BAMS Essay

Can We Do Better at Teaching Mathematics to Undergraduate Atmospheric Science Students?

Elizabeth M. Page, Samuel S. P. Shen, and Richard C. J. Somerville

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At the 2019 annual Members Meeting of UCAR, an informal survey was made during a well-attended breakout session organized by the UCAR COMET program. The following polling question was posed to an audience of department chairs and educational leaders of atmospheric and oceanic science programs at universities in the United States and Canada, "What is the greatest challenge students have when entering an atmospheric science program?" The majority of participants in the breakout session answered the question, and the dominant responses of these leading atmospheric science educators can be summarized with a single short word: "math." These responses included the topics of relating math and physics as well as computer programming and coding skills. The subsequent discussion explored the participants' experiences in greater detail and the nuances of the obstacle that math presents for many students entering atmospheric science programs.

The conclusion that can be drawn from this one poll of atmospheric science educators is unequivocal. Mathematics, according to this poll, is by far the greatest challenge faced by undergraduate university students when they enter an atmospheric science program.

Mathematics is often taught by mathematics faculty, not by atmospheric scientists

Why did mathematics stand out? To answer this question, we rely primarily on our own familiarity with university-level education in the United States in atmospheric science and closely related fields. A typical atmospheric science major will devote considerable time and effort to taking required mathematics courses. This student would typically have to take about 6 mathematics courses, with the following list being representative: Calculus I (4–5 units), Calculus II (4–5 units), Calculus III (3–4 units), Differential Equations (3 units), Introduction to Statistics (3 units), and Computer Programming (3 units). These courses represent a substantial number of credit hours: 20–23 semester units, equivalent to almost one-fifth of a typical required total of 120 credit units for graduation. No other single subject area outside the major has as many required courses as mathematics.

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Corresponding author: Richard C. J. Somerville, rsomerville@ucsd.edu
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AFFILIATIONS: Page—University Corporation for Atmospheric Research, Boulder, Colorado; **Shen**—San Diego State University, San Diego, California; **Somerville**—Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California

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In a typical U.S. university, these math courses will not be taught by an instructor who belongs to the department in which the student is majoring, such as an atmospheric science department. Instead, that department will often outsource the math teaching to another department. The instructors will typically be members of a department such as mathematics, statistics, or computer science. An advantage of this policy is that the instructors will almost surely be knowledgeable about the mathematical topics they teach. However, there are also disadvantages to outsourcing the teaching in this way. A major disadvantage is that the instructor is likely to lack a detailed knowledge of the field in which the students are majoring and will not relate how the mathematics they are being taught will be applied in their major courses and in their careers. In fact, the students may ultimately find, to their dismay, that much of the mathematics they are being taught is indeed not useful to them in their major courses. At the same time, some relevant, useful, and modern (*RUM*) mathematics they will need in their coursework and their careers has never been taught.

Thus, when students begin the usual required first-year physics class, they are likely to find important gaps between the math that they have learned, and the math actually used in physics. For example, they may be unfamiliar with the divergence theorem used in connecting the differential and integral forms of the law relating the flux of an electric field out of an arbitrary closed surface to the electric charge enclosed by the surface. This relationship is known as Gauss's law of electricity, also called Gauss's flux theorem. It is part of Maxwell's equations of electromagnetism. They may not know the line integral method used to describe Faraday's law of induction, another fundamental principle of electromagnetism, which describes how electric current along the closed path of the line integral is generated by a varying magnetic field inside the path. Their fluid dynamics instructor may well find it necessary to teach or reteach the divergence theorem or Stokes's theorem, because the students do not recognize it, or have not yet learned the theorem typically included in Calculus III. When the same students take atmospheric science courses, they are likely to discover other shortcomings in their mathematical education. For example, as they begin to analyze typical atmospheric science datasets, they will find that they need to use a variety of methods they have not learned in their math courses, such as the analysis for a large rectangular data matrix, and computer visualization of a variety of data as line graphs or maps. Consequently, the large number of credit units of mathematics courses the students took and passed did not properly equip them for their major courses, research, and career.

Another scenario is that after completing an undergraduate degree, students attending graduate school are likely to discover other deficiencies in their mathematics education. For example, the students may have never been exposed to the topics of empirical orthogonal functions and singular-value decomposition. Yet these techniques and tools are routinely used by the more advanced graduate students in atmospheric science and oceanography, and they are often important in the research presentations that students will attend. In graduate school, students thus are confronted with the remedial need to learn the missing mathematics, either from senior colleagues or by self-study or by taking additional math courses. We emphasize that some advanced math topics indeed can and should be learned in graduate school. However, we ask, should some of these topics have been covered in their undergraduate math courses?

Biology and other scientific fields often face the same issues

Atmospheric science is not unique in this respect. Many science departments often outsource the undergraduate mathematics education of their students to other university departments. The courses are often taught in a manner disconnected from the science in which the students are majoring. The instructors are typically faculty who themselves come from varying backgrounds in mathematics or statistics or computer science. Thus, it should not be surprising

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if some science students in varying fields are puzzled by not having been shown examples of how the math they are taught will be used in their major courses.

For example, the abstract of O'Leary et al. (2021) begins with the following: "Calculus is typically one of the first college courses encountered by science, technology, engineering, and mathematics (STEM) majors. Calculus often presents major challenges affecting STEM student persistence, particularly for students from groups historically underrepresented in STEM. For life sciences majors, calculus courses may not offer content that is relevant to biological systems or connect with students' interests in biology."

These words are certainly relevant to our topic. However, they are from a paper (O'Leary et al. 2021) in a journal devoted to life sciences education. The title of the paper is, "Reimagining the introductory math curriculum for life sciences students." Few scientists or educators in atmospheric sciences are likely to know about this life sciences journal, and even fewer are likely to have seen this paper in it. Yet the problems addressed by O'Leary et al. (2021) are almost identical to problems we face in atmospheric sciences and closely related fields, problems that we discuss at length in this essay. There are similar examples from other STEM fields, all devoted to similar problems, but they are scattered about in journals devoted to those fields and are not likely to be seen by readers outside those disciplinary silos.

Furthermore, we should not overlook the fact that for many university students, mathematics has a reputation of being difficult. Those of us in academia and in other branches of the atmospheric sciences community are likely to have met some prospective atmospheric science students who decided not to pursue the field, simply because the required math courses appeared to be both difficult and of limited relevance to their major field. It also seems likely that other STEM fields have lost prospective students because of the mathematics courses that their students are required to take (Hatfield et al. 2022).

We recognize that the issues expressed here might best be regarded in the context of specific course prerequisites, or with teaching approaches in a different department (one that the major department does not control), or with students simply not being confident in their understanding. Additionally, math anxiety is a very real phenomenon and is likely to be a contributing factor to student performance (e.g., Ashcraft 2002; Maloney et al. 2013; Dowker et al. 2016). In brief, there are many possible contributing factors that could affect why mathematics is such a dominant obstacle for many students.

RUM is a useful paradigm

We must therefore ask, What are the shortcomings of the mathematics courses included in the atmospheric science curriculum supporting the major coursework? A student who has already spent a great deal of time learning math may well discover a need to learn additional math. Much of the math recalled from math courses may sometimes turn out not to be the math needed. Of course, many career paths for atmospheric scientists do not involve solving and manipulating partial differential equations. However, familiarity with mathematical methods and techniques is needed to understand many important theoretical concepts.

The importance of mathematics for students in atmospheric science and closely related fields is also clear in the requirements for atmospheric scientists employed by U.S. government agencies such as NOAA and in this AMS information statement: *Bachelor's Degree in Atmospheric Science* (American Meteorological Society 2023). The federal government GS-1340 requirements (U.S. Office of Personnel Management 2023) state that 3 semester hours of ordinary differential equations are required. They importantly include the language that "there is a prerequisite or corequisite of calculus for course work in atmospheric dynamics and thermodynamics, physics, and differential equations. Calculus courses must be appropriate for a physical science major." Our main point is worth emphasizing: *all* the math courses

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that atmospheric science students take "must be appropriate for a physical science major." Similarly, the AMS statement cited above (American Meteorological Society 2023) lists these areas of competency in mathematics, scientific computing, and data analytics: differential and integral calculus, vector and multivariable calculus, an introductory programming and coding course, application of numerical and statistical methods to atmospheric science problems, principles of measurement and uncertainty, statistical analysis of observations, familiarity with emerging technologies for data acquisition, and scientific computing and data analytics, modeling, and visualization to make inferences about and analyze the atmosphere. We agree completely, and in this essay, we offer concrete suggestions as to how instruction in these areas can be improved. We also note and agree with these additional items from the AMS statement: scientific software development in a suitable computing environment; exposure to commonly used programming tools within the atmospheric sciences (e.g., Python, MATLAB, R, NCL, Fortran, and IDL), and practices of open-source software development and FAIR data principles.

We are convinced that efficient mathematics education for science students is characterized by the acronym RUM. Here "R" is for "relevant" to both courses and research in the science; "U" is for "useful" to not only the courses and research, but also to future jobs; and "M" is for "modern," incorporating recent progress and tools, especially progress in computing and data science. Improving this situation will not happen overnight. Meeting these RUM criteria will require time and commitment to changing the curriculum to better support learning. Progress will necessarily be incremental. Mathematics courses and instructors can both evolve to help meet the objective of well-educated students prepared for successful careers.

RUM-based practice, ideas, and suggestions are valuable

We followed a convoluted path in deciding to emphasize "relevant" and "useful" and "modern" as criteria to identify and characterize the portion of mathematics that is most valuable for undergraduate students majoring in atmospheric science and closely related fields. In part, we were inspired by the discussion that took place at the polling experiment done at the 2019 UCAR Members Meeting, which is described above. In part, two of us recently spent three years writing an undergraduate mathematics textbook for students in fields such as atmospheric science (Shen and Somerville 2019, 2020). While writing the book, we extensively discussed, both among ourselves and with colleagues, the crucial decisions of which topics to include and which to leave out. During this process, we gradually came to realize that the RUM criteria were an accurate summary of how we based these decisions. The RUM criteria were applied in the recent book by Shen and North (2023) on climate statistics and data visualization.

In addition, our discussions with many atmospheric and climate students and educators in seminars and other forums have strengthened our conviction that the RUM criteria are a concise description of the mathematics that best meets the needs of these students. In brief, we devised the RUM criteria ourselves, and we are not aware of any other source of the RUM acronym. The view that these criteria are appropriate in this context is simply our opinion.

Are atmospheric science faculty members familiar with these issues? Yes! Experienced faculty are likely to have known about them for many years and may have encountered them as students themselves. Of course, a sound atmospheric science curriculum should be designed to meet the needs of students. Optimizing the mathematical portion of such a curriculum requires effective codesign among several groups of people: students, atmospheric science faculty, mathematics faculty, computer science faculty, and some typical possible future employers of the atmospheric science students. However, the communication channels connecting these groups are often inadequate, and this lack of consistent and continuing communication of needs and potential solutions may be attributed at least partly to the

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problems we have identified. We think that encouraging and facilitating this interdisciplinary communication deserves greater attention in the future.

Why has math education in atmospheric science become what it is? We attribute its shortcomings to at least one conspicuous defect: In the traditionally outsourced mathematics courses, the examples given in mathematics textbooks and the questions posed in their problem sets are, unsurprisingly, not related to atmospheric science or oceanography or climate science. Thus, from a purely pedagogical point of view, these textbooks miss a promising opportunity to show the student how mathematics is applied in the major field. Every important equation can correspond to an atmospheric science story, and every example can lead to a relevant insight. Thus, when mathematically educating a student majoring in atmospheric science, it is well worthwhile to seek connections between mathematical topics and atmospheric science applications. After all, there are many good applications of mathematical principles in atmospheric science. Pedagogically, this methodology is consistent with how novices frequently learn material (e.g., Anderson et al. 1997; Atkinson et al. 2000; Renkl 2014; Roebber 2005). We are encouraged by recent developments that are already being incorporated into some atmospheric science courses. Clear discussions and excellent worked examples are available (e.g., Davenport 2019; Petters 2021). Also, effective interventions are being developed to reduce math anxiety (e.g., Headley 2022).

These resources provide interesting and instructive examples of teaching mathematics based on the RUM principle for our students. The identified issues, the UCAR survey results, and the examples we have cited in the above references all point to a direction of change. If this change is taken to heart and successfully implemented following the RUM principle, then our students should have more successful undergraduate experiences and should also be better prepared for both graduate school and the job market. The change should not be designed to "dumb-down" the math portion of the curriculum to make it easy, but rather to "smarten-up" the mathematics courses and adapt them to the needs of the modern atmospheric science courses that are key components of the major. Whether intentionally or not, it will be unfortunate if some qualified students continue to be "weeded out" or "filtered out" by disliking or by performing poorly in the traditional required mathematics courses that provide little support for their major courses and careers.

Future activities include holding workshops and developing discussion questions

This article was stimulated by a poll taken at a UCAR Members Meeting during a breakout session originating in the COMET program of UCAR. The UCAR Members Meetings offer opportunities for UCAR staff to engage with faculty from a wide variety of universities. The COMET program, one of the UCAR community programs, engages in providing professional development opportunities for weather forecasters, hydrologists and other professionals in environmental forecasting. As part of activities to support weather forecasters in developing nations and small island states, COMET has identified math skills as an area that should be improved to better prepare students entering programs in atmospheric science. COMET plans to convene workshops to extend the discussion related to the topics in this essay.

We endorse the concept of a broad range of efforts to identify, study, and resolve the problems in mathematics education that have been highlighted by representatives of the UCAR members, as shown by the poll at a UCAR Members Meeting discussed in this essay. There are many potential discussion questions. We provide a few here that may help audiences start the discussion. Can the department of mathematics or computer science have a designated faculty member who has atmospheric science research experience or knowledge to teach the math courses for atmospheric science majors? Can the math courses be cotaught by the faculty members from both atmospheric science and mathematics? Would some atmospheric science departments wish to stop outsourcing the courses of

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mathematics, statistics, and scientific computing/computer programming? What are the sufficient indices to reflect "modern" in the RUM principle? These may include the following: modern interactive textbooks using recent datasets and computer codes; modern examples involving the math used in remote sensing, new instruments, and artificial intelligence; and modern pedagogy of progressive education in the digital era, such as (i) collaborative learning through group projects, social media platforms like Discord and WhatsApp, and code development platforms like GitHub and Colab; (ii) individualized learning through cultivating each student's interest and strength through research; and (iii) learning-by-doing through experimenting with mathematical theory, computer codes, real data, visualizations, and storytelling.

We strongly advocate exploring and testing a variety of actions and approaches, rather than a one-size-fits-all route to improving mathematics education for students majoring in atmospheric science and closely related fields. We hope this essay will help trigger a wider-ranging discussion about undergraduate atmospheric science education, including the evolving roles of mathematics, big data, and computer programming.

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