Remote Tracking of Distributed Dynamic Sources Over a Random Access Channel With One-Bit Updates

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***Abstract*—In this work, we consider a network, where distributed information sources whose states evolve according to a random process transmit their time-varying states to a remote estimator over a shared wireless channel. Each source generates packets in a decentralized manner and employs a slotted random access mechanism to transmit the packets. In particular, we are interested in networks with a large number of low-complexity devices that share low-capacity random access channels. Accordingly, we investigate update strategies for remote tracking of source states that require each update to constitute as few bits as possible. To that end, we develop update strategies requiring only one-bit of information per update that employ a local cancellation strategy. We further analytically compare the performance of the cancellation-enabled update policy to the optimal policy that does not restrict the number of bits for each update, which show that an asymptotic upper bound of the optimality ratio is $\frac{13\sqrt{2}}{12}$. Through simulations, we compare the proposed cancellation-enabled one-bit update policy with zero-wait sampling and threshold-based sampling policies that require more than one-bit of information per update. The comparisons show that the cancellation-enabled update policy at its optimal threshold level outperforms the multi-bit update policies.**

***Index Terms*—Asymptotic analysis, distributed scheduling, internet of things, random walks, remote estimation.**

I. Introduction

THE Internet of Things (IoT) has attracted significant attention resulting in an ever growing number of applications such as traffic monitoring and healthcare monitoring systems [1]. In such systems, where distributed IoT devices/sensors are connected to a remote monitor/controller, the sensors send update packets with time-varying (sensing) information to the monitor so that the monitor can track the state of the monitoring objects. To this end, it is crucial to send timely updates to keep the monitor maintaining fresh information. The timely updates can be challenging in an IoT network where many IoT devices are communicating over a shared channel. This article tackles this problem by developing strategies that require each update[1] to constitute as few bits as possible so that a large device population can be served.

Age of Information (AoI) has been introduced and studied to measure the freshness of information [2], [3], [4], [5], [6], which is defined as the time that has elapsed since the latest packet received at a remote monitor (or a receiver) was generated at a source. In [3], the authors investigate the cases when the zero-wait sampling is not age-optimal with a single source-receiver pair. Networks with multiple sources updating a common receiver over a shared wireless channel are considered in [4], [5], [6]. Centralized update policies with throughput constraints are studied in [4], and decentralized update policies employing a slotted random access with channel collision feedback are studied in [5]. In [6], a sleep-wake update policy when each source has a limited battery capacity is developed. None of these designs apply to our setting since they characterize the timely updates via age, whereas in our setting the timely updates are characterized via the estimation error.

Recently, remote estimation has attracted much attention to characterize the timely updates in IoT networks, which is also the focus of our work. In this scenario, instead of AoI, the value of information may be measured in terms of an estimation error, which is an error between the actual state at a source and the estimate at a receiver [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21]. In [7], optimal sampling policies for a Wiener process are developed to minimize the Mean Squared Error (MSE) with the frequency sampling constraints. This problem is also studied when a communication channel has random delay in [8] and it is shown that an optimal policy is a threshold-type. Optimal sampling policies for an Ornstein-Uhlenbeck (OU) process are investigated with a channel having random delay [9] and with average power constraint [10]. In [11], [12], [13], a source whose state x_t evolves as $x_{t+1} = ax_t + w_t$, where $a \in \mathbb{R}$ and w_t is an independent and identically distributed (i.i.d.) random variable, are considered. In [11], update policies to minimize the MSE subject to a sampling frequency constraint

[1]Throughout this article, we use 'each update' as a short-hand for 'each update packet'.

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are investigated. In [12] and [13], it is assumed that each update pays a communication cost and update policies to minimize estimation error plus communication costs.

In [14], [15], [16], [17], [20], [21], [22], a network where n sources updating a common receiver is considered when the state of each source is modeled as a Linear Time Invariant (LTI) system with an independent zero-mean Gaussian noise [14], [15], [16], [17], the Ornstein-Uhlenbeck (OU) process [22], a zero-mean independent and identically distributed random process [20] or a random walk with Gaussian steps [21]. In [14] and [15], time-based (centralized) scheduling policies at the receiver are investigated to minimize the average estimation error covariance when at most one source can update the receiver at a time [14] or when at most m out of n sources can update the receiver at a time and the communication channel has a packet drop probability [15]. In [22], a centralized scheduling policy is investigated to minimize the mean squared error (MSE) using the fact that the MSE of the OU process is proportional to the variance of the OU process and the AoI. In [16] and [17], decentralized scheduling policies are investigated, where each source's objective is to minimize its estimation error covariance at the receiver subject to transmission power constraint. This problem is modeled as a multi-player game, and a Nash equilibrium (NE) is found in [16]. In [15], a concept of correlated equilibrium (CE) where the estimation performance can be improved compared with NEs is introduced, and a strategy that achieves the performance at the CE is proposed. In [20] and [21], distributed update policies for minimizing the expected estimation error are investigated. In [20], each source makes sampling and transmission decisions with or without local communication, i.e., whether sources can communicate with each other or not when the state of each source is a zero-mean independent and identically distributed random variable. In [21], the authors design distributed update policies depending on whether each transmitter can observe the exact state of the source when the state of each source is a random walk process with Gaussian steps.

In [18], [19], a network with n independent source-receiver pairs communicating over a shared channel is considered. In [18], a centralized scheduling policy is proposed when each transmission incurs a communication cost to minimize the average MSE plus communication costs. In [19], a decentralized scheduling policy is investigated to minimize the transmission power subject to a lower bound constraint on the successful transmission probability.

In this work, we consider a network with n distributed sources updating a common receiver over a shared wireless channel and investigate decentralized update policies to minimize the estimation error. Our work is different from other groups of works, in which centralized (e.g., [14], [15], [18]) or game theoretic (e.g., [16], [17] settings are considered. Further, we are interested in networks with a large number of low-complexity devices that share low-capacity random access channels. Such a setting is becoming increasingly important in massive IoT networks with an increasing number of low-complexity devices being connected to the networks such as remote health monitoring or smart architecture. Accordingly, we investigate update policies (i.e., sampling and scheduling policies) that require each update to constitute as few bits as possible. Thus, it is unsuitable for the sampling policies proposed in [7], [8], [9], [10], [11], [12], [13], [20] and [21] to be directly applied in this setting since those sampling policies do not carefully deal with the number of bits per sampling/transmission in the existence of transmission failures. We also remark that part of the results in this work was present in the conference version [23].

Our contributions can be summarized as follows.

- We formulate the remote tracking problem to minimize the estimation error with a large number of low-complexity devices updating a common receiver over a low-capacity random access channel when the state of each information source evolves according to a symmetric random walk.
- We develop update strategies that require one-bit of information per update as a case of particular interest. We first consider a natural benchmark update policy and reveal that the benchmark policy will not be able to make the system stable in terms of estimation error under some conditions.
- We then introduce an improvement on the benchmark policy that employs a local cancellation strategy, which makes the system always stable. We further compare the performance of the cancellation-enabled update policy to the optimal policy that does not restrict the number of bits for each update.
- We suggest how the proposed one-bit update policy can be applied to more general source models.
- We compare the proposed one-bit update policy with zero-wait sampling and threshold-based sampling policies that require more than one-bit of information per update through simulations. Numerical results show that the proposed one-bit update policy outperforms the multi-bits update policies, which implies that the proposed one-bit update policy is more beneficial when we consider transmission power that is usually increasing as the packet size (i.e., the number of bits per update) increases.

The rest of the paper is organized as follows. In Section II, we describe the system model and formulate the problem. In Section III, we develop and analyze update strategies that require only one-bit of information per update. In Section IV, we extend our results to more general source models. In Section V, we compare the proposed one-bit update policy with other update policies through simulations. In Section VI, we conclude our work.

II. System Model and Problem Formulation

A. Network Model

We consider a fundamental scenario of n distributed information sources (e.g., sensors) whose states evolve according to a random process, and one remote estimator (e.g., sink or collector) that aims to remotely track the time-varying state of the sources over a shared wireless channel, as shown in Fig. 1. In this work, we are interested in developing strategies for remote tracking of source states that require one-bit of information per update as a particular interest, which will be explained in Section II-B.

Considering a time-slotted system operation, we let $x_{i,t}$ denote the state of source i at the beginning of time t, which

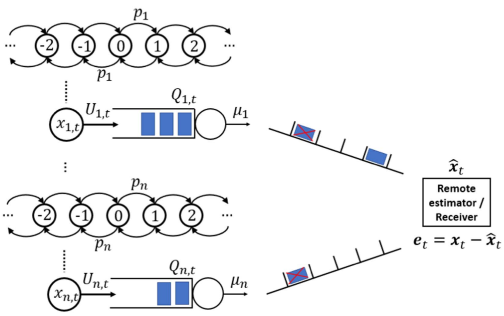


Fig. 1. System model.

evolves over integer values according to a simple random walk. In particular, $x_{i,t}$ evolves as

$$x_{i,t+1} = x_{i,t} + w_{i,t}, \text{ for } t \geq 0, \tag{1}$$

where $w_{i,t}$ is given by

$$w_{i,t} = \begin{cases} 1, & \text{with probability } p_i, \\ 0, & \text{with probability } 1-2p_i, \\ -1, & \text{with probability } p_i, \end{cases} \tag{2}$$

for some $p_i \in [0, 0.5]$. The transition probability p_i is known to each source. Note that the noise $w_{i,t}$ is independent and identically distributed (i.i.d.) with a zero-mean and finite variance, and that it is symmetric, i.e., $\mathbb{P}(w_{i,t}=1) = \mathbb{P}(w_{i,t}=-1)$. We note that such a basic evolution lies at the foundation of many important estimation and control mechanisms. By varying the p_i parameter, this process can capture more and less variable source evolution. After developing our results for this model, we will also discuss more general state evolution in Section IV.

Let $U_{i,t} \in \{0,1\}$ denote the packet generation (or sampling) decision of source i at time slot t, where $U_{i,t}=1$ implies that source i generates a new packet at time slot t. At the end of time slot $t-1$, the packet generation decision $U_{i,t}$ is made in a decentralized manner by each source based on their own observations up to time slot $t-1$. Each source maintains a First-Come First-Served (FCFS) queue, and the newly generated packet is stored in the queue. The queue length of source i at time slot t is denoted by $Q_{i,t}$.

In view of the low-complexity nature of communication capabilities of these devices, we assume a slotted random access channel for wireless updates whereby if more than one sources transmit packets simultaneously, then all the transmissions fail due to a packet collision. Let $Z_{i,t} \in \{0,1\}$ denote the indicator variable for the successful transmission of source i at time slot t. The source i transmits the packet with probability $\mu_i \in (0,1]$ (which is to-be-determined), and idles with probability $1-\mu_i$. We assume that if queue i is empty (i.e., $Q_{i,t}=0$) then source i transmits a dummy packet.[2] Then, we have

$$\gamma_i := \mathbb{E}[Z_{i,t}] = \mu_i \textstyle\prod_{j\neq i}(1-\mu_j). \tag{3}$$

[2]This assumption makes the mathematical analysis more tractable. In practical operation, letting source i idle when it has no packet to send can give more transmission opportunities to the other sources and improve the system performance.

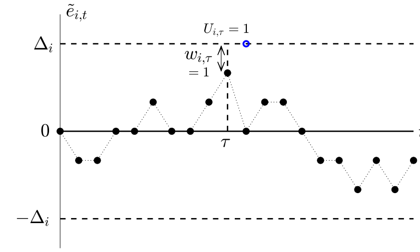


Fig. 2. A trajectory of the virtual error $\tilde{e}_{i,t}$ of source i.

If source i is the only source transmitting a packet at time slot t, then the packet is successfully transmitted to the estimator (i.e., $Z_{i,t}=1$). We assume that the communication channel is error-free and each transmission is done within a time slot.

Let $\hat{x}_{i,t}$ denote the estimated state of source i at the estimator at time slot t, which can be updated using information received by time slot t. Let $e_{i,t}$ denote the information mismatch (or error) between $x_{i,t}$ and $\hat{x}_{i,t}$, i.e.,

$$e_{i,t} = x_{i,t} - \hat{x}_{i,t}. \tag{4}$$

We assume that $x_{i,0} = \hat{x}_{i,0}$ for all $i \in \{1,\ldots,n\}$.

B. One-Bit Update Policy At the Sources

In this work, we consider a low-overhead sampling policy, whereby each update constitutes *one-bit* of information so that the shared channel load is minimized for each transmission. This is especially important for wireless channels that serve a large population, as expected in future IoT networks. This motivates us to consider a threshold-type packet generation policy, whereby $\Delta_i \in \mathbb{N}$ denotes the (state) threshold used for sampling. To describe this policy more explicitly, let $\tilde{e}_{i,t}$ denote the virtual error of source i, which is a variable being held by each source i and is updated as

$$\tilde{e}_{i,t+1} = \begin{cases} 0, & \text{if } U_{i,t}=1, \\ \tilde{e}_{i,t}+w_{i,t}, & \text{if } U_{i,t}=0. \end{cases} \tag{5}$$

Here, the packet generation decision $U_{i,t}$ under the above threshold-base policy at time slot t is given by

$$U_{i,t} = \begin{cases} 1, & \text{if } |\tilde{e}_{i,t}+w_{i,t}| = \Delta_i, \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

In other words, when $\tilde{e}_{i,t}+w_{i,t}$ hits the threshold Δ_i or $-\Delta_i$, a packet with one-bit information is generated and sent to its queue with the value $+1$ for Δ_i or -1 for $-\Delta_i$, and the value $\tilde{e}_{i,t+1}$ is reset to 0. Fig. 2 shows a trajectory of virtual error $\tilde{e}_{i,t}$, where a new packet with the value $+1$ is generated at time slot τ. We will provide an explanation of the relationship between the error $e_{i,t}$ and the virtual error $\tilde{e}_{i,t}$ in Section II-C.

Next, we provide a few interesting facts about the absolute estimation error performance of such a threshold-based one-bit update rule. These are interesting in explicitly characterizing how the error relates to the threshold level Δ_i and the source dynamics p_i.

Theorem 2.1: Under the threshold-based one-bit update policy with threshold Δ_i, the long-term expectation of virtual error $\tilde{e}_{i,t}$ of source i is given by

$$\mathbb{E}[|\tilde{e}_{i,\infty}|] = \frac{\Delta_i^2-1}{3\Delta_i}. \quad (7)$$

Further, the long-term expectation of update decision $U_{i,t}$ of source i is given by

$$\mathbb{E}[U_{i,\infty}] = \frac{2p_i}{\Delta_i^2}. \quad (8)$$

Proof: The virtual error $\tilde{e}_{i,t}$ is a finite-state Markov chain with $2\Delta_i - 1$ states from (5) and (6). Thus by solving global balance equations, we can obtain its stationary distribution

$$\pi_{i,k} = \frac{\Delta_i-|k|}{\Delta_i^2} \text{ for } k \in \{-\Delta_i+1, \ldots, \Delta_i-1\}, \quad (9)$$

from which we can obtain the long-term expected virtual error $\mathbb{E}[|\tilde{e}_{i,\infty}|]$:

$$\mathbb{E}[|\tilde{e}_{i,\infty}|] = \sum_{k=-\Delta_i+1}^{\Delta_i-1} k\pi_{i,k} = \frac{\Delta_i^2-1}{3\Delta_i}. \quad (10)$$

Further, since each source independently generates a packet, we can consider $\tilde{e}_{i,t}$ as an independent renewal process, which is reset to 0 upon every packet generation. In [24], it is shown that

$$\mathbb{E}[U_{i,\infty}] = \lim_{t\to\infty} \mathbb{P}(U_{i,t}=1) = \frac{2p_i}{\Delta_i^2} \quad (11)$$

using Blackwell's renewal theorem (Theorem 4.6.2 in [25]). ■

C. Estimation At the Receiver

Now that we described the policy at the sources, we turn to the corresponding estimation process at the receiver. We denote $V_{i,t}^k \in \{-1,1\}$ for $k \in \{1,\ldots,Q_{i,t}\}$ as the value of k-th packet in queue i at time slot t with $V_{i,t}^0 = 0$, where $k=1$ is the index for the head of the queue. If $Z_{i,t} = 1$, then the packet with value $V_{i,t}^1$ is successfully sent to the receiver and we have $V_{i,t+1}^k = V_{i,t}^{k+1}$. Then, at the receiver, the estimate $\hat{x}_{i,t}$ is updated as

$$\hat{x}_{i,t+1} = \hat{x}_{i,t} + V_{i,t}^1 Z_{i,t}\Delta_i. \quad (12)$$

In other words, when a new packet is received from source i, the estimated $\hat{x}_{i,t}$ is either increased by Δ_i if the received information is 1, or decreased by Δ_i if -1 is received. Thus, the virtual error $\tilde{e}_{i,t}$ is the (actual) error after the last generated packet is delivered to the receiver. By the definition of the error $e_{i,t}$ in (4) and the virtual error $\tilde{e}_{i,t}$ in (5), we have that

$$e_{i,t} = \tilde{e}_{i,t} + \Delta_i \sum_{k=1}^{Q_{i,t}} V_{i,t}^k \quad (13)$$

with $e_{i,0} = \tilde{e}_{i,0} = 0$. This implies that the error $e_{i,t}$ at time t can be measured using the virtual error $\tilde{e}_{i,t}$ plus the sum of values of the packets stored in the queue at time t. We refer to Appendix A for detailed proof.

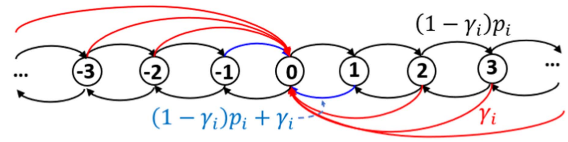


Fig. 3. A Markov chain generated by the evolution of estimation error $e_{i,t}$ under the optimal policy.

D. Distributed Remote-Estimation Problem

Given the one-bit update policy at the sources and the estimation policy at the receiver, the goal of the remote tracking problem is to optimize the choices of thresholds $\mathbf{\Delta} \triangleq \{\Delta_1,\ldots,\Delta_n\}$, and the probabilities $\boldsymbol{\mu} \triangleq \{\mu_1,\ldots,\mu_n\}$ for random access transmissions that minimize the mean absolute estimation error. Mathematically, our objective is to design $(\mathbf{\Delta},\boldsymbol{\mu})$ given the source dynamics $\mathbf{p} \triangleq (p_1,\ldots,p_n)$ to minimize the expected average absolute-error over infinite time horizon:

$$\min_{\mathbf{\Delta},\boldsymbol{\mu}} J(\mathbf{\Delta},\boldsymbol{\mu}) = \lim_{t\to\infty} \frac{1}{tn} \sum_{s=1}^{t}\sum_{i=1}^{n} \mathbb{E}_\pi\left[|e_{i,s}|\right]. \quad (14)$$

III. Design and Analysis of One-Bit Update Policies for Remote Estimation

In this section, we attack the problem formulated in the previous section by designing one-bit update policies for distributed remote tracking. At the outset, it is even unclear whether there exists a policy that can guarantee a bounded absolute estimation error. In fact, in Section III-B, we investigate a class of First-Come-First-Serve (FCFS) policies to find a condition on the (source-dynamics, threshold-level) pairs, $(\mathbf{p},\mathbf{\Delta})$, that can be stabilized by such policies. The negative result from this design motivates us in Section III-C to propose an improved class of policies that employ a *cancellation* strategy within the transmission queues in order to guarantee stability for all possible source dynamics $\mathbf{p}$.

A. Optimal Sampling Without Constraints on Information Size

We first consider the estimation error minimization problem over a random access channel without constraints on information size. That is, the source can generate a packet with the exact state information at the time the packet is generated. Since transmission time is not stochastic, an optimal update policy is to generate a packet with value $x_{i,t}$ (or $e_{i,t}$) and make a transmission with probability μ_i at every time slot. Hence, letting $\gamma_i = \mu_i \prod_{j\neq i}(1-\mu_j)$ be the probability of successful transmission for source i, the evolution of the estimation error $e_{i,t}$ can be viewed as a Markov chain with $+1$ or -1 with probability $(1-\gamma_i)p_i$ and returning to 0 with probability γ_i as shown in Fig. 3.

It is not difficult to see that the error evolution process $\{e_{i,t}\}_t$ is an ergodic Markov chain since it returns to 0 with probability $\gamma_i > 0$ from all states. Hence, there exists a unique steady state distribution. Let $e_\infty^{opt}(\boldsymbol{\mu})$ denote the long-term estimation error under the optimal sampling policy with activation probabilities $\boldsymbol{\mu}$. The next theorem provides the long-term expected absolute error $\mathbb{E}[|e_\infty^{opt}(\boldsymbol{\mu})|]$ and the entropy $H(e_{i,\infty}(\boldsymbol{\mu}))$ of the estimation error $e_{i,\infty}$ for each source i under the optimal sampling policy given a set of activation probabilities $\boldsymbol{\mu}$.

Theorem 3.1: The long-term expected absolute error of the optimal sampling policy with activation probabilities $\boldsymbol{\mu}$ is given by

$$\mathbb{E}[|e_\infty^{opt}(\boldsymbol{\mu})|] = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{\sqrt{\beta_i^2+2\beta_i}}, \tag{15}$$

and the entropy of the estimation error $e_{i,\infty}^{opt}(\boldsymbol{\mu})$ for source i is given by

$$H(e_{i,\infty}^{opt}(\boldsymbol{\mu})) = \log\left(\sqrt{1+\frac{2}{\beta_i}}\right) + \frac{1}{\sqrt{\beta_i^2+2\beta_i}}\log\left(1+\beta_i+\sqrt{\beta_i^2+2\beta_i}\right), \tag{16}$$

where $\beta_i = \frac{\gamma_i}{2(1-\gamma_i)p_i}$ *and* $\gamma_i = \mu_i\prod_{j\neq i}(1-\mu_j)$.

The expected estimation error $\mathbb{E}[|e_\infty^{opt}(\boldsymbol{\mu})|]$ and the entropy $H(e_{i,\infty}^{opt}(\boldsymbol{\mu}))$ can be obtained from the steady-state distribution of $e_{i,\infty}$, which is obtained by solving global balance equations for the Markov chain represented in Fig. 3. The detailed proof is in Appendix B. Note that source i generates and transmits an update packet with the exact value of the error $e_{i,t}^{opt}(\boldsymbol{\mu}) \in \mathbb{Z}$, which implies that the average information size (i.e., the number of bits required to deliver $e_{i,t}^{opt}(\boldsymbol{\mu})$) is lower-bounded by the entropy $H(e_{i,t}^{opt}(\boldsymbol{\mu}))$ by Shannon's source coding theorem.

Note that γ_i is the probability of successful transmission for source i, which becomes very small as the number n of sources becomes large in general. Then, as can be expected, both the error $\mathbb{E}[|e_\infty^{opt}|]$ and the entropy $H(e_{i,\infty})$ increase as n is increasing. In the following sections, we will design an update policy that requires one bit of information and compare the estimation error between the optimal policy and the proposed policy.

B. Benchmark Analysis for First-Come First-Serve Updates for a Single Source

To develop a basic understanding of the system operation, let us consider the operation of the one-bit update and random-access service policy in a single source case. Suppose that the source uses a threshold level of Δ and achieves a transmission success probability of μ in each transmission. The next theorem establishes a condition between Δ, p, and μ that would make the FCFS update policy unstable.

Theorem 3.2: Under the threshold-based one-bit sampling and the First-Come First-Serve update policy, if $\Delta \leq \sqrt{\frac{2p}{\mu}}$, then the system is unstable, i.e.,

$$\lim_{t\to\infty}\mathbb{E}[|e_\infty|] = \infty. \tag{17}$$

This follows from the fact that, to make the system stable, the source has to make the queue stable and the condition for queue stability is that, in the long-term, the arrival rate must be less than the service rate, i.e., $\frac{2p}{\Delta^2} < \mu$, [28]. The detailed proof using [29] is in Appendix C. In the next section, we shall show that this deficiency can be eliminated through a cancellation mechanism within the transmission queue of each source.

C. One-Bit Update Policies With Packet Cancellation

The performance of FCFS update policy revealed that the estimation error will be unbounded if $\frac{2p_i}{\Delta_i} > \gamma_i$, where $\gamma_i = \mu_i\prod_{j\neq i}(1-\mu_j)$. In this subsection, we introduce an improvement on these benchmark policies with substantial improvement. To that end, we first note that the dynamics of $x_{i,t}$ in (2) is symmetric, i.e., $\mathbb{P}(x_{i,t_0+t} = x \mid x_{i,t_0} = 0) = \mathbb{P}(x_{i,t_0+t} = -x \mid x_{i,t_0} = 0)$, due to symmetry of noise $w_{i,t}$. Using this symmetry of the dynamics, we can manipulate the FCFS queue if the information of packets in the queue can be accessed. If the values of the newly generated packet and the packet at the tail of the queue are the opposite, then those two packets *cancel* each other and are discarded from the queue before transmission. Let $D_{i,t} \in \{0,1\}$ be the indicator variable for this event, where $D_{i,t} = 1$ indicates the packet cancellation occurs. Note that $\mathbb{E}[D_{i,t}] = \frac{1}{2}\mathbb{E}[U_{i,t}]\mathbb{P}\{Q_{i,t} > 0\}$ since $\mathbb{P}(x_{i,t} = \Delta_i \mid U_{i,t} = 1) = \mathbb{P}(x_{i,t} = -\Delta_i \mid U_{i,t} = 1) = \frac{1}{2}$ from symmetry of the dynamics of $x_{i,t}$.

Under this cancellation-enabled policy, the values of all the packets at queue i must be the same at all times, i.e., $V_{i,t}^1 = \cdots = V_{i,t}^{Q_{i,t}}$. We assume that departure happens after arrival. Under this queueing discipline, the queue length $Q_{i,t}$ evolves as

$$\begin{aligned} Q_{i,t+1} = Q_{i,t} + U_{i,t} - 2D_{i,t} - Z_{i,t}\mathbb{I}\{Q_{i,t} > 1\} \\ - Z_{i,t}\mathbb{I}\{Q_{i,t} = 1\}((1-U_{i,t}) + U_{i,t}(1-D_{i,t})) \\ - Z_{i,t}\mathbb{I}\{Q_{i,t} = 0\}U_{i,t}. \end{aligned} \tag{18}$$

Note that $D_{i,t} = 1$ implies that $U_{i,t} = 1$ and $Q_{i,t} > 0$ by its definition. Further, since we are assuming departure-after-arrival, $Z_{i,t}$ can be 1 only if (a) $Q_{i,t} > 1$, (b) if $Q_{i,t} = 1$, either a new packet is not generated ($U_{i,t} = 0$) or a packet is generated ($U_{i,t}$) and the packet cancellation does not occur ($D_{i,t} = 0$), or (c) if $Q_{i,t} = 0$, a new packet is generated ($U_{i,t} = 1$).

D. Analysis of One-Bit Updates With Cancellation

In this subsection, we present fundamental results on the error performance of cancellation-enabled one-bit update policies that is introduced in the previous subsection. We start with the next lemma that establishes the strongly ergodic (non-stationary) nature of the transmission queue-length $\{Q_{i,t}\}_t$.

Lemma 3.1: For each source i, the queue length process $\{Q_{i,t}\}_{t\geq 0}$ under the cancellation-enabled one-bit update policy described in (18) forms a strongly ergodic Markov Chain for any $\Delta_i > 0$, $\mu_i > 0$, and $p_i \in [0, 1/2]$.

Note that the non-stationary property of the queue length process $\{Q_{i,t}\}_{t\geq 0}$ comes from the packet generation probability $\lambda_{i,t} = \mathbb{P}(U_{i,t} = 1)$, which converges to $\lambda_i = \frac{2p_i}{\Delta_i^2}$. Hence, the non-stationary Markov chain generated by $\{Q_{i,t}\}$ converges to a (stationary) Markov chain shown in Fig. 4 and it can be shown that the Markov chain is ergodic. The detailed proof using [30], [31] is in Appendix D.

In contrast to the FCFS policy performance (see Theorem 3.2), Lemma 3.1 proves that cancellation-enabled update policy can stabilize the error level for any $\Delta_i > 0, \mu_i > 0$ and any feasible

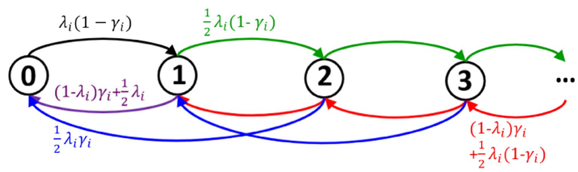


Fig. 4. A Markov chain generated by the queue length process $\{Q_{i,t}\}_{t\geq 0}$ with $\lambda_i = \mathbb{E}[U_{i,\infty}] = \frac{2p_i}{\Delta_i^2}$ and $\gamma_i = \mu_i \prod_{j\neq i}(1-\mu_j)$.

p_i.[3] Specifically, it proves that there exists a unique steady-state distribution for the queue length process $\{Q_{i,t}\}_{t\geq 0}$ under the cancellation-enabled update policy.

It is intractable to solve global balance equations for the Markov chain in Fig. 4. Thus, we instead investigate the asymptotic behavior of the Markov chain and obtain the steady-state distribution for the large number n of sources. Note that the probability γ_i of successful transmission for each source i decreases as the number n of sources increases since $\mu_i \in (0,1)$. Now, we consider a behavior of threshold Δ_i to achieve an optimal estimation error. Lemma 3.1 implies that the queue will be stable for any $\Delta_i > 0$, $\mu_i > 0$ and $p_i \in [0, 1/2]$, and the long-term expected virtual error $\mathbb{E}[|\tilde{e}_{i,\infty}|]$ in (10) is finite for $\Delta_i < \infty$. Since, under the cancellation-enabled policy, we can write the error $e_{i,t}$ as

$$e_{i,t} = \tilde{e}_{i,t} + \Delta_i V_{i,t}^1 Q_{i,t}, \tag{19}$$

the network will be stable for any Δ_i in terms of the estimation error. However, when the number n of sources is large, a small threshold Δ_i will result in a large queue length $Q_{i,t}$ since the (steady state) probability $\lambda_i = \frac{2p_i}{\Delta_i^2}$ of packet generation is relatively larger than the probability γ_i of successful transmission. Hence, increasing Δ_i as n becomes large is necessary to achieve an optimal estimation error.

Note that λ_i and γ_i are dependent on the number n of sources, so for the following discussion, we use $\lambda_{n,i}$ and $\gamma_{n,i}$, respectively, to clarify their dependency on n. With $\lambda_{n,i}$ and $\gamma_{n,i}$ decreasing as $n \to \infty$ and the assumption that $\gamma_{n,i}/\lambda_{n,i} \to c_i$ for some $c_i > 0$ as $n \to \infty$, we can observe, in Fig. 4, that the transition probability from state k to $k-2$ (i.e., $\frac{1}{2}\lambda_{n,i}\gamma_{n,i}$) is dominated by the transition probabilities from state k to $k-1$ and from state k to $k+1$ as $n \to \infty$, which consist of $\lambda_{n,i}(1-\gamma_{n,i})$ and $\gamma_{n,i}(1-\lambda_{n,i})$ terms. Hence, the Markov chain asymptotically becomes a birth-death process as $n \to \infty$, which is tractable to obtain the steady-state distribution as in the following lemma.

Lemma 3.2: Assume that $\lim_{n\to\infty}\lim_{t\to\infty}\frac{\gamma_{n,i}}{\lambda_{n,i,t}} = c_i$ for some $c_i > 0$. Then, when the number n of sources is sufficiently large, the steady-state distribution $\boldsymbol{\theta}_{n,i} = (\theta_{n,i,k})_0^\infty$ of the queue length process $\{Q_{i,t}\}_{t\geq 0}$ for source i under the cancellation-enabled one-bit update policy described in (18) is given by

$$\theta_{n,i,0} \sim \left(1 + \frac{\lambda_i(1-\gamma_i)\big((1-\lambda_i)\gamma_i + \frac{1}{2}\lambda_i(1-\gamma_i)\big)}{(1-\lambda_i)\gamma_i\big((1-\lambda_i)\gamma_i+\frac{1}{2}\lambda_i\big)}\right)^{-1},$$

$$\theta_{n,i,k} \sim \frac{\lambda_i(1-\gamma_i)\theta_{i,0}}{(1-\lambda_i)\gamma_i+\frac{1}{2}\lambda_i}\left(\frac{\frac{1}{2}\lambda_i(1-\gamma_i)}{(1-\lambda_i)\gamma_i+\frac{1}{2}\lambda_i(1-\gamma_i)}\right)^{k-1} \tag{20}$$

for $k = 1, 2, \ldots,$ *where* $\lambda_i = \lambda_{n,i} = \lim_{t\to\infty}\mathbb{P}(U_{i,t}=1) = \frac{2p_i}{\Delta_i^2}$, $\gamma_i = \gamma_{n,i} = \mu_i\prod_{j\neq i}(1-\mu_j)$ *and* $x_n \sim y_n$ *means that* $\lim_{n\to\infty}\frac{x_n}{y_n} = 1$.

The detailed proof is in Appendix E. For the rest of the paper, we omit the subscript n to save space.

From (19), (9), (20) and the fact that $\mathbb{P}(V_{i,t}^1 > 0 \,|\, Q_{i,t} > 0) = \mathbb{P}(V_{i,t}^1 < 0 \mid Q_{i,t} > 0) = \frac{1}{2}$ by the symmetry of dynamics, we can obtain the steady state distribution of $e_{i,\infty}$ and further the expected estimation error $\mathbb{E}[|e_{i,\infty}|]$. However, it is intractable to optimize μ_i and Δ_i that minimize $\mathbb{E}[|e_{i,\infty}|]$ mainly due to (20). Hence, in the next section, we propose an alternative choice of μ_i and Δ_i and compare the expected estimation error $\mathbb{E}[|e_{i,\infty}|]$ between the proposed policy and the optimal policy studied in Section III-A.

E. Comparison of Optimal and Cancellation-Enabled Updates

The intractability of minimizing $\mathbb{E}[|e_{i,\infty}|]$ mainly comes from the steady state distribution $\boldsymbol{\theta}_i$ of each source i in (20). Hence, we instead propose an alternative choice of $\boldsymbol{\mu}$ and $\boldsymbol{\Delta}$ and compare its estimation error to that of the optimal policy. Let $e_\infty^{opt}(\boldsymbol{\mu})$ and $e_\infty^{cxl}(\boldsymbol{\mu},\boldsymbol{\Delta})$ denote the long-term estimation error under the optimal policy with parameter $\boldsymbol{\mu}$ and the cancellation-enabled policy with parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Delta}$, respectively.

1) Activation probabilities $\boldsymbol{\mu}$: We consider $\boldsymbol{\mu}$ that minimizes $\mathbb{E}[|e_\infty^{opt}|]$ instead of $\mathbb{E}[|e_\infty^{cxl}|]$ since $\mathbb{E}[|e_\infty^{opt}|]$ depends only on $\boldsymbol{\mu}$ given $\boldsymbol{p}$. Note, in Theorem 3.1, that $\beta_i = \frac{\gamma_i}{2(1-\gamma_i)p_i} \to 0$ as $\gamma_i \to 0$, i.e., as $n \to \infty$, and thus β_i^2 is dominated by β_i as $n \to \infty$. Further, we have that $\sqrt{\frac{1}{2\beta_i}} = \sqrt{\frac{p_i}{\gamma_i} - p_i} \to \sqrt{\frac{p_i}{\gamma_i}}$ as $n \to \infty$. From this asymptotic behavior for a large number n of sources, we use activation probabilities $\boldsymbol{\mu}^{asym}$ that solves the following convex optimization problem:

$$\boldsymbol{\mu}^{asym} := \arg\min_{\boldsymbol{\mu}} \frac{1}{n}\sum_{i=1}^n \sqrt{\frac{p_i}{\mu_i\prod_{j\neq i}(1-\mu_j)}}. \tag{21}$$

The convexity of the objective function can be shown by showing that the leading principal minors of the Hessian matrix of $\sqrt{\frac{p_i}{\mu_i\prod_{j\neq i}(1-\mu_j)}}$ are positive. For completeness, we provide the detailed proof in Appendix F.

2) Thresholds $\boldsymbol{\Delta}$: Given a set $\boldsymbol{p}$ of state transition probabilities and a set $\boldsymbol{\mu}$ of activation probabilities, let

$$\Delta_i^{\boldsymbol{\mu}} = \left\lfloor\sqrt{\frac{2p_i}{\gamma_i}}\right\rfloor \text{ or } \left\lceil\sqrt{\frac{2p_i}{\gamma_i}}\right\rceil, \tag{22}$$

where $\gamma_i = \mu_i\prod_{j\neq i}(1-\mu_j)$. Note that both choices of $\Delta_i^{\boldsymbol{\mu}}$ in (22) result in the same asymptotic performance since both of them become close to $\sqrt{\frac{2p_i}{\gamma_i}}$ as $n \to \infty$, and

[3]There is an intuition about the stability of the cancellation-enabled update policy. Note that the cumulative arrival process evolves as a symmetric random walk with the cancellations of packets since, given a packet arrival, the packet is equally likely to have a positive or negative value. This grows at the rate $O(\sqrt{t})$. On the other hand, the cumulative service process with any positive probability of transmission grows at the rate of $O(t)$. Hence, the queue length process remains stable for any positive probability of transmission from each source.

that $\lambda_i \approx \gamma_i$ as $n \to \infty$ since $\lambda_i = \frac{2p_i}{(\Delta_i^{\mu})^2}$, which makes the steady state distribution $\boldsymbol{\theta}_i$ of the queue length process $\{Q_{i,t}\}$ of source i in (20) simpler[4]:

$$\theta_{i,0} \sim \frac{3-2\gamma_i}{6-5\gamma_i},\ \theta_{i,k} \sim \frac{2-2\gamma_i}{6-5\gamma_i}\frac{1}{3^{k-1}} \text{ for } k \geq 1. \quad (23)$$

Next, we compare the expected long-term estimation error $\mathbb{E}[|e_{\infty}^{cxl}(\boldsymbol{\mu}^{asym}, \boldsymbol{\Delta}^{\boldsymbol{\mu}^{asym}})|]$ of the cancellation-enabled update policy with parameters $\boldsymbol{\mu}^{asym}$ and $\boldsymbol{\Delta}^{\boldsymbol{\mu}^{asym}}$ to that of the optimal policy in the following theorem.

Theorem 3.3: The optimality ratio of the cancellation-enabled one-bit update policy with parameters $\boldsymbol{\mu}^{asym}$ and $\boldsymbol{\Delta}^{\boldsymbol{\mu}^{asym}}$ is asymptotically upper bounded by $\frac{13\sqrt{2}}{12}$ as $n \to \infty$, i.e.,

$$\lim_{n\to\infty} \frac{\mathbb{E}[|e_{\infty}^{cxl}(\boldsymbol{\mu}^{asym}, \boldsymbol{\Delta}^{\boldsymbol{\mu}^{asym}})|]}{\mathbb{E}[|e_{\infty}^{opt}(\boldsymbol{\mu}^{asym})|]} \leq \frac{13\sqrt{2}}{12} \approx 1.5321. \quad (24)$$

Note, from (19), that we can obtain

$$\mathbb{E}[|e_{i,t}|] \quad \leq \mathbb{E}[\tilde{e}_{i,t}] + \Delta_i \mathbb{E}[Q_{i,t}] \text{ for all } t, \quad (25)$$

and that $\mathbb{E}[|\tilde{e}_{i,\infty}|]$ is given in (10) and $\mathbb{E}[Q_{i,\infty}]$ for large n can be obtained using (23), from which we can obtain the upper bound of the optimality ratio. The detailed proof is in Appendix G.

Theorem 3.3 implies that the cancellation-enabled one-bit policy is not far from the optimal policy in terms of the estimation error. However, from Theorem 3.1, we can see that $H(e_{i,\infty}) \to \infty$ as $n \to \infty$, i.e., the average packet length becomes longer. Therefore, in terms of transmission power, the update policy with one bit of information becomes more beneficial than the optimal policy.

IV. Extension to More General Source Dynamics

A. Symmetric Dynamics With Finite Variance

In this section, we investigate the estimation error minimization problem described in (14), but with a different type of source, where the state evolution of each source is a Gaussian random walk. This problem has also been studied in [21], but our work is different from [21] in that we consider a scenario where each update must constitutes a limited number of bits.

Suppose that the state $x_{i,t}$ of source i changes as

$$x_{i,t+1} = x_{i,t} + w_{i,t}, \text{ for } t \geq 0, \quad (26)$$

where $w_{i,t}$ is a Gaussian random variable with zero mean and finite variance σ_i^2. A new packet is generated (i.e., $U_{i,t} = 1$) if $|\tilde{e}_{i,t} + w_{i,t}| \geq \Delta_i$ for $\Delta_i \in (0, \infty)$, and the virtual error $\tilde{e}_{i,t}$ is updated as

$$\begin{aligned}\tilde{e}_{i,t+1} = \tilde{e}_{i,t} + w_{i,t} - \Delta_i \mathbb{I}\{\tilde{e}_{i,t} + w_{i,t} \geq \Delta_i\} \\ + \Delta_i \mathbb{I}\{\tilde{e}_{i,t} + w_{i,t} \leq -\Delta_i\}. \quad (27)\end{aligned}$$

Also, the source randomly accesses the channel with the successful transmission probability of $\mu_i \in (0,1)$. Then, the next theorem provides the long-term expected absolute error performance under the cancellation-enabled one-bit update policy.

Theorem 4.1: Under the cancellation-enabled one-bit update policy with parameter $(\boldsymbol{\mu}, \boldsymbol{\Delta})$ when a noise of source i is a Gaussian random variable with zero mean and finite variance σ_i^2, we have

$$\mathbb{E}[|e_{i,\infty}|] \leq \frac{\sigma_i^2 + \mathbb{P}(|\tilde{e}_{i,\infty}| \geq \Delta_i)\Delta_i^2}{2\Delta_i \mathbb{P}(|\tilde{e}_{i,\infty}| \geq \Delta_i)} + \frac{\Delta_i \mathbb{E}[U_{i,\infty}]}{2\gamma_i} + \frac{\Delta_i}{2}, \quad (28)$$

where $\gamma_i = \mu_i \prod_{j\neq i}(1-\mu_j)$.

To prove this, we first show that the virtual error process $\tilde{e}_{i,t}$ with a Gaussian noise with zero mean and finite variance σ_i^2 forms a positive Harris recurrent Markov chain with a unique invariant distribution. If one can show that the virtual error process $\tilde{e}_{i,t}$ with an arbitrary symmetric noise with zero mean and finite variance σ_i^2 forms a positive Harris recurrent Markov chain with a unique invariant distribution, then Theorem 4.1 holds for the particular symmetric noise. The detailed proof using [32], [33], [34] is in Appendix H.

Note that $\mathbb{P}(|\tilde{e}_{i,\infty}| \geq \Delta_i) > 0$ for $\Delta_i \in (0, \infty)$; otherwise, i.e., $\mathbb{P}(|\tilde{e}_{i,\infty}| \geq \Delta_i) = 0$ for $\Delta_i \in (0, \infty)$, the system is naturally stable with $\mathbb{E}[|e_{i,\infty}|] < \Delta_i$. Further, from Theorem 4.6.2 in [25], we have $\mathbb{E}[U_{i,\infty}] = \lim_{t\to\infty} \mathbb{P}(U_{i,t} = 1) = \frac{1}{\mathbb{E}[T]}$, where T is the packet generation period. Since $\mathbb{P}(|\tilde{e}_{i,\infty}| \geq \Delta) > 0$, we have $\mathbb{E}[T] \in [1, \infty)$. Thus, the upper bound in (28) is finite, which implies that the system is always stable for any $\sigma \in (0, \infty)$. Further, if one can analytically obtain the long-term probability $\mathbb{P}(|\tilde{e}_{\infty}| \geq \Delta)$ of packet generation and the long-term expected packet generation period $\mathbb{E}[U_{\infty}]$, then one can optimize the upper bound in (28) and have a sub-optimal update policy.

B. Asymmetric Dynamics

In this section, we consider an asymmetric noise and apply the cancellation-enabled one-bit update policy. Suppose that the state $x_{i,t}$ of source i changes as

$$x_{i,t+1} = x_{i,t} + w_{i,t}, \text{ for } t \geq 0, \quad (29)$$

where

$$w_{i,t} = \begin{cases} 1, & \text{with prob. } p_i, \\ 0, & \text{with prob. } 1 - p_i - q_i, \\ -1, & \text{with prob. } q_i, \end{cases} \quad (30)$$

where $p_i, q_i \in [0,1]$ such that $p_i + q_i \leq 1$ and $p_i - q_i = \alpha_i$.

Note that $\mathbb{E}[x_{i,t+1} - x_{i,t} \,|\, x_{i,t}] = \alpha_i$, i.e., the state $x_{i,t}$ is drifted by α_i. We assume that the amount α_i of drift is known to the receiver. Then, the receiver updates the estimate $\hat{x}_{i,t}$ for source i as

$$\hat{x}_{i,t+1} = \hat{x}_{i,t} - \alpha_i + V_{i,t}^1 Z_{i,t}\Delta_i, \quad (31)$$

where $Z_{i,t} = 1$ if a packet is arrived from source i and $V_{i,t}^1$ is the sign of the received information. That is, the receiver makes a correction by the amount of drift at each time slot. Then, the estimation error $e_{i,t} = x_{i,t} - \hat{x}_{i,t}$ evolves as

$$e_{i,t+1} = e_{i,t} + w_{i,t} - \alpha_i - V_{i,t}^1 Z_{i,t}\Delta_i, \quad (32)$$

[4]It can be easily shown that $|\gamma_i - \frac{2p_i}{(\Delta_i^{\mu})^2}| \to 0$ as $n \to \infty$. The choice of this specific threshold Δ_i is for obtaining analytical results in Theorem 3.3 by simplifying the steady-state distribution of the queue length process in (20).

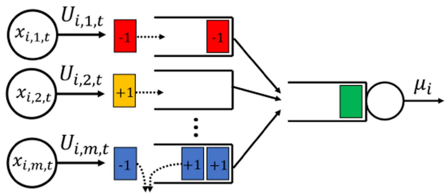


Fig. 5. Cancellation-enabled policy for m-dimensional source dynamics, where the packet cancellation occurs at the source m's queue.

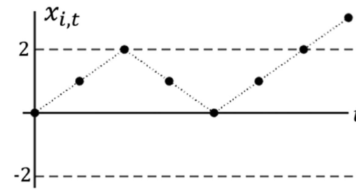


Fig. 6. A trajectory of the state $x_{i,t}$ of source i.

and the virtual error $\tilde{e}_{i,t}$ evolves as

$$\tilde{e}_{i,t+1} = \tilde{e}_{i,t} + w_{i,t} - \alpha_i - \Delta_i \mathbb{I}\{\tilde{e}_{i,t} + w_{i,t} - \alpha_i \geq \Delta_i\} + \Delta_i \mathbb{I}\{\tilde{e}_{i,t} + w_{i,t} - \alpha_i \leq \Delta_i\}. \quad (33)$$

Since $w_{i,t} - \alpha_i$ is an asymmetric random variable with mean 0, we may not have $\mathbb{P}(\tilde{e}_{i,t} > 0 \,|\, |\tilde{e}_{i,t}| \geq \Delta_i) = \mathbb{P}(\tilde{e}_{i,t} < 0 \mid |\tilde{e}_{i,t}| \geq \Delta_i)$, which is the property that the cancellation-enabled update policy is built on. However, we show that the symmetric property holds for a large number n of sources in the following theorem.

Theorem 4.2: For the virtual error process $\tilde{e}_{i,t}$ defined in (33), we have

$$\lim_{n\to\infty} \mathbb{P}(\tilde{e}_{i,t} > 0 \mid |\tilde{e}_{i,t}| \geq \Delta_i) = \lim_{n\to\infty} \mathbb{P}(\tilde{e}_{i,t} < 0 \mid |\tilde{e}_{i,t}| \geq \Delta_i) = 1/2. \quad (34)$$

Note that $\Delta_i \to \infty$ as $n \to \infty$ by the choice of $\boldsymbol{\mu}$ and $\boldsymbol{\Delta}$ in Section III-E. Then, it can be shown that the virtual error $\tilde{e}_{i,t}$ is equally likely to be positive or negative when it exceeds threshold Δ_i for a sufficiently large n (i.e., large Δ_i) using analysis of Martingales [26]. The detailed proof is in Appendix I.

From Theorem 4.2, we can use the cancellation-enabled update policy with drift adjustment and obtain the result on the optimality ratio represented in Theorem 3.3.

C. Multi-Dimensional States

Lastly, we consider a problem of multi-dimensional states. Suppose that each source i is observing m_i different dynamics, where each dynamics is one-dimensional as we have investigated throughout this article. Let $x_{i,k,t}$ denote the state of k^{th} dynamics observed by source i at time t, and let $\boldsymbol{x}_{i,t} = [x_{i,1,t}, \ldots, x_{i,m_i,t}]^T$ for $m_i \in \mathbb{N}$, where $x_{i,k,t} \in \mathbb{R}$. The objective is to minimize:

$$\lim_{t\to\infty} \frac{1}{tn} \sum_{s=1}^{t} \sum_{i=1}^{n} \frac{1}{m_i} \sum_{k=1}^{m_i} \mathbb{E}[|e_{i,k,t}|]. \quad (35)$$

We assume that $x_{i,1,t}, \ldots, x_{i,m_i,t}$ are independent each other. Then, the source can locally use the cancellation-enabled for each $x_{i,k,t}$ with threshold $\Delta_{i,k}$, which can be $\Delta^{\boldsymbol{\mu}^{asym}}_{i,k}$ if $x_{i,k,t}$ is a symmetric random walk with parameter $p_{i,k}$, generate a packet containing the values of m_i local queues at every time slot, and if the packet is not successfully transmitted to the receiver then the packet is discarded at the end of the time slot as shown in Fig. 5.

Since $x_{i,1,t}, \ldots, x_{i,m_i,t}$ are independent, we can obtain the optimality ratio obtained in Theorem 3.3 (i.e., asymptotic upper bound of $\frac{13\sqrt{2}}{12}$). However, in the multi-dimensional case, each local queue has three types of information (i.e., three quantization bins): $+1$, 1 and 0, where $+1$ and -1 are the value of packet if exists, and 0 means that the error does not exceed the threshold.[5] Hence, by the Shannon's entropy theorem [27], the average data length is upper-bounded by $\log_2 3m_i$ since there is $3m_i$ number of quantization bins.

V. Numerical Results

In this section, we verify the performance of our threshold-based one-bit update policies. We first compare four different one-bit update policies: updates without packet cancellation proposed in Section III-B (denoted by No-pck-cancel), cancellation-enabled updates (denoted by Pck-cancel) proposed in Section III-C, threshold-based updates with one bit inspired by [8] (denoted by Th-based (1 b)), and one-bit updates with freshest information inspired by the optimal policy in Section III-A (denoted by Fresh-info (1 b)). Given (Δ, μ), the Th-based (1 b) policy tries to generate a new packet after a successful transmission thus the queue being empty. If $\tilde{e}_t \geq \Delta$ (or $\leq -\Delta$), then a packet having Δ (or $-\Delta$) is generated and the virtual error $\tilde{e}_t$ decreases (or increases) by Δ, i.e., $\tilde{e}_{t+1} = \tilde{e}_t - \Delta$ (or $\tilde{e}_{t+1} = \tilde{e}_t + \Delta$). If $|\tilde{e}_t| < \Delta$, then it waits until $\tilde{e}_t$ hits the thresholds Δ or $-\Delta$. The Fresh-info (1 b) policy generates a packet if $|\tilde{e}_t| \geq \Delta$ with the corresponding sign at the beginning of each time slot, and if the packet is not successfully transmitted to the receiver, then the packet is discarded at the end of the time slot[6]. For example, suppose that $x_{i,0} = \hat{x}_{i,0} = 0$, $(x_{i,t})_{t=1}^{7} = (1, 2, 1, 0, 1, 2, 3)$ and $\Delta_i = 2$ as in Fig. 6, and that no packets have been successfully delivered to the receiver for $t = 1, \ldots, 7$. Then, under the Pck-cancel policy, the queue has three packets with the value $+1$, -1 and $+1$ generated at time 2, 4 and 6 at the end of time 7, and the virtual error $\tilde{e}_{i,7}$ at time 7 is 1. On the other hand, the queue has one packet with the value $+1$ generated at time 2 under the Th-based (1 b) policy at the end of time 7, and the queue has one packet with the value $+1$ generated at time 7 under the Fresh-info (1 b) before discarding the packet at the end of time 7.

We first consider remote tracking of a single source. The source has transition probability $p = 0.4$ and activation probability $\mu = 0.04$, and the simulations run for $T = 10^5$ time slots

[5] For 1-dimensional case, the 0 can be replaced by not sending a packet.

[6] Under Th-based (1 b) policy, the generated packets are not discarded. However, under Fresh-info (1 b) policy, the generated packets are discarded whenever, at the end of the time slot, the generated packet at the beginning of the time slot is not delivered to the receiver. Thus, they can be viewed as non-preemptive and preemptive policies, respectively. In addition, the cancellation-enabled one-bit update policy can be viewed as a preemptive policy.

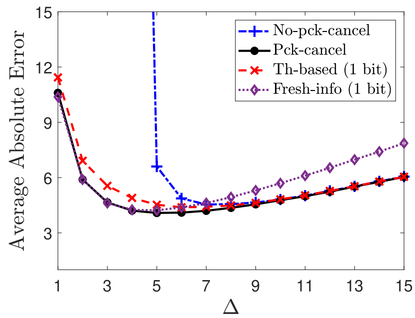
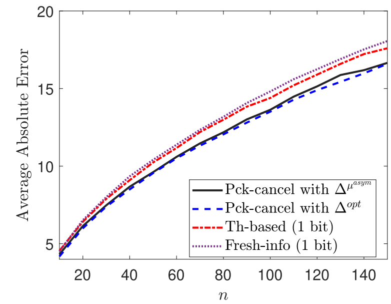


Fig. 7. Average absolute error of four different one-bit update policies for a single source with different thresholds Δ given $p = 0.4$ and $\mu = 0.04$.

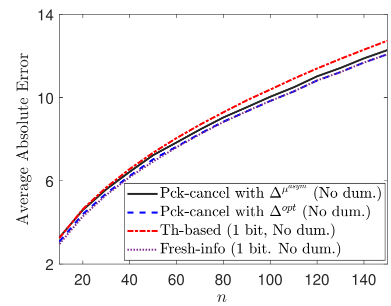
and are averaged over 200 repetitions. Fig. 7 shows the average absolute error of four different one-bit update policies with respect to threshold Δ. For No-pck-cancel policy, the thresholds $\Delta > \sqrt{\frac{2p}{\mu}} \approx 4.4741$ is the stability condition as stated in Theorem 3.2, while the other three policies (Pck-cancel, Th-based and Fresh-info) make the system always stable. Further, Pck-cancel policy outperforms the other one-bit update policies for all Δ.

Next, consider remote tracking of multiple homogeneous sources with $p_i = p = 0.4$ for all i. Since the sources have the same dynamics, it is reasonable to set the activation probabilities $\mu = \frac{1}{n}$ for all the sources given n number of sources in the system. For the cancellation-enabled one-bit updates, we use two different thresholds: one is the threshold $\Delta^{\mu^{asym}} = \lfloor \sqrt{\frac{2p}{\frac{1}{n}(1-\frac{1}{n})^{n-1}}} \rfloor$, which is the threshold obtained in Section III-E, and another one is the optimal threshold Δ^*, which is numerically found through exhaustive search. For Th-based (1 b) and Fresh-info (1 b) policies, the optimal thresholds Δ are also numerically found. The simulations run for $T = 10^5$ time slots and are averaged over 200 repetitions.

Fig. 8 shows the average absolute error of four different one-bit update policies with respect to the number n of sources with and without dummy packets, which are assumed for analytical simplicity. Under no dummy packet assumption (denoted by *No dum.*), each source tries a transmission only when it has an update packet in its queue. Fig. 8(a) shows that, under the dummy packet assumption, the gap between the cancellation-enabled one-bit updates with thresholds $\Delta^{\mu^{asym}}$ and Δ^{opt} is unnoticeable, and Pck-cancel policies with $\Delta^{\mu^{asym}}$ and Δ^{opt} outperform the other two update policies. On the other hand, Fig. 8(b) shows that, without the dummy packet assumption, the gap between Pck-cancel with Δ^{opt} and Fresh-info (1 b) is unnoticeable. In the numerical simulations, it is observed that, at the optimal threshold obtained by exhaustive search, Fresh-info policy generates update packets less frequently than Pck-cancel policy. Note that if a source sends update packets too frequently then the source generates too much traffic on the network resulting in the performance degradation. On the other hand, if a source sends update packets too occasionally, then its estimation error will be large, which also results in the overall performance degradation. With dummy packets, a source under Fresh-info policy cannot use the benefit giving more transmission chances to the other sources. Further, note that removing dummy packets improves the error performance



(a) Update policies with dummy packet assumption.



(b) Update policies without dummy packet assumption.

Fig. 8. Average absolute error of four different 1-bit update policies for homogeneous sources with the different number n of sources given $p = 0.4$.

for all update policies. As mentioned in Section II-A, the dummy packet assumption is made for the tractability of the mathematical analysis, but it would be more beneficial not to use the dummy packets in practical operation. It will also be an interesting open problem to analyze the performance of the system without the dummy packet assumption.

Next, we compare the cancellation-enabled one-bit update policy with three different update policies with perfect information: the optimal policy in Section III-A, which keeps the queue with the freshest packet (denoted by Fresh-info (perf. info.)), threshold-based update policy in [8] (denoted by Th-based (perf. info.)), and zero-waiting update policy (denoted by ZW (perf. info.)). Note that “perfect information” means that the policy do not restrict the number of bits for information, i.e., the packet can have the exact value at the time it is generated. The Zero-waiting policy generates a new packet with the actual state value after successful transmission. The Th-based (perf. info.) policy is similar with the Th-based (1 b) policy except that, if $\tilde{e}_t \geq \Delta$ (or $\leq -\Delta$), a packet having the actual value $\tilde{e}_t$ is generated and the virtual error $\tilde{e}_t$ becomes 0. If $|\tilde{e}_t| < \Delta$, then it waits until $\tilde{e}_t$ hits the thresholds Δ or $-\Delta$.

Fig. 9 shows the average absolute error of the four different update policies with respect to threshold Δ with a single source having transition probability $p = 0.4$ and activation probability $\mu = 0.2$. The simulations run for $T = 10^5$ time slots and are averaged over 100 repetitions. As can be seen, the Pck-cancel policy outperforms the zero-waiting and threshold-based update policies with perfect information at its optimum threshold level.

Fig. 10 shows the optimality ratio of average absolute error with respect to the number n of homogeneous sources with $p = 0.4$. The simulations run for $T = 10^5$ time slots and are averaged over 500 repetitions. It can be seen that the optimality

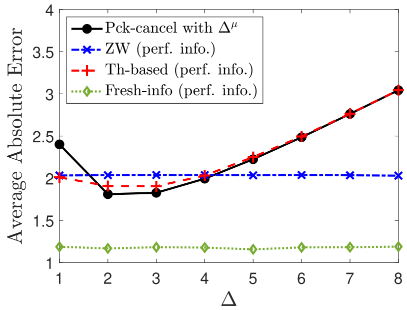


Fig. 9. Average absolute error of the 1-bit Pck-cancel policy and three different M bits update policies for a single source with different thresholds Δ given $p = 0.4$ and $\mu = 0.2$.

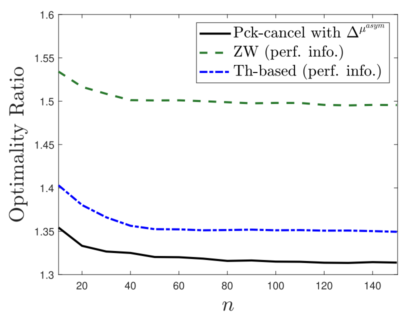


Fig. 10. Optimality ratio of three different update policies for homogeneous sources with the different number n of sources given $p = 0.4$.

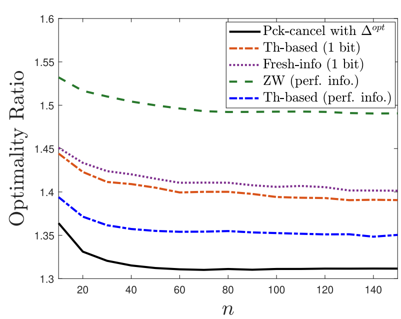


Fig. 11. Optimality ratio of five different update policies for homogeneous sources with different number n of sources when a noise is a zero-mean Gaussian random variable with variance 4.

ratio converges to some constant as the number n of sources becomes large for all three update policies. In general, transmission time and power increase as the packet size (i.e., the number of bits for the state information) increases. This suggests that the cancellation-enabled one-bit update policy could be greatly beneficial for applications where transmission power or shared channel capacity is limited.

Next, we consider the general source dynamics studied in Section IV: random walks with (i) a Gaussian noise and (ii) an asymmetric noise. Figs. 11 and 12 show the optimality ratio of five different update policies when a noise is a zero-mean Gaussian random variable with variance 4 and when a noise is an asymmetric noise with parameters $p = 0.5$ and $q = 0.3$, respectively. The simulations run for $T = 10^5$ time slots and are averaged over 500 repetitions. As can be seen in Figs. 11 and 12, the optimality ratio converges to some constant and

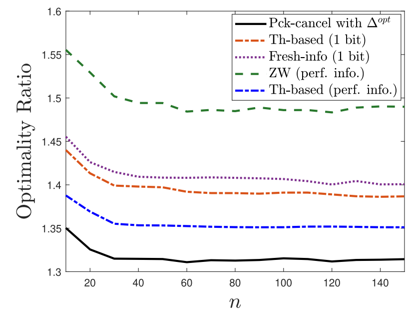


Fig. 12. Optimality ratio of five different update policies for homogeneous sources with different number n of sources when a noise is asymmetric with $p = 0.5$ and $q = 0.3$.

the Pck-cancel policy outperforms the others at its optimum threshold level.

VI. Conclusion

Motivated by massive IoT network applications, we considered the scenario of a large number of low-complexity devices updating their evolving state to a receiver over low-capacity random access channels. In particular, we developed decentralized update policies that require one-bit of information per update for minimizing the expected absolute (estimation) error when states of sources evolve according to symmetric random walks. We first studied a benchmark first-come first-serve (one-bit) update policy and showed that this policy will fail to stabilize the system under some conditions. Then, we introduced a cancellation-enabled one-bit update policy that improves the performance of the benchmark policy and makes the system always stable. We proposed a choice of parameters for the cancellation-enabled policy and showed that the cancellation-enabled policy with the sub-optimal parameters has optimality ratio $\frac{13\sqrt{2}}{12}$ to the optimal policy that does not restrict the number of bits for each update. Through simulations, we identified that the sub-optimal parameters are robust to errors compared with the optimal parameters obtained through exhaustive search, and compared the cancellation-enabled one-bit update policy with zero-wait sampling and threshold-based sampling policies that require more than one-bit of information per update. The numerical comparison showed that the cancellation-enabled update policy at its optimal threshold level outperforms the multi-bits update policies. This suggests that the cancellation-enabled one-bit update policy could be greatly beneficial for applications where transmission power or shared channel capacity is limited. Further, analytical comparison between update policies used in the simulations can be an interesting open problem, especially a comparison between the cancellation-enables one-bit update policy and the one-bit update policy with the freshest information.

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