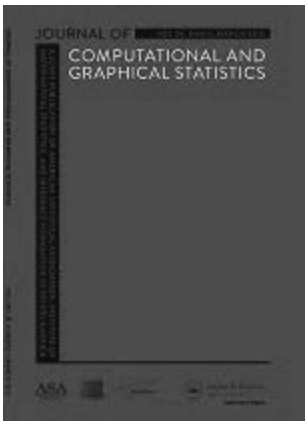


Scaled Torus Principal Component Analysis

Journal:	*Journal of Computational and Graphical Statistics*
Manuscript ID	Draft
Manuscript Type:	Original Article
Keywords:	Multidimensional scaling, Directional statistics, Statistics on manifolds, Dimension Reduction

Abstract

artifacts can create spurious cluster structures in the scores of datasets with non-existent cluster structures (see Figure 5). A final limitation of T-PCA, presented in Remark 2.3 of Eltzner et al. (2018), is that T-PCA is applicable whenever there is a structural data gap in all angles except for at most two.

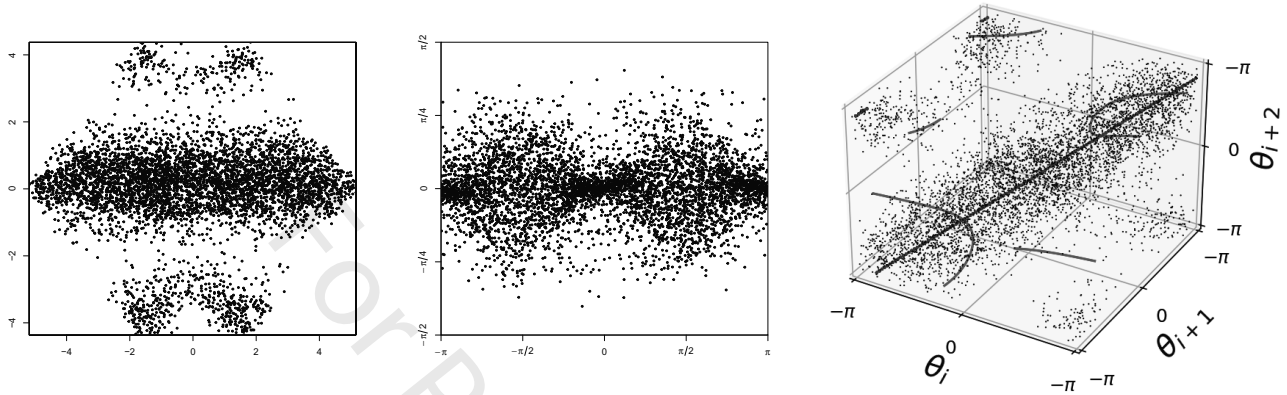
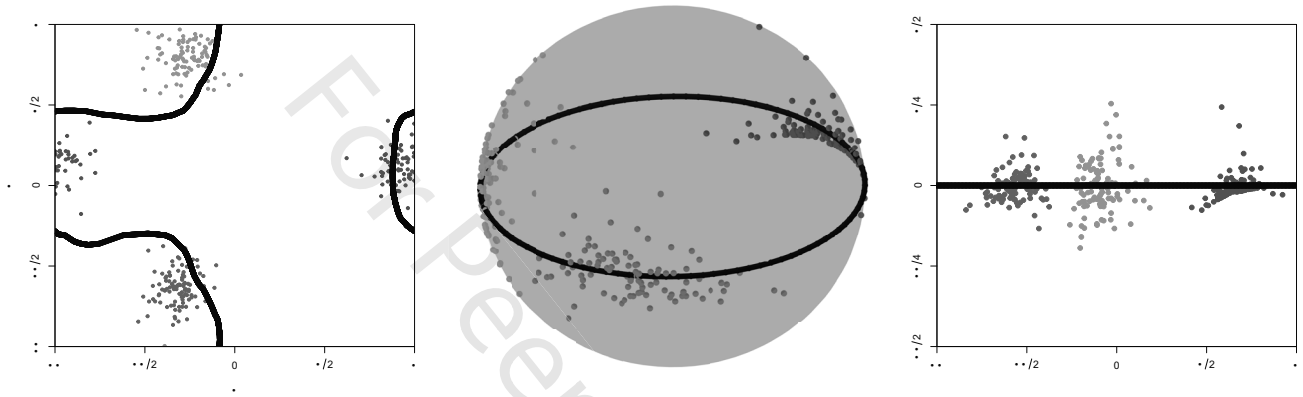


Figure 1: Scores (scatterplots (a) and (b)) and modes of variation (panel (c)) for the sunspots data; panels (a) and (b) show the first (horizontal axis) and second (vertical axis) scores for PCA and ST-PCA, respectively. In panel (c), the original data is shown with black points, while the T-PCA and ST-PCA first modes of variation are represented as red and blue solid lines, respectively. The figure illustrates that ST-PCA gives a more efficient mode of variation than standard PCA and T-PCA.

In this paper we propose a toroidal PCA that is based on embedding the sample on $\mathbb{T}^d$ into $\mathbb{S}^d$ by using a data-driven map aimed at preserving the pairwise geodesic distances of the data. Our dimension-reduction method is summarized in three steps. First, we transform the original sample on $\mathbb{T}^d$ to a similar configuration of points on a d-dimensional sphere by using Spherical MultiDimensional Scaling (SMDS). SMDS is an optimization procedure designed to minimize the squared differences between the sample pairwise geodesic distances on $\mathbb{T}^d$ and the corresponding pairwise geodesic distances of the transformed sample on a d-sphere. The radius of the arrival d-sphere is also chosen automatically in a further attempt to minimize distortions. Second, we apply PNS to the transformed data to find the sequence of best-fitting subspheres and the scores associated to them. Finally, we show how to optionally invert the SMDS map, in a step usually referred to as *prediction*, to produce a low-dimensional representation of the data in the original torus via the inverted PNS scores. Due to the multidimensional scaling operation, we henceforth refer to our method as Scaled Torus PCA (ST-PCA). An illustration of the adequacy of ST-PCA for data analysis is advanced in Figure 1. Figure 1c shows astronomical data on the appearance of sunspots, which lie on $\mathbb{T}^3$ in a roughly diagonal pattern, and are later described in detail in Section 5.1. Figure 1a displays the scores from standard PCA, which disregards the periodicity of the data, while Figure 1b shows that ST-PCA gives more sensible scores. Figure 1c shows that while T-PCA honors the periodicity of the data, it yields a less efficient mode of variation (red curve) when compared to ST-PCA (blue curve). In this case study, and among other competitors, ST-PCA performs the best in the sense of preserving data fidelity and explaining the most variance in the first principal component. A second case study to an RNA dataset in $\mathbb{T}^7$ that has lately

$\boldsymbol{\mu}_1 = (-1, -2)'$, $\boldsymbol{\mu}_2 = (3, 0.5)'$, and $\boldsymbol{\mu}_3 = (-0.8, 2.5)'$, each with $n = 100$, has been considered. The dataset is displayed in Figure 2a. We performed Step 1 to obtain a configuration of the data on $\mathbb{S}^2$, displayed in Figure 2b. For this specific dataset, we have obtained $r^* = 1.47$ as an initial value from (7), which is then modified to $\widehat{r} = 1.35$ through (6). In Step 2, we obtained the PNS scores plot for this spherical configuration, displayed in Figure 2c. The black curve in Figure 2b shows S^1, the best fitting $\mathbb{S}^1$ to the spherical data, which corresponds to the horizontal axis of the scores plot in Figure 2c.



(a) (b) (c)

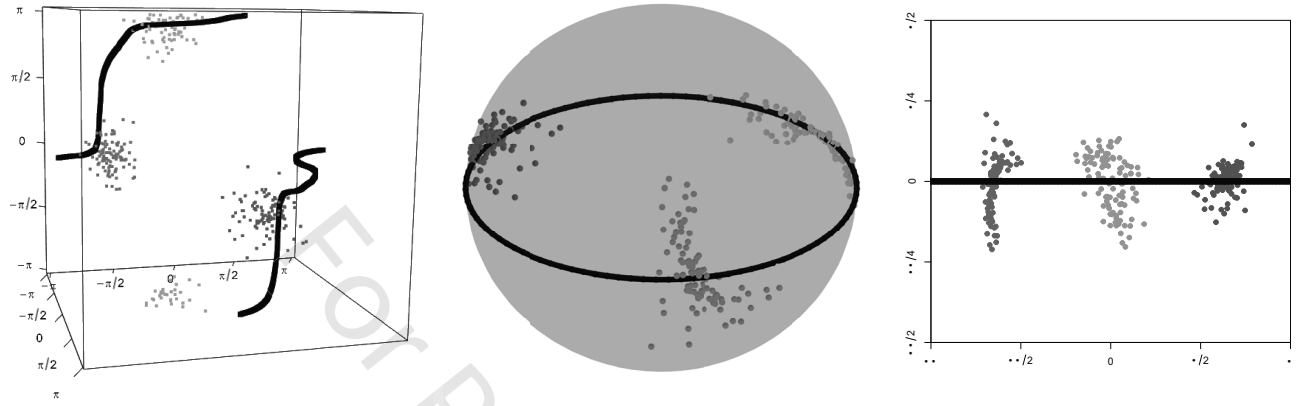
Figure 2: Panel (a) shows the three clusters of simulated data on $\mathbb{T}^2$. Panel (b) presents the sample SMDS-mapped to $\mathbb{S}^2$. Panel (c) displays the PNS scores. The principal mode of variation on the torus, sphere, and scores is displayed in all panels as a black curve. The first ST-PCA mode of variation clearly captures the three clusters.

The resulting $\widetilde{S}^1$, computed in Step 3 of ST-PCA, is shown in the black curve in Figure 2a. The figure reveals that the first toroidal component efficiently adapts to the clustered structure of the data (percentage of variance explained: 95%). $\widetilde{S}^1$ was obtained by predicting an equispaced grid $\{\boldsymbol{\psi}_j\}_{j=1}^{100} \subset S^1$ and then linearly interpolating the resulting predictions on $\mathbb{T}^2$ to yield a smooth curve.

4.2 Clusters toy data on $\mathbb{T}^3$

We now simulate data from an isotropic mixture of three wrapped normal distributions in $\mathbb{T}^3$. We simulated $n = 300$ points equally distributed in the three components with centers $\boldsymbol{\mu}_1 = (-1, -2, 0)'$, $\boldsymbol{\mu}_2 = (1, 1, -1)'$, and $\boldsymbol{\mu}_3 = (0, -1, 3)'$. Figure 3a displays the three clusters on $\mathbb{T}^3$, where each side of the cube is glued to the opposite side. After running SMDS in Step 1, we have obtained a configuration on points $\widehat{\boldsymbol{y}}_1, \ldots, \widehat{\boldsymbol{y}}_n \in \mathbb{S}^3$. For visualization purposes, we show in Figure 3b the spherical view of the configuration's projections to $S^2 \cong \mathbb{S}^2$. For this dataset we have obtained $r^* = 1.79$, which then yielded $\widehat{r} = 1.63$. Figure 3c shows the scores plot after applying PNS to $\widehat{\boldsymbol{y}}_1, \ldots, \widehat{\boldsymbol{y}}_n \in \mathbb{S}^3$ (Step 2). Finally, we inverted S^1, the black curve in Figure 3b, to produce $\widetilde{S}^1$ as displayed in the black curve of Figure 3. We did so with an equispaced grid $\{\boldsymbol{\xi}_j\}_{j=1}^{100} \subset S^1$ complemented by a periodic linear interpolation on $\mathbb{T}^3$. The entire cluster structure is captured by

the first mode of variation, which clearly exploits the periodicity of $\mathbb{T}^3$ for efficiently explaining the data variability. The percentage of variance explained by the first and second ST-PCA components is 94% and 1%, respectively.



(a) (b) (c)

Figure 3: Panel (a) shows the three clusters of simulated data on $\mathbb{T}^3$. Panel (b) presents the projection to $S^2 \cong \mathbb{S}^2$ of the sample SMDS-mapped to $\mathbb{S}^3$. Panel (c) displays the first PNS scores. The principal mode of variation on the torus, sphere, and scores is displayed in all panels as a black curve. The first ST-PCA principal component successfully separates the three clusters.

4.3 Anisotropic wrapped normal on $\mathbb{T}^2$

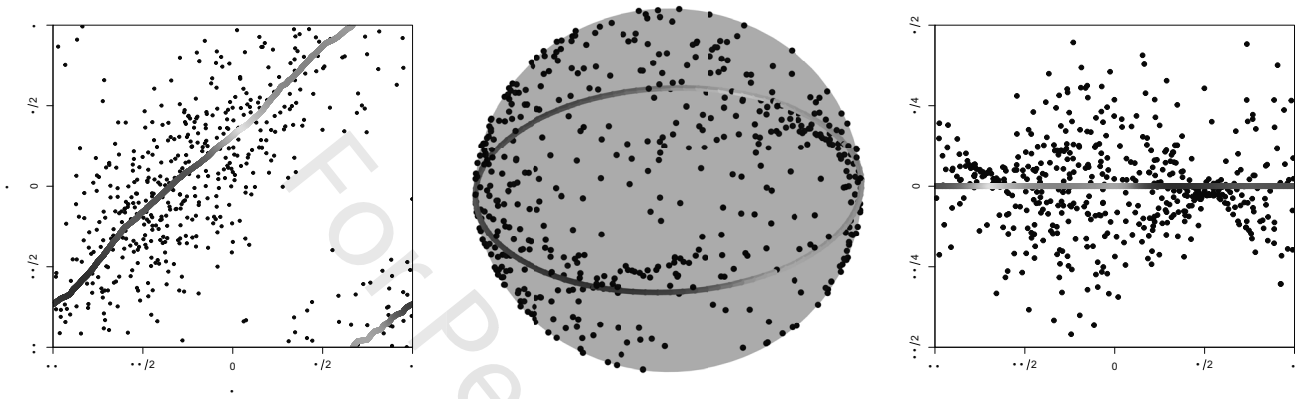
In the final numerical experiment we simulated $n = 500$ observations from a wrapped normal distribution on $\mathbb{T}^2$ with mean $\boldsymbol{\mu} = (-1, 0)'$ and covariance matrix $\boldsymbol{\Sigma} = (2, 2; 2, 3)$. Figure 4 shows the analogous analysis to that carried out in Figure 2, this time using a rainbow palette for easier identification of the first ST-PCA principal component and with $r^* = 1.48$ and $\widehat{r} = 1.39$. The mode of variation successfully utilizes the periodicity of the torus to optimally capture the data variability, attaining a percentage of variance explained of 89%.

5 Real data applications

5.1 Sunspots

Sunspots are cooler regions of the Sun's photosphere that are related with solar magnetic field concentrations. They have been used as a measure of the Sun's solar activity and are speculated to have an impact on Earth's long-term climate (see, e.g., Haigh, 2007). The solar magnetic field generating the sunspots varies according to a nearly periodic, 11-year period that is referred to as a solar cycle. Sunspots observations have been well documented in the Debrecen Photoheliographic Data and the Greenwhich Photoheliographic Results. From the latter source, García-Portugués et al. (2020) give a curated dataset with the centers of groups of sunspots. The locations of the centers refer to the first-ever observations of such sunspots (henceforth referred to as "births").

We focus on the last cycle that has been fully observed with curated data, the *23rd solar cycle*. It includes a total of 5373 records ranging from August of 1996 to November of 2001. Even though the main generator of sunspots, the twisting of the solar magnetic field, has a rotationally symmetric nature, the existence of *preferred longitudes* has been speculated for the recurrent appearance of sunspots (Babcock, 1961; Bogart, 1982; Pelt et al., 2010).



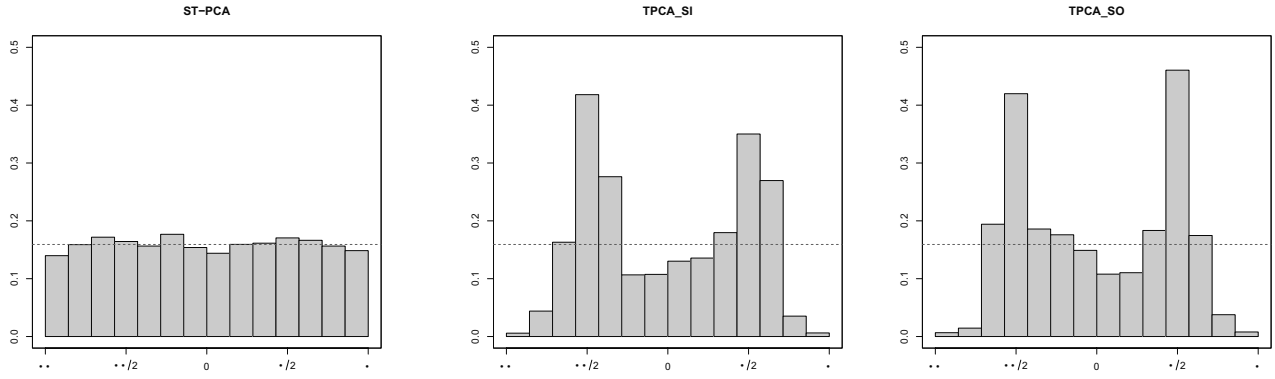
(a) (b) (c)

Figure 4: Panel (a) shows the sample from a wrapped Gaussian distribution on $\mathbb{T}^2$; panel (b): spherical view of the configuration of points obtained by MDS. Panel (c) displays the first PNS scores. The principal mode of variation from the sphere is displayed in all panels in a rainbow palette. The first ST-PCA mode of variation spans along the presumed center of the banded data-dense region.

We explore the dependence structure of the longitudes $\theta \in [-\pi, \pi)$ of sunspots births. Since the Earth rotates around the Sun, sunspots visibility is limited by which side of the Sun is facing the Earth. Therefore, if there existed significantly preferred longitudes, a significant deviation from a linear dependence structure in the series of sunspots longitudes should be expected. To investigate this serial structure, we jointly consider the series of longitudes and its lagged versions of order one and two,

$$\boldsymbol{\Theta}_i := (\theta_i, \theta_{i+1}, \theta_{i+2})' \in \mathbb{T}^3, \quad i = 1, \ldots, 5371,$$

the main driver behind that selection being showing graphically the output of ST-PCA on $\mathbb{T}^3$. The principal mode of variation for the longitudes of the births of three consecutive groups of sunspots is shown in Figure 1c. The figure reveals that the data are concentrated around the (periodic) diagonal line and that ST-PCA (with $\widehat{r} = 1.81$) correctly identifies this line as the first mode of variation, $\widetilde{S}^1$. Recall that the points on the edges of the cube belong to the cloud of points around the diagonal, thus rendering it as the most sensible choice for a principal mode of variation. $\widetilde{S}^1$ is remarkably linear: a suitably-adapted linear fit to the grid of 100 points defining $\widetilde{S}^1$ yields $R^2 = 1 - 3 \times 10^{-5}$. Therefore, the apparent linearity of the innovations in the series of sunspots births longitudes points towards the non-existence of (at least) major preferred longitudes during the 23rd solar cycle, a result that is coherent with the non-rejection of rotational symmetry for such cycle reported in García-Portugués et al. (2020).



(a) (b) (c)

Figure 5: Histograms of the first scores of ST-PCA and T-PCA: (a) ST-PCA; (b) T-PCA with SI ordering; (c) T-PCA with SO ordering. Both versions of T-PCA suggest the existence of spurious clusters (the marginals of the data are not significantly non-uniform), which are not present in ST-PCA.

Method	Component 1	Component 2	Component 3
dPCA*	38.48	34.06	7.67
aPCA*	41.80	37.06	21.14
dPCA+*	52.32	26.99	20.69
Euclidean PCA*	55.43	24.48	20.09
Complex-dPCA*	72.51	14.59	12.90
T-PCA (SI)†	88.27	6.97	4.76
T-PCA (SO)†	89.32	6.43	4.25
ST-PCA†	90.46	3.09	6.45

Table 2: Percentage of variance explained by components of dimension reduction methods on the torus, sorted decreasingly according to the percentage of variance explained by their first components. ST-PCA and T-PCA explain the most percentage of variance in their first component. *Variance calculations based on standard PCA. †Variance calculations based on torus variance as defined in Section 3.5.

5.2 An RNA dataset

This RNA dataset consists of nine angles, out of which six are dihedral $(\alpha, \beta, \gamma, \delta, \epsilon, \zeta)$, one corresponds to the base (χ), and two to the pseudotorsion angles (η, θ). This dataset was put together by Duarte and Pyle (1998) and updated by Wadley et al. (2007) in an endeavor to cluster the complex structure of RNA nucleotides. The pseudotorsion angles were found to approximate well their folding structure. Sargsyan et al. (2012) then curated a subset that consists of 190 observations and three clusters (C3'-endo clusters I, II, and V of Wadley et al. (2007), containing 59, 88, and 43 points respectively), henceforth referred to as the *small RNA dataset*. The η–θ plot of the pseudotorsion angles of the small RNA set reveals a clear cluster structure, which is not visible by the pairwise scatterplots of the other seven backbone and base angles (see, e.g., Figure 7 in Eltzner et al. (2018)). We want to investigate whether it is possible to predict the cluster structure revealed in the η–θ plot by using the remaining seven variables only. Following the analyses by

Sargsyan et al. (2012), Eltzner et al. (2018), and Nodehi et al. (2020), the small RNA dataset has become a de facto benchmark for dimensionality reduction methods on the torus.

Figures 3 and 6 of Sargsyan et al. (2012) reveal that dPCA fails at accomplishing any separation by utilizing the dihedral and the base angles, while GeoPCA succeeds in separating cluster I from II in the first two principal components. However, as it can be seen from the η–θ plot, cluster V is close to cluster II and GeoPCA can offer no further insight on such a separation based on the first two components. An analysis by TPPCA on a subset of the small RNA dataset consisting of 181 nucleotides can give a decent separation using the first two components. Once more, however, the distinction between the two nearby clusters is not very clear (see Figure 2 of Nodehi et al. (2020)). Out of torus dimensionality reduction techniques, T-PCA on a 181 nucleotides subset, followed by mode hunting, has been the most successful at revealing the cluster structure of all 3 clusters by using only the first principal component (see Figure 8 of Eltzner et al. (2018)).

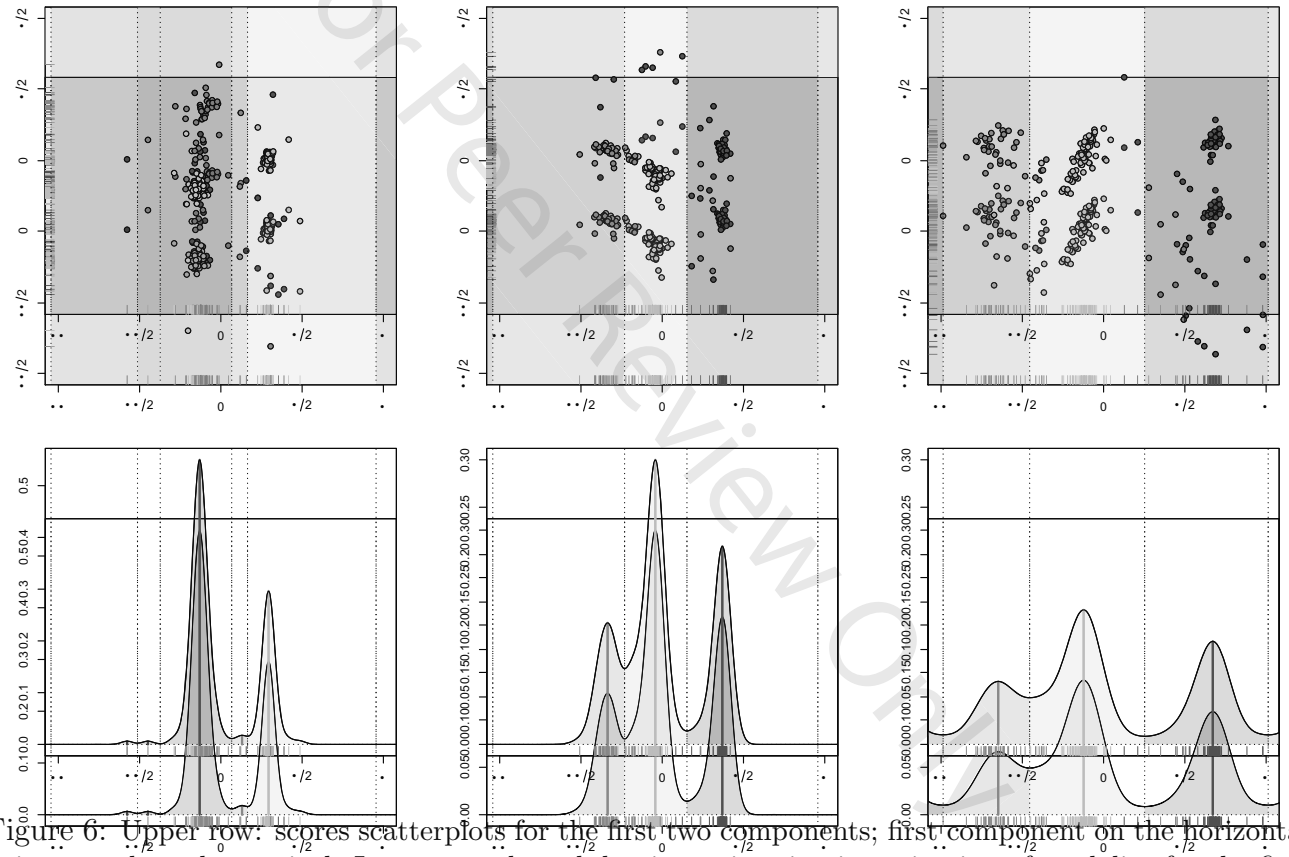


Figure 6: Upper row: scores scatterplots for the first two components; first component on the horizontal axis, second on the vertical. Lower row: kernel density estimation investigation of modality for the first scores. From left to right: T-PCA with SI ordering; T-PCA with SO ordering; ST-PCA. Shows that ST-PCA gives the best separation of these clusters in the first scores.

To produce a clear comparison of ST-PCA's performance to T-PCA, we obtain the spherical scores of the 190 nucleotides dataset with ST-PCA ($\widehat{r} = 1.49$), as well as T-PCA with SI and SO ordering (version "20200518" as provided by the authors). Then, we proceed with a modified clustering scheme (to accommodate for the periodicity of the data) based on kernel density estimation, on the scores of the first principal components. The bandwidth has been determined using the plug-in selector for density derivative estimation (Wand and Jones, 1995, pages 49 and

170.6518	159.1396	56.93419	86.37911	-135.57	-79.217	177.1992	19.20521	-134.515
69.16577	147.834	38.91342	86.54883	-144.399	-73.1093	-177.054	-56.8344	-129.782
162.3669	-173.799	51.07122	83.8322	-141.614	-63.8967	-160.872	29.95225	-128.826
44.86654	179.9128	62.9364	82.55719	-142.303	-77.2195	179.9042	-55.3981	-142.695
-107.27	106.0227	135.8617	83.58057	-140.667	56.74097	-179.539	162.9016	-18.8185
177.8141	147.2034	60.0656	89.32308	-126.196	-88.7378	-179.539	27.78268	-130.257
171.1126	148.1455	52.5461	83.4499	-132.519	-71.8295	-179.539	27.33371	-121.664
166.4159	171.8045	53.30551	83.40437	-132.708	-70.6479	-161.459	31.44748	-126.875
-117.841	-174.882	60.93731	82.35619	-152.096	40.92043	-157.498	172.6817	-29.3516
130.5243	-132.467	66.11368	85.9074	-155.951	-61.0241	-169.334	35.07116	-132.61
176.5316	152.5912	53.21276	89.56549	-129.339	-84.7416	-169.334	16.54162	-129.746
64.18303	152.3337	40.10394	81.90556	-152.694	-69.619	-169.334	-60.798	-139.262
165.6466	167.4834	50.24255	86.37732	-139.922	-65.1556	-170.325	25.25611	-124.667
81.49251	-153.704	67.26839	89.64093	-142.652	-59.2554	-175.693	27.6292	-116.885
-155.812	159.2475	49.8379	87.23268	-137.001	-77.0785	170.6687	32.65499	-133.12
163.3436	60.44921	-173.307	83.22511	-179.038	63.17266	-154.756	-39.5833	-115.472
33.01065	-172.453	44.45755	81.62513	-149.932	-66.4392	179.042	-60.8148	-141.979
157.5995	-178.855	53.68228	85.94795	-144.816	-63.1708	-169.859	26.46799	-127.386
116.4615	151.5809	31.95604	83.33955	-140.276	-53.9059	-173.734	27.5921	-114.004
116.4615	151.5809	31.95604	83.33955	-140.276	-53.9059	-173.734	27.5921	-114.004
116.4615	151.5809	31.95604	83.33955	-140.276	-53.9059	-173.734	27.5921	-114.004
116.4615	151.5809	31.95604	83.33955	-140.276	-53.9059	-173.734	27.5921	-114.004
116.4615	151.5809	31.95604	83.33955	-140.276	-53.9059	-173.734	27.5921	-114.004
116.4615	151.5809	31.95604	83.33955	-140.276	-53.9059	-173.734	27.5921	-114.004
147.8432	-174.729	64.07107	91.29146	-131.075	-75.1105	-165.39	33.95004	-125.614
-45.4653	149.862	64.46398	85.95075	168.1027	66.71454	-156.253	179.9032	-15.6108
170.2879	138.327	51.2796	90.28259	-130.539	-85.4324	-177.557	17.93301	-127.863
41.61935	173.9293	49.45854	85.89087	-149.644	-73.8799	178.1502	-32.9008	-142.115
62.51083	-141.492	132.1737	85.87563	-125.265	-76.386	166.4227	36.76424	-125.138
167.6141	170.822	48.26789	80.83275	-140.318	-59.8277	-148.204	22.95172	-126.211
-58.3396	172.1408	43.60672	85.73865	-157.153	38.16407	-140.539	164.7155	-23.5619
-174.73	120.4502	53.31717	86.69181	-130.457	-80.911	-179.275	29.32579	-126.078
147.5623	179.4416	70.59235	90.47656	-131.757	-79.6677	-175.504	33.80071	-131.419
147.0368	177.8082	-172	74.17668	-150.47	67.44023	-171.234	160.2085	-24.79
41.62032	161.1428	56.29386	88.7779	-97.4685	-138.605	-142.555	-45.1504	-114.954
152.6493	-175.998	62.46497	91.23656	-150.539	-65.7495	-156.611	30.56859	-128.77
171.0935	142.0085	54.24112	82.10729	-140.459	-64.3573	-167.917	20.9618	-123.673
-175.719	158.9818	171.4501	83.55071	-143.45	74.49644	168.137	174.4546	-14.1616
29.89749	151.5054	66.37798	79.23347	-138.494	-79.8303	-174.998	-48.9268	-139.315
155.1089	163.7202	61.91898	86.83338	-134.348	-71.4588	-168.629	25.15497	-126.508
-169.199	178.1393	52.39021	88.27013	-148.357	-67.7092	-178.648	35.11157	-132.743
167.4242	162.1611	52.55598	83.26883	-138.861	-64.4224	-160.081	25.06954	-124.88
82.90724	-173.181	50.69964	85.68318	-145.792	-67.5868	-168.993	15.90936	-134.067
-163.615	139.553	43.38307	90.13762	-144.638	-78.5359	177.9282	18.10452	-130.77
51.6708	-157.355	72.60922	83.80393	-151.276	-63.2747	179.7519	-54.4836	-134.045
157.9374	-178.852	61.27164	80.39882	-128.772	-72.8384	-165.32	36.35782	-130.5
108.0561	-147.747	60.50112	88.40934	-145.883	-61.325	-164.96	42.48646	-123.945
179.2293	177.5343	55.34209	89.37527	-149.147	-70.133	179.9016	36.47603	-134.231
47.80427	171.9251	49.04879	86.82791	-147.819	-70.9348	-169.567	-50.6899	-135.537
169.4187	150.6053	49.70815	83.72892	-140.819	-63.7587	-170.285	20.72117	-123.376
116.5336	-153.12	54.73237	90.62179	-159.707	-69.7641	-167.862	27.50615	-109.438
177.1957	140.3013	47.42921	88.04495	-147.107	-65.9813	-172.363	19.2544	-124.125
-98.8473	-133.09	109.3859	63.10138	-132.707	-90.7586	-159.693	-57.4717	-116.696
-65.2	-176.051	50.60962	80.88511	-152.137	46.54247	-156.79	173.021	-29.5123
-63.1626	169.3499	51.74882	82.91373	-150.113	59.23564	-142.614	163.9484	-19.1818
-63.0729	167.3714	55.87457	84.46952	-153.056	59.22218	-152.692	168.8032	-15.0148

-179.664	152.4096	49.13105	85.38079	-127.635	-84.2148	-170.882	36.83487	-132.28
-69.5162	177.766	60.00282	88.06099	-179.246	71.49491	-156.557	170.6645	-28.0371
-66.389	174.6375	58.0445	83.14072	-151.273	50.31301	-150.044	171.6927	-20.4083
155.5566	-168.88	56.0783	88.98668	-146.074	-60.1128	-170.509	32.71078	-124.272
64.84644	-153.746	179.127	82.07113	-144.517	51.62119	-179.974	147.4293	-19.8175
150.0832	-173.596	60.16505	88.19678	-138.885	-74.022	-173.261	29.45414	-132.195
168.5818	174.0532	51.81897	85.64217	-143.1	-73.0295	-163.925	31.74514	-134.037
-76.0672	172.3302	60.52346	83.36937	-147.079	49.71061	-157.522	164.2151	-17.7472
168.8855	167.0487	51.85694	84.10875	-135.84	-69.0659	-160.755	32.59177	-127.517
77.09416	-177.572	53.55295	88.83246	-150.12	-65.1562	-177.747	-53.0331	-131.041
-66.0427	178.4429	57.108	82.46808	-154.299	85.39847	-162.324	170.7684	-3.2
168.1229	138.9896	59.69094	81.9053	-132.166	-68.971	-161.571	19.35673	-121.481
107.4395	-112.343	-172.252	85.4549	-167.584	59.40938	-171.996	157.3585	-23.54
177.5939	148.1655	52.85104	82.31917	-133.143	-77.6061	-168.744	30.92886	-130.623
160.1433	179.733	51.66158	82.99252	-143.944	-62.6456	-167.908	25.95606	-129.508
-64.0281	176.6825	54.49192	79.38133	-148.136	46.18228	-165.461	171.5225	-26.8649
161.3802	174.5784	52.20645	83.27746	-139.536	-68.4912	-166.884	21.48594	-130.677
-66.6854	175.7021	54.56178	87.86018	-157.655	45.57922	-146.715	169.0051	-21.6644
-66.1426	178.9373	52.84037	78.10849	-167.068	34.16354	-154.166	167.0708	-25.869
79.62865	135.8339	45.89947	82.58531	-126.274	-84.9673	-145.378	-56.3952	-127.579
48.91294	177.4874	53.31516	84.96621	-147.86	-69.5703	-179.888	-55.3086	-137.582
-59.7244	172.1518	48.96137	77.19291	-150.751	49.4319	-158.768	164.0574	-24.9964
35.8505	-146.243	56.0953	82.31676	-155.168	-59.4548	10.66923	-38.2173	-135.85
178.1884	158.2567	44.84857	84.80076	-162.497	-60.8832	-148.586	35.93556	-116.304
165.0964	158.0364	51.53558	83.56571	-125.951	-76.0571	-156.71	21.52079	-126.769
-64.1437	173.5304	52.40359	83.83297	-163.229	64.92364	-146.969	163.8906	-14.3582
171.7779	171.5914	51.35859	82.12413	-145.219	-61.9712	-157.694	35.96513	-129.871
-53.6684	165.4355	51.25852	83.4353	-151.791	53.84223	-149.149	168.3825	-17.0975
168.6048	163.3021	53.19662	82.87697	-144.527	-64.0603	-164.344	29.41823	-129.093
51.22741	-166.203	56.08736	89.24088	-111.824	-80.6471	164.931	-55.1591	-115.173
-79.4791	162.7224	58.01281	86.39236	3.846636	-126.182	-165.94	-179.572	-9.86152
153.2411	174.0469	55.52294	81.64593	-138.541	-61.2944	-170.185	15.58223	-124.556
172.7651	148.4995	48.3667	85.45597	-132.908	-76.0823	-140.107	24.99756	-127.502
-69.025	-178.596	51.97886	81.23015	-154.194	50.16061	-147.021	167.3871	-26.4186
173.0777	166.9322	46.90246	80.24502	-140.941	-65.3347	-168.122	32.05884	-131.167
-60.6389	173.0337	51.10287	84.41814	-150.443	58.83851	-148.547	168.4134	-19.1713
176.9727	143.3269	49.35222	86.20903	-145.882	-65.0494	-167.891	20.57143	-123.764
-59.5184	168.7888	53.77261	82.64995	-142.068	49.33168	-146.18	168.2203	-18.9372
-60.6489	169.3743	55.6336	74.81399	-168.12	56.03	-168.824	166.0605	-2.34169
165.9289	171.3268	51.14728	83.49275	-132.436	-70.1834	-162.838	30.74706	-127.118
-70.1535	174.2832	55.62414	82.93035	-159.208	51.70385	-165.784	160.9207	-14.9021
169.4431	147.1274	51.0215	83.48576	-139.048	-72.3263	-173.429	20.6211	-129.351
170.2855	164.8464	51.81113	85.18661	-142.157	-70.5511	-171.178	31.20497	-131.42
175.1823	148.5321	53.81205	85.66118	-140.482	-70.6856	-159.461	30.39458	-127.758
-62.9444	169.8522	54.42579	82.35607	-146.165	46.04775	-159.119	167.3546	-26.1411
-73.6399	163.0858	59.59931	83.50044	-148.016	51.2838	-151.972	152.1962	-15.8267
158.4472	164.355	51.14839	87.93469	-122.598	-82.0499	-146.794	23.77697	-127.788
-104.362	-135.546	160.0195	81.35373	-135.013	-66.1142	173.4279	-45.7567	-136.843
-61.7063	-179.588	54.88487	82.85154	-158.695	54.92937	-163.474	177.1273	-23.3529
54.16521	-152.485	58.89671	88.98542	-137.339	-67.7003	174.8351	-36.5492	-124.011
56.46962	178.9829	48.28218	85.51642	-159.668	-63.5811	-168.684	-53.4403	-140.058
-62.0159	166.2869	54.6109	80.70417	-148.515	33.78919	-154.951	165.7887	-28.295
176.3414	133.1807	56.66044	84.75572	-138.845	-68.241	-175.896	19.93437	-122.413
56.77585	136.064	50.06296	83.17585	-137.247	-67.7274	-171.87	-60.6024	-120.717
-57.6098	167.8959	53.8299	80.85689	-149.998	49.42181	-164.564	171.2032	-22.7008
-64.481	167.6914	57.64768	84.44554	-153.304	51.52963	-147.016	166.2271	-17.517

-56.3749	162.818	52.75431	81.34275	-155.401	64.40867	-152.703	159.1634	-17.9393
166.6598	151.5511	52.98709	86.56023	-142.818	-68.0207	-163.087	19.40126	-126.036
80.08184	145.9439	49.85339	81.80753	-139.517	-72.0756	-172.861	-37.2198	-129.792
87.47241	-131.064	-163.465	82.69277	-163.776	123.7002	-158.828	157.3212	-2.10443
-70.5433	172.4892	42.75329	74.33085	-173.918	58.21791	-155.249	150.8691	-8.95145
68.9241	-168.414	55.87373	85.23538	-142.242	-61.59	179.238	-51.7167	-124.091
167.2591	165.2985	48.52122	86.60335	-131.931	-76.8748	-140.081	29.78576	-129.436
168.3219	139.7086	54.65417	82.85469	-134.31	-68.58	-155.102	15.64777	-122.227
-68.621	176.2357	54.38499	84.30078	-145.6	61.94035	-151.945	168.2969	-17.3445
-55.238	171.5034	53.07193	80.60486	-144.172	54.93058	-154.111	169.2665	-18.1671
158.6071	148.9077	60.47415	86.50948	-131.247	-73.1688	-174.405	18.90144	-124.563
-71.0486	-178.865	53.40438	76.39438	-163.297	41.30381	-154.448	165.2078	-17.2423
115.5835	165.0856	50.8491	86.32635	-153.33	-62.7214	-163.943	-39.3738	-130.841
-61.652	170.7638	56.92295	82.8693	-148.793	48.9052	-151.55	175.9915	-22.4944
-62.7788	158.6105	56.12058	81.76068	-154.987	58.22978	-149.264	160.2818	-17.8643
177.7513	134.7843	-178.503	80.26397	-147.728	108.478	-166.291	168.7025	-18.9558
49.4211	177.21	51.77789	83.65248	-164.502	-56.573	-175.662	-47.2591	-132.803
159.0682	175.71	56.7215	79.56553	-121.313	-66.8639	-171.832	30.71519	-125.848
67.688	-145.34	73.0975	94.65506	-145.801	-63.2278	175.2435	21.56148	-112.992
173.1698	130.5074	53.66037	88.87732	-122.43	-77.2833	-168.711	17.2818	-118.714
98.78209	142.5761	70.07518	87.9906	-168.311	-58.9843	174.6208	-39.6706	-134.522
171.6211	169.2941	53.63623	83.25831	-145.254	-63.8661	-153.078	36.56911	-130.08
176.0461	149.8265	52.64155	83.52225	-131.896	-76.0859	-176.037	34.81085	-130.958
179.5058	150.1054	39.92065	86.83193	-142.795	-68.1175	-162.759	25.99743	-127.67
171.9152	176.6275	47.53141	83.61806	-130.361	-72.0929	-175.745	36.7966	-129.796
53.07935	138.0518	57.27358	85.06986	-149.332	-60.2374	175.9695	-52.9867	-122.081
176.6574	146.8322	43.33171	82.63655	-138.427	-67.2669	175.0152	17.8871	-125.718
175.0734	136.2518	53.03364	85.17551	-144.192	-67.5659	-162.853	19.66441	-124.226
167.9707	-175.119	49.37105	89.20364	-148.817	-65.6231	-154.728	38.40319	-131.968
40.10382	-167.704	52.80215	84.5391	-145.314	-74.354	177.3841	-37.9479	-141.736
169.1697	167.435	50.37074	82.2411	-136.687	-71.207	-161.477	22.28003	-130.887
62.45	166.1832	75.97783	87.43393	-147.114	-75.8779	177.2553	-38.0317	-140.719
-115.83	-131.972	53.04442	81.63938	-179.583	95.65614	-159.827	149.452	-12.3446
142.8314	-127.778	165.0141	88.46297	-159.823	50.58353	-155.344	152.2838	-7.85984
-57.0788	164.7236	53.81199	86.283	-151.018	30.89924	-149.083	167.3764	-24.851
-66.5893	175.841	49.82845	80.80563	-153.726	44.07372	-152.246	162.402	-25.3151
-60.898	154.2416	54.34762	84.4742	-154.19	20.59823	-139.4	158.6541	-25.6905
-66.3237	169.2029	48.67437	81.99366	-157.044	45.46477	-144.952	155.1916	-27.0657
109.0317	164.4464	57.45768	86.27968	-146.422	-77.1078	-178.833	-38.1692	-140.342
43.35622	174.9491	52.55484	87.00692	-137.781	-69.878	-143.234	-51.9518	-127.125
115.8349	145.6064	53.36837	80.59218	-143.784	-77.7663	177.0317	-39.4748	-140.129
-65.4618	167.9525	53.99573	85.92269	-139.109	40.92437	-148.996	158.986	-25.2057
165.2029	160.1364	54.00786	87.5405	-125.691	-81.8638	-155.461	32.70158	-129.446
-71.5458	179.4683	54.59543	85.42263	-153.444	48.95067	-156.233	163.0965	-23.401
-179.528	131.6291	51.5503	84.81616	-137.259	-71.5393	-154.076	20.96086	-123.74
174.5002	151.4585	45.06948	90.29445	-123.339	-87.4258	-160.367	29.58489	-128.571
121.3671	124.6593	47.21155	79.76204	-150.537	-67.3518	-173.738	-46.3458	-130.804
-66.6388	-179.799	55.55054	87.51268	-146.979	47.02942	-153.082	175.5309	-24.8822
-71.7978	-166.889	54.19335	84.35243	-174.813	95.71878	-153.811	176.2376	-13.0225
76.0503	163.9463	56.02039	86.01763	-122.26	-78.3667	-144.443	-54.8209	-124.86
-57.9591	160.2034	54.89957	83.18871	-146.112	46.50361	-151.481	157.5619	-19.6471
-178.291	135.312	55.23386	89.14931	-120.498	-90.2477	-138.276	37.61381	-131.366
-57.7625	168.6167	47.19077	85.46681	-167.785	48.66114	-132.74	166.0867	-25.8613
-60.8358	157.0037	60.01226	82.34888	-162.822	45.78711	-143.775	156.2652	-28.9773
178.7717	143.0028	48.84289	85.28521	-135.425	-75.2884	-165.683	27.5235	-128.538
178.126	130.5369	52.06682	87.29909	-132.573	-79.6617	-173.951	20.37568	-125.18

178.013	169.7201	50.0685	80.66193	-147.312	-63.3244	-153.615	39.2577	-134.517
119.3617	138.8477	44.55074	83.47398	-155.224	-47.0905	-162.414	-42.2298	-115.02
-55.4255	174.681	52.33867	83.5191	-141.417	62.89298	-146.287	179.3033	-19.0702
154.5486	157.6875	60.06283	80.74405	-135.464	-61.1675	-163.134	15.82036	-123.023
-55.954	170.788	49.47675	83.35658	-145.048	45.37292	-126.622	172.178	-23.6783
59.49049	156.7405	52.42442	83.57819	-140.922	-76.2937	-179.9	-36.3346	-136.39
-166.305	123.2774	59.66114	93.61968	-102.134	-153.232	-179.9	30.66119	-116.711
-79.0748	-166.389	142.2364	90.94914	-74.3786	-152.489	-179.9	-48.9679	-139.954
104.9675	-168.753	94.41336	88.22871	-112.469	-68.15	173.4698	16.33009	-122.46
153.8708	174.5624	-179.329	79.89219	-152.675	66.80798	-170.034	159.4896	-20.8347
69.43521	144.7162	49.6072	85.21569	-151.49	-60.8381	-156.95	-59.9243	-129.637
69.43521	144.7162	49.6072	85.21569	-151.49	-60.8381	-156.95	-59.9243	-129.637
69.43521	144.7162	49.6072	85.21569	-151.49	-60.8381	-156.95	-59.9243	-129.637
69.43521	144.7162	49.6072	85.21569	-151.49	-60.8381	-156.95	-59.9243	-129.637
69.43521	144.7162	49.6072	85.21569	-151.49	-60.8381	-156.95	-59.9243	-129.637
32.64327	170.8713	54.26971	87.11634	-149.831	-61.5175	178.3487	-57.6152	-129.76
155.2207	-170.843	52.18176	84.1593	-132.186	-79.259	-169.114	15.04631	-133.842
-124.434	-177.597	69.51292	97.69679	-34.5831	-54.8805	-143.774	155.1691	-28.1965
152.2521	-178.097	57.73937	86.09247	-145.451	-61.7553	-173.373	16.0373	-126.189
-29.967	-125.114	76.74432	89.54398	-160.662	-63.6244	179.824	-52.3649	-128.056
166.6033	166.663	47.54887	85.57427	-130.454	-70.6724	-153.314	27.21253	-120.852
122.6223	-177.065	63.69893	88.58902	-126.209	-73.5963	-178.875	23.25659	-119.561

V1	V2	V3	V4	V5	V6	V7	c(rep(1, 59), rep(2, 88)	
1	########	-23.6589	9.220699	-27.4195	-59.3771	########	103.8568	1
2	3.858881	5.323688	-4.62384	3.126138	-6.43771	1.172271	-46.7768	1
3	-3.28876	-2.6803	-4.03909	-20.54	62.85895	12.49649	-28.0764	1
4	########	########	5.066994	-2.34467	-3.05128	15.05918	25.1404	1
5	########	########	-7.29564	-9.66923	6.632428	13.92973	21.62465	1
6	-1.41561	-3.39754	28.12018	25.15485	5.574687	36.74477	-69.9256	1
7	-1.68817	1.227302	8.283223	########	-9.2414	18.43078	30.5543	1
8	1.195328	-2.01186	4.316761	3.106728	-4.81627	18.04108	25.33001	1
9	1.120216	-1.21454	-3.76986	-6.04187	1.300861	17.18114	26.39082	1
10	########	-3.09446	-3.24524	-8.96405	-2.40082	23.57478	-40.8384	1
11	########	########	-8.79417	-22.2059	23.28258	-6.56242	17.60935	1
12	-1.96289	1.552165	2.852436	-1.3609	-7.97254	19.05407	28.57014	1
13	########	-1.26921	-2.22074	-2.53275	1.81558	15.71531	22.06281	1
14	-1.72992	-5.50856	-1.41033	-10.277	15.9625	-29.4274	19.48165	1
15	########	2.51433	8.288027	-5.86142	-13.5079	18.93297	14.09256	1
16	########	########	-4.19152	-6.57331	7.781489	11.41262	21.21231	1
17	########	-7.23617	-7.07331	14.10283	2.802154	-5.7363	22.83564	1
18	########	-7.23617	-7.07331	14.10283	2.802154	-5.7363	22.83564	1
19	########	-7.23617	-7.07331	14.10283	2.802154	-5.7363	22.83564	1
20	########	-7.23617	-7.07331	14.10283	2.802154	-5.7363	22.83564	1
21	########	-7.23617	-7.07331	14.10283	2.802154	-5.7363	22.83564	1
22	########	-7.23617	-7.07331	14.10283	2.802154	-5.7363	22.83564	1
23	-2.62233	-2.90435	-2.96147	-10.156	9.091862	7.380557	31.0597	1
24	-2.63295	11.41377	2.522689	16.60585	########	-5.78997	-50.664	1
25	-2.39405	########	5.332967	6.063093	-9.10416	16.82407	29.81954	1
26	########	-1.41893	19.27078	-26.6886	25.79298	-42.877	28.90665	1
27	1.977866	########	-8.88702	-3.31159	3.34767	20.18422	22.88738	1
28	-1.62046	3.289056	-2.18869	########	-16.4182	-5.01563	-50.2636	1
29	########	1.795641	10.35193	11.78452	-14.555	23.91958	27.81704	1
30	-2.08147	-1.93824	2.530626	-8.90932	8.331197	6.601361	31.73542	1
31	3.020248	-1.01274	11.23161	1.064693	65.55567	34.64998	-55.1845	1
32	-3.09834	2.856191	-5.66726	-6.99907	12.96791	10.7768	24.14904	1
33	1.436657	########	2.514722	7.332129	-1.79149	21.74977	24.19141	1
34	-1.43594	-6.22593	17.31024	6.301423	48.03545	40.18206	-63.6693	1
35	########	-1.84436	########	-1.27254	5.038406	13.00969	29.72443	1
36	-1.21362	3.745216	1.995163	-10.5975	-2.40051	18.37943	12.10229	1
37	########	########	-3.21138	########	1.748	19.29888	24.27868	1
38	########	-2.26075	-4.34415	-1.25806	8.840562	-26.2284	21.21017	1
39	-2.48498	5.479368	6.514439	5.128119	-13.8311	20.27378	16.23735	1
40	2.826441	-3.09499	-2.70138	-9.89648	5.760438	12.93528	29.63939	1
41	-1.35156	-3.87694	-8.65017	-13.3257	19.31562	-16.7771	22.58134	1
42	-1.75618	3.27858	1.807637	-8.97423	1.194155	15.77734	15.82594	1
43	########	########	########	4.401037	-1.26215	19.09525	22.3379	1
44	-2.65265	2.112963	-8.31124	-11.6733	17.00927	-11.6454	19.20991	1
45	-1.60218	3.08084	2.778618	8.216966	-4.78398	20.95978	19.51633	1
46	1.157839	-2.22866	-1.51738	-5.24097	-11.3679	-1.67016	-50.4557	1
47	########	-1.79327	-3.53189	2.388336	-13.1813	########	-55.858	1
48	-1.08008	-2.24951	########	3.50562	-10.8879	########	-54.7589	1
49	########	########	2.356408	-1.56017	-10.3573	19.56418	27.11164	1
50	-3.22878	5.653232	-3.52292	1.775542	########	########	-50.8032	1
51	########	-1.36548	########	-1.6136	-12.0022	########	-54.5414	1
52	-1.5939	-1.72473	-5.08018	-10.3364	10.92128	9.817316	19.9008	1
53	########	-6.55072	12.26364	-7.83157	57.92634	-41.8042	-57.6773	1
54	-1.04626	-1.42584	-2.23098	-9.94736	8.469583	6.762426	26.0736	1
55	########	2.555189	-3.40363	-6.3047	1.885841	15.75984	22.44693	1

56	########	-3.9431	1.95663	-1.94233	-11.7503	6.282096	-53.324	1
57	########	########	-3.44581	-3.79624	########	18.56327	24.93296	1
58	########	-5.87721	-7.30175	2.277141	-2.0362	########	-55.4239	1
59	1.507363	########	2.831509	7.294389	-1.83645	22.84114	30.69909	1
60	-1.78875	########	-1.87449	-26.0432	62.05383	-21.0489	-40.9457	2
61	1.542624	1.628569	2.987175	1.573834	-6.92391	21.09564	26.32084	2
62	1.317815	########	-4.87297	-6.21743	6.581314	12.8184	20.97002	2
63	2.045953	-4.92791	1.853134	-2.316	-11.8578	-1.30229	-50.7136	2
64	1.199042	########	-3.78842	-5.29139	3.972406	13.65978	23.82344	2
65	-2.66564	1.895477	########	-2.06098	-12.1039	########	-52.2941	2
66	2.374776	5.448601	1.464746	-3.48348	-10.412	-1.41129	-47.589	2
67	2.90877	-3.23468	########	########	-13.0472	-3.17416	-51.0731	2
68	########	9.930093	-5.67934	2.890409	1.296246	21.78893	14.32578	2
69	########	-1.535	-3.36499	########	-2.75813	18.26547	31.57571	2
70	########	1.609327	-4.42093	2.357516	-7.3843	########	-53.2463	2
71	1.464686	2.234429	-4.99845	-4.44895	3.113879	19.76444	19.94868	2
72	########	-1.87659	########	4.835397	-13.5693	-5.51348	-54.3109	2
73	1.161769	1.540922	-1.98565	########	2.56965	18.49887	21.65535	2
74	2.091732	-3.72717	-3.22859	-3.30694	7.679516	11.28909	24.44907	2
75	########	4.099138	-5.76724	2.966443	-5.51833	23.68159	30.34458	2
76	########	########	-2.93039	-4.22552	-11.454	########	-52.4455	2
77	2.633715	########	-2.82979	-3.08211	########	18.13308	19.85806	2
78	-1.00392	-2.83973	-3.24093	1.178353	-12.0097	-2.28573	-54.531	2
79	########	3.053046	1.80883	6.715566	-3.71981	22.00015	20.27031	2
80	########	-4.09951	########	########	-16.7453	-2.79646	-56.4279	2
81	3.856911	1.364046	1.111839	4.355454	-5.36312	-2.05877	-49.2186	2
82	1.101207	-1.78378	-4.06936	-5.55431	########	16.61867	25.93719	2
83	########	-1.29128	########	########	-8.02776	1.827304	-49.5126	2
84	########	########	2.400259	4.755346	-3.43234	17.92628	24.6682	2
85	########	1.321642	########	-2.5668	########	16.66534	21.84043	2
86	########	3.474401	########	3.220143	-2.98752	22.68931	24.7767	2
87	########	-4.42952	2.224487	########	-14.4466	-1.11775	-52.5018	2
88	########	-2.56595	2.083039	2.462426	-13.7058	6.140753	-54.8846	2
89	-1.36789	-1.02336	-7.08325	-3.27188	-1.60553	14.96085	35.62405	2
90	########	-1.80217	########	-1.74527	-7.08559	-2.64388	-50.7818	2
91	1.247116	-1.00943	4.720244	########	-17.4872	-1.95937	-51.3134	2
92	########	1.04266	7.264532	9.405176	-5.35756	22.77077	24.35457	2
93	1.135274	-4.48209	2.286293	2.84594	-12.2964	-3.61076	-51.5631	2
94	-1.07851	########	########	1.906588	-12.7907	########	-55.1928	2
95	########	-2.03734	-2.00555	7.358303	-10.3231	-3.01712	-54.6659	2
96	########	2.008844	########	4.057466	########	18.80459	24.7581	2
97	-1.49815	4.633046	-2.26331	-18.0239	57.47244	-13.3499	-75.1394	2
98	########	2.702475	-8.81241	-3.44732	-1.71536	19.51407	30.30571	2
99	########	1.025349	########	7.75849	-2.43103	23.04367	29.87816	2
100	########	-5.38358	-3.35788	-1.161	-11.055	1.890042	-55.3119	2
101	1.105781	-5.70742	########	1.492393	-13.4816	-5.02941	-55.1763	2
102	########	-2.44866	4.196479	3.922847	########	15.09083	30.68509	2
103	3.225875	3.216943	########	-4.38435	-9.57436	1.276435	-48.7721	2
104	########	-2.42517	1.198784	########	-13.5745	-1.73355	-54.7539	2
105	########	########	########	7.335503	-12.3553	########	-55.3875	2
106	-1.16616	########	12.56818	20.0825	48.5255	38.15892	-87.6884	2
107	3.445724	-7.95741	-1.88569	-7.80662	2.942987	14.00209	29.1323	2
108	-4.14166	-3.50367	2.507863	-15.4025	14.45607	-33.7795	14.26771	2
109	-1.82305	-2.1011	5.213228	8.451416	-9.84352	22.70895	32.77082	2
110	########	3.461082	-5.05929	-3.78658	3.269749	20.79995	21.59369	2
111	1.120361	########	4.083357	1.10308	-6.5813	19.37178	25.55731	2

112	########	2.787978	-2.49397	3.680753	-6.69541	21.08396	19.38302	2
113	1.342786	-3.30495	-2.08002	-9.071	-2.48093	15.15332	22.44403	2
114	1.54069	-1.42768	3.201	4.862458	-7.15096	17.56941	19.36991	2
115	########	3.872819	2.57322	9.341999	-3.7929	23.55851	23.84811	2
116	-2.08316	3.355576	-8.03583	-9.35575	5.672707	15.88932	19.24639	2
117	1.595707	########	-3.59973	-3.95505	########	17.85954	24.60605	2
118	########	########	-17.5254	-19.1309	15.2291	5.179578	-44.4478	2
119	-1.66968	1.142066	4.53017	1.43163	-18.4323	-5.14559	-51.7272	2
120	1.051628	########	########	-2.10819	-13.454	########	-50.6001	2
121	-1.02719	6.301944	6.285228	4.254223	-22.7884	-2.34453	-51.7493	2
122	########	2.126833	-1.3433	1.172379	-14.4141	########	-51.4509	2
123	-1.42775	-4.36428	2.089768	########	-19.2305	########	-54.6546	2
124	-1.07579	########	-2.79165	-2.75189	-3.00991	17.45972	33.04808	2
125	-1.22188	-1.7135	########	-3.78991	-10.6015	2.329555	-51.1383	2
126	########	4.293218	1.754299	9.279242	-8.25203	27.15244	26.64386	2
127	-2.44132	########	-1.5926	########	-10.8613	18.84663	30.98086	2
128	-2.20871	-3.37933	########	-4.74381	-12.9502	########	-53.23	2
129	-1.47639	1.927431	-12.1798	-2.43753	5.237905	########	-51.7177	2
130	########	-3.0105	2.77472	4.89369	-16.8156	-2.97277	-54.9719	2
131	-2.48677	5.531764	########	3.387035	-13.5803	27.48774	37.8032	2
132	-1.93693	7.208174	-3.63686	3.597618	-12.2111	-4.35777	-52.1588	2
133	########	5.069865	3.390196	6.952557	-13.1529	-1.0924	-54.6762	2
134	########	2.173289	1.931513	4.450484	-8.04182	22.186	25.10671	2
135	-1.09911	1.657876	6.569583	8.890134	-10.4728	21.78915	27.93018	2
136	2.059442	4.594821	-5.08061	-4.54003	########	22.07735	18.31176	2
137	########	-6.50021	-4.3683	########	-12.9997	-4.82884	-57.6627	2
138	2.296836	-3.15399	-1.43317	3.099012	6.613544	15.53399	28.9075	2
139	########	########	-4.02516	########	-19.1741	-5.85459	-57.9821	2
140	-4.60304	10.77866	10.3979	2.527547	-32.552	7.797071	51.71599	2
141	########	-12.7799	10.45365	-11.7828	18.84794	-17.0905	38.44627	2
142	########	-1.45165	10.46947	2.504175	61.58366	37.62645	-50.7708	2
143	########	-2.04169	-5.24989	-12.4999	3.934855	8.053692	26.94003	2
144	-5.84489	-20.9005	6.949333	-37.2619	-45.5046	63.03599	36.32448	2
145	########	-1.80436	-2.86608	-6.16818	10.18922	9.150666	21.7391	2
146	########	-1.17636	-6.63953	-3.53655	-1.07631	18.41859	27.63996	2
147	########	-7.87299	########	-6.34777	9.750572	-6.14631	32.14648	2
148	########	########	1.336474	14.92949	-4.24146	-26.2731	20.69373	5
149	1.781473	1.464736	8.313901	-2.57121	-1.20924	-37.6797	15.39615	5
150	1.421336	2.41584	########	14.51586	-1.6882	-29.2409	17.01739	5
151	-2.8606	10.83616	20.38339	56.21203	39.6622	72.89433	-55.0753	5
152	2.130524	1.411884	2.707804	-4.1172	-4.20034	-35.999	3.127871	5
153	########	2.606387	5.477411	1.450861	-4.01656	-34.6097	10.00806	5
154	-1.83889	5.729119	2.666262	-5.96617	-25.3757	-34.4241	50.43863	5
155	3.146087	4.283532	15.73904	8.010872	-10.9135	-41.5702	15.99002	5
156	1.0996	########	5.500368	-10.0164	9.925625	-38.2799	9.60723	5
157	########	1.990787	2.607669	3.887934	-2.34189	-36.7283	12.96921	5
158	11.11448	16.05174	16.62998	-54.1894	-15.35	9.224364	-8.24425	5
159	-1.51701	-1.59938	########	########	8.300447	-27.0717	17.87512	5
160	########	########	-2.63313	18.94935	-5.71768	-26.5141	39.06061	5
161	########	########	4.715388	########	########	-35.1123	12.04097	5
162	-1.76392	28.95941	-41.3692	-20.875	########	-60.7351	1.881747	5
163	########	-10.5226	6.351211	-12.7731	-5.90407	-35.2182	23.09668	5
164	1.855708	11.88586	29.44888	-56.8432	19.93858	16.30165	-21.0455	5
165	-1.16298	-4.77644	2.070128	-13.4414	4.844687	-36.2221	12.91494	5
166	########	3.633936	########	3.176983	4.312815	-33.781	9.90501	5
167	########	-2.30696	7.041942	19.83838	-6.50812	-33.885	22.74417	5

168	1.732325	-2.47203	2.582051	15.58371	########	-24.2005	27.3662	5
169	########	-5.02894	########	-4.08034	7.564044	-31.3944	16.338	5
170	########	########	-5.25605	7.875825	11.80122	-7.79686	24.58821	5
171	########	4.125668	2.526033	4.037636	4.642775	-34.0471	4.520804	5
172	-1.75194	4.376046	8.632679	18.46597	14.0133	-13.2996	19.81179	5
173	########	########	11.5274	19.61582	-2.39552	-32.3034	14.91347	5
174	########	########	4.095411	-7.39604	-1.92124	-36.5236	9.186402	5
175	########	2.293772	12.13872	4.289825	5.927985	-32.7343	22.31402	5
176	########	########	1.286051	5.060848	7.726955	-11.3124	27.45074	5
177	-1.11867	2.289947	-1.16296	2.86047	-4.12218	-46.1303	16.75327	5
178	2.42876	########	3.460148	12.40529	3.106671	-6.1948	28.99158	5
179	2.243208	########	1.954006	23.75745	2.614195	########	28.13603	5
180	########	-3.77921	-3.99974	6.908768	1.363007	-32.7669	38.04951	5
181	########	-1.40112	-4.71423	21.4656	9.965966	-2.32257	23.20071	5
182	1.077539	########	6.256657	9.223432	-3.18859	-31.4269	21.19546	5
183	-3.06198	13.56703	29.27082	-52.5855	-26.5436	-13.1873	80.74563	5
184	########	1.501796	########	19.68236	3.035898	-31.2595	21.42216	5
185	########	1.501796	########	19.68236	3.035898	-31.2595	21.42216	5
186	########	1.501796	########	19.68236	3.035898	-31.2595	21.42216	5
187	########	1.501796	########	19.68236	3.035898	-31.2595	21.42216	5
188	########	1.501796	########	19.68236	3.035898	-31.2595	21.42216	5
189	########	1.206216	8.689081	3.14916	-4.3443	-37.3546	3.955968	5
190	-1.97623	8.707165	8.768637	-34.4664	-3.47512	-30.2924	-22.0188	5

```
), rep(5, 43))
```

V1	V2	V3	V4	V5	V6	V7	c(rep(1, 59), rep(2, 88)
1	########	3.874078	-4.6367	13.32982	11.69879	-25.7309	-52.2557	1
2	########	1.247533	1.428244	-3.84273	-9.63335	-20.6136	-42.148	1
3	1.209728	-4.05185	########	1.85813	########	-4.48091	-75.3256	1
4	########	########	########	-1.0077	-7.26291	-2.98979	-68.0784	1
5	-3.25356	4.945606	-5.74756	########	16.15203	-59.6699	18.43347	1
6	########	3.248316	1.555407	4.830497	18.93139	-50.1796	-15.0411	1
7	########	########	1.831995	-1.91953	-4.3667	-6.93633	-65.12	1
8	########	1.419159	########	########	-6.64167	-8.34046	-68.8128	1
9	########	########	########	########	-4.2865	-8.55933	-67.8311	1
10	1.743569	-3.34434	1.515953	########	-3.9707	-13.3241	-67.6143	1
11	########	########	########	-1.15584	-5.20938	-8.30098	-65.631	1
12	########	3.429268	-1.14307	-3.40167	6.464406	-33.8546	68.46646	1
13	########	1.623142	########	########	-4.55661	-11.0102	-61.1035	1
14	########	########	4.280173	########	-1.53459	-13.1722	-69.8365	1
15	1.047665	-1.84416	-2.49426	-9.25322	13.30753	-51.4196	53.65912	1
16	-1.09355	########	2.07805	########	-1.94771	-6.56281	-64.8042	1
17	1.680311	########	########	-1.44966	-6.49445	-7.43372	-65.2613	1
18	########	-2.77894	########	-1.61364	-5.23406	-5.45315	-63.5909	1
19	-1.48488	########	1.771953	########	-3.31746	-5.61987	-67.3804	1
20	########	########	########	########	-5.94041	-9.91854	-69.0378	1
21	########	########	########	########	-3.91012	-4.57621	-70.801	1
22	-1.7915	3.79907	9.502438	########	19.03019	30.71679	-56.6604	1
23	########	########	########	-2.03679	-6.98555	-8.79752	-65.0493	1
24	########	########	1.27754	########	-5.07444	-7.48193	-69.0454	1
25	########	2.207705	########	########	-5.09115	-5.55583	-68.3298	1
26	-1.47423	-3.07103	1.67466	########	-1.03329	-7.3682	-67.356	1
27	########	-1.48219	2.066569	########	-2.67699	-9.48946	-63.1434	1
28	########	########	1.244872	########	-3.13399	-6.16007	-65.5684	1
29	########	2.025884	########	########	-5.70111	-10.4979	-62.7912	1
30	########	-1.45535	1.944792	########	-2.11741	-7.50693	-67.0093	1
31	########	########	########	########	-3.90171	-3.88282	-64.7792	1
32	########	########	1.575401	########	-1.46884	-5.15033	-67.7011	1
33	########	########	########	########	-5.63168	-7.88095	-66.7795	1
34	########	########	1.111752	1.040391	-3.4445	-7.01418	-70.9154	1
35	########	1.996373	7.086896	########	1.37658	-52.4658	73.71527	1
36	-1.63295	-3.62254	2.546223	########	-6.54368	-10.414	-65.3558	1
37	########	1.637632	2.193051	########	-5.14907	-10.5969	-66.3289	1
38	########	########	1.263816	########	-2.94854	-5.20134	-69.9617	1
39	-1.68185	-2.13117	########	-1.80507	-5.80432	-8.07845	-62.5179	1
40	########	1.05038	########	########	-4.25322	-6.49852	-67.0357	1
41	########	########	########	1.390055	-5.01952	-8.32442	-68.1613	1
42	########	4.851663	3.208798	8.725231	9.908939	-66.0982	-22.2464	1
43	########	-5.45218	10.49162	-2.73752	-7.05243	-34.4336	-56.8186	1
44	2.85393	1.410177	-9.33382	-16.0887	10.41101	-57.4439	22.48355	1
45	1.033279	########	-1.23694	########	-4.32454	-2.0456	-66.8637	1
46	########	########	########	-1.08618	-5.76407	-6.79765	-64.5746	1
47	########	########	-3.66982	########	-6.72415	########	-65.0224	1
48	########	########	########	########	-7.49672	-6.84595	-65.5461	1
49	########	2.956245	########	########	-5.47467	-5.65882	-64.7155	1
50	########	########	1.47171	-1.46237	-5.02921	-9.39747	-62.9694	1
51	1.286391	1.228796	1.115617	-1.94091	-4.72629	-7.54722	-64.8286	1
52	########	-2.44532	4.855222	########	-3.77578	-17.2994	-71.3621	1
53	########	1.362921	########	########	-3.88368	-4.75916	-68.1068	1
54	1.108283	-1.01895	-1.47427	########	-8.34269	-4.88465	-70.5219	1
55	########	########	-2.60335	########	-5.69626	-6.09379	-67.8165	1

56	########	1.823645	1.430025	########	-4.17608	-6.32653	-71.568	1
57	########	2.60093	-1.5419	-1.50509	-8.31654	-3.33835	-71.0789	1
58	-1.08692	5.253749	-6.64366	########	14.87921	-58.8313	10.64955	1
59	3.328624	26.97732	14.67118	-8.19519	7.728222	22.23871	-32.5983	1
60	########	-1.6954	2.337282	1.835928	8.079002	6.490363	6.971616	2
61	########	########	########	-7.09934	-3.34588	6.159815	10.20971	2
62	1.597603	########	1.475872	4.186108	10.42837	11.14565	4.406772	2
63	-1.12783	########	2.206436	4.577103	5.006093	6.130978	6.239551	2
64	-1.20433	1.810883	-1.20247	-3.43617	########	9.044568	8.836104	2
65	########	-7.24131	########	-15.0091	########	-2.70793	26.49676	2
66	1.977896	########	########	2.315495	5.306708	13.07385	5.005776	2
67	########	########	########	-1.01286	########	5.317904	8.45748	2
68	1.354026	########	1.889989	-7.11672	########	-7.8502	49.69506	2
69	########	-3.66848	4.885402	2.482124	9.996565	8.123124	-10.1712	2
70	########	-2.00493	########	-4.6572	########	3.110723	12.22792	2
71	-1.01843	-1.05488	5.730387	4.088742	-6.07239	-1.81163	31.82525	2
72	-1.01843	-1.05488	5.730387	4.088742	-6.07239	-1.81163	31.82525	2
73	-1.01843	-1.05488	5.730387	4.088742	-6.07239	-1.81163	31.82525	2
74	-1.01843	-1.05488	5.730387	4.088742	-6.07239	-1.81163	31.82525	2
75	-1.01843	-1.05488	5.730387	4.088742	-6.07239	-1.81163	31.82525	2
76	-1.01843	-1.05488	5.730387	4.088742	-6.07239	-1.81163	31.82525	2
77	3.037472	2.078737	########	-7.11016	2.852131	6.196987	18.44677	2
78	2.119453	########	2.153142	6.455114	7.185218	10.82293	7.994904	2
79	########	1.073364	########	-6.20605	9.081794	-11.2226	62.52644	2
80	-2.39898	1.649617	-3.99496	-4.11027	-8.95523	8.778537	7.498528	2
81	########	-1.03841	1.394801	11.0022	7.911046	11.03455	########	2
82	2.417702	########	########	-4.48308	7.557199	4.484831	18.69178	2
83	4.044416	-1.59174	-5.27985	-7.59932	-2.81399	4.957247	15.36974	2
84	-1.44413	########	-2.3743	5.577899	########	6.263003	5.832321	2
85	########	1.832138	-1.87668	########	2.469695	5.546997	14.25387	2
86	1.697576	-4.634	1.992653	-2.83381	4.452222	5.958365	-3.85435	2
87	########	1.161928	-2.67907	########	-3.14571	7.431992	7.740305	2
88	########	-1.0265	1.691	-3.56815	-3.7282	-3.75945	48.9872	2
89	2.211969	-4.89649	3.415893	6.423586	6.584829	10.45843	-5.91214	2
90	-3.07973	2.30218	########	-6.01957	2.275856	7.017662	13.18072	2
91	1.399245	-2.23095	########	-11.1525	-2.8301	-2.99212	37.14038	2
92	2.222514	-4.72198	1.754013	-2.61854	5.320378	4.863498	2.032959	2
93	########	########	########	3.51133	########	5.644675	6.51477	2
94	2.511517	-6.27194	########	-8.70251	-1.77375	########	33.44728	2
95	1.825074	-2.91343	########	6.220613	########	6.925007	2.903303	2
96	########	########	1.404914	2.648051	5.172457	13.31235	2.985477	2
97	2.142485	-2.50464	########	-7.25054	########	1.391956	13.04055	2
98	1.380453	-1.28963	########	-5.87166	3.827509	4.261166	16.89266	2
99	########	-1.66913	-1.27894	-3.53189	########	9.085404	7.899697	2
100	########	1.267928	-1.7057	-2.0761	-1.76184	9.077518	7.46775	2
101	-1.87167	3.303963	-4.6322	5.960359	########	8.822417	7.960052	2
102	-1.64523	########	########	3.619818	3.220073	11.11092	3.804155	2
103	-1.03371	-1.80969	########	-4.42987	-1.52809	3.892141	10.96645	2
104	-1.03827	########	########	-3.34156	########	6.284382	10.92098	2
105	########	-5.08723	-6.08142	########	-9.93398	9.975633	2.297667	2
106	-1.30703	3.963632	-1.97099	########	-1.21483	12.63054	10.25941	2
107	-1.37837	########	-3.15408	-3.49958	-4.73388	7.227609	5.392348	2
108	########	########	-2.37814	########	-1.93598	6.104207	7.008069	2
109	-1.96862	########	########	-2.94973	########	2.154881	14.15268	2
110	########	4.070203	-6.69863	1.550297	-7.05776	18.03364	7.025401	2
111	-2.59608	-1.53354	########	########	-1.16752	6.920904	4.872569	2

112	########	-1.86517	-1.50105	5.180913	########	7.391248	2.994424	2
113	-1.18435	1.543151	########	-3.00122	########	8.786935	8.995835	2
114	########	-1.21721	########	4.388566	2.539507	7.469719	7.219672	2
115	########	-1.74577	########	########	1.607968	6.984283	6.642627	2
116	########	########	-4.20706	2.916926	-1.49133	10.57909	4.578211	2
117	1.155685	5.510696	-3.14424	-2.73777	-2.80955	16.41387	14.76188	2
118	########	########	########	8.184862	3.865533	6.234886	3.492775	2
119	1.097636	########	-2.9424	2.451614	-1.57956	7.872549	8.420066	2
120	########	4.023496	-5.46928	-3.55665	-6.86735	17.26498	9.91312	2
121	-1.21613	3.161406	-5.21197	5.42563	-3.25058	10.68037	7.984377	2
122	########	1.730977	########	3.748752	4.505178	5.722381	12.63487	2
123	-3.87165	3.587126	2.409181	-3.51675	2.554409	4.816216	11.93427	2
124	2.222747	-1.61837	2.899609	-6.08907	########	-9.59204	57.0487	2
125	1.623783	3.315628	########	8.574638	3.76341	11.73202	6.207715	2
126	########	########	-5.14754	-3.52936	-5.40998	8.644694	5.761327	2
127	-1.09759	########	1.323819	3.775783	5.088281	8.696859	4.246054	2
128	1.039877	-1.68875	########	3.243252	-3.3447	10.8938	2.133121	2
129	-1.27724	########	4.214596	-2.67646	3.416637	7.285513	5.936568	2
130	-1.35869	-2.98163	5.053806	5.583312	4.475475	4.771345	3.017927	2
131	########	########	-3.9787	6.707051	-1.46248	9.024837	4.307738	2
132	2.653447	-2.35368	-2.39011	-7.26144	-5.36033	8.709174	7.79151	2
133	-1.68319	########	-1.34352	-2.00589	-1.37763	9.712811	7.510657	2
134	1.006077	3.945562	-2.59681	########	########	14.53367	10.88628	2
135	########	1.629867	-5.36385	7.574852	-3.05677	13.72123	2.242657	2
136	2.166145	2.171564	########	2.154346	2.161595	17.283	6.679453	2
137	2.179807	6.650476	-7.79278	4.853429	-1.23649	24.94428	4.195389	2
138	########	########	-1.02831	4.914606	1.020819	11.80929	3.113248	2
139	########	########	########	8.409555	5.27872	11.09005	3.688499	2
140	-2.0968	-1.31593	-4.38266	-3.32538	-5.89175	9.325281	2.508726	2
141	-2.58536	2.886869	-3.54347	########	-1.82969	3.708097	13.69151	2
142	########	-6.33959	5.445714	5.721894	26.95035	29.38533	########	2
143	########	5.408415	2.1341	-5.95914	9.796566	-6.50365	39.49538	2
144	########	########	2.305227	-6.61299	2.916191	8.405282	15.04836	2
145	########	-2.0619	########	-4.59645	########	########	14.6499	2
146	########	3.180599	-1.61991	-2.50975	-4.37025	11.95372	9.037351	2
147	########	1.852536	2.941798	-4.1325	4.57817	########	30.17052	2
148	########	########	3.311205	3.179311	-3.27682	-4.32874	55.93348	5
149	########	########	1.585401	########	-2.00814	-8.10984	67.77687	5
150	########	-1.3568	1.690587	2.194772	-5.36135	-4.74803	58.25501	5
151	2.253362	-1.9188	-15.0121	20.28314	15.49178	-68.5754	2.222083	5
152	########	########	2.795587	########	-4.4536	-9.64142	73.44768	5
153	########	########	2.474462	########	-3.23744	-8.46211	69.25044	5
154	########	2.052807	########	########	########	1.625967	74.62229	5
155	-1.05178	1.384814	########	1.309233	-3.37259	-8.7738	75.19917	5
156	########	########	1.100175	-3.65448	-1.7198	-10.0161	64.42654	5
157	########	########	1.424141	########	-4.68434	-6.98007	66.15482	5
158	-7.836	-3.95832	-7.02506	-16.5061	18.05577	17.33933	-42.5327	5
159	1.053438	-1.62903	2.298407	-1.97433	-2.2898	-6.3371	51.54467	5
160	-1.25673	3.544218	-1.85251	3.961629	-6.13687	4.642731	53.38841	5
161	########	########	2.202606	########	-3.08772	-8.27841	65.51148	5
162	########	4.427947	-7.36833	-6.02941	-17.3801	-3.95319	91.35864	5
163	########	1.356992	5.78752	-1.32839	########	-8.10611	65.39637	5
164	########	-2.33716	-6.67513	-12.8593	29.86394	2.140113	-42.6886	5
165	########	########	3.986023	-3.65296	-1.29513	-8.8793	63.36738	5
166	########	-1.65105	1.098674	-1.14501	-5.56559	-7.2673	61.77336	5
167	########	1.2952	1.214117	4.053954	-4.19929	-6.19666	61.81729	5

168	-1.2872	########	1.200911	3.480586	-2.60609	-3.09157	50.53159	5
169	########	########	3.185363	-2.95487	-2.0669	-8.08945	55.50085	5
170	########	-2.11968	-1.42845	########	-5.16155	########	32.76894	5
171	########	-1.76914	1.35714	########	-5.06317	-9.68847	65.04849	5
172	1.732894	-4.9491	-1.6043	4.80535	1.448813	-8.241	40.99634	5
173	########	########	1.482904	3.838358	-2.8608	-9.42042	63.34809	5
174	########	########	2.785876	-1.25754	-2.90234	-8.76811	70.12473	5
175	########	########	########	########	########	-7.58984	59.27094	5
176	########	-2.14274	1.03692	########	2.141939	########	36.72804	5
177	########	2.557083	########	-1.79476	-7.56965	-5.39658	69.13183	5
178	-1.86536	-2.53829	2.031806	4.681204	3.223526	########	33.60141	5
179	-1.9052	-3.35615	########	9.100904	-1.87676	########	30.6269	5
180	########	5.533902	-2.1327	-1.52014	-5.92891	1.775247	54.2426	5
181	########	-1.96715	-1.19174	6.104316	-9.54552	-3.03566	30.11958	5
182	########	########	2.121477	1.700025	-1.82329	-6.09762	60.56789	5
183	########	-2.9005	1.185015	-2.40823	26.63104	25.23988	-55.968	5
184	########	########	-1.61695	2.870217	-7.81246	-4.6051	55.60516	5
185	########	########	-1.61695	2.870217	-7.81246	-4.6051	55.60516	5
186	########	########	-1.61695	2.870217	-7.81246	-4.6051	55.60516	5
187	########	########	-1.61695	2.870217	-7.81246	-4.6051	55.60516	5
188	########	########	-1.61695	2.870217	-7.81246	-4.6051	55.60516	5
189	########	########	1.965637	########	-4.26202	-10.3055	73.43248	5
190	########	-4.5531	-1.18897	-4.73311	7.768473	11.09359	-75.6689	5

```
), rep(5, 43))
```