Here, we present a systematic analysis the noise-induced decay feature of quantum magnons when it is simultaneously coupled to a lossy cavity. Our results agree with the experimental observation for enhanced cavity photon loss. More importantly, they predict the Purcell effect of magnon excitations of which experiment confirmation is yet to be carried out.

The cavity magnonic system under consideration is shown in Fig. 1(a). A magnet (yttrium iron garnet sphere), placed inside a microwave cavity, is coherently coupled to the cavity mode via magnetic dipole interaction [3]. It is assumed that only the Kittel mode [4] of the magnet is excited by the magnetic component of the microwave field and a bias magnetic field B is applied on the magnet. The Hamiltonian of the system reads

$$H = \hbar\omega_c c^\dagger c + \hbar\omega_m m^\dagger m + \hbar g(c^\dagger m + m^\dagger c), \tag{1}$$

where $\hbar$ is the reduced Planck constant, $c^\dagger$ (c) and $m^\dagger$ (m) are the creation (annihilation) operators of cavity photons with frequency ω_c and magnons with frequency ω_m, respectively, g is the coupling rate between photons and magnons. The magnon frequency is mediated by a bias magnetic field B, i.e., $\omega_m = \gamma B$, where γ is the gyromagnetic ratio.

Purcell effect describes the enhancement of decay which originates from the coupling to the noisy reservoirs. In this cavity magnonic system, we consider two independent broadband reservoirs coupled to mangon and cavity respectively. With Markovian treatment, the dynamics of the system state ρ can be described by the master equation

$$\dot{\rho} = \frac{1}{i\hbar}[H,\rho] - \kappa_c(c^\dagger c\rho - 2c\rho c^\dagger + \rho c^\dagger c) - \kappa_m(m^\dagger m\rho - 2m\rho m^\dagger + \rho m^\dagger m), \tag{2}$$

where κ_c and κ_m are the independent decay rates of the cavity and magnon respectively. With the master equation, the rate equation of any state can be obtained. In contract to conventional two-level systems or qubits, magnon is an oscillator with infinite energy levels. Therefore, the decay of magnon (or cavity mode) exciations has to include radiations from all levels. To account for such an effect, we derive equations of motion for average exciation number $\mathrm{Tr}[\hat{O}\rho]$ where $\hat{O} = c^\dagger c, m^\dagger m, c^\dagger m$ and $m^\dagger c$ based on master equation (2). Four coupled equations can be achieved

$$\begin{aligned}
\langle \dot{c^\dagger c}\rangle &= -ig\langle c^\dagger m\rangle + ig\langle m^\dagger c\rangle - 2\kappa_c\langle c^\dagger c\rangle,\\
\langle \dot{m^\dagger m}\rangle &= ig\langle c^\dagger m\rangle - ig\langle m^\dagger c\rangle - 2\kappa_m\langle m^\dagger m\rangle,\\
\langle \dot{c^\dagger m}\rangle &= i\omega_c\langle c^\dagger m\rangle - i\omega_m\langle c^\dagger m\rangle - ig\langle c^\dagger c\rangle + ig\langle m^\dagger m\rangle - \kappa_c\langle c^\dagger m\rangle - \kappa_m\langle c^\dagger m\rangle,\\
\langle \dot{m^\dagger c}\rangle &= -i\omega_c\langle m^\dagger c\rangle + i\omega_m\langle m^\dagger c\rangle + ig\langle c^\dagger c\rangle - ig\langle m^\dagger m\rangle - \kappa_c\langle m^\dagger c\rangle - \kappa_m\langle m^\dagger c\rangle.
\end{aligned} \tag{3}$$

In the experimental demonstration [3], Purcell enhancement is investigated under the conditions: $g^2 > \kappa_c\kappa_m$ and $\kappa_c < g < \kappa_m$. The system is excited by a microwave pulse in the experiment initially, giving the initial conditions: $\langle c^\dagger c\rangle = N_c$ and $\langle m^\dagger m\rangle = \langle c^\dagger m\rangle = \langle m^\dagger c\rangle = 0$. The exponential decay of microwave cavity mode excitations is observed in the experiment with parameters: $\omega_c/2\pi = 7.58$ GHz, $g/2\pi = 15$ MHz, $\kappa_c/2\pi = 1.08$ MHz and

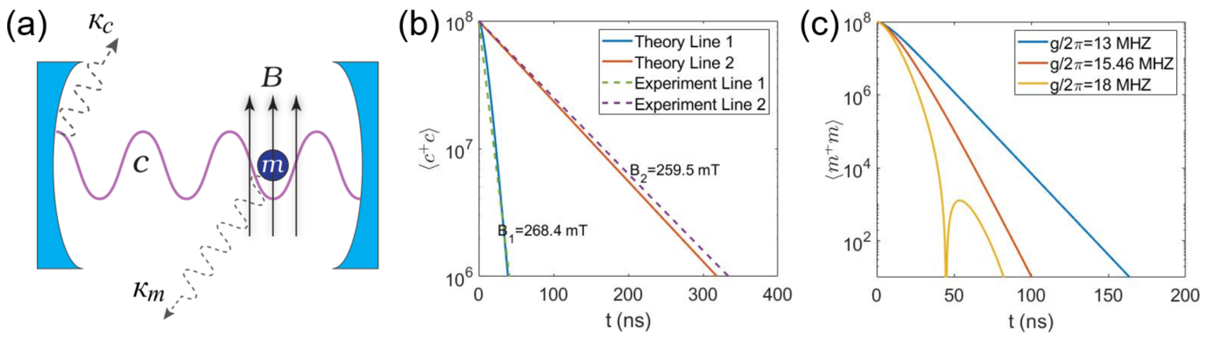

Fig. 1. (a) The schematic diagram of cavity magnon system. (b) The decay curves plotted with parameters using in the experiments. (c) Magnon decay with three different coupling rates.

$\kappa_m/2\pi = 32$ MHz. As shown in Fig. 1 (b), the two dashed lines are experimental fit of photon number decay for two respective bias magnetic field $B_1 = 268.4mT$ and $B_2 = 259.5mT$ with initial average photon number $N_c = 10^8$ [3]. The solid lines are the theoretical behaviors achieved from Eq. (3) with the exact same parameters used in the experiment. It is found that the dynamical behavior match the experimental data very well.

To analytically analyze the decay of magnon, one assumes N_m magnons are initially excited, corresponding to the conditions: $\langle m^\dagger m \rangle = N_m$ and $\langle c^\dagger c \rangle = \langle c^\dagger m \rangle = \langle m^\dagger c \rangle = 0$ at resonance. Then the solutions of Eq. (3) can be achieved as

$$\begin{aligned} \left\langle m^\dagger m \right\rangle (t) &= N_m \left[c_0 e^{-t(\kappa_c+\kappa_m)} + c_1 e^{-t\left(\kappa_c+\kappa_m+\sqrt{\delta}\right)} + c_2 e^{-t\left(\kappa_c+\kappa_m-\sqrt{\delta}\right)} \right], \\ \left\langle c^\dagger c \right\rangle (t) &= \frac{1}{2} N_m c_0 \left[-e^{-t(\kappa_c+\kappa_m)} + e^{-t\left(\kappa_c+\kappa_m+\sqrt{\delta}\right)} \right], \end{aligned} \tag{4}$$

where $c_0 = \frac{-2g^2}{\delta}$, $c_1 = \frac{-2g^2+(\kappa_c-\kappa_m)\left(\kappa_c-\kappa_m-\sqrt{\delta}\right)}{2\delta}$, $c_2 = \frac{-2g^2+(\kappa_c-\kappa_m)\left(\kappa_c-\kappa_m+\sqrt{\delta}\right)}{2\delta}$, and $\delta = -4g^2 + (\kappa_c - \kappa_m)^2$. As shown in Eq. (4), magnons will decay via three channels when the parameter $\delta = -4g^2 + (\kappa_c - \kappa_m)^2 > 0$. This condition limits the parameters in weak coupling regime [5]. If $\delta < 0$, the dynamics of magnons will have an oscillating feature, indicating the entering of the strong coupling regime. The dynamics of average magnon number with three different coupling rates is plotted in Fig. 1(c) with parameters: $\omega_c/2\pi = \omega_m/2\pi = 7.58$ GHz, $\kappa_c/2\pi = 1.08$ MHz and $\kappa_m/2\pi = 32$ MHz. Due to exponential decay, the red-shift third term $\left(\kappa_c + \kappa_m - \sqrt{\delta}\right)$ in Eq. (4) dominates the long time decay. In the weak coupling regime, the average magnon number decays faster as the coupling constant g increases, implying the coupling to the cavity will enhance the spontaneous emission of magnon excitations.

In conclusion, we have analyzed the Purcell effect of excitation (both magnon and cavity mode) decays in a cavity magnonic system. The magnon-cavity weak coupling regime is analytically identified for decay enhancement. Our analytical result of cavity photon mode decay agrees very well with existing experimental observations, confirming the validity of our theoretical treatment. More importantly, our results predicts for the first time the Purcell effect of magnon excitations by revealing three decay channels for the dynamics process. Our findings provide valuable insights into the Purcell effect in cavity magnonic systems and contribute to the understanding of coherent coupling between magnons and microwave photons in the context of cavity QED. Further investigations can be explored for the implications of the Purcell effect in magnonic devices and their potential applications in information processing and quantum technologies.

References

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