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In search of the maximum lost momentum*

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We performed a series of 1381 full numerical simulations of high energy collision of two black holes to search for the maximum recoil velocity after their merger. We studied equal mass binaries with opposite spins pointing along the orbital plane to maximize asymmetric gravitational radiation and performed a search of spin orientations in the plane, impact parameters, and initial linear momenta to find a maximum recoil velocity extrapolated to the extreme spinning case of $28,562 \pm 342\,\text{km/s}$, thus tightly bounding recoil by 10% the speed of light.

Keywords: Binary black holes; numerical simulations.

1. Introduction

The search for the maximum recoil the remnant of a binary black hole merger could achieve from the radiation reaction of the gravitational waves emitted is a problem that attracted researchers since at least 35 years,[1] and can only be solved with full numerical relativity techniques,[2] since most of the asymmetric radiation takes place during the formation of a common horizon, a highly nonlinear process.

Ever since the discovery through full numerical simulations[3,4] that the merger of binary black holes may lead to large (astrophysically speaking) gravitational recoil velocities, a fascinating search for such events in nature takes place.[5,6] Since the first modeling of large recoils,[7] it was clear that the spins of the black holes played a crucial role in their merger remnant reaching up to several thousand km/s speeds.

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It was next found a configuration[8] that maximized the recoil nearing 5,000 km/s. This configuration combined the opposite spins of Ref. 7 that maximized asymmetry with the hangup effect[9] that maximized radiation. All those configurations assumed negligible eccentricities at the time of merger, when most of the asymmetric radiation takes place. While this is the most plausible astrophysical scenario, new gravitational waves observations show the potential for large residual eccentricity in some binary black hole merger events.[10]

The growth of structure seeded by primordial black holes has been studied in Ref. 11, and the effects of gravitational-wave recoil on the dynamics and growth of supermassive black holes has been studied in Ref. 12. While the scenario of supermassive rotating black holes potentially accelerating orbiting black holes to high energies was discussed in Ref. 13.

Here we will explore the extreme scenario of high energy collisions of black holes, in the realm of high-energy colliders,[14,15] to discover the fundamental laws of nature,[16–18] with applications to gauge/gravity duality, holography,[19] primordial black hole collisions in the early universe,[20–22] and as tests of the radiation bounds theorems and the cosmic censorship conjecture in General Relativity.[23–25]

This high energy collision of black holes scenario was studied in Ref. 26 to compute the energy radiated by equal mass, nonspinning black holes in an ultra-relativistic headon collision. This first study was then followed up by the claim in Ref. 27 that the spin effects did not matter for these collisions. Non-headon high energy collisions have also been studied in Ref. 28, and analytically in Ref. 29 leading to a wide range of maximum recoil velocity estimates from simulations of 10,000 km/s and 15,000 km/s to potential extrapolations up to 45,000 km/s.[30,31] Some of the early reviews on the subject are Refs. 16 and 18, and more recent ones are Refs. 32–34.

Here we perform studies with much larger data sets obtained by numerically solving General Relativity field equations on supercomputers, and focusing on the computation of the maximum achievable gravitational recoil from grazing, high energy collisions of binary black holes, where the holes' spin orientation and magnitude play a crucial role.

2. Numerical Techniques

The full numerical simulations were performed using the LazEv code[35] implementation of the moving puncture approach.[36] We use here the general relativistic BSSNOK evolution system formalism.[37–39] The LazEv code uses the Cactus[40]/Carpet[41]/EinsteinToolkit[42,43] infrastructure. The Carpet mesh refinement driver provides a "moving boxes" style of mesh refinement. To compute the numerical (Bowen–York) initial data, we use the TwoPunctures[44] code. We use AHFinderDirect[45] to locate apparent horizons and measure the magnitude of the horizon spin S_H, using the *isolated horizon* algorithm as implemented in Ref. 46. We measure radiated energy, linear momentum, and angular momentum, in terms

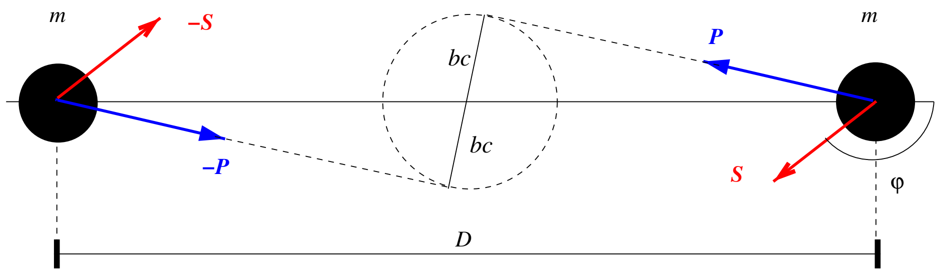

Fig. 1. Maximum high energy collision kicks binary black hole initial configurations. On the orbital plane of equal mass m black holes with opposing spins $\pm\vec{S}$ and momentum $\pm\vec{P}$ at the critical impact parameter b_c and initial separation $D = 50M$.

of the radiative Weyl scalar ψ_4, using the formulas provided in Refs. 47 and 48. As described in Ref. 49, we use the Teukolsky equation to analytically extrapolate expressions for ψ_4 from a finite observer location ($R_{\rm obs} > 100M$) to infinity ($\mathscr{I}^+$).

One can argue on asymmetry properties that the maximum recoil can be searched for in black holes configurations with equal masses and opposite spins on the orbital plane, as displayed in Fig. 1. The compromise with maximizing the energy radiated via the hangup effect[9] that we needed for quasicircular orbits is here replaced by the determination of the critical impact parameter, b_c and momentum P_c, separating direct merger from scattering. Here we study this problem in detail with our specially designed set of simulations to explicitly model the problem in terms of the Bowen–York dimensionless initial momentum of the holes, γv, impact parameter, b, and spin, $\mathbf{s} = \mathbf{S}_H/m_H^2$ (where $m_H = m_{1,2}$ is the horizon mass of each hole), i.e. a four-dimensional parameter search.

3. Simulations' Results

Our simulations families consist of a choice of an initial (Bowen–York) data spin magnitude, here $s = 0.40, 0.70, 0.80, 0.85, 0.90$, and for each of them an initial momentum per irreducible mass, γv, and impact parameter, bM, as measured at the initial separation of the holes $D = 50M$ (with $M = m_1 + m_2$ the addition of the horizon masses of the system). We then vary the orientation of the spins pointing on the orbital plane by an angle φ with respect to the line initially joining the black holes. This allows us to model the leading φ-dependence of the recoil velocity as a $\cos\varphi$.[50] In practice one needs about 4–7 simulations to fit this dependence and to determine the amplitude of the curve leading to the value of the maximum recoil for this configuration.

This process is repeated now for each impact parameter b to find the value $b_{\rm max}$ that leads to the largest recoil velocity. In practice, the $b_{\rm max}$ corresponds closely to the critical value of the impact parameter b_c separating the direct merger from the scattering of the holes.

Table 1. All simulations have equal mass $m_1 = m_2$, and are initially placed at $x_{1,2} = \pm 25M$. The relaxed spin magnitude, $|s_r|$, are used for the final fit. Measured maximal recoil velocities and its extrapolation (order) to infinite resolution are given on the right panel.

# Runs	$\pm s$	$|s_r|$	$b_c^{\max}$	$(\gamma v)_{\max}$	$V_{\max}^{n100}$ [km/s]	$V_{\max}^{n120}$ [km/s]	$V_{\max}^{n144}$ [km/s]	$V_{\max}^{\infty}$ [km/s]	Order
72	0.40	0.400	2.38	1.20	11,637 $\pm$ 67	11,827 $\pm$ 67	11,944 $\pm$ 64	12,133 $\pm$ 189	2.7
233	0.70	0.699	2.38	1.10	19,832 $\pm$ 267	20,163 $\pm$ 267	20,360 $\pm$ 262	20,649 $\pm$ 289	2.9
472	0.80	0.789	2.38	1.10	22,212 $\pm$ 228	22,583 $\pm$ 226	22,800 $\pm$ 217	23,104 $\pm$ 304	3.0
305	0.85	0.838	2.38	1.10	23,291 $\pm$ 514	23,666 $\pm$ 486	23,892 $\pm$ 482	24,231 $\pm$ 339	2.8
299	0.90	0.885	2.38	1.09	24,172 $\pm$ 579	24,609 $\pm$ 565	24,870 $\pm$ 552	25,256 $\pm$ 386	2.8

A similar analysis can be done to complete the two-dimensional search, by varying the initial velocity, v, or rather the linear momentum per irreducible mass of the holes, $\gamma v = P/m_{\rm irr}$, with $\gamma = (1 - v^2)^{-1/2}$, the Lorentz factor, and $A_H = 16\pi m_{\rm irr}^2$ the measured horizon area. We observe the same feature of maximization of the recoil velocity for values about the critical momentum, P_c, separating the direct merger from the scattering of the holes.

The final results of the maximum recoil velocities for each individual spin $s_{1,2}$ value and the (more physical) corresponding relaxed (at around $t = 30M$) spin magnitude, $|s_r|$ are summarized in Table 1. Those results are used for the fit in Fig. 2, where we also display the measurement error bars of each point and a fit to a quadratic dependence on s_r to extrapolate to the ultimate recoil velocity, finding $28,562 \pm 342$ km/s for the extremely spinning binary black holes case, $s_r = 1$.

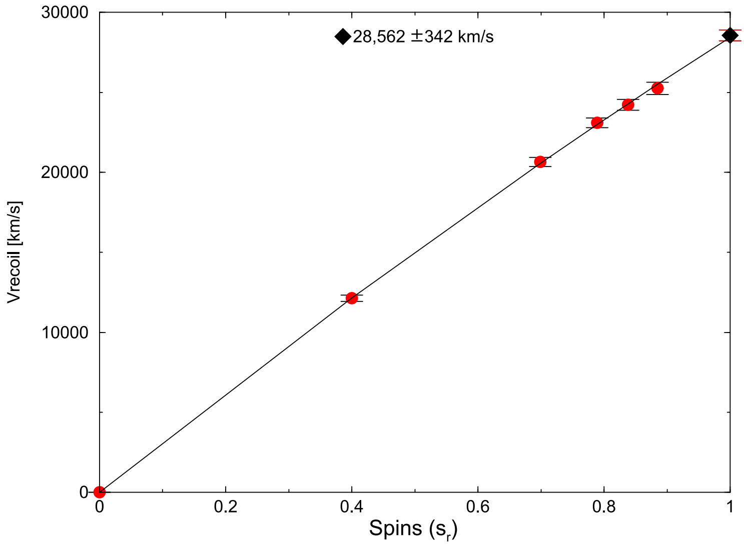

Fig. 2. Maximum recoil velocity versus the settled spins value s_r and its extrapolation to maximal spin $s_r = 1$.

For most of the simulations (for spins $s = 0.4$ to $s = 0.85$), we use a grid, labeled as $n100$, with 10 levels of refinement, the coarsest of which has resolution of $4M$ and outer boundary of $r = 400M$, with each successive grid with twice the previous resolution. If we label the coarsest grid by $n = 0$, and the finest grid by $n = 9$, the resolution on a given level is $M/2^{(n-2)}$. The wavezone is $n = 2$ with a resolution of $M/1$ and boundary out to $r = 125M$. The finest grid has a resolution of $M/128$ with a size of $0.5M$ centered around each black hole. The spin $s = 0.9$ case has an additional refinement level around each black hole with a resolution of $M/256$ and a radius of $r = 0.3M$.

To evaluate the finite differences errors and extrapolation of our simulations, we have performed two additional sets of simulations with increasingly global resolutions by factors of 1.2 ($n120$, $n144$) with respect to our base resolution, $n100$, for the peak velocity cases with $b_c = 2.38$, $(\gamma v)_{\max}$, and four $\varphi = 0°, 45°, 90°, 150°$ degrees for each of the spins, $s = 0.40, 0.70, 0.80, 0.85, 0.90$. The resulting measured recoil velocities are given on the right panels of Table 1. Infinite resolution extrapolation leads to $V^{\infty}_{\max}$ values representing about a 3% increase from the $n100$ results. The near third-order convergence rate found for the net recoil (computed as large differences of anisotropic radiation), is what one expects from the fourth Runge–Kutta time integrator used by our code.

As a further check of our numerical accuracy, we have recalculated a set of cases for the spin 0.8 with the extra refinement level and increased grid sizes as we used for the spin 0.9 runs. We then recalculated the sequence that gives a maximum value for spin 0.8 of $21,802 \pm 191\,\mathrm{km/s}$. Compared to the original grid computation of $21,903 \pm 213\,\mathrm{km/s}$, this leads to a difference of $101\,\mathrm{km/s}$ or 0.46%.

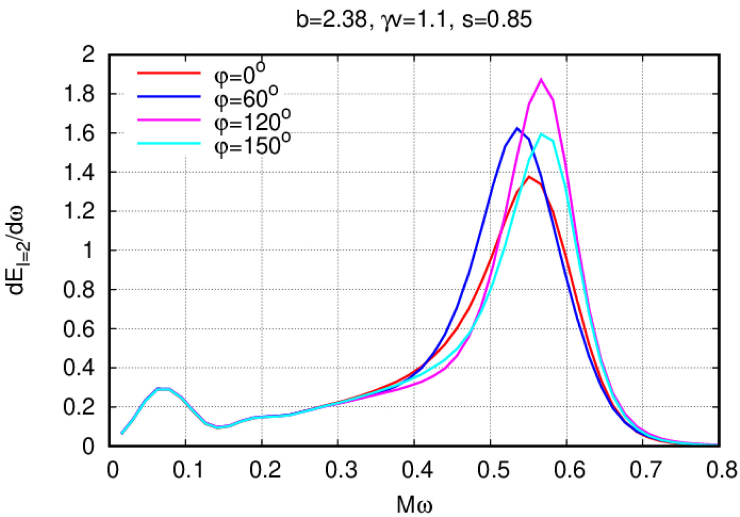

Fig. 3. The spectrum of the ($\ell = 2$-modes) energy radiated $dE_{\ell=2}/d\omega$ by a representative set of simulations (with $b = 2.38$, $s = 0.85$, $\gamma v = 1.1$) for different orientation angles, φ, of the spin.

4. Conclusions

In Fig. 3, we display the spectrum of radiated energy from the leading ($\ell = 2, m = 0, \pm 2$)-modes for one of the peak recoil cases ($b = 2.38$, $s = 0.85$, $\gamma v = 1.1$) for different orientation angles, φ of the spin. We observe a bulge at low frequencies, corresponding to the initial and "bremsstrahlung-like" radiation of the holes approaching each other from $D = 50M$ and that the different spin orientations do not produce notable differences in this part of the spectrum. Meanwhile, at higher frequencies (by an order of magnitude), corresponding to when the holes reach the critical separation $2b_cM = 4.76M$ and the subsequent merger, the spectrum shows a strong dependence on the spin orientations.

In summary, we have been able to provide an accurate estimate of the ultimate recoil, product of the high energy collision of two black holes. In order to perform the four-dimensional search (momentum γv, impact parameter b, spin orientation φ and magnitude s) we performed a total of 1381 simulations to look for the critical b_c marginally leading to merger and the corresponding value of P_c that maximized the recoil, all as a function of φ for each s. Extrapolation to maximum spins has led us to estimate the value of $28,562 \pm 342\,\mathrm{km/s}$ for the ultimate recoil, placing the bound just below 10% the speed of light.

We thus note the crucial relevance of the holes' spin magnitude and orientation in the determination of the high energy collision kicks. These accurate results point towards challenging mathematical relativists to put forward new bound hypothesis not only for the maximum radiated energy and final spin of the merger of two black holes,[34,51,52] but also for the net linear momentum radiated, perhaps from a horizon computation like in Ref. 53.

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