Phys. Lett. B 849 (2024) 138479

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

Letter

Coalescence sum rule and the electric charge- and strangeness-dependences of directed flow in heavy ion collisions

Kishora Nayak [a,b], Shusu Shi [b], Zi-Wei Lin [c,*]

[a] Department of Physics, Panchayat College, Sambalpur University, Bargarh 768028, Odisha, India
[b] Key Laboratory of Quark & Lepton Physics (MOE), Institute of Particle Physics, Central China Normal University, Wuhan, 430079, China
[c] Department of Physics, East Carolina University, Greenville, NC, 27858, USA

ARTICLE INFO

Editor: F. Gelis

Keywords:
Directed flow
Quark coalescence
Coalescence sum rule
Quark-gluon plasma

ABSTRACT

The rapidity-odd directed flows (v_1) of identified hadrons are expected to follow the coalescence sum rule when the created matter is initially in parton degrees of freedom and then hadronizes through quark coalescence. A recent study has considered the v_1 of produced hadrons that do not contain u or d constituent quarks. It has constructed multiple hadron sets with a small mass difference but given difference in electric charge Δq and strangeness ΔS between the two sides, where a nonzero and increasing Δv_1 with Δq has been proposed to be a consequence of electromagnetic fields. In this study, we examine the consequence of coalescence sum rule on the Δv_1 of the hadron sets in the absence of electromagnetic fields. We find that in general $\Delta v_1 \neq 0$ for a hadron set with nonzero Δq and/or ΔS due to potential v_1 differences between $\bar{u}$ and $\bar{d}$ and between s and $\bar{s}$ quarks. We further propose methods to extract the coefficients for the Δq- and ΔS-dependences of the direct flow difference, where a nonzero constant term would indicate the breaking of the coalescence sum rule. The extraction methods are then demonstrated with transport model results.

1. Introduction

The properties of the quark-gluon plasma produced in relativistic heavy ion collisions can be studied with the directed flow (v_1) [1–4]. For example, v_1 is found to be a sensitive probe of the equation of state of the produced matter [5,6], and v_1 of heavy flavors [7] is expected to be sensitive to the strong electromagnetic field in the early stage of noncentral heavy ion collisions.

The coalescence sum rule is often found to describe well the relations of anisotropic flows of different hadron species in heavy ion collisions at high energies [8–13]. For collisions where the dynamics of anisotropic flows is dominated by parton interactions, quark coalescence relates the hadron flow directly to the flows of the hadron's constituent quarks [8,14,15]. When the constituent quarks in a hadron are comoving with each other and the quark coalescence probability is small, the hadron elliptic flow v_2 follows the coalescence sum rule at leading order [8,15,16]. The same formulation can be extended to the directed flow. When we neglect the mass difference of the constituent quarks [15], the coalescence sum rule is simply given by [8]

$$v_n^H(p_T^H) = \sum_j v_{n,j}(p_T), \text{ with } p_T^H = N_{cq}\, p_T. \tag{1}$$

In the above, $n=1$ for v_1 and $n=2$ for v_2, $v_{n,j}(p_T)$ represents the flow v_n of constituent quark j at the quark transverse momentum p_T, while N_{cq} is the number of constituent quarks (NCQ) of the hadron species H. Furthermore, if the quark $v_n(p_T)$ is the same for each constituent quark of hadron species H, Eq. (1) reduces to the most used form of the NCQ scaling: $v_n^H(N_{cq}\, p_T) = N_{cq}\, v_n(p_T)$.

It has been proposed [17] that the direct flows of hadrons whose constituent quarks are all produced quarks can be properly combined to better test the coalescence sum rule. In contrast to produced quarks, hadrons containing u and/or d quarks get contributions from slowed-down (or transported) u and d quarks in the incoming nuclei [18,19], which complicate the flow analysis. Our study here has been motivated by a recent study [20], which further considered the v_1 difference of various combinations of hadron sets consisting of seven produced hadron species: $K^-, \phi, \bar{p}, \bar{\Lambda}, \bar{\Xi}^+, \Omega^-$, and $\bar{\Omega}^+$. For example, one of the combinations is $v_1[\bar{\Lambda}] - (v_1[\phi]/2 + 2v_1[\bar{p}]/3)$. That study focused on the dependence of the v_1 difference on the electric charge difference

* Corresponding author.
E-mail address: linz@ecu.edu (Z.-W. Lin).

https://doi.org/10.1016/j.physletb.2024.138479
Received 8 November 2023; Received in revised form 17 January 2024; Accepted 18 January 2024
Available online 22 January 2024

Table 1
List of several hadron sets, where the left side and right side have the same total number of $\bar{u}$ and $\bar{d}$ quarks and the same total number of s and $\bar{s}$ quarks (after including the weights). Δq, Δq_{ud} and ΔS represent the difference in the electric charge, electric charge from $\bar{u}$ and $\bar{d}$ quarks, and strangeness number, respectively, between the two sides. Note that sets 1 to 4 and set 5A are independent of each other, while set 5B is not independent of them.

Set #	Δq_{ud}	ΔS	Δq	L (left side)	R (right side)
1	0	0	0	$v_1[K^-(\bar{u}s)]+v_1[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})]$	$v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]+v_1[\phi(s\bar{s})]$
2	0	0	0	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]+\frac{1}{2}v_1[\overline{\Xi}^+(\bar{d}\bar{s}\bar{s})]$
3	0	0	0	$\frac{1}{3}v_1[\Omega^-(sss)]+\frac{1}{3}v_1[\overline{\Omega}^+(\bar{s}\bar{s}\bar{s})]$	$v_1[\phi(s\bar{s})]$
4	0	1	1/3	$\frac{1}{2}v_1[\phi(s\bar{s})]$	$\frac{1}{3}v_1[\Omega^-(sss)]$
5A	1/3	1	2/3	$\frac{1}{2}v_1[\phi(s\bar{s})]+\frac{1}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$	$v_1\left[K^-(\bar{u}s)\right]$
5B	1/3	1	2/3	$v_1\left[\overline{\Lambda}(\bar{u}\bar{d}\bar{s})\right]$	$\frac{1}{2}v_1[\phi(s\bar{s})]+\frac{2}{3}v_1[\bar{p}(\bar{u}\bar{u}\bar{d})]$

Δq and the strangeness difference ΔS of the hadron set combinations. A nonzero v_1 difference at nonzero Δq was considered as the breaking of the coalescence sum rule and proposed to be a consequence of the electromagnetic fields [20,21], especially if the v_1 difference increases with Δq. The study also recognized the need for further investigation if a systematic dependence of the v_1 difference on ΔS is observed [20].

In this study, we examine in detail the v_1 difference of various combinations of these seven hadron species. Note that the v_1 throughout this study refers to the rapidity-odd directed flow, although v_1 contains both rapidity-odd and rapidity-even components where the rapidity-even directed flow originates from event-by-event fluctuations. In addition, since we only consider light quarks, which constituent masses are not too different, we neglect the effect of different quark masses on the coalescence sum rule [15] and thus start the analysis from Eq. (1). The paper is organized as follows. In Sec. 2, we derive the coalescence sum rule relationships between the v_1 difference of each hadron set and the quark v_1. In Sec. 3, we present two methods to extract the dependences of the v_1 difference on the electric charge difference Δq and the strangeness difference ΔS, and in Sec. 4 we demonstrate the extraction methods with the numerical v_1 results from a multi-phase transport (AMPT) model. Finally, we summarize in Sec. 5.

2. Coalescence sum rule relations for the v_1 difference of a hadron set

In this study, we only consider produced hadrons whose constituent quarks consist of $\bar{u}$, $\bar{d}$, s and $\bar{s}$ quarks. Table 1 lists several such hadron sets, where for each combination the left side and the right side have the same total number of $\bar{u}$ and $\bar{d}$ quarks and the same total number of s and $\bar{s}$ quarks (after including the weighting factors). For a given hadron set, let N_i^L and N_i^R be the total number of constituent quarks of flavor i in each hadron multiplied by the weighting factor of the hadron on the left side and right side, respectively. We then write

$$\Delta N_i \equiv N_i^L - N_i^R \tag{2}$$

as the difference of N_i between the two sides. Then each hadron set in Table 1 satisfies the following relations:

$$\Delta N_{\bar{u}} + \Delta N_{\bar{d}} = 0,\ \Delta N_s + \Delta N_{\bar{s}} = 0. \tag{3}$$

For example, set 5A has $N_{\bar{u}}^L = 2/3$, $N_{\bar{d}}^L = 1/3$, $N_s^L = N_{\bar{s}}^L = 1/2$, $N_{\bar{u}}^R = 1$, and $N_s^R = 1$. Similar to Eq. (2), we can define the differences of the total electric charge in $\bar{u}$ and $\bar{d}$ quarks (q_{ud}), the total strangeness S, and the total electric charge q, between the two sides as

$$\begin{aligned}
&\Delta q_{ud} \equiv q_{ud}^L - q_{ud}^R = \Delta N_{\bar{d}},\\
&\Delta S \equiv S^L - S^R = 2\Delta N_{\bar{s}},\\
&\Delta q \equiv q^L - q^R = \Delta q_{ud} + \frac{1}{3}\Delta S,
\end{aligned} \tag{4}$$

respectively. The values of Δq_{ud}, ΔS, and Δq for each hadron set are given in Table 1, where the left side and right side are shown with the constituent quark content and the weighting factor of each hadron. Because of Eq. (3), the mass difference (after including the weighting factors) between the two sides is small for most of these hadron sets. Note that sets 1, 2, and 3 each have identical constituent quark content on the left and right sides and thus satisfy $\Delta q_{ud} = \Delta q = \Delta S = 0$. On the other hand, sets 4, 5A and 5B each have a nonzero charge difference and/or a nonzero strangeness difference between the two sides. One can show that the conditions of Eq. (3) lead to the following general hadron set:

$$\begin{aligned}
&a_1 K^- + a_2\,\phi + a_3\,\bar{p} + a_4\,\bar{\Lambda} - (a_1 + 3a_3 + 2a_4)\,\bar{\Xi}^+\\
&+a_5\,\Omega^- + \left(\frac{a_1}{3} - \frac{2a_2}{3} + 2a_3 + a_4 - a_5\right)\bar{\Omega}^+ = 0,
\end{aligned} \tag{5}$$

where a_i are arbitrary constants; as a result, there are only five sets of independent hadron sets[1] [20]. Sets 1 to 4 and 5A in Table 1 give one example of the five independent sets; so do sets 1 to 4 and 5B. However, sets 1, 5A, and 5B are not independent of each other, since the v_1 difference between the two sides of set 5B can be written as that of set 5A plus that of set 1. With sets 1 to 4 and 5A (or 5B) in Table 1, one can construct all the hadron sets of earlier studies [20,21].

We now apply the coalescence sum rule in Eq. (1) to evaluate the difference between the v_1 from two sides of a given hadron set. Since we neglect the mass difference of $u/d/s$ constituent quarks, the quarks coalescing to form a hadron have the same $p_{\rm T}$. If we only consider quarks at a given $p_{\rm T}$, then they will form mesons at $p_{\rm T}^M = 2p_{\rm T}$ and (anti)baryons at $p_{\rm T}^B = 3p_{\rm T}$; this is why we have chosen the $p_{\rm T}$ range as $[0,2]$ GeV/c for mesons and $[0,3]$ GeV/c for (anti)baryons for the analysis of the model calculations in Sec. 4. The difference between the v_1 from two sides of a given hadron set is then given by

$$\Delta v_1 \equiv v_1^L - v_1^R = \sum_i \Delta N_i\, v_{1,\mathrm{i}}\,, \tag{6}$$

where $v_{1,\mathrm{i}}$ represents the v_1 of quark flavor i with $i \in \{\bar{u}, \bar{d}, s, \bar{s}\}$ and we have skipped the $p_{\rm T}$ argument in the $v_1(p_{\rm T})$ notations for brevity. Note that although the above relation is written for a given quark $p_{\rm T}$, it still applies when quarks are selected within a given $p_{\rm T}$ range, in which case $v_{1,\mathrm{i}}$ just represents the average v_1 of quark flavor i within that $p_{\rm T}$ range. With Eqs. (3)-(4), we further obtain

$$\begin{aligned}
&\Delta v_1 = (v_{1,\bar{d}} - v_{1,\bar{u}})\Delta q_{ud} + \left(\frac{v_{1,\bar{s}} - v_{1,s}}{2}\right)\Delta S\\
&= (v_{1,\bar{d}} - v_{1,\bar{u}})\Delta q + \left(\frac{v_{1,\bar{s}} - v_{1,s}}{2} - \frac{v_{1,\bar{d}} - v_{1,\bar{u}}}{3}\right)\Delta S.
\end{aligned} \tag{7}$$

[1] We realized that there are only five independent hadron sets under the constraint of Eq. (3) in August 2021.

v_1 observables such as those appearing in Eqs. (6)-(7) are functions of the hadron rapidity y. The rapidity-odd v_1 around mid-rapidity is often fit with a linear function in rapidity, with the only parameter being the slope (v_1'). If we assume that the rapidity of a hadron formed by quark coalescence is the same as that of the coalescing quarks (which have the same rapidity due to the comoving requirement), we can then take the derivative with respect to y and obtain

$$\Delta v_1' = \sum_i \Delta N_i \, v_{1,\mathrm{i}}' \, , \text{ with } v_1' \equiv \frac{dv_1}{dy}\Big|_{y=0}. \tag{8}$$

The above just relates the difference of the v_1 slope parameters from two sides of a hadron set to the quark v_1 slope parameters. We also have

$$\Delta v_1' = (v_{1,\bar{d}}' - v_{1,\bar{u}}')\Delta q_{ud} + \left(\frac{v_{1,\bar{s}}' - v_{1,s}'}{2}\right)\Delta S$$
$$= (v_{1,\bar{d}}' - v_{1,\bar{u}}')\Delta q + \left(\frac{v_{1,\bar{s}}' - v_{1,s}'}{2} - \frac{v_{1,\bar{d}}' - v_{1,\bar{u}}'}{3}\right)\Delta S. \tag{9}$$

Therefore, the difference of the v_1 slope parameters of a hadron set depends linearly on both Δq_{ud} and ΔS, where the corresponding coefficient is given by the difference of the quark-level v_1 slope parameters. It is also clear that the interpretation of the coefficients is simpler if we use Δq_{ud} instead of Δq for the electric charge difference. When one assumes that $\bar{u}$ and $\bar{d}$ quarks have the same v_1 slope and that s and $\bar{s}$ have the same v_1 slope [20], all the coefficients in Eq. (9) would be zero. However, $\Delta v_1' \neq 0$ in general according to the coalescence sum rule when Δq and/or ΔS is nonzero, which is the case for sets 4, 5A, and 5B in Table 1.

3. Extracting coefficients for the Δq and ΔS dependences

Since there are five independent sets, e.g., sets 1 to 4 and 5A, one will get five independent $\Delta v_1'$ data points from the experimental measurement (for a given event class of a given collision system). One can then extract the Δq and ΔS coefficients, which reflect the quark-level v_1 slope differences. One way to extract the coefficients is to simply fit the five data points; this is the 5-set method. Alternatively, since sets 1 to 3 all have $\Delta q_{ud} = \Delta q = \Delta S = 0$, we can combine these three data points into one and then fit three data points (the combined point plus sets 4 and 5); this is the 3-set method.

For certain collision systems, the coalescence sum rule may not be satisfied, e.g., if v_1 is not dominated by parton dynamics or the flows are affected by other effects such as the electromagnetic field. Since Eq. (9) based on the coalescence sum rule gives $\Delta v_1' = 0$ for $\Delta q_{ud} = \Delta S = 0$ (and for $\Delta q = \Delta S = 0$), we use the following modified equations to fit the 5-set or 3-set $\Delta v_1'$ values:

$$\Delta v_1' = c_0 + c_q \Delta q_{ud} + c_S \Delta S \tag{10}$$
$$= c_0^\star + c_q^\star \Delta q + c_S^\star \Delta S. \tag{11}$$

This way, a nonzero value of the new intercept term c_0 or $c_0^\star$ would mean the breaking of coalescence sum rule. According to Eq. (9), the coalescence sum rule predicts the following:

$$c_0 = c_0^\star = 0,$$
$$c_q = c_q^\star = v_{1,\bar{d}}' - v_{1,\bar{u}}' \, ,$$
$$c_S = \frac{v_{1,\bar{s}}' - v_{1,s}'}{2}, \quad c_S^\star = c_S - \frac{c_q}{3}. \tag{12}$$

In the 3-set method, we combine the three $\Delta v_1'$ points (from sets 1 to 3) into one point. Because these three data sets can have very different statistical errors (e_i) or hadron counts, we average the central values of the three $\Delta v_1'$ data points by using $1/e_i^2$ as the weight, and we calculate the statistical error of the combined data point as $1/\sqrt{1/e_1^2 + 1/e_2^2 + 1/e_3^2}$. Let us denote the combined data point as $\Delta v_{1,1-3}'$; we also denote the data point from sets 4 and 5 (5A or 5B) as $\Delta v_{1,4}'$ and $\Delta v_{1,5}'$, respectively. Eq. (10) then leads to $\Delta v_{1,1-3}' = c_0, \Delta v_{1,4}' = c_0 + c_S, \Delta v_{1,5}' = c_0 + c_q/3 + c_S$. Therefore, the coefficients in Eq. (10) for the 3-set method can be extracted as

$$c_0 = \Delta v_{1,1-3}' \, , \quad c_q = -3(\Delta v_{1,4}' - \Delta v_{1,5}'),$$
$$c_S = -\Delta v_{1,1-3}' + \Delta v_{1,4}'. \tag{13}$$

Similarly, the coefficients in Eq. (11) for the 3-set method can be extracted as

$$c_0^\star = \Delta v_{1,1-3}' \, , \quad c_q^\star = -3(\Delta v_{1,4}' - \Delta v_{1,5}'),$$
$$c_S^\star = -\Delta v_{1,1-3}' + 2\Delta v_{1,4}' - \Delta v_{1,5}'. \tag{14}$$

4. Tests with a transport model

We now use the AMPT model [22] as an example to demonstrate the v_1 analysis and extraction of the Δq and ΔS coefficients. We use the default version of the AMPT model to simulate mid-central (10-50%) Au + Au collisions at $\sqrt{s_{\mathrm{NN}}} = 7.7$, 14.5, 27, 54.4, and 200 GeV. Note that the string melting version of the AMPT model with a dominant parton phase does not provide a good description of direct flow observables [19], although it well describes higher-order flows such as elliptic flow and triangular flow. This failure of the string melting AMPT model is related to its neglection of the nonzero longitudinal width of the incoming nuclei at low to moderate collision energies, which affects the tilt of the created dense matter and consequently the direct flow pattern [23]. The focus of this study is the analytical dependences of the v_1 difference on Δq and ΔS as well as the proper way to extract the corresponding coefficients. Therefore, we choose to use the default AMPT model to demonstrate the extraction methods while leaving quantitative investigation of the coefficients with the string melting AMPT model to future studies. In the following, the event centrality is determined from the multiplicity of charged hadrons within the pseudorapidity range $|\eta| < 1/2$. For simplicity, we calculate v_1 with respect to the reaction plane angle (Ψ_{RP}) as $v_1 = \langle \cos(\phi - \Psi_{RP}) \rangle$, where ϕ is the azimuthal angle of a hadron's momentum [24,25].

As an example, Fig. 1 shows the rapidity dependence of v_1 for hadron set 2, where $v_1^L = v_1[\bar{\Lambda}]$ and $v_1^R = v_1[\bar{p}]/2 + v_1[\bar{\Xi}^+]/2$. We then fit their difference Δv_1 (circles) within $|y| < 1.5$ at each energy with a rapidity-odd linear function of y to obtain the slope difference $\Delta v_1'$. Note that for hadron set 2 with $\Delta q_{ud} = \Delta q = \Delta S = 0$, we expect $\Delta v_1' = 0$ from Eq. (9). However, this is not the case for the default-AMPT model results at low energies in Fig. 1.

Fig. 2 shows the slope difference $\Delta v_1'$ of each set at the five energies as functions of (a) ΔS, (b) Δq_{ud}, and (c) Δq. Since $\Delta v_1'$ depends linearly on both Δq and ΔS, one cannot determine the coefficient c_q (or c_S) by simply performing a one-dimensional linear fit of the Δq plot such as Fig. 2(c) (or the ΔS plot such as Fig. 2(a)) [21]. Note that a one-dimensional linear fit as a function of Δq performed at the same ΔS value [20] would be better. Here, we propose to extract the c_q and c_S coefficients by describing the $\Delta v_1'$ data with a two-dimensional plane (over the Δq-ΔS space). We can use the 5-set method by fitting five independent data points with the relation of Eq. (10). As a demonstration, Fig. 3(a) shows the fitting of five data points (from sets 1 to 4 and set 5A) from the AMPT model at $\sqrt{s_{\mathrm{NN}}} = 14.5$ GeV with the 5-set method. Alternatively, we can use the 3-set method, where we fit the combined data point for sets 1 to 3 and the data points from set 4 and set 5A (or 5B). This is demonstrated in Fig. 3(b), where the data point at $\Delta q_{ud} = \Delta S = 0$ represents the average of the three corresponding data points shown in Fig. 3(a) (from the three hadrons sets with identical constituent quark content on the two sides). The resultant coefficients obtained from the 5-set method and the 3-set method are practically the same, as we can see from the almost identical planes in Fig. 3(a)

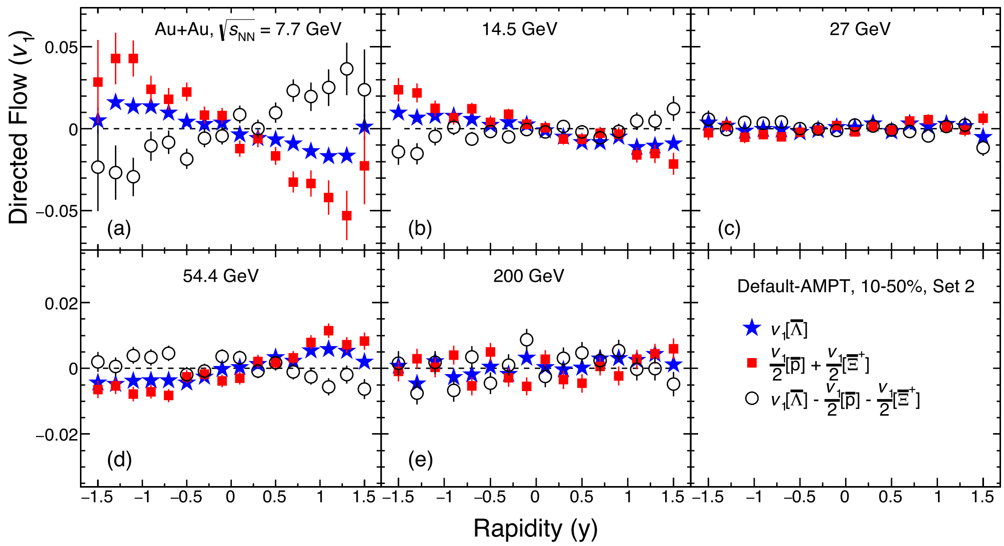

Fig. 1. Directed flows of hadron set 2 in Table 1: $v_1[\bar{\Lambda}]$, $v_1[\bar{p}]/2 + v_1[\bar{\Xi}^+]/2$, and their difference as functions of rapidity from the default AMPT model for 10-50% central Au + Au collisions at $\sqrt{s_{NN}}$ = 7.7, 14.5, 27, 54.4 and 200 GeV.

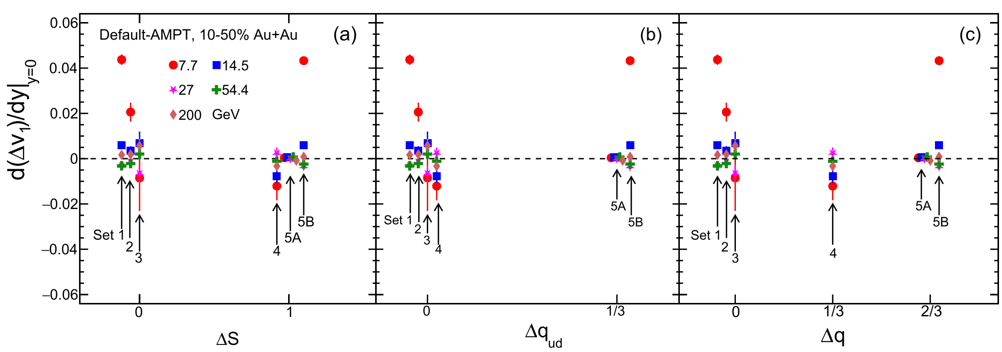

Fig. 2. The difference of the v_1 slopes at mid-rapidity for each hadron set in Table 1 versus (a) ΔS, (b) Δq_{ud}, and (c) Δq from the default AMPT model for 10-50% central Au + Au collisions at several energies. Data points at the same horizontal value are often slightly shifted horizontally for better visibility.

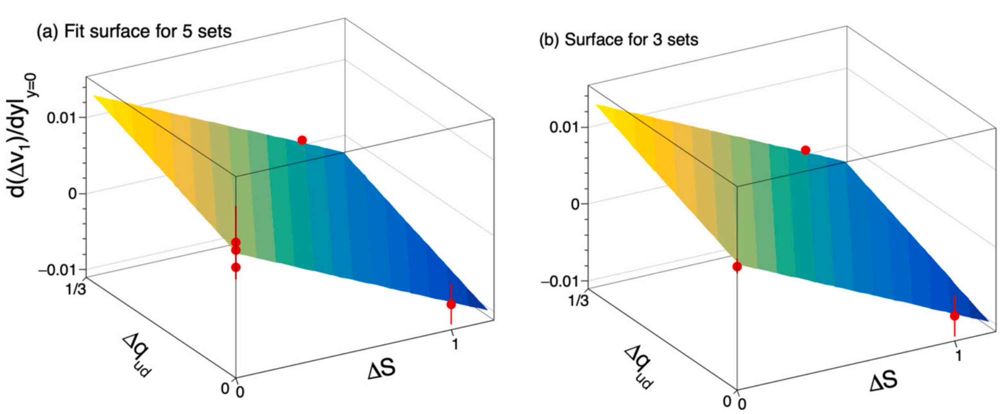

Fig. 3. The fit plane to extract the $\Delta v_1'$ dependences on Δq_{ud} and ΔS as shown in Eq. (10) using (a) the 5-set method and (b) the 3-set method. The data points, corresponding to sets 1 to 4 and set 5A in Table 1, come from the default AMPT model for mid-central (10-50%) Au + Au collisions at $\sqrt{s_{NN}}$ = 14.5 GeV.

and (b). On the other hand, the 3-set method has an advantage in that the coefficients can be determined by Eq. (13) or Eq. (14) without the need to perform a fit.

In Fig. 4, we compare the coefficients extracted from the AMPT model results for semi-central Au + Au collisions versus the colliding energy. Fig. 4(a) compares c_0, c_q, c_S in Eq. (10) (filled symbols) with $c_0^\star, c_q^\star, c_S^\star$ in Eq. (11) (open symbols) extracted with the 5-set method using sets 1 to 4 and 5A. We confirm the relations of Eqs. (13)-(14) in that fitting the data versus Δq_{ud} or Δq does not affect the c_0 and c_q coefficients but gives different c_S values. We also see that the coefficients here exhibit a clear energy dependence, especially at low energies. In particular, at 7.7 GeV the nonzero intercept c_0 indicates the breaking of the coalescence sum rule; as a result, one cannot trust Eq. (12) and interpret the c_q and c_S coefficients as quark-level v_1' differences there.

Although there are only five independent hadron sets for this study, they can be written in different combinations [20,21]. For example, one

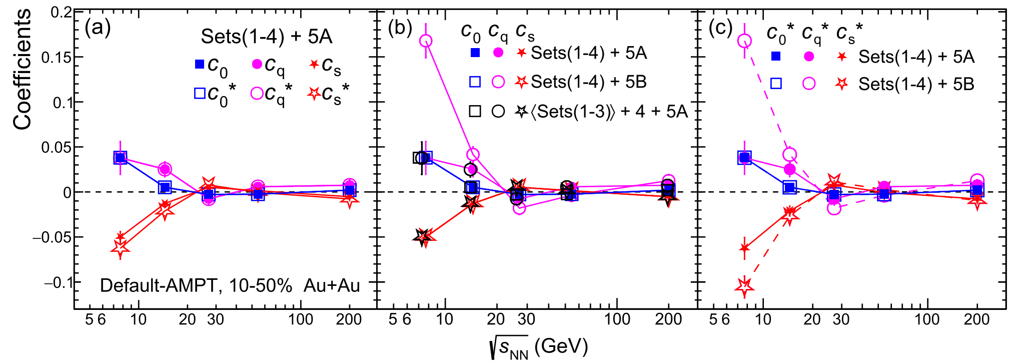

Fig. 4. Comparisons of the coefficients extracted from AMPT results with the 5-set method as functions of colliding energy: (a) c_0, c_q, c_S compared with $c_0^\star, c_q^\star, c_S^\star$ from sets 1 to 4 and 5A, (b) c_0, c_q, c_S values, and (c) $c_0^\star, c_q^\star, c_S^\star$ values from sets 1 to 4 and 5A compared with those from sets 1 to 4 and 5B. Panel (b) also shows the coefficients from the 3-set method (black symbols).

can choose them as sets 1 to 4 and set 5B (instead of 5A). The corresponding coefficients are shown in Fig. 4(b) for c_0, c_q, c_S and in Fig. 4(c) for $c_0^\star, c_q^\star, c_S^\star$, in comparison with those extracted from sets 1 to 4 and set 5A. We see in Fig. 4(b) that the c_q value depends on the choice of the five sets, while c_0 and c_S values do not. This is expected from Eq. (13), which shows that set 5 only affects the c_q value. We also show in Fig. 4(b) the coefficients extracted with the 3-set method of Eq. (13) for hadron sets 1 to 4 and set 5A; they are essentially the same as those extracted with the 5-set method. Note that, since hadron set 5B in Table 1 is a combination of set 1 and set 5A, the difference in the c_q value from using set 5B and that from using set 5A is given by (three times) the $\Delta v_1'$ value of set 1, which is shown in Fig. 2 to be nonzero at low energies. In Fig. 4(c), we see that both the $c_q^\star$ and $c_S^\star$ values depend on the choice of using set 5A or 5B. This is consistent with the expectations of Eq. (14), and the nonzero differences are again due to the nonzero $\Delta v_1'$ of set 1 (which would be zero if the coalescence sum rule were exact). Therefore, getting different coefficient values from different choices of five independent hadron sets, like a nonzero c_0 value, indicates the breaking of the coalescence sum rule.

5. Conclusions

In this study, we start from the coalescence sum rule and derive the relations between the rapidity-odd directed flows (v_1) of different hadron sets. Following earlier studies, we consider seven species of produced hadrons (those without u or d constituent quarks): $K^-, \phi, \bar{p}, \bar{\Lambda}, \bar{\Xi}^+, \Omega^-$, and $\bar{\Omega}^+$, where the two sides of each hadron set have the same total number of $\bar{u}$ and $\bar{d}$ quarks and the same total number of s and $\bar{s}$ quarks after including the weighting factors. Earlier studies have proposed that a nonzero directed flow difference (Δv_1) between the two sides of the hadron sets, especially a dependence on the electric charge difference Δq, means the breaking of the coalescence sum rule and would indicate the effect of the electromagnetic fields. Here we show that the coalescence sum rule only leads to zero Δv_1 for a hadron set if its two sides have identical constituent quark content (or equivalently if $\Delta q = \Delta S = 0$). In general, Δv_1 depends linearly on both ΔS and Δq, or on both ΔS and Δq_{ud} (the electric charge difference in $\bar{u}$ and $\bar{d}$ constituent quarks). The same is true for $\Delta v_1'$, the difference of the v_1 slopes around mid-rapidity (v_1'). For $\Delta v_1'$, the coefficient c_q for its Δq_{ud} dependence reflects the $\bar{d}$ and $\bar{u}$ quark v_1' difference, while the coefficient c_S for its ΔS dependence reflects half the $\bar{s}$ and s quark v_1' difference.

Since there are only five independent such hadron sets, there will be five independent $\Delta v_1'$ data points from the measurement of a given collision system. We propose to fit the data points with a two-dimensional plane in the functional form of $c_0 + c_q \Delta q_{ud} + c_S \Delta S$ to extract the three coefficients, where a nonzero intercept c_0 indicates the breaking of the coalescence sum rule. In the 5-set method, one simply fits the five data points with this function. In the more elegant 3-set method, we combine the data points from the three sets at $\Delta q_{ud} = \Delta S = 0$ into one and then obtain the coefficients analytically. We have also used results from the default version of the AMPT model for mid-central Au + Au collisions at various energies to demonstrate the extraction methods. The 5-set method and the 3-set method are shown to extract essentially the same coefficients. In addition, we show that the extracted coefficients may depend on the choice of the five independent hadron sets, and getting different coefficients from different choices indicates the breaking of the coalescence sum rule. This work provides the baseline relations for the v_1 difference of various hadron sets from the coalescence sum rule. Further work is needed to consider the possible effect of the electromagnetic fields.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgements

S.S. is supported in part by the National Key Research and Development Program of China under Grant No. 2022YFA1604900, 2020YFE0202002, and the National Natural Science Foundation of China under Grant No. 12175084, 11890710 (11890711). K.N. thanks Prof. Bedangadas Mohanty for providing computational facilities and hospitality to stay in NISER during the working of this paper. K.N. is supported by OSHEC, Department of Higher Education, Government of Odisha, Index No. 23/EM/PH/124 under MRIP 2023. Z.-W.L. is supported by the National Science Foundation under Grant No. 2012947 and 2310021.

References

[1] B.-A. Li, C.M. Ko, Phys. Rev. C 58 (1998) R1382, https://link.aps.org/doi/10.1103/PhysRevC.58.R1382.
[2] H. Stöcker, Nucl. Phys. A 750 (2005) 121, https://doi.org/10.1016/j.nuclphysa.2004.12.074.
[3] Y. Nara, H. Niemi, A. Ohnishi, H. Stöcker, Phys. Rev. C 94 (2016) 034906, https://link.aps.org/doi/10.1103/PhysRevC.94.034906.
[4] X. Luo, S. Shi, N. Xu, Y. Zhang, Particles 3 (2020) 278.
[5] H. Sorge, Phys. Rev. Lett. 78 (1997) 2309, https://link.aps.org/doi/10.1103/PhysRevLett.78.2309.
[6] N. Herrmann, J.P. Wessels, T. Wienold, Annu. Rev. Nucl. Part. Sci. 49 (1999) 581, https://doi.org/10.1146/annurev.nucl.49.1.581.

[7] S.K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina, V. Greco, Phys. Lett. B 768 (2017) 260.
[8] D. Molnár, S.A. Voloshin, Phys. Rev. Lett. 91 (2003) 092301, https://link.aps.org/doi/10.1103/PhysRevLett.91.092301.
[9] J. Adams, et al., STAR Collaboration, Phys. Rev. Lett. 92 (2004) 052302, https://link.aps.org/doi/10.1103/PhysRevLett.92.052302.
[10] B.I. Abelev, et al., STAR Collaboration, Phys. Rev. C 75 (2007) 054906, https://link.aps.org/doi/10.1103/PhysRevC.75.054906.
[11] S.S. Adler, et al., PHENIX Collaboration, Phys. Rev. Lett. 91 (2003) 182301, https://link.aps.org/doi/10.1103/PhysRevLett.91.182301.
[12] A. Adare, et al., PHENIX Collaboration, Phys. Rev. Lett. 98 (2007) 162301, https://link.aps.org/doi/10.1103/PhysRevLett.98.162301.
[13] B. Abelev, et al., J. High Energy Phys. 06 (2015) 190, https://doi.org/10.1007/JHEP06(2015)190.
[14] Z.-W. Lin, C.M. Ko, Phys. Rev. Lett. 89 (2002) 202302.
[15] Z.-W. Lin, D. Molnár, Phys. Rev. C 68 (2003) 044901, https://link.aps.org/doi/10.1103/PhysRevC.68.044901.
[16] Z.-W. Lin, J. Phys. G 38 (2011) 075002.
[17] L. Adamczyk, et al., STAR Collaboration, Phys. Rev. Lett. 120 (2018) 062301, https://link.aps.org/doi/10.1103/PhysRevLett.120.062301.
[18] J.C. Dunlop, M.A. Lisa, P. Sorensen, Phys. Rev. C 84 (2011) 044914, https://link.aps.org/doi/10.1103/PhysRevC.84.044914.
[19] K. Nayak, S. Shi, N. Xu, Z.-W. Lin, Phys. Rev. C 100 (2019) 054903, https://link.aps.org/doi/10.1103/PhysRevC.100.054903.
[20] A.I. Sheikh, D. Keane, P. Tribedy, Phys. Rev. C 105 (2022) 014912, https://link.aps.org/doi/10.1103/PhysRevC.105.014912.
[21] STAR, arXiv:2304.02831, 2023.
[22] Z.-W. Lin, C.M. Ko, B.-A. Li, B. Zhang, S. Pal, Phys. Rev. C 72 (2005) 064901, https://link.aps.org/doi/10.1103/PhysRevC.72.064901.
[23] C. Zhang, Z.-W. Lin, Phys. Rev. C 106 (2022) 054910.
[24] S. Voloshin, Y. Zhang, Z. Phys. C, Part. Fields 70 (1996) 665, https://doi.org/10.1007/s002880050141.
[25] A.M. Poskanzer, S.A. Voloshin, Phys. Rev. C 58 (1998) 1671, https://link.aps.org/doi/10.1103/PhysRevC.58.1671.