Quantum-Inspired Optimal Power Flow

Farshad Amani, *Student Member, IEEE*, Amin Kargarian, *Senior Member, IEEE*

***Abstract*—This paper addresses the critical challenge of Optimal Power Flow (OPF) in electrical engineering, emphasizing the intersection of complexity and technological advancements. Leveraging quantum computation, particularly the Primal-Dual Interior Point Method (PD-IPM) and the Harrow-Hassidim-Lloyd (HHL) algorithm, we explore enhanced solutions for both Alternating Current OPF (ACOPF) and Direct Current OPF (DCOPF). PD-IPM generates linear systems during optimization, aligning with the selected quantum techniques that offer exponential speedup. We investigate the scalability and challenges of HHL, emphasizing the current limitations for larger power systems. Additionally, we introduce a hybrid approach that dynamically transitions between quantum and classical methods to optimize convergence. Our study evaluates the proposed methodology's performance under diverse error conditions, emphasizing its potential to revolutionize solving times for OPF in power systems.**

***Index Terms*—Quantum computing, optimal power flow, Harrow-Hassidim-Lloyd algorithm, ACOPF, DCOPF.**

I. Introduction

ACCURACY and speed of the mathematical methodologies in addressing complex engineering problems have consistently been given significant attention [1], [2]. Among these challenges, Optimal Power Flow (OPF) stands out as a pivotal concern in electrical engineering, with its complexity increased by the integration of advanced technologies [3]. The advent of quantum computation, surpassing conventional methods in various instances, presents an opportunity to accelerate solutions to power system-related problems. By leveraging quantum computation, traditional optimization problems like OPF can be efficiently transformed and resolved at a quicker speed than previously achievable [4], [5].

Various approaches for solving OPF have been proposed, ranging from linear to non-linear and convex to non-convex formulations [6], [7]. The Alternating Current OPF (ACOPF) provides a detailed description of power systems, but it is non-convex and computationally demanding [8]. In contrast, although Direct Current OPF (DCOPF) is less accurate, it is less complex than ACOPF, featuring linear equations that enhance computational efficiency [9]. The Primal-Dual Interior Point Method (PD-IPM) emerges as a promising approach for achieving optimal solutions in both ACOPF and DCOPF [10]. A salient feature of PD-IPM is the generation of a linear system of equations during the optimization process [11], [12].

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The authors are with the Department of Electrical and Computer Engineering, Louisiana State University, Baton Rouge, LA 70803 USA (e-mail: famani1@lsu.edu, kargarian@lsu.edu).

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This linear system holds significance since certain quantum computation techniques have been developed to solve it [13], [14]. Leveraging these quantum approaches can reduce the time required for solving the system, particularly considering the iterative nature of PD-IPM that needs multiple attempts.

Several advanced traditional methods exist for solving systems of linear equations, but their speed intensively relies on the dimension of the system [15]. Quantum computation often introduces a logarithmic time complexity, surpassing the fastest traditional approach, which typically exhibits a cubic time complexity [16]. Therefore, the shift from traditional to modern quantum methods appears inevitable to improve the runtime of critical problems like OPF, which forms the core of numerous challenges in power systems. The Harrow-Hassidim-Lloyd (HHL) algorithm has recently gained attention for its ability to compute the inverse of a square matrix exponentially faster than conventional algorithms [17]. It uses a phase estimation stage to guess the eigenvector and subsequently rotates it $180°$ to determine the inverse, leading to the solution of the linear system [18]. While [19] uses HHL to solve the DC Power Flow, [20] employs a quantum algorithm to solve AC Power Plow, both of which are considered a small test system. [5] moves further and reveals the scalability of HHL and solves ACOPF for larger systems. The author shows that HHL cannot be implemented for larger power systems due to noises and an increase in the depth of the circuit needed for the implementation of HHL. [18] uses the Lagrangian function method to address the Karush-Kuhn-Tucker (KKT) conditions for obtaining the DCOPF solution. [21] applies an advanced noise-tolerant quantum IPM to solve the DCOPF using HHL as a linear solver.

This study uses PD-IPM to address DCOPF and ACOPF using HHL. In both cases, we encounter Newton's direction which is a system of linear equations. Solving this system imposes many challenges due to imperfections in current quantum algorithms and hardware. We examine the performance of the proposed approach under various noise and error conditions, to realize the effectiveness and inefficiencies of our approach.

II. Methodology

A. IPM Based OPF

OPF problem is a critical optimization task in power systems that can be shown as follows:

$$\begin{aligned} &\text{minimize} \quad f(x) \\ &\text{subject to} \quad \begin{cases} g(x) = 0 \\ h_l \le h(x) \le h_u \end{cases} \end{aligned} \tag{1}$$

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OPF is often expressed in the form of either ACOPF or DCOPF. Although their general structure is the same, these formulations differ in their constraints. ACOPF addresses various constraints, including active and reactive power balance, voltage magnitude and angle, line flow limitations, generator output limits, and nonlinear network equations. These network equations capture the intricate relationships among bus voltages, power injections, and power flows. In contrast, DCOPF is a simplified version of ACOPF, neglecting reactive power considerations and assuming uniform voltage magnitudes. This simplification results in a linearized OPF, facilitating a less complex solution process.

PD-IPM stands out as an efficient mathematical approach capable of determining optimal solutions for both ACOPF and DCOPF. Given the familiarity of both ACOPF and DCOPF within the power system community, we neglect the explicit representation of their formulations and instead focus on the general form of the OPF problem in the subsequent discussion. Interested readers can easily substitute these equations with well-established formulations.

PD-IPM involves a four-step process to solve optimization problems. The initial step entails transforming inequality constraints into equality constraints through the introduction of positive slack variables. Subsequently, in the second step, non-negativity conditions are incorporated into the objective function in the form of a logarithm barrier function. This addition penalizes the objective function when variable values approach zero. Following these two steps, the resulting problem takes the following form:

$$\begin{aligned} &\text{minimize} \quad f(x) - \mu(\ln s_l + \ln s_u) \\ &\text{subject to} \quad \begin{cases} g(x) = 0 \\ h(x) - h_l - s_l = 0 \\ -h(x) + h_u - s_u = 0 \end{cases} \end{aligned} \tag{2}$$

The third step aims to convert the constrained optimization problem established in the previous steps into an unconstrained optimization problem. This transformation is facilitated through the formulation of the Lagrangian function, denoted as:

$$\begin{aligned} \mathcal{L} = f(x) - \mu(\ln s_l + \ln s_u) - \lambda^T g(x) \\ - \pi_l^T(h(x) - h_l - s_l) - \pi_u^T(-h(x) + h_u - s_u) \end{aligned} \tag{3}$$

where the Lagrange multipliers λ, π_l, and π_u are called dual variable. This process results in a problem with only equality constraints.

Moving on to the fourth step, we aim to identify the optimal solution using the perturbed Karush-Kuhn-Tucker (KKT) first-order optimality conditions. This leads us to a system equation, which can be efficiently solved using the Newton-Raphson method. The Hessian matrix, which represents the second-order derivatives, can be expressed as a linear symmetric system (4). While this system shows the KKT condition, the symmetry of the matrix enables a reduction in dimensionality, simplifying the problem-solving process:

$$\begin{bmatrix} 0 & -\Delta g(x) \\ -\Delta g(x)^T & H \end{bmatrix} \begin{bmatrix} \Delta\lambda \\ \Delta x \end{bmatrix} = - \begin{bmatrix} \Delta_\lambda \mathcal{L} \\ \eta \end{bmatrix} \tag{4}$$

where:

$$H = \Delta_x^2 \mathcal{L} + \mu \Delta_x h(x)^T (S_l^{-2} + S_u^{-2}) \Delta_x h(x) \tag{5}$$

$$\begin{aligned} \eta = \Delta_x \mathcal{L} + \Delta_x h(x)^T [\mu(S_u^{-2} \Delta_{\pi_u} \mathcal{L} - S_l^{-2} \Delta_{\pi_l} \mathcal{L} \\ + \Delta_{s_l} \mathcal{L} - \Delta_{s_u} \mathcal{L}] \end{aligned} \tag{6}$$

In PD-IPM, we guess an initial number for positive scaler μ, called the barrier parameter, and then gradually diminish it by each iteration of the Newton step. System (4) is a Hermitian matrix that can be solved by a quantum linear solver. The approach that we use to find the solution of OPF is outlined in Algorithm I.

Algorithm I OPF.

Input: Objective function and constraints
Output: Solution of OPF
1: Form the OPF formulation
2: Use positive slack variable to make $h(x)$ as equality constrains
3: Using the logarithmic barrier function bring the non-negativity constraint to the objectives function
4: Form unconstraint optimization problem (3) using Lagrange theory
5: **While** PD-IPM is not converged or number of iterations $< k^{\max}$
6: Form (4) using KKT condition
7: Solve system (4) using quantum Algorithm II
8: update variables and μ
9: **end**
10: **Return** solution of OPF

B. *Quantum Linear Solver*

The HHL algorithm has caught the attention of researchers for its ability to solve small-scale linear systems of equations, albeit with some errors. HHL works by estimating the eigenvector of a Hermitian matrix using a Fourier Transformer. This involves rotating phases and multiplying them with the vector b (in $Ax = b$), ultimately yielding the solution vector x. Notably, the time required for this quantum calculation is significantly faster than classical approaches.

In the context of ACOPF and DCOPF, where we encounter system (4), that is Hermitian, we can leverage the HHL algorithm directly. This entails feeding the Hermitian matrix from the system (4) into the HHL circuit during each iteration of the PD-IPM. In this system, we assume the matrices are defined as follows:

$$\begin{aligned} &A = \begin{bmatrix} 0 & -\Delta g(x) \\ -\Delta g(x)^T & H \end{bmatrix}, \quad x = \begin{bmatrix} \Delta\lambda \\ \Delta x \end{bmatrix}, \\ &b = - \begin{bmatrix} \Delta_\lambda \mathcal{L} \\ \eta \end{bmatrix} \end{aligned} \tag{7}$$

The first step in applying this system to quantum hardware involves encoding these classical matrices into a quantum state. This encoding process serves a critical role, enabling the translation of classical information into a quantum state, and paving the way for the efficient or even inefficient use of HHL.

In the quantum computation framework, the transformed system (4) is represented as $|\tilde{A}\rangle|\tilde{x}\rangle = |\tilde{b}\rangle$. Following the phase estimation stage, the quantum state is expressed as $|\omega\rangle = \sum_j |0\rangle \otimes |\tilde{b}_j\rangle|\lambda_j\rangle \otimes |u_j\rangle$, where $|\lambda_j\rangle$ denotes the eigenvalue and $|u_j\rangle$ represents the normalized eigenbases of the matrix $\tilde{A}$. The vector $\tilde{b}$ is encoded as quantum state $|\tilde{b}\rangle$. Applying a rotation gate around the y-axis and subsequently an inverse of the Fourier Transformer leads to the quantum state $|\omega\rangle = |0\rangle \otimes \sum_j \left(\sqrt{1 - \frac{C_0^2}{\lambda_j^2}}|0\rangle + \frac{C_0}{\lambda_j}|1\rangle\right) \otimes \tilde{b}_j |u\rangle_j$. Measuring the state $|1\rangle$ allows the identification of the quantum state $|\omega\rangle = C_0 \sum_j \frac{\tilde{b}_j}{\lambda_j}|u_j\rangle$. Normalizing this state provides the solution to the linear system or the vector x for the system (4). This resulting vector serves as the step size or the Newton direction in the PD-IPM. This quantum-enhanced approach accelerates the computation of the Newton direction, contributing to the enhanced efficiency of the overall optimization process in systems like ACOPF and DCOPF. The approach used to solve the solution for the system (4) is outlined below:

Algorithm II Quantum linear solver.

Input: Matrix $\tilde{A}$ and column vector $\tilde{b}$
Output: A solution vector $\tilde{x}$
1: Encode vector $b = -(\Delta_\lambda, \eta)^T as|\tilde{b}\rangle$
2: Initialize unitary operation $U = e^{(-i\tilde{A}t)}$
3: Perform quantum Fourier transform and rotate the obtained phases
4: Uncompute the work registers
5: Measure state $|1\rangle$ from the ancilla qubit
6: Using the normalization proportionality factor, convert $|\tilde{x}\rangle$ to classical vector x
7: **Return x**

The outcome of this equation relies on various factors, including quantum hardware performance, matrix condition, initialization, encoding method, and the HHL circuit. While these challenges currently limit the effectiveness of the HHL algorithm in many power systems, it remains applicable for smaller systems, particularly those with fewer than 9 buses [5].

C. Hybrid quantum classic IPM (HIPM)

To ensure both speed and accuracy compared to the classical IPM, we introduce Algorithm III.

$$Sp_i = |f_{i+n} - f_i| \quad \forall i = 1, 2, ..., k^{\max} - (n-1) \tag{8}$$

$$In_j^{\text{Conv}} = \left|\text{Sp}_{i+1} - \text{Sp}_i\right| \quad \forall i = 1, 2, ..., k^{\max} - (n-2) \tag{9}$$

Algorithm III HIPM.

Input: Matrix $\tilde{A}$ and column vector $\tilde{b}$
Output: A solution vector X
1: **Do**: Algorithm I
2: Compute In_j^{Conv} as the objective function improvement index
3: **If** $In_j^{\text{Conv}} < \epsilon$
4: Use the last iteration data as the initial point for IPM
5: **Do**: IPM
6: **Else** Iteration number $< k^{\max}$
7: **Continue**: Algorithm I
Return: Solution x

Although not as fast as Algorithm I, Algorithm III significantly improves upon the classical IPM. The core concept of Algorithm III revolves around dynamically assessing the convergence rates of QIPM. If QIPM demonstrates a slow convergence rate, Algorithm III stops it and uses the last obtained information as an initial guess for the classical IPM. Subsequently, the classical IPM continues from this point, benefiting from the proximity of the initial guess to the exact solution. We anticipate observing fewer iterations than the pure IPM approach without the integration of the HHL. By reducing the computational burden of some of the iterations by using HHL, Algorithm III aims to reduce the overall solving time for OPF.

III. RESULTS

A. Settings

This study employs four systems including 3-bus, 5-bus, 118-bus, and 300-bus systems to evaluate the proposed approach. To test the method on the first two smaller systems, we utilize the Qiskit simulator to implement the HHL [22]. For larger systems, we simulate errors and noise on classical computers to demonstrate the effectiveness of our approach. The simulations use Python for Qiskit and the Matpower package in MATLAB for classical computation. Both DCOPF and ACOPF are performed for all systems under consideration. Using HHL as a linear solver on real quantum computers or quantum simulators imposes various challenges due to errors and noises inherent in current quantum systems and algorithms. Regarding algorithmic imperfections, HHL suffers from the preparation and initialization stages to achieve a satisfying solution. Additionally, the phase estimation process introduces its own set of challenges. The number of required qubits and the amount of auxiliary quantum resources needed by the algorithm can pose scalability concerns. Moving beyond algorithmic aspects, the real-world implementation of the HHL on quantum hardware introduces additional challenges. Quantum gate noise, arising from imperfections in gate operations, can compromise the accuracy of quantum computations. Gate errors, such as deviations in rotation angles, further contribute to inaccuracies in quantum operations. Readout and measurement errors represent another significant challenge, where inaccuracies in the measurement process can distort the results. To consider the impacts of these errors, we simulate them classically. In the context of error simulation, we introduce a random uniform error within the range of $\pm e\%$. We use three values for e, namely 5, 10, and 20, to examine the impact of errors on the proposed approach. To validate the accuracy of our error approximation, we conduct both Qiskit and classical simulations for the two smaller systems. This comparative analysis allows us to assess how closely our error approximation aligns with the Qiskit simulation results.

B. Qiskit simulation

Using Algorithm I and Algorithm II, the solution for the OPF in the 3-bus system, considering the DCOPF, is as

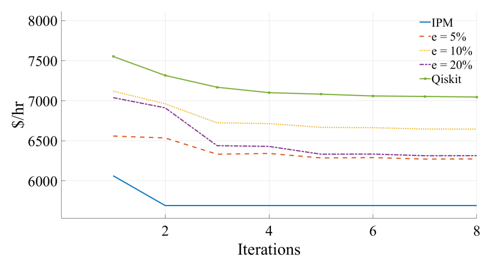


Fig. 1: DCOPF objective function for the 3-bus system.

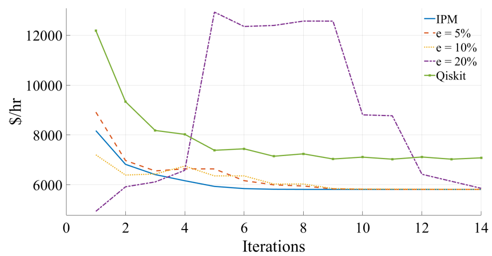


Fig. 2: ACOPF objective function for the 3-bus system.

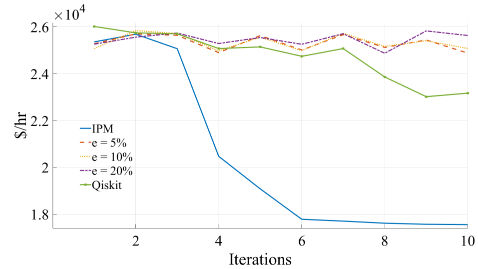


Fig. 4: ACOPF objective function for the 5-bus system.

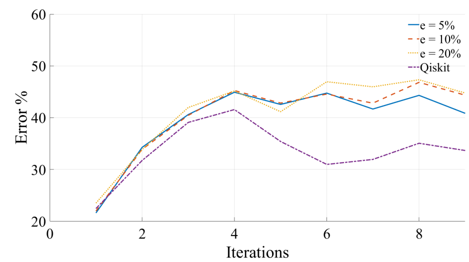


Fig. 5: Relative error percentage for the 5-bus system.

follows: When the ACOPF constraints are considered the following figures result: Fig. 2 Shows that even with the presence of nonlinear constraints, QIPM follows a trajectory similar to the classical IPM, as shown in Fig. 1. However, the high linearity nature of the ACOPF introduces increased complexity for the QIPM, resulting in a more challenging convergence towards a solution closely resembling the classical IPM. In both Fig. 1 and Fig. 2, the trend observed from Qiskit closely aligns with the error simulation in both cases.

Fig. 2 highlights the sensitivity of ACOPF to errors, where an error of 20% leads to an overshooting trend that results in a deviation from the classical IPM solution within a few iterations. In the case of DCOPF, although the magnitude of errors influences the relative convergence error between QIPM and classical IPM, its impact on the convergence trend is minor. This phenomenon is attributed to the linear nature of DCOPF, resulting in lower complexity compared to ACOPF.

In the case of the 5-bus system, the results for the DCOPF scenario are presented in Fig. 3. Notably, the number of iterations required to achieve an objective function close to the classical IPM is greater than the 3-bus system. This indicates that the system type exerts an influence on the outcome trend.

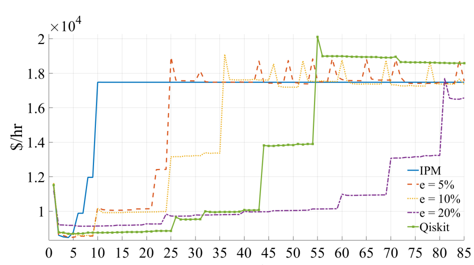


Fig. 3: DCOPF objective function for the 5-bus system.

However, it is noteworthy that by adjusting the maximum number of iterations ($k^{\max}$) to a higher value, a satisfactory solution can be obtained. Generally, a larger error tends to need more iterations for approaching the IPM solution. Continuing our study with the ACOPF scenario, the subsequent figure, Fig. 4, is obtained. The results for the 5-bus system in the case of ACOPF reveal that the algorithm faces challenges in moving close to the IPM solution. Despite increasing the maximum iteration number $k^{\max}$ to higher values, no noticeable improvement is observed. When examining the relative error between the QIPM and IPM, Fig. 5, it becomes evident that the error percentage remains unacceptably high, even with a large maximum iteration number. This underscores the difficulty of achieving convergence in the ACOPF scenario for the 5-bus system, highlighting potential limitations of the approach in certain system configurations.

C. *Error simulation*

In this section, our focus shifts to simulating errors and determining solutions for larger systems. To observe the behavior of the proposed approach under extreme conditions, we have selected two large systems. It's worth noting that current quantum computers or simulators face limitations in providing suitable solutions for larger matrices in the case of HHL. However, we anticipate that with the ongoing development of HHL and quantum hardware, the proposed method outlined in this paper could be applied to find the OPF for power systems in these challenging scenarios.

We conducted error simulations and obtained solutions for larger systems, selecting two larger systems to examine the behavior of our approach in extreme scenarios. While current quantum computers or simulators face limitations in finding suitable solutions for larger matrix dimensions in linear

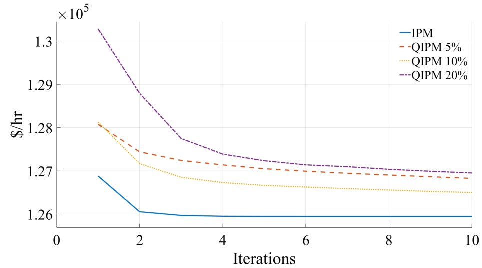


Fig. 6: DCOPF objective function for the 118-bus system.

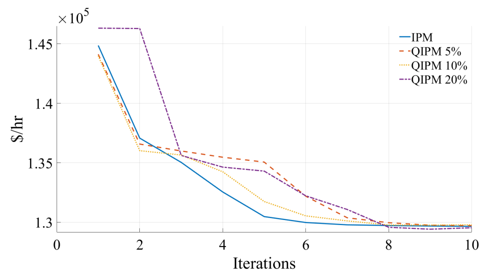


Fig. 7: ACOPF objective function for the 118-bus system.

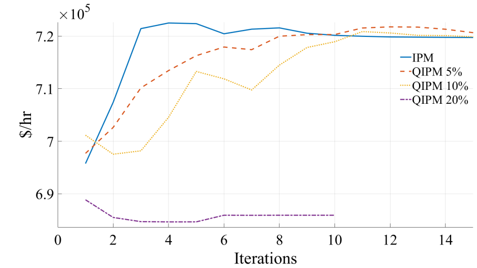
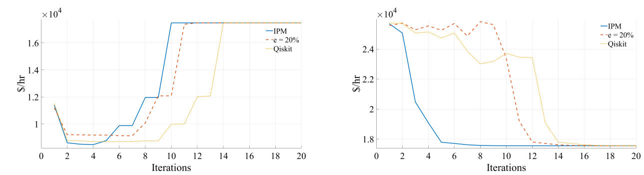


Fig. 9: ACOPF objective function for the 300-bus system.



(a) (b)

Fig. 10: HIPM-based for 5-bus system a) DCOPF and b) ACOPF.

systems, we anticipate that with further advancements in the HHL and quantum hardware, the method proposed in this paper could be applied to address OPF.

The performance of QIPM for the IEEE 118-bus system in solving the DCOPF is shown in Fig. 6. Despite not observing apparent convergence, it is noteworthy that QIPM consistently progresses towards the classical IPM. While there may be a slight error, its impact can be considered negligible, as the output values closely align with the exact ones. Executing the algorithm for ACOPF yields Fig. 7. The results exhibit a trend converging towards the classical IPM solution within a few iterations. However, convergence was not observed under the default error convergence setting of IPM in Matpower.

In the context of the 300-bus system DCOPF, Fig. 8 illustrates an exceptional approach to the classical value even under varying degrees of errors. Conversely, when considering the case of ACOPF, a distinct behavior emerges, as depicted in Fig. 9. For smaller errors, a proper convergence towards the exact value is evident. However, with an increase in the error magnitude (e.g., $e = 20$), a numerical failure occurs, indicating a transition to an infeasible region. In such instances, QIPM struggles to approach any valid solution. Based on our observations, particularly in the nonlinear ACOPF scenario, it is necessary to use a more reliable method to ensure secure convergence towards exact values. This underscores the necessity for enhanced methodologies when dealing with nonlinear optimization problems in power systems.

D. Hybrid Quantum Classic IPM

In this section, our objective is to use Algorithm III to guarantee a reliable solution for all types of OPF. We have selected the worst-case scenarios, represented by the 5-bus system and the 300-bus system, as our case studies. For the 5-bus system, ACOPF and DCOPF are executed using Qiskit, alongside error simulation with $e = 20$. For the 300-bus system, only the most challenging error scenario is simulated on a classical computer. The obtained results are presented in Fig. 10a and Fig. 10b, respectively, for DCOPF and ACOPF. These figures showcase the performance and effectiveness of Algorithm III in ensuring reliable solutions under extreme conditions. The proposed HIPM is anticipated to outperform classical algorithms in terms of computational speed. This is due to each iteration of the classical algorithm requiring more time compared to all iterations of QIPM. The polynomial time

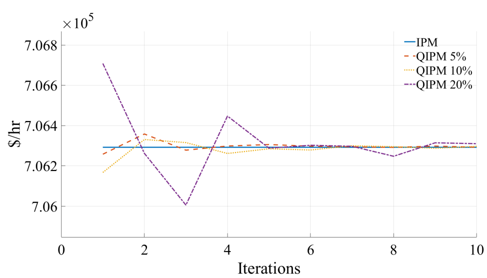
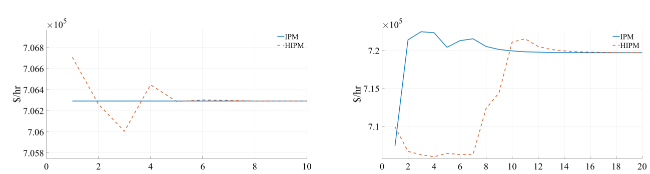


Fig. 8: DCOPF objective function for the 300-bus system.



(a) (b)

Fig. 11: HIPM-based for 300-bus system a) DCOPF and b) ACOPF.

complexity of the classical algorithm becomes notably extensive, especially when dealing with high matrix dimensions. In contrast, the quantum counterpart, leveraging quantum parallelism, is expected to provide a computational advantage, potentially leading to faster convergence and solution times for large-scale problems.

IV. Conclusion

In this study, we proposed a novel approach to address OPF challenges, leveraging quantum computation techniques, particularly PD-IPM and HHL algorithms. We examined the performance of the proposed approaches under various scenarios through comprehensive evaluations of various systems. Classical IPM, QIPM, and HIPM were the optimization approaches used for solving the DCOPF and ACOPF.

Both DCOPF and ACOPF were successfully addressed using Qiskit for the two small systems. The results demonstrated that, despite the presence of nonlinear constraints in ACOPF, QIPM closely followed a trajectory similar to the classical IPM. However, the increased linearity in ACOPF added complexities, which made the convergence process more challenging, particularly in the presence of errors. ACOPF presented notable challenges in approaching the classical IPM solution and showed potential limitations in certain systems such as the 5-bus system.

Moving to larger systems, such as the 118-bus and 300-bus scenarios, we conducted error simulations on classical computers due to current limitations in quantum hardware for larger matrices in HHL. Despite not achieving apparent convergence in some instances, the QIPM consistently progressed towards the classical IPM, showcasing its resilience under challenging conditions.

The introduction of HIPM further enhances the reliability of our approach. By dynamically evaluating convergence rates and transitioning between quantum and classical methods, Algorithm III ensures reliable solutions even under extreme conditions. The results for the 5-bus and 300-bus systems using HIPM underscore its potential to outperform classical algorithms, offering faster convergence and solution times, particularly for large-scale problems.

Future work should focus on refining quantum hardware and algorithms to overcome current limitations, paving the way for practical implementations in larger power systems.

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