
2024

Instructional Decision Making in a Gateway Quantitative Reasoning Course

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Recommended Citation

Budhathoki, Deependra, Gregory D. Foley, and Stephen Shadik. "Instructional Decision Making in a Gateway Quantitative Reasoning Course." *Numeracy* 17, Iss. 1 (2024): Article 2. DOI: <https://doi.org/10.5038/1936-4660.17.1.1451>

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Instructional Decision Making in a Gateway Quantitative Reasoning Course

Abstract

Many educators and professional organizations recommend Quantitative Reasoning as the best entry-level postsecondary mathematics course for non-STEM majors. However, novice and veteran instructors who have no prior experience in teaching a QR course often express their ignorance of the content to choose for this course, the instruction to offer students, and the assessments to measure student learning. We conducted a case study to investigate the initial implementation of an entry-level university quantitative reasoning course during fall semester, 2018. The participants were the course instructor and students. We examined the instructor's motives and actions and the students' responses to the course. The instructor had no prior experience teaching a QR course but did have 15 years of experience teaching student-centered mathematics. Data included course artifacts, class observations, an instructor interview, and students' written reflections. Because this was a new course—and to adapt to student needs—the instructor employed his instructional autonomy and remained flexible in designing and enacting the course content, instruction, and assessment. His instructional decision making and flexible approach helped the instructor tailor the learning activities and teaching practices to the needs and interests of the students. The students generally appreciated and benefited from this approach, enjoyed the course, and provided positive remarks about the instructors' practices.

Keywords

instructional decision-making, instructional autonomy, gateway course, quantitative reasoning, instruction, assessment

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Cover Page Footnote

Deependra Budhathoki is an Assistant Professor of Teacher Education at Defiance College. His research focuses on teaching and learning in Quantitative Reasoning courses. In his doctoral dissertation *Formative Assessment in Postsecondary Quantitative Reasoning Courses*, he explored how QR instructors at public colleges and universities in Ohio implement formative assessment. Gregory D. Foley is the Morton Professor of Mathematics Education at Ohio University. Foley is the principal investigator for QuantNet Ohio: Developing a Statewide Professional Development Network for Effective Teaching of Undergraduate Quantitative Reasoning Courses and for Assessing Quantitative Reasoning, and he is the lead author of *Advanced Quantitative Reasoning: Mathematics for the World Around Us*. Stephen Shadik is an Assistant Professor of Instruction in the Department of Mathematics at Ohio University. His teaching and research include student-centered, inquiry-based teaching practices in entry-level postsecondary (i.e. gateway) mathematics courses.

Introduction

Mathematics is an interconnected set of cognitive tools that humans have developed to support their thinking and reasoning (Skemp 1987). We humans reason using mathematics in our professional and everyday lives; we use numbers and quantities to understand the world around us and make daily decisions. Though the terms *number* and *quantity* often are used interchangeably, we see a *quantity* as a number used in a context that includes an appropriate unit of measurement (Foley et al. 2017). Individuals use quantities to calculate, estimate, measure, evaluate, analyze, explain, and communicate their quantitative thinking. Quantitative information comes in many forms, including natural language, tables, graphs, diagrams, and specialized symbolic representations. Quantitative reasoning embraces all these uses of quantities expressed in these many forms (Lutsky 2008).

Quantitative reasoning (QR) encompasses an individual's understanding, use, and thinking about numbers in a given context. Elrod (2014) described QR as the intersection of critical thinking, real-world context, and mathematics, and Alhammouri (2018) identified real-world contexts and critical thinking as fundamental to one's proficiency in mathematical modeling. Combining the ideas of Elrod and Alhammouri, we define *quantitative reasoning* as thinking critically about a real-world context and about the mathematics and statistics needed to make sense of and analyze the context. Figure 1 is a model of quantitative reasoning that emphasizes critical thinking (Foley & Wachira 2021). It shows critical thinking as the *sine qua non* of quantitative reasoning: A person engaged in quantitative reasoning thinks deeply about the context, the related mathematics, and the interaction between the two.

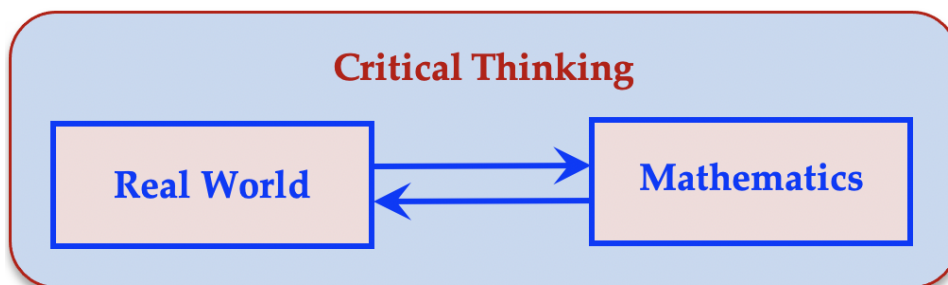


Figure 1. A model of quantitative reasoning that centers on critical thinking.

Quantitative reasoning is a matter of life and death. A person's QR affects their comprehension, decisions, and outcomes related to everyday situations, including their health and wealth. Moreover, *numeric self-efficacy* (confidence in one's quantitative reasoning)—both separately and in interaction with QR—affects the individual's engagement and persistence with quantitative tasks. In turn, QR and

numeric self-efficacy influence the person's financial and health-maintenance behaviors (Peters et al. 2019; Peters & Shoots-Reinhard 2023).

In the United States, there is a growing consensus that high schools, colleges, and universities should develop their students' QR skills and even offer separate QR courses (Madison 2019). Several professional organizations, including the American Mathematical Association of Two-Year Colleges (AMATYC), the Mathematical Association of America (MAA), and the National Council of Teachers of Mathematics (NCTM), have stated that QR is an important educational goal. They have recommended that every student learn how to develop mathematical models to solve real-world problems. All three organizations have published documents emphasizing advancing quantitative reasoning skills: *Quantitative Reasoning for College Graduates* (Sons 1994), *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025* (Saxe and Braddy 2015), *IMPACT: Improving Mathematical PROWESS and College Teaching* (AMATYC 2018), and *Catalyzing Change in High School Mathematics* (NCTM 2018).

Despite this consensus that QR is essential, opinions vary about the nature, purpose, and place of QR within the postsecondary curriculum (Tunstall et al. 2019). Some professional organizations and educators argue that QR skills are interdisciplinary in nature (Mayes and Shader 2012), that such skills cannot be mastered in a single course (Grawe 2011), and that they should be a focus across the curriculum (Elrod 2014). Others emphasize the importance of applying QR skills to socioeconomic and sociopolitical issues (e.g., Wilder 2012). Still, others focus on addressing authentic situations collaboratively and developing evaluative arguments (Lutsky 2008; Stump 2017).

QR is still a fairly new course at many institutions in the United States. Consequently, some instructors express their confusion regarding the content selection and best approaches for instruction and assessment in this course. Additionally, they seek support on how to use this course to address students' everyday and professional needs (Budhathoki 2022). Therefore, this study aims to investigate the initial implementation of an entry-level postsecondary (i.e., gateway) QR course to determine how students respond to the course.

This case study focuses primarily on the instructor's decision making and reflection regarding curricular design and the teaching and assessment approaches the instructor uses and to a lesser extent on how the students respond to such practices. The study centers on specific behavioral aspects of the instructor's decision making concerning topics, instructional approaches, and assessments, as well as the reflective process used to make these decisions. This study addresses the following questions:

- How did the instructor make decisions about course content, teaching methods, technology use, and assessment techniques?

- How did these instructional decisions support the instructor in making the course approachable for the students?
- How did the students respond to the instructor's implemented practices?

The findings from this study may prove useful to QR practitioners, especially to instructors with the autonomy to design and implement the content and assessments used in the QR course that they teach.

QR Competencies, Instruction, and Assessment

Core QR Competencies

Drawing on and further refining the frameworks of Sons (1994), the American Association of Colleges and Universities (AAC&U 2009), and Boersma et al. (2011), we view the core components of a gateway QR course as the interaction among six interrelated competencies that require the deep cognitive engagement and persistence of the students:

1. *Interpretation*. Ability to read critically and to glean and explain quantitative information presented in various forms (e.g., paragraphs, tables, graphs, diagrams, and equations)
2. *Representation*. Ability to convert quantitative information from one form (e.g., paragraphs, tables, graphs, diagrams, and equations) into another
3. *Calculation*. Ability to perform arithmetical, mathematical, and statistical computations
4. *Analysis*. Ability to develop conclusions based on quantitative information and critical thinking
5. *Assumptions*. Ability to recognize, make, and evaluate underlying suppositions in estimation, modeling, and data analysis
6. *Explanation*. Ability to organize, contextualize, synthesize, and present thoughts and processes using mathematical and statistical evidence both orally and in writing

The first three competencies refer to students' capacity to understand, translate, and calculate quantitative information, while the other three focus on reason-based communication (Budhathoki 2022). These six core competencies can serve as instructional goals and as a framework for assessing student proficiency in a QR course or program.

Aligning Instruction and Assessment

QR assessment and instruction should work in tandem to support student learning. The core QR competencies can best be achieved when the instruction and assessment are tied together (Sundre 2003; MAA 2018). For example, the MAA recommends that instructors use daily formative assessments and authentic summative assessments that align with instructional goals. The MAA suggests that instructors engage students in written and oral communication and provide frequent feedback to reinforce course goals. Giving feedback to students also helps

instructors reflect on their teaching, which in turn contributes to improved instruction, hence improved student achievement (Spector et al. 2016).

Instructional Decision Making

Professional autonomy is an individual's personal freedom to make decisions about routine tasks in their work (Friedman 1999; Kasher 2005). In educational contexts, professional autonomy is *instructional autonomy* and is closely related to *responsive teaching*. We will use the term *instructional decision making* in this paper to mean an instructor's capacity to make decisions about their day-to-day teaching, including identifying students' instructional needs and determining and implementing subsequent teaching moves and assessments to address these needs (Pardis et al. 2019). Some authors, including Alsup (2004), refer to it as instructors' *empowerment* to make decisions about curricular content, instruction, and assessment.

Instructors' power to make decisions can ignite nontraditional, innovative practices. Instructional decision making allows for contextualized teaching, rather than relying on standardized content, instruction, and assessments. It requires the instructor to diagnose the learning needs of students in a given context and to design instructional tasks and assessments that best support student learning.

Diagnosis is an integral part of instructional decision making (Ketterlin-Geller and Yovanoff 2009). The instructor must design diagnostic tasks, analyze the student responses, and use the results to inform the next steps of instruction, including just-in-time remediation as needed (Hamilton et al. 2009). Such diagnosis should focus on student thinking and use it as the basis for instructional decision making (Lesseig and Hine 2019).

Instructional autonomy in collegiate mathematics is a relatively new and open area of research. In undergraduate mathematics, standardized learning objectives, instruction, and assessment across multiple sections often limit instructors in exercising professional autonomy and instructional decision making. However, instructional autonomy may create opportunities for instructors to contextualize their teaching in two ways—based on their knowledge, experience, and vision of mathematics teaching and on their knowledge of students and the classroom context. Mason (2019) listed two factors that may allow for instructional decision making in mathematics: (a) their vision of high-quality mathematics instruction and (b) their views of students as mathematically capable. Individual characteristics of the instructor and the teaching context both influence instructional decision making. Instructor characteristics, according to Sullivan et al. (2021), include the instructor's *knowledge of content and teaching*, which, according to Hill et al. (2008), is one aspect of the instructor's pedagogical content knowledge (Shulman 1986). Likewise, the teaching context includes knowledge of students as well as the

classroom environment. Moreover, research framed by instructional autonomy may help the field understand its impact and generate knowledge for improved implementation of reform practices in the mathematics classroom (Lande 2015).

Context and Methods

QR Contexts

National-level context. Early QR work by Sons (1994), Steen (2004), and others have led to the current second wave of QR implementation associated with the mathematical pathways movement. In the past decade, AMATYC, the Carnegie Foundation for the Advancement of Teaching, the Dana Center at the University of Texas, the MAA, the National Organization for Student Success, and Transforming Post-Secondary Education in Mathematics have advocated for multiple mathematical pathways—including Quantitative Reasoning—that better align mathematical coursework with students’ programs of study at institutions of higher education across the United States. These organizations recommend offering Quantitative Reasoning courses, especially to non-STEM majors, and contextualizing the content of these courses to meet students’ professional needs and postsecondary objectives (Barker et al. 2004; Kazis and Cullinane 2015).

State-level context. In May 2013, the Ohio Department of Higher Education (ODHE) held a state-level summit to encourage the state’s public colleges and universities to rethink their gateway mathematics courses, thus launching the Ohio Mathematics Initiative. Within this initiative, a group of mathematics faculty from institutions across Ohio developed guidelines for a new gateway mathematics course known as TMM011–Quantitative Reasoning (ODHE 2015). Foley and Wachira (2021) recommend that this serve as the default gateway mathematics course for non-STEM majors. TMM011 calls for a focus on critical thinking within the broad topic areas of numeracy, probability and statistics, and mathematical modeling. These state guidelines focus on meaningful intellectual tasks, enhancing students’ communication and reasoning skills. However, the ODHE guidelines remain silent on how to teach and how to assess student learning in this course. So far, 27 of 36 public institutions in Ohio have had their QR courses approved at the state level to be transferrable across institutions. The number of students enrolled in this transferable QR course across Ohio has increased from 251 in 2015–2016 to 9,759 in 2021–2022 (P. K. Compton, personal communication, 21 March 2023).

Institutional context. The university reported in this study is a public, research university with a liberal arts tradition. It enrolls more than 25,000 students, including some 5,000 graduate students. Undergraduate students at this university can use Quantitative Reasoning as one of the options to satisfy their quantitative

skills requirement for most of the 250 possible undergraduate programs of study—but not for STEM, business, or several other specific program areas. As alternatives to this course, most students may choose from courses in the Departments of Mathematics (College Algebra, Introductory Statistics, etc.), Philosophy (Principles of Reasoning), or Psychology (Elementary Statistical Reasoning).

The study reported herein focused on one section of Quantitative Reasoning offered at this university during the fall semester of 2018. This was the first time the course had been offered at the university. This three-credit course met three times a week.

Research Team

The authors of this paper served as the research team. All three had some role in the first implementation of this course at the selected university. Collectively, at the time of the study, they had some 75 years of mathematics teaching experience. They are advocates for—and *connoisseurs* (Patton 2015) of—quantitative reasoning and student-centered instruction.

Participants

The initial population for this study was the instructor and students enrolled in the only section of Quantitative Reasoning at the university during the fall semester of 2018. The instructor had never taught a QR course. However, he possessed a master's degree in statistics and had more than 15 years of prior experience teaching other upper-level high school and entry-level university mathematics courses, mainly using student-centered approaches. He also had participated in professional development programs that focused on teaching mathematical modeling.

A total of 10 students enrolled in the course initially. However, 1 student dropped the course during the fifth week of the semester. Of the 9 remaining students, 4 were men and 5 were women, and they varied from freshmen through seniors. Among them, 2 were freshmen, 2 were seniors, and the rest were sophomores or juniors. All of the students were pursuing non-STEM majors. Most of them had had negative prior experiences in mathematics, and for some, this course was the only remaining mathematics requirement for their graduation.

Data Sources, Collection, and Analysis

The data sources included classroom observations, an interview with the instructor, course documents, student-written reflections, course grades, and the standard departmental course evaluation. One of the authors served as the primary researcher and observed the teaching of this course throughout the semester. This researcher wrote daily field notes of his classroom observations while informally assisting with instruction during group work. Roughly halfway through the semester, he conducted a 30-minute interview with the instructor using a semistructured

questionnaire to explore the instructor's course design, reasons for choosing classroom activities, plans for assessing student learning, and general reflections about teaching the course. The interview was audio-recorded, and the researcher conducting the interview wrote some field notes. Course documents—the syllabus, day-to-day plans, and student assignments—were used to evaluate the extent to which the instructor's plans aligned with his practices.

For each class meeting, the primary researcher either typed or handwrote and then typed field notes of his observations. In addition, he typed the notes from the instructor interview and retyped the textual data from the students' written reflections and course evaluations. He transcribed the audio data from the interview. He then created a narrative describing the course by combining the daily field notes, the interview notes, the interview transcription, and data from the course documents. Next, he provided this narrative to the instructor to check its accuracy. The instructor generally agreed with the narrative but suggested some corrections and clarifications, which the researcher then addressed.

Near the end of the semester, the lead author, Budhathoki, used 5 minutes of a class period to explain the study's objectives and to invite all nine students to participate in the study by completing an optional written reflection on the course. Later, he followed up once via email. Budhathoki used the institutional review board (IRB) approved recruitment document during the in-class explanation and follow-up emails. The instructor agreed to provide bonus points to students who completed the optional written reflections. Six of nine students provided consent to participate in the study and submitted their reflections. Budhathoki provided eight IRB-approved questions to students; the questions served as writing prompts for their reflections. The questions focused on their reasons for enrolling in this course, the nature of their experiences with the content and instruction of the course, and their perceptions about whether and how learning in this course would help them in other courses and future endeavors. The questions also focused on the student's overall evaluation of the course and suggestions for the future teaching of this course. For anonymity, students submitted these reflections by dropping them in a box in the classroom, and they were available to the instructor only after the end of the semester. In addition, five students completed the standard departmental course evaluation.

Once all the data were collected, organized, and checked, we moved to the analysis phase. Using the narrative describing the course (explained above), the students' written reflections, and the course evaluation data, Budhathoki developed initial codes for these data using an open coding method (Saldana 2016), for which he classified the important textual information into some main groups, like instructor's preparedness, his classroom practices, class activities, student engagement, students' responses. Then, the research team collaboratively reviewed

these codes, synthesized them into themes, and developed the findings and conclusions.

Findings

The key theme from the instructor interview, which was supported by the other data sources, was that the instructor intentionally used a flexible and emergent design. Because this was the first time the course was offered at the selected university, the instructor could not rely on his past QR teaching or the prior experiences of colleagues who have taught QR. Moreover, the instructor did not feel responsible for covering specific chapters or teaching prescribed content. This situation allowed him to choose any quantitative issues as the learning contexts and adopt any teaching strategy to support student learning. Therefore, the instructor planned to remain flexible in his lesson design and did not have a predetermined plan for what and how he would teach during the semester.

Flexible Course Content Made the Course Approachable

Instructional decision making was central to the instructor's praxis. The instructor reported that he had great flexibility in teaching Quantitative Reasoning because this was a stand-alone course. In contrast to teaching the Precalculus course that he taught for several years at the same university, during which he was responsible for covering the specified content, preparing students for departmental exams, and readying them for the Calculus sequence, in QR he could determine the content and assessments and set his own pace. In keeping with the findings of Hill et al. (2008) and Mason (2019) that an instructor's knowledge of content and teaching and their vision of high-quality mathematics instruction influence instructional decision making, the instructor in the present study focused on the depth of the topics, rather than widespread coverage. He said, "I am covering topics at a much slower rate than I am used to covering. ... We are covering fewer topics, but I'm trying to cover them at a deeper level." He added, "Instead of doing four or five things, we might only do one or two things. ... I think that's something where things can change without me being able to anticipate until I am in [the] classroom."

The instructor had the full authority to determine course content and instruction; nonetheless, in planning and teaching he considered the QR content and practices recommended by the state agency in its TMM011—Quantitative Reasoning (ODHE 2015). Based upon the analysis of the instructor interview, the syllabus, and the class-by-class field notes, Table 1 shows that the course focused mainly on three content areas: dealing with data, statistics and probability, and mathematical modeling. The first two content areas received greater emphasis than mathematical modeling, which was interspersed throughout the semester and included in a capstone modeling project that was used in place of the final exam.

These three content areas match those suggested by the TMM011—Quantitative Reasoning, except that the ODHE uses Numeracy as a category label instead of Dealing with Data.

Table 1
Content Outline for the QR Course

Dealing with Data	Statistics and Probability	Mathematical Modeling
Problem-solving strategies	Weighted averages	Modeling with sequences
Proportional reasoning	Conditional probability	Financial modeling
Personal finance	Statistical reasoning	Capstone modeling project

The students provided positive feedback concerning the instructor's curricular and instructional practices. They appreciated the flexible pacing of the course and its positive impact on their learning. They mainly explained that the content in this course was unlike that in other mathematics courses and was approachable to them. They also appreciated how the instructor slowed down the pace to ensure everyone understood the concepts. One student wrote, "It's very approachable while still maintaining a level of difficulty that makes me study and think about the subjects presented to me ... not super common to most math courses I've taken." Another student wrote, "I love this class much more than any other math class I've taken." The other students also stated that the contents helped them with the math tools necessary to succeed in figuring out solutions to problems with numbers. On the other hand, some students wished that this course had been more planned and structured, which, according to them, would have made the content and their learning more manageable. One student expressed concern about the unstructured nature of the course: "It could have been slightly more structured."

Emergent Tasks Focused on Student Thinking

The instructor's flexibility in designing the course allowed him to choose everyday quantitative situations as the basis for his learning activities. The *Advanced Quantitative Reasoning* course textbook (Foley et al. 2017) was a source for many real-world contexts and student activities. In addition, the instructor often designed what Bikner-Ahsbahr and Janßen (2013) call *emergent tasks*; that is, the instructor translated the immediate learning needs of the students into a task either by picking an activity from other sources or by designing tasks on his own using contemporary real-world situations. For instance, he once shared a promotional email that he had received from an airline company, which stated that the company's wait time for flights had decreased by 60% on average. The instructor encouraged students to discuss the possible past and current wait times and their impact on the company's business and then to analyze these issues from a passenger's point of view. In addition, the instructor frequently brought socioeconomic issues into the class and allowed students to discuss and develop their opinions about them. Other instructor-

developed activities addressed (a) the 2018 US midterm elections, the associated issues, and the impact of choosing to vote or not; (b) the increasing economic dependency ratio; and (c) the scores of basketball games. He once asked students to design carnival games based on the probability and statistics they had learned in the course. Bikner-Ahsbabs and Janßen (2013) identified three requirements for the effective design and implementation of emergent tasks: (a) sufficient mathematical knowledge to extend the lesson content, (b) focus on student learning, and (c) willingness to move away from the planned lesson to follow unexpected directions of students' activity. The instructor possessed sufficient mathematical knowledge and regularly exhibited the focus and flexibility needed to satisfy all three of these requirements.

The instructor also kept student thinking and the students' mathematical capacities at the center while designing the learning activities. For this, as he claimed during the interview and as was confirmed through class observations, the instructor diagnosed the students' learning needs and designed progressions of tasks to meet these needs. As Lesseig and Hine (2019) suggested, the instructor considered student thinking as the basis of his diagnosis and instructional decision making. Also, as suggested by Ketterlin-Geller and Yovanoff (2009), the instructor used information from such diagnosis in making appropriate decisions about the progression of content to be examined by the students. Moreover, per Bikner-Ahsbabs and Janßen (2013) and Mason (2019), the instructor emphasized developing students' expertise in using prior mathematics knowledge and skills (largely middle-grade content) to explore real-world contexts with explicit reasoning. During the interview while answering about his preparation for teaching the course and orchestrating problems for students, the instructor focused on what students could do and reach with a given problem; he said, "I try to think about the ways that come with problems, ..., beforehand, before class, trying to think about the ways that the students can do." The research team confirmed his plans through analysis of the course documents and classroom observations.

All six students reported that the knowledge and skills they learned in this course would help them in their future lives. They appreciated the instructor's emphasis on group work and his use of real-world contexts, citing specifically personal finance, taxes, voting, budgeting, and billing, which they thought would help them in other courses and their personal lives. One student wrote, "Many of the problems we talked about are real-life concerns for every person, whether it is taxes or voting." The student added that experiences working with such concerns would support them to work effectively in their everyday world, including personal finance, budgeting, or setting up spreadsheets. Though only about half of them mentioned that the learning from this course would help them in other courses, almost everyone agreed that knowledge and skills from this course would help address real-life situations.

Inductive Teaching and Infrequent Lecturing

In addition to this flexibility concerning course content and contexts, the instructor was flexible in his instructional practices. He often used an inductive approach to help students construct knowledge by asking them to consider specific contexts to build toward general techniques, and this sometimes took a longer time than he expected. He chose anchor activities for particular days and then crafted follow-up activities as needed for subsequent days. He let students discuss the issue for each activity, facilitated their discussion, and decided whether to “stretch” the issue and its discussion for the next day. He explained that this flexible design helped him select tailor-made activities and modify his instructional practices to address students’ learning needs. He explained that he did not start teaching this course with a semester-long plan for content other than wanting to address the content domains listed in Table 1. Typically, the instructor did not even have a week-long plan. He reflected, “I probably don’t do a good job [about] what I expect at the end of every week.” He added, “I know what I am gonna do tomorrow with them, but I don’t know what I am gonna do the following day with them. I might have some ideas, but I am really gonna see what happens tomorrow and then go build from there.”

Nevertheless, his inductive approach always included real-world aspects, which contributed to students’ active engagement. For example, he once asked students to work in groups to identify which one was better—an ACT score of 25 or an SAT score of 1200 (scores on two standard university admission exams). At first, the students had diverse answers based on various arguments. Later, the instructor suggested that they confirm their answers using the means and standard deviations of the scores. Upon researching, the students found that the ACT and SAT had means (and standard deviations) of 20.8 (4.8) and 1055 (195), respectively. Then, the students agreed that the ACT score was better because it was 0.88 standard deviations above its mean, whereas the SAT score was just 0.74 standard deviations above its mean. The students shared in class discussions that this helped them understand how their scores had been evaluated when they applied to the university. Later, the instructor presented the concept and calculation of *z*-scores more formally and explained that 0.88 and 0.74 were the *z*-scores for an ACT score of 25 or an SAT score of 1200, respectively. As the semester continued, he often referred to earlier topics to help students link them to their current learning.

The instructor claimed himself to be a big believer in students working together, talking together, and figuring things out together. He characterized his teaching as *infrequent lecturing*, where he orchestrated opportunities for students and facilitated students’ discussions to share their thoughts even if they were wrong. To provide opportunities for students to collaborate and communicate, the instructor posed problems, organized students into groups to solve them, and then orchestrated student discussions. Occasionally, the instructor would reorganize the collaborative groups, depending on how they interacted in their previous groups.

He remained flexible as much as possible and made changes to support student learning. As another example of flexibility, on a particular day, he realized that one of the students, an introvert, was reluctant to share her answers and reasoning. The next day the instructor started using Padlet software so that students could post their answers anonymously.

The students reported that the instructor's pedagogical practices encouraged student learning in an inclusive manner. The students especially appreciated the activities that involved small-group and whole-class discussions; they reported that learning mathematics by exploring real-world contexts and discussing what they found was enjoyable. One student wrote, "The instructor is personable and easy to talk to. I find the more relaxed discussion-based approach makes it easier; I want to speak in class." They also mentioned that the instructor used various approaches to ensure that everyone understood the concept. Another student wrote, "My instructor's pace was phenomenal with a lot of group work ethic ... we moved at a good pace, and the instructor always made sure everyone is on the same page." The instructor's flexible pacing supported every student in learning the content; MAA (2018) listed flexible design as a characteristic of an inclusive mathematics course. The students even stated that using multiple teaching strategies helped their proficiency at working with others and advanced their thinking and reasoning skills. Many students reported that the instructor pressed them to explain things thoroughly, and they enjoyed doing so.

Technology Addressing Students' Needs and Interests

The instructor incorporated several technologies in his teaching. He frequently engaged the students in using Excel spreadsheets to represent, organize, and analyze data. He was strategic in using student-friendly and student-accessible technological tools and software in his instruction. The instructor explained that he chose Excel because this software is available for free to students through the university. He added that Excel is relevant to many jobs that may interest the students. Therefore, as he stated in the interview, the instructor used Excel at least 50% of the time. As with other aspects of the course, the instructor remained flexible in his choice of technology to address students' needs and interests. In addition to engaging students in using mathematical-action technology to solve problems, he employed technology to support small-group collaboration. To ensure that all students had opportunities to model real-world problems using technology, especially spreadsheets, the instructor moved the class from a regular classroom to a computer lab around the middle of the semester.

The students reported having a positive experience with using technology in this course. Most of them said that this was their first experience learning mathematics with technology and appreciated learning to use Microsoft Excel. One student wrote, "This was brand new to how I perceive math." The student added,

“I think this amount of technology is just right because it is not overpowering.” However, some other students reported insufficient time for Excel in this course. Indeed, three of six students expressed their frustrations with using technology at some level. For example, one student said, “Some more time without Excel could help the class, maybe at the beginning of the course to help understand basic ideas or thinking processes.” Likewise, another student mentioned that using Excel required them to remember certain equations; the instructor frequently required students to create formulas in Excel and not use templates created by others. Those who considered this learning helpful indicated the resulting problem-solving skills from using Microsoft Excel would be important in other courses. One student in the opposing group wrote, “I don’t know if another class I take will value explanation as much as this.”

Diagnostic Assessments

As recommended by Hamilton et al. (2009), the instructor used his professional autonomy to choose a wide variety of assessments. His assessments supported his instruction in two ways:

- The assessments helped him diagnose what students knew and could do as well as to identify gaps in their knowledge and skills.
- Consequently, the instructor was able to customize instructional and assessment practices for his students and modify these practices as needed.

Unlike other mathematics courses of the same level, the instructor did not use quizzes, midterm tests, or final examinations. Instead, he gave take-home assignments and collaborative projects and presentations. As discussed by Hamilton et al. (2009), the instruction used the information from such assessments to measure students’ existing mathematical knowledge and know-how and then to make decisions about what types of tasks and activities would maximize their learning (Ketterlin-Geller and Yovanoff 2009). When interviewed about his assessments, the instructor stated, “I’m trying to figure out what they know and what they don’t know. So, then I can pattern my response [to] that.”

The instructor thought his assessments were formative. He referenced his projects as an example, “I assess them, I will provide feedback for them and give them suggestions [on] how they can improve next time we do a project.” He orchestrated the assignments as opportunities for students to use their current knowledge and skills to solve real-world problems and enhance their communication skills. Each such assignment was followed by students reporting their findings and explaining their thinking processes orally, in writing, or both. For example, at the beginning of the course, the instructor assigned students to work in groups on a card trick, demonstrate it to the rest of the class, and explain its embedded quantitative aspects. Students seemed to enjoy the activity and learned some related mathematics. The instructor used a collaborative project in place of

the final exam. For this summative assessment, the students worked in pairs to select one real-world Fermi problem from a list and made in-class presentations. Each group's final presentation included their approach to solving the problem, the mathematical knowledge they employed, their findings, and their conclusions. In addition, they submitted written group reports. The students appreciated these assessments, mentioning that they had not had to worry about the exams and tests. One student stated, "I love where the grades come from rather than strictly exam grades."

Discussion

The instructor did not have prior experience in teaching a QR course. However, he leveraged his professional autonomy as well as his expertise in freshman-level mathematical content and experience in student-centered teaching by using a flexible design in his content selection, instructional design, and assessment methods. In particular, he based QR activities on real-world events and emphasized students' critical thinking about these events and related mathematical and statistical concepts. Moreover, he asked students to make their thinking seen and heard in small groups and via whole-class discussions. The instructor engaged the students in the six core QR competencies of interpretation, representation, calculation, analysis, assumptions, and explanation, as well as many other desirable aspects of a QR course. In addition, his emergent task design approach, as Ainley and Margolinas (2021) discussed, helped him to remain flexible and support students in their mathematical thinking.

These findings align with and support our previously published QR research (Budhathoki 2022; Foley et al. 2023) and the associated theoretical frameworks. Budhathoki (2022) listed the 4 Cs of Quantitative Reasoning: quantitative Content and skills, real-world Contexts, Critical thinking, and Collaboration. Building on Foley and Wachira's (2021) model of QR student engagement and Budhathoki's 4 Cs of Quantitative Reasoning, Foley et al. (2023) presented the 5 C Model of Quantitative Reasoning, putting Critical thinking at the heart of quantitative reasoning to provide an environment for students to connect the given Context to mathematical and statistical Concepts while using QR Competencies and Collaboration to support their critical thinking.

Moreover, the activities were interdisciplinary (Grawe 2011) and promoted social justice by addressing socioeconomic and sociopolitical issues (Wilder 2012). The students actively constructed mathematical and statistical knowledge and used their knowledge and skills to solve real-world problems involving everyday contexts (Mayes and Shader 2012). The instructor gave students opportunities to collaborate to solve problems and organize, contextualize, and revise their thinking through student communication and instructor feedback (MAA 2018). The

instructor embraced Steen's (2004) "sophisticated reasoning with elementary mathematics" (p. 9) approach to QR content.

In this case study, the instructor aligned the course content with students' prior and proximate knowledge and modified his instructional practices to support student engagement; these factors contributed to student motivation and achievement. Moreover, as suggested by Ketterlin-Geller and Yovanoff (2009), the instructor diagnosed the students' learning needs in the given context and chose optimal instructional and assessment approaches to support their learning. The students responded positively to the genuine contexts due to their relevance. In general, we would expect that remaining flexible in content selection would provide opportunities to welcome emerging issues that are relevant to the students as learning contexts.

In the local setting, the QR course was—and is—open to a wide variety of majors. This makes flexible content and a wide variety of contexts a desirable feature of the course. In similar settings, employing such flexibility and variety would allow the instructor to tailor the content, instruction, and assessment to students' prior knowledge and particular interests.

Regardless of these suggestions, all participating students reported enjoying learning in this class, and the performance in the course among all students was strong. All 9 students who remained in the course passed it with a grade of C or higher. On a 5-point scale, the overall instructor evaluation was 4.65, and the overall course evaluation was 4.75, both of which were well above the respective departmental means.

In addition, the findings of the present study align with Budhathoki's (2022) post hoc analysis, which discovered that instructional autonomy was a critical variable in support of high-quality QR content, instruction, and assessment. Budhathoki thus recommended that instructors be granted the autonomy to make decisions concerning tasks, instructional strategies, and assessments for the QR classes that they teach. The findings of the present study reinforce this recommendation.

Conclusion

Instructional decision making in Quantitative Reasoning allows instructors to offer customized learning activities and instructional practices tailored to the learning context and students' needs and interests. This gives power to instructors to modify their teaching practices to accommodate student interests and classroom dynamics, which has the potential to enhance students' engagement in learning activities and improve their achievement. Furthermore, explicitly practicing collaboration and communication in a QR classroom creates opportunities for students to construct mathematical and statistical proficiency and apply their knowledge and skills in

real-world contexts. By prioritizing communication, student reasoning and thinking become visible for inspection and reflection. This helps students build relevant mathematical skills and allows them to critique their quantitative reasoning and that of their classmates. These skills and experiences put the students on a firm footing to perform well in other academic pursuits and their future lives.

Because this study was based on the practices of one instructor with a few students enrolled, the findings may not be generalizable. However, like other qualitative study findings, our findings may be transferable or adaptable to other QR instructional environments with similar or analogous contexts. We wish to emphasize that the instructor's extensive experience in freshman-level mathematics content and student-centered teaching contributed significantly to his instructional decision making about content, instruction, assessment, his selection of real-world tasks, and especially his design of emergent tasks. In addition, his strong belief in the power of collaborative learning and formative assessment further influenced and strengthened his instructional decision making. Nonetheless, our findings may be helpful or aspirational to other QR programs and to other novice and experienced QR instructors. Lande (2015) discussed that understanding the beneficial impact of instructional autonomy on student learning may give insight into and encourage the use of reform practices. Future related research involving larger class sizes and instructors with varying experience levels could prove useful and insightful.

Current Practice at the University

Quantitative Reasoning at the university has grown substantially since the case reported herein. Of particular interest, the university received its Ohio Transfer-36 approval only in the summer of 2019. The course has grown both in the number of sections offered each term and in student enrollment. In Fall 2023, the university is offering seven sections of QR students for more than 200 students and one section of a corequisite course for students who wish to enroll in the main QR course but do not meet the university's mathematics placement requirement.

The mathematics department at the university is committed to the sustainability of the course and anticipates that over time student enrollment in QR will surpass that of other gateway mathematics courses. The department has hired a designated QR course coordinator, mainly to coordinate academic and administrative activities related to QR teaching, including facilitating weekly meetings of the QR instructors. At these meetings, the QR instructors discuss current issues and challenges in teaching their QR section(s) as well as ways to improve the teaching of this course. The QR instructors at the university still have the autonomy to make individual decisions about tasks, teaching, technology, and assessment. In particular, the instructors have the option to adopt, adjust, amend, or drastically change the coordinator-developed course syllabi, instructional

strategies, and assessment materials; generally, the instructors teaching this course for the first time closely follow the course coordinator's lead.

Another key factor influencing the change of the Quantitative Reasoning course at the university is its membership in an NSF-funded statewide network of colleges and universities: *Developing a Statewide Professional Development Network for Effective Teaching of Undergraduate Quantitative Reasoning*. This project is known colloquially as QuantNet Ohio. The QuantNet Ohio project will affect the course at the selected university as well as its instructors as they participate in its professional development and interact with QR colleagues throughout Ohio. Currently, the state of Ohio is a center of activity for QR research and development, which will influence all QR teaching for the foreseeable future.

Funding Information

The research reported herein was supported by the Ohio University research incentive fund, by the Patton College of Graduate Study and Educational Research fund, and by National Science Foundation award no. 2216197. The findings and conclusions are those of the authors and do not necessarily represent those of the funding agencies.

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