

Effect of Antenna Pattern on Time-Domain Canceling of Interference from Satellites

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Abstract—Signals from satellites are a source of interference to radio telescopes. One possible scheme for mitigation of this interference is coherent time-domain canceling. Using a simple but broadly-applicable model for the antenna pattern, we show how the antenna pattern combined with the motion of the satellite limits the time available to compute an accurate estimate of the interference waveform, which subsequently limits the extent to which interference can be canceled in the output. We suggest a simple remedy to the problem.

I. INTRODUCTION

Interference to radio telescopes from satellites is a long-standing problem with limited options for unilateral mitigation [1]. The dramatic increase in the number of satellites posing a risk to radio astronomy motivates the search for effective means by which observations can continue even when satellites are present in the main lobe.

Coherent time-domain canceling (CTC) in particular is a potential solution [2]. Most CTC systems can be described as shown in Figure 1. Here, the interference signal $z(t)$ is estimated using a filter whose primary input is the telescope output $x(t)$, and whose output is $\hat{z}(t)$, an estimate of $z(t)$. The filter response is determined adaptively by comparing the input $x(t)$ to a reference signal $d(t)$ (typically a signal model or the signal from a low-gain “auxiliary” antenna) over an interval τ . In principle, performance improves with increasing τ . However, this is not necessarily true since the $z(t)$ includes a time-varying multiplicative factor associated with the motion of the satellite through the antenna pattern, which will be missing or absent in $d(t)$.

In this paper we derive a limit on interference canceling performance due to this effect (Equation 7), and, in Section IV, suggest a remedy.

II. THEORY

For the purposes of this study, it is reasonable to model $z(t)$ as a sinusoid having magnitude A which is approximately

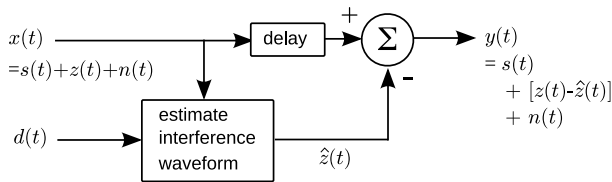


Fig. 1. Coherent time-domain canceling. $s(t)$ is the desired astrophysical signal and $n(t)$ is noise. The “estimate interference waveform” block is a filter whose response is computed by comparing $x(t)$ to $d(t)$ over a time interval τ , and whose response is held constant over the next interval of length τ .

constant over any interval τ that we care to consider. The waveform estimation filter develops an estimate which is also a sinusoid but with a biased magnitude $\hat{A}$ because the satellite is moving through the antenna pattern. (The phase of the waveform estimate is also potentially biased; however, the impact of this effect is negligible in comparison [3].) The ratio of interference power in the output to interference power in the input will therefore be

$$\frac{(A - \hat{A})^2/2}{A^2/2} = \left[1 - \frac{\hat{A}}{A}\right]^2 \quad (1)$$

An upper bound I_{UB} on this quantity imposed by the unmodulated antenna pattern variation is simply

$$I_{UB} = \left[1 - \frac{f(t+\tau)}{f(t)}\right]^2 \quad (2)$$

where $f(t)$ is the pattern value in the direction of the satellite at time t . For τ sufficiently short, the numerator of the second term can be approximated as $f(t) + f'(t)\tau$ where $f'(t)$ is the time derivative of $f(t)$; thus

$$I_{UB} \approx \left[\frac{f'(t)\tau}{f(t)}\right]^2 \quad (3)$$

The antenna pattern is modeled as $f(\theta) = 2J_1(\xi)/\xi$ where $\xi = (\pi D/\lambda) \sin \theta$, D is diameter, λ is wavelength, and θ is the time-varying angle measured from the center of the main lobe. This is exact for a circular uniformly-illuminated aperture, but is a reasonable approximation for other large antennas, including reflector antennas with complex optics and phased arrays. It is also worst case in the sense that any other antenna which can be described as an aperture with maximum dimension D will have a wider main lobe, and will therefore exhibit smaller $f'(t)$. We ignore the orthogonal coordinate ϕ since the variation with θ is typically much greater, and we assume the worst case polarization alignment. In the main lobe, $f(\theta)$ can accurately be approximated as follows:

$$f(\theta) \approx 1 - 0.112 \left(\frac{\pi D}{\lambda}\right)^2 \sin^2 \theta \quad (4)$$

This is accurate for $\theta \leq 1.15\theta_{HP}$, where θ_{HP} is half-power angle. Substitution into Equation 3 yields:

$$I_{UB} \approx \left[\frac{2.211(D/\lambda)^2(\sin \theta \cos \theta)\omega_a\tau}{1 - 1.105(D/\lambda)^2 \sin^2 \theta}\right]^2 \quad (5)$$

where $\omega_a = d\theta/dt$, which is addressed in Section III. Solving for τ and invoking the small angle approximations $\sin \theta \approx$

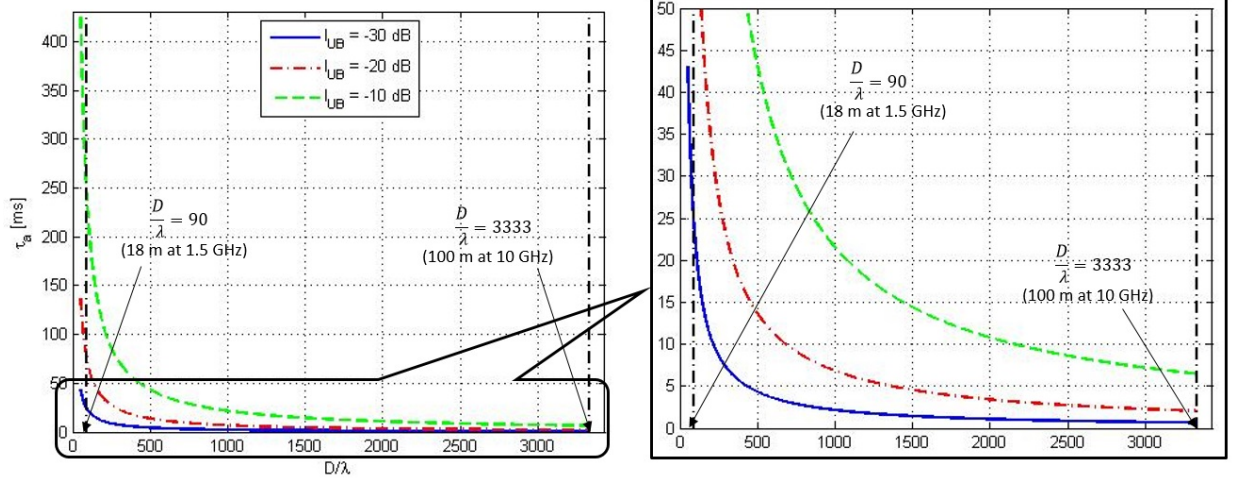


Fig. 2. Maximum estimation interval τ_a (Equation 7) as a function of antenna size and I_{UB} , assuming a typical LEO satellite ($\omega_a = 0.55^\circ/\text{s}$).

$\tan \theta \approx \theta$, we find:

$$\tau \approx I_{UB}^{1/2} \left[\frac{0.904}{(D/\lambda)^2 2\theta} - \frac{\theta}{2} \right] \frac{1}{\omega_a} \quad (6)$$

Note that this expression is not valid at $\theta = 0$; this is because $f'(t)$ changes sign at $\theta = 0$, and so the linearization of $f(t)$ is not appropriate if $\theta = 0$ at any time within the estimation interval. However, for sufficiently large D , $\theta_{HP} \approx \lambda/2D$, so the small-angle approximations apply. If we consider an estimation interval which is centered on the time at which $\theta = \theta_{HP}$, and which is not so long that θ encroaches on zero, then Equation 6 is valid. With this in mind, we define the metric τ_a to be Equation 6 evaluated at $\theta = \theta_{HP}$. Substituting $\theta = \lambda/2D$ into Equation 6 and simplifying without further approximation, we find:

$$\tau_a \approx \frac{0.654}{\omega_a (D/\lambda)} I_{UB}^{1/2} \quad (7)$$

This metric is particularly appropriate since both $f(\theta_{HP})$ and $f'(\theta_{HP})$ are relatively large at this point on the main lobe, making this approximately worst case for impact of interference.

III. APPARENT ANGULAR SPEED OF A SATELLITE

The quantity ω_a in Equation 7 is the angular speed of the satellite from the perspective of the radio telescope, which is assumed to be on the ground. The worst case (highest ω_a , so smallest τ_a) occurs when the satellite is directly overhead. Using a simple geometrical argument, it can be shown that the apparent angular speed at this moment is

$$\omega_a \approx \frac{h + R_E}{h} \frac{2\pi}{T} \quad (8)$$

where h is the altitude of satellite, $R_E \cong 6378$ km is the mean radius of the Earth, and T is the orbital period [3]. As an example, typical parameters for a satellite in low Earth orbit (LEO) are $h = 781$ km and $T = 100$ min, yielding $\omega_a = 9.6 \times 10^{-3}$ rad/s ($0.55^\circ/\text{s}$).

IV. RESULTS

We now consider a scenario in which the satellite described in the previous section is present in the main lobe of a zenith-pointing antenna. Figure 2 shows values of τ_a obtained over a range of antenna diameters D/λ for three values of I_{UB} . For example: Consider a scenario in which the ratio of interference power in the output divided by interference power in the input is $I_0 < -30$ dB for a specified estimation interval $\tau = \tau_0 = 25$ ms when there is no variation due to the motion of the satellite through the antenna pattern. For a reflector antenna with $D = 18$ m operating at 1.5 GHz, τ_a is found to be about 24 ms for $I_{UB} = -30$ dB. Since $\tau_a < \tau_0$, the variation in the antenna pattern is sufficient (in the worst case) to limit the achieved suppression to -30 dB as opposed to I_0 . Furthermore, any attempt to improve performance by increasing τ_0 would be unsuccessful.

To do better requires that the CTC system account for the change in $f(t)$ over the estimation interval. A simple way to do this is to divide the input to the waveform estimation filter by $f(t)$ (thereby eliminating the variability due to antenna pattern), and then to multiply the output of the waveform estimation filter by $f(t)$. In [3] we demonstrate that this dramatically increases the effective value of τ_a even when a simple generic antenna pattern model for $f(t)$ is used.

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