Market Impacts of Relaxed Incumbent Protection in Spectrum Sharing

Kangle Mu and Randall Berry
Electrical and Computer Engineering, Northwestern University, Evanston IL 60208
kangle.mu@northwestern.edu, rberry@northwestern.edu

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Abstract—**Spectrum sharing approaches such as that adopted in the Citizens Broadband Radio Service (CBRS) provide strong protection to incumbent users of the spectrum. This can lower the value of such spectrum for commercial service providers (SPs). This paper considers the impact of relaxing this protection on the market for secondary spectrum services. We assume that SPs must reduce their traffic when the incumbent is present. We find that consumers always benefit from relaxing incumbent protection but SPs' revenue and social welfare exhibit subtle behavior. Depending on the extent of relaxation, the market might respond negatively implying that regulators need to carefully choose such policies.**

I. Introduction

The proliferation of mobile services is driving demand for commercial wireless spectrum. Considering the high cost of relocating incumbent services, spectrum sharing is increasingly being considered for meeting this demand. Examples of sharing include the U.S. Federal Communications Commission's (FCC) 2015 policy for shared use of the 3.5 GHz Citizens Broadband Radio Service (CBRS) band [1] and the 2020 ruling for shared use of the 6 GHz band [2]. These sharing approaches provide strong protection to incumbent users. For example, CBRS follows a three-tiered access framework in which the incumbents belongs to the highest tier. A Spectrum Access System (SAS) prohibits lower tier access whenever an incumbent is active. The existence of incumbent users makes the shared spectrum intermittent to the commercial service providers (SPs). The intermittency can lower the value of such spectrum for SPs and also disadvantage smaller SPs [3].

In this paper, we study the impact of relaxing the strong incumbent protection for commercial SPs using shared spectrum. Namely, when an incumbent is present we assume that a SP must reduce its traffic on the shared band, but not fully reduce it to zero. This relaxed sharing approach has some parallels with the U.S. 6 GHz policy, which allows unlicensed indoor access points to operate at lower power even when incumbents are present [4]. Compared to CBRS, 6 GHz incumbent activity is largely static, but one could envision an approach in other bands in which a SAS determines at any time if users could operate at higher or lower power.

Our objective is to study the *market impacts* of such a policy, i.e., to what degree does relaxing incumbent protection increase the value of shared spectrum to SPs? How do any benefits depend on the size of the SPs? What impact does it have on consumer surplus?

Our approach builds on [3] which studied a model where SPs compete via Cournot competition using shared spectrum assuming strong incumbent protection. Here, we instead consider a model with relaxed incumbent protection. We assume that SPs must account for a given level of "incumbent activity" on the shared band when the incumbent is active, which increases a congestion cost. This could represent an actual incumbent activity or a form of non-linear pricing imposed on a SP for the traffic it generates when the incumbent is active.

Our main results are summarized as follows:

1) Consumer surplus is a non-decreasing function of the degree of relaxation. In other words, consumers always benefit from relaxing incumbent protection.
2) Allocating additional shared bands to smaller SPs tends to achieve higher consumer surplus in most cases. If this allocation is determined by a 2nd-price auction, we observe that larger SPs are more likely to win the auction.
3) A too conservative relaxation can hurt the overall revenue of SPs. Specifically, the marginal revenue gain of a SP due to such relaxation could be overshadowed by more significant revenue losses incurred by other SPs that do not have shared spectrum.
4) Typically, as the degree of relaxation increases, we observe a corresponding increase in overall social welfare. But social welfare sometimes presents a complex dynamic and may not be a monotonic function.

In terms of related work, our work is based on modeling a wireless market using the framework of competition with congestible resources [5]. This framework has been used to study competition under different spectrum sharing models in a number of papers including [6]–[8]. Other work studying market aspects of spectrum sharing includes [9]–[14]. This paper differs from prior work with its relaxation of incumbent protection.

The rest of the paper is organized as follows: Section II introduces the model. Section III characterizes the equilibrium in different settings. Section IV shows the main results in terms of consumer surplus, the SPs' revenue, and social welfare. Finally, Section V concludes the paper.

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II. The Model

We begin with a similar model to [3] in which a set of N SPs use both proprietary spectrum and licensed shared spectrum[1] to serve their customers, where the licensed shared spectrum is intermittently available due to the presence of incumbent users. The SPs compete via Cournot competition for a pool of infinitesimal users (consumers), i.e., each SP specifies the quantity of customers it will serve. Without loss of generality, we assume that the total amount of users (i.e., the market size) is 1. This in turn leads to a market clearing *delivered price* given by

$$p_d = P(x_{total}) := 1 - x_{total}, \tag{1}$$

where x_{total} is the total quantity of customers served in the market. The corresponding consumer welfare generated in the market is then given by

$$CS(x_{total}) = \int_0^{x_{total}} \big(P(z) - P(x_{total})\big)dz = \frac{x_{total}^2}{2}, \tag{2}$$

where $P(z) - P(x)$ represents the surplus of the zth user.

The *service price* that a SP receives from its customers is given by the difference between p_d and a *latency cost* incurred by that SP's customers. This cost accounts for the quality of service that the SP delivers. Each SP seeks to maximize its revenue given by the product of its service price and the quantity of users served. This results in a game as the delivered price will depend on the actions of all the SPs.

The latency cost for each SP i in turn depends on the amount of proprietary spectrum (B_i) and licensed shared spectrum (W_i) owned by that SP, as well as the probability that the shared band is available (α). In [3], the overall latency cost was calculated assuming that a SP must off-load any customers on the shared band to its proprietary band, whenever the shared band was not available (with probability $1 - \alpha$). This paper departs from this assumption by allowing SPs to use shared bands with reduced traffic.

Next, we define the latency cost on proprietary bands and shared bands. We assume that when SP i is serving x customers on its proprietary band, then these customers experience a latency cost of

$$l_i(x) = \frac{x}{B_i}. \tag{3}$$

For the licensed shared band, we define the *equivalent incumbent activity*, C_i, to be an amount of user traffic added to the shared band whenever the incumbent is active. C_i could represent an actual level of incumbent activity or an equivalent incumbent user mass when a certain amount of channel resources are reserved for incumbent users.[2] This can also be viewed as a non-linear pricing term imposed on a SP for the traffic it generates when the incumbent is active.[3] We define the latency on the shared band of bandwidth W_i while serving w amount of users as

$$l_i^w(w) = \begin{cases} \frac{w}{W_i} & \text{if incumbent users are inactive,} \\ \frac{w+C_i}{W_i} & \text{otherwise.} \end{cases} \tag{4}$$

For SP i, let x_i and w_i be the user mass served on the proprietary band and licensed shared band when incumbent users are inactive (with probability α). When incumbent users are active, we assume that SP i off-loads Δw_i users to its proprietary band to reduce congestion on the shared band.

We further assume that users are sensitive to the expected latency cost, averaged over the incumbent's activity level. Therefore, the latency cost for x_i users served on the proprietary band is given by

$$l_i^x = \alpha\frac{x_i}{B_i} + (1-\alpha)\frac{x_i + \Delta w_i}{B_i}. \tag{5}$$

Similarly, the latency cost for the $w_i - \Delta w_i$ users (who are served on the shared band all the time) is given by

$$l_i^{w-\Delta w} = \alpha\frac{w_i}{W_i} + (1-\alpha)\frac{w_i - \Delta w_i + C_i}{W_i}, \tag{6}$$

and the latency cost for Δw_i users (who are served on both bands) is given by

$$l_i^{\Delta w} = \alpha\frac{w_i}{W_i} + (1-\alpha)\frac{x_i + \Delta w_i}{B_i}. \tag{7}$$

The total loss due to the congestion for SP i is then

$$x_i l_i^x + (w_i - \Delta w_i) l_i^{w-\Delta w} + \Delta w_i l_i^{\Delta w}. \tag{8}$$

Given a total number of users $\tilde{x}_i = x_i + w_i$ served by SP i in any equilibrium, $x_i^\star$, $w_i^\star = \tilde{x}_i - x_i^\star$, and $\Delta w_i^\star$ must minimize the loss in (8). This can be formulated as

$$\min_{x_i, w_i, \Delta w_i} \; x_i l_i^x + (w_i - \Delta w_i) l_i^{w-\Delta w} + \Delta w_i l_i^{\Delta w} \tag{9a}$$

$$\text{s.t.} \quad x_i + w_i = \tilde{x}_i \tag{9b}$$

$$w_i - \Delta w_i \geq 0 \tag{9c}$$

$$x_i, w_i, \Delta w_i \geq 0 \tag{9d}$$

where (9c) enforces that the users served on the shared band can not be negative. The solution is given by

$$\Delta w_i^\star = \begin{cases} \frac{B_i C_i}{2(B_i + W_i)}, & \text{if } \tilde{x}_i \geq \frac{B_i C_i}{2W_i}, \\ w_i^\star, & \text{otherwise,} \end{cases} \tag{10a}$$

$$x_i^\star = \frac{B_i}{B_i + W_i}\tilde{x}_i, \tag{10b}$$

$$w_i^\star = \frac{W_i}{B_i + W_i}\tilde{x}_i. \tag{10c}$$

The break point in (10a) comes from the constraint $w_i^\star - \Delta w_i^\star \geq 0$. Note that $\Delta w_i^\star$ is equal to $w_i^\star$ for large enough C_i, which means the SP off-loads all of the traffic on the shared band to its proprietary band. In this case, no SP traffic is on the

[1] A SP with licensed shared access to spectrum has exclusive secondary access to this spectrum. Thus it only shares the spectrum with the incumbent, similar to the Priority Access (PA) tier in CBRS.

[2] For example, this could represent the channel resources needed for an incumbent radar.

[3] This price corresponds to the increased latency cost imposed on the band due to C_i

shared band when incumbent users are active and so incumbent users are fully protected.

According to the solution in (10), with the optimal allocation, the revenue loss in (8) can be characterized by the total user mass $\tilde{x}_i$. Thus, we can derive an equivalent model in which each SP decides on serving $\tilde{x}_i$ customers. The resulting revenue of SP i is

$$r_i(x_i, w_i, \Delta w_i) = \tilde{x}_i \left(p_d - p_i^{latency}\right) \tag{11}$$

where

$$p_d = P\left(\sum_{j=1}^{N} \tilde{x}_j\right) = 1 - \sum_{j=1}^{N} \tilde{x}_j \tag{12}$$

is the delivered price as in (1) and

$$p_i^{latency} = \frac{x_i}{\tilde{x}_i} l_i^x + \frac{w_i - \Delta w_i}{\tilde{x}_i} l_i^{w-\Delta w} + \frac{\Delta w_i}{\tilde{x}_i} l_i^{\Delta w_i}. \tag{13}$$

If we substitute the optimal allocation (10) into the revenue (11), we can get a simplified but equivalent model:

Theorem 1 (Equivalent Model 1): Given total user mass $\tilde{x}_i$ that SP i wants to serve, there exists a unique optimal user allocation given by (10) under which, the optimal revenue of SP i when $\tilde{x}_i \geq \frac{B_i C_i}{2W_i}$ is given by

$$\begin{aligned} r_i(\tilde{x}_i) = \tilde{x}_i &\left(p_d - \frac{\tilde{x}_i}{B_i + W_i} - \frac{(1-\alpha)C_i}{B_i + W_i}\right) \\ &+ \frac{(1-\alpha)B_i C_i^2}{4W_i(B_i + W_i)}. \end{aligned} \tag{14}$$

Otherwise,

$$r_i(\tilde{x}_i) = \tilde{x}_i \left(p_d - \frac{1 + (1-\alpha)\frac{W_i}{B_i}}{B_i + W_i} \tilde{x}_i\right). \tag{15}$$

The revenue $r_i(\tilde{x}_i)$ is a piece-wise defined function of the total user mass $\tilde{x}_i$ since $\Delta w_i^\star$ in (10a) is piece-wise. As mentioned before, the relaxed incumbent protection may turn into full incumbent protection for large enough C_i (or small enough $\tilde{x}_i$). Large C_i means the shared band is crowded thus SPs choose not to use the shared band to avoid high congestion cost. For small $\tilde{x}_i$, SPs might have enough proprietary bandwidth to serve their users so there is no need to use the shared band. For ease of analysis, we refer to (14) as the *SP's revenue with relaxed incumbent protection* (or *RIP* for short), and (15) as the *SP's revenue with incumbent protection* (or *IP* for short). Note that the IP case is identical to the Shared Licensed Access Model in [3].

III. Equilibrium

A. Many SPs

Consider a market with multiple SPs that compete via Cournot competition. Note that since we have already derived the equivalent models in which total user mass $\tilde{x}_i$ is sufficient to characterize the revenue of SP i, we can simply focus on this quantity and ignore how this is divided among the available bands. The following theorem shows that there always exists a unique equilibrium of quantities announced by the SPs.

Theorem 2 (Existence and uniqueness of equilibrium): Given a market with any number of SPs competing in a Cournot game by announcing the total user masses they want to serve, there is a unique Nash equilibrium.

The proof of Theorem 2 can be established by showing the underlying game is a potential game and the potential function is strictly concave. The potential function is a piece-wise function due to (10a), and it is easy to verify its piece-wise concavity. To prove it is globally concave, we then use the approach in [15].

For tractability, our analysis will primarily focus on competition between two SPs.

B. Two SPs

Suppose there are only two SPs in the market. To obtain the user mass at equilibrium, we consider the first-order optimality conditions $\frac{\partial r_i(\tilde{x}_i)}{\partial \tilde{x}_i} = 0,\ i \in \{1, 2\}$.

From (14) and (15), the revenue $r_i(\tilde{x}_i)$ depends on the value of $\tilde{x}_i$ and $\frac{B_i C_i}{2W_i}$. Thus the equilibrium has 4 types for two SPs.

Theorem 3 (Equilibrium): Suppose there are two SPs with parameters B_i, W_i, and C_i for $i \in \{1, 2\}$. Denote the user mass at the equilibrium by $\tilde{x}_i^\star$. The unique equilibrium is of the following four types:

1) Type 1 (RIP and RIP) if $\tilde{x}_i^\star \geq \frac{B_i C_i}{2W_i}$, $\forall i \in \{1, 2\}$.
2) Type 2 (IP and RIP) if $\tilde{x}_1^\star < \frac{B_1 C_1}{2W_1}$ and $\tilde{x}_2^\star \geq \frac{B_2 C_2}{2W_2}$.
3) Type 3 (RIP and IP) if $\tilde{x}_1^\star \geq \frac{B_1 C_1}{2W_1}$ and $\tilde{x}_2^\star < \frac{B_2 C_2}{2W_2}$.
4) Type 4 (IP and IP) if $\tilde{x}_i^\star < \frac{B_i C_i}{2W_i}$, $\forall i \in \{1, 2\}$.

We can further give closed-form expressions for each of these cases, but omit them due to space considerations. The expressions of these types are obtained by using different combinations of (14) and (15). Some basic properties are summarized as follows. WLOG, we state these from the perspective of SP1.

1) The more resources SP1 possesses, the larger the relative market share of SP1 (i.e., $\tilde{x}_1^\star / \tilde{x}_2^\star$). Specifically, $\tilde{x}_1^\star / \tilde{x}_2^\star$ is a increasing function of B_1 and W_1, and a non-increasing function of C_1.
2) C_1 matters only when SP1 belongs to the RIP case (i.e., type 1 and type 3) since if SP1 is in the IP case, it does not use the shared band when incumbent users are active thus C_1 is an irrelevant variable.
3) If the equilibrium hits the edge case, i.e., $\tilde{x}_1^\star = \frac{B_1 C_1}{2W_1}$, we would get the same $\tilde{x}_1^\star$ if we apply the first order condition to another formula of $r_1(\tilde{x}_1)$. The reason is that the edge case $\tilde{x}_1^\star = \frac{B_1 C_1}{2W_1}$ actually belongs to both RIP and IP since in this case $\Delta w_i^\star = \frac{B_i C_i}{2(B_i + W_i)} = w_i^\star$ (10a), which means the optimal $\Delta w_i^\star$ happens to result in incumbent protection. In other words, SP1 is indifferent to RIP and IP in the edge case.

Since the condition for each type in Theorem 3 depends on the equilibrium $\tilde{x}_i^\star$ itself, it is still challenging to find the true equilibrium by using Theorem 3 alone. But Theorem 3 provides us with four candidates. We next give an algorithm for determining the equilibrium.

The reason for having 4 candidate equilibria is that the revenue of each SP has 2 different expressions (14) and (15). To determine whether the equilibrium falls into the range of (14) or (15), we design an iterative approach: First, we assume that both SPs belong to RIP and compute an equilibrium candidate $(\tilde{x}_1^\star, \tilde{x}_2^\star)$. We then check if $(\tilde{x}_1^\star, \tilde{x}_2^\star)$ satisfies the condition in type 1. If both satisfy the condition, then this $(\tilde{x}_1^\star, \tilde{x}_2^\star)$ is the true equilibrium. Otherwise, for any invalid $\tilde{x}_i^\star$, switch its model from RIP to IP and compute the equilibrium candidate again by using the corresponding formula of the switched type and again verify the new candidate. Repeat this process until we get both valid $\tilde{x}_1^\star$ and $\tilde{x}_2^\star$. The description of this process can be found in Algorithm 1.

Algorithm 1 Algorithm for finding the equilibrium with 2 SPs

Suppose both SP1 and SP2 belong to RIP.
repeat
  Compute the equilibrium $\tilde{x}_1^\star$ and $\tilde{x}_2^\star$ according to the model combination.
  for $i = 1, 2$ **do**
    if SP i is of RIP and $\tilde{x}_i^\star < \frac{B_i C_i}{2W_i}$ **then**
      Switch the model of SP i to IP.
    end if
  end for
until No one switches its model
return $\tilde{x}_1^\star$ and $\tilde{x}_2^\star$

It can be proved that this algorithm can find the true equilibrium within a maximum of 4 iterations.

IV. Main Results

A. Consumer Surplus

1) Consumer Surplus versus C: The following theorem shows the relationship between C and consumer surplus.

Theorem 4 (Consumer surplus and C): Consider a market with two SPs. The consumer surplus at the equilibrium $CS(\tilde{x}_1^\star + \tilde{x}_2^\star)$ is a non-increasing function of both C_1 and C_2.

If we consider increasing C_i as reducing resources of SP i, then, unsurprisingly, fewer resources result in less consumer surplus. As with fewer resources, SP i would serve fewer users to reduce the potentially large congestion loss.

A numerical example is shown in Fig. 1. Notice that there is a flat area where the consumer surplus does not depend on C_1 and C_2. As we have discussed in Sect. III-B, C_1 and C_2 become inactive when both SPs are in M1-IP. The equivalent incumbent activity is so large that no SPs wants to use their shared bands when incumbent users are present, thus consumer surplus does not vary with C_1 and C_2.

2) Consumer Surplus with an Additional Band: Next we consider the scenario in which two SPs do not own any licensed shared band initially and there is a shared band of bandwidth W to be allocated. We consider to which SP should this shared band be allocated to maximize consumer surplus?

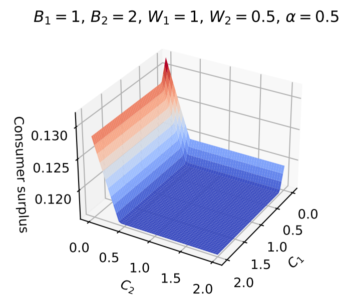


Fig. 1: Consumer surplus versus C_1 and C_2 with the indicated parameters.

Theorem 5 (Shared band allocation considering consumer surplus): Consider a market with 2 SPs. Suppose they only have access to their proprietary band with bandwidth $B_1 < B_2$ (and $W_1 = W_2 = 0$). If there is an additional shared band available with parameters W and C, the optimal allocation in terms of consumer surplus is:

1) If $C = 0$, allocating W to SP1 results in a larger consumer surplus.
2) For large enough C (such that it reaches the flat area as shown in Fig. 1), allocating W to SP2 results in a larger consumer surplus if the following inequality holds

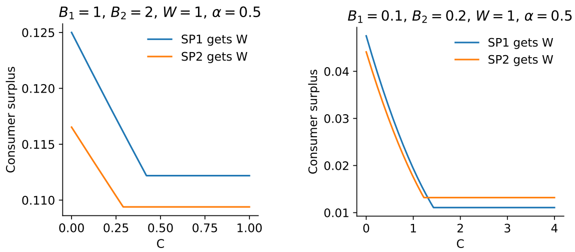
$$\frac{1(1-\alpha)W}{B_1 + B_2} - (B_1 + B_2 + W + 4) > 0. \qquad (16)$$

3) For other cases, if (16) holds, there exists a threshold $C^\dagger$ such that allocating W to SP1 is optimal for all $C \le C^\dagger$. Conversely, allocating to SP2 is optimal for all $C > C^\dagger$.

$C = 0$ means there is no incumbent activity thus the extra band W just acts like a proprietary band. Therefore, intuitively, allocating W to a smaller SP results in greater competition among SPs, benefiting consumers more. For large C, a larger SP is better able to use the shared band because it can use its larger proprietary bandwidth to better absorb the off-loaded traffic. The inequality in (16) gives a condition under which the larger SP would be preferred as C increases. It is worth noting that for (16) to hold, W needs to be significantly larger than B_i's — i.e., the amount of shared bandwidth added to the market would need to be much larger than the SP's existing proprietary bandwidths.[4]

Fig. 2 shows an example of the resulting consumer surplus in two different settings. We observe that as C decreases, the rate of increase in consumer surplus is greater when W is allocated to a smaller SP. This implies that regulators might aim to allocate shared bands to smaller SPs if they wanted larger consumer surplus boosts.

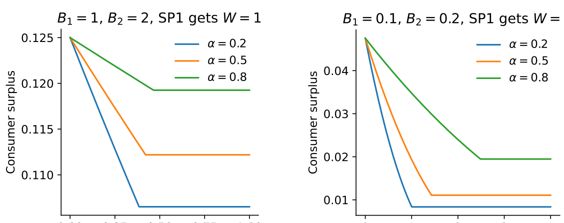
[4]For example, in the shared spectrum allocated in the CBRS band, this condition would not hold as only 70 MHz of spectrum was allocated as Priority Access Licenses (PALs), which is much less than the proprietary spectrum holding of commercial service providers.



(a) Smaller SP is preferred.

(b) Preference depends on C.

Fig. 2: Resulting consumer surplus with different allocation choices.



(a) $W \approx B_1, B_2$

(b) $W \gg B_1, B_2$

Fig. 3: Improvement in consumer surplus by allowing relaxed incumbent protection versus the availability of the shared band α.

3) Efficiency of Relaxed Protection: For large enough C, the relaxed incumbent protection is actually reduced to full incumbent protection. Next, we consider how much consumer surplus is gained by decreasing the value of C.

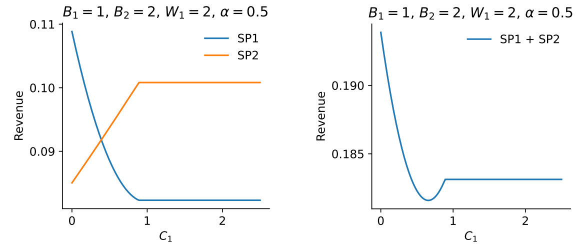
Fig. 3 shows the consumer surplus with $W \approx B_1, B_2$ and $W \gg B_1, B_2$ under different availability settings. Compared to the full incumbent protection (i.e., the flat part in Fig. 3), the gains are up to 5–17% and 144–468%, respectively. It is obvious that relaxation matters for small α since commercial users and incumbent users coexist for $1-\alpha$ portion of time. But for large enough W, large α's can still achieve a non-trivial gain in consumer surplus.

B. SP Revenue

Next we turn to how SP revenue changes if incumbent protection is relaxed.

Theorem 6 (Non-monotonicity of total SP revenue): Consider a market with two SPs and full incumbent protection (i.e., the revenue is given by the IP case in (15)). If the incumbent protection of one of them is relaxed, (the revenue is given by the RIP in (14)), there always exists a threshold denoted by $C^\dagger$ such that the total revenue will be lower than the status quo for all $C > C^\dagger$.

Theorem 6 reveals that a too conservative relaxation might hurt the total revenue. As illustrated in Fig. 4, the flat parts correspond to revenue with full incumbent protection, and the protection is relaxed as C_1 decreases. SP1 is the beneficiary of this relaxation so it gradually obtains more profits as C_1 decreases. However, SP2 is becoming less competitive so it keeps losing profits. Fig. 4b shows the non-monotonicity of the



(a) Individual revenue.

(b) Total revenue.

Fig. 4: Total revenue is not a monotonic function of C.

total revenue. At around 0.6–0.9, the total revenue is lower than the full incumbent protection case. We are essentially injecting more resources into the market as C_1 decreases but the market responds negatively. From the perspective of each SP:

- SP1 makes more profits since equivalently it is receiving more resources, thus it tends to serve more users than the status quo.
- SP2 loses profits since SP1 tends to serve more users resulting in a drop in the delivered price which directly hurts SP2's revenue.

As shown in Fig. 4a, SP2 loses profits at an approximately constant rate whereas SP1's rate of gaining profits increases gradually from a small value. Therefore, at some point, SP2's losses dominate. The reason is that the delivered price drop experienced by SP2 has a more direct impact on its revenue. It can be proved that the rate of losing profits is approximately constant thus SP2's revenue loss in Fig. 4a is approximately linear. However, SP1's revenue gain is not linear. To see this, we rearrange SP1's revenue (14) as follows

$$r_1 = \tilde{x}_1 \left(1 - \tilde{x}_1 - \tilde{x}_2 - \frac{1}{B_1 + W_1}(\tilde{x}_1 + x_C)\right), \quad (17)$$

$$x_C = C_1(1-\alpha)\left(1 - \frac{B_1 C_1}{4W_1\tilde{x}_1}\right), \quad (18)$$

where we can interpret x_C as the *equivalent additional user mass* introduced by C_1 as if the total bandwidth of SP1 is $B_1 + W_1$. SP1 does not serve these x_C users but they are accounted in SP1's latency. Note that x_C is valid only for $C_1 \leq \frac{2W_1\tilde{x}_1}{B_1}$ which follows from the domain of (14). For any $C_1 > \frac{2W_1\tilde{x}_1}{B_1}$, incumbent users are fully protected. It is clear that x_C is a quadratic function of C_1 and reaches the maximum at $C_1 = \frac{2W_1\tilde{x}_1}{B_1}$ as illustrated in Fig. 5. Since x_C decreases as C_1 decreases, SP1 serves fewer extra users than the status quo resulting in more profits. However, at around $C_1 = \frac{2W_1\tilde{x}_1}{B_1}$, $\frac{\partial x_C}{\partial C_1} \approx 0$ which means that changing C_1 does not have a large impact on SP1's revenue. Therefore, SP1's revenue gain will be dominated by SP2's revenue loss for C_1 close enough to $\frac{2W_1\tilde{x}_1}{B_1}$.

We can use the SP's revenue changes to gain insight into the outcome if a single shared band of spectrum was allocated via a second price auction [16]. Since in a second price auction truthful bidding is an equilibrium, we assume that each SP bids the revenue difference between winning and losing; the

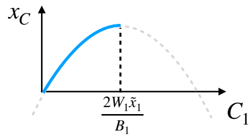
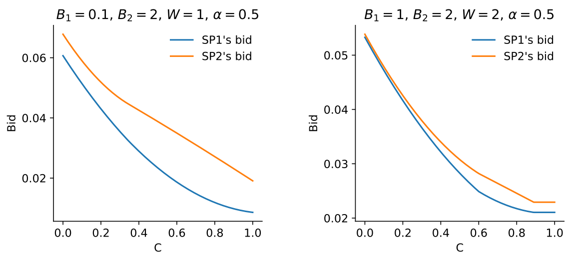


Fig. 5: Equivalent additional user mass introduced by incumbent activity.



(a) $B_1 \ll B_2$ (b) $B_1 \approx B_2$

Fig. 6: SP's bids versus C.

firm with the larger bid wins. Two numerical results are shown in Fig. 6. The SP with larger proprietary bandwidth wins the auction in both cases. As noted in Sect. IV-A2, allocating the band to the smaller SP will achieve higher consumer surplus. If maximizing consumer surplus is a goal, a regulator could subsidize smaller bidders.[5] The amount of subsidy should be at least the difference between the two SPs' bids, which from Fig. 6 would increase for larger values of C.

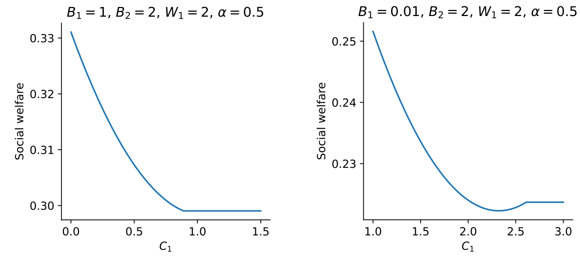
C. Social Welfare

Social welfare is the summation of consumer surplus and the total revenue of all SPs. As shown in Sect. IV-A1, consumer surplus is a non-increasing function of C. The total revenue, however, is not a monotonic function of C. Two examples of social welfare are shown in Fig. 7, representing the case of $B_1 \approx B_2$ and $B_1 \rightarrow 0$, respectively. In Fig. 7a, SP2's revenue loss is offset by SP1's revenue gain and the increasing consumer surplus, resulting in monotonically increasing social welfare. However, Fig. 7b shows that permitting relaxed protection might hurt the social welfare for some choices of C. In this example, though consumers always benefit from the relaxed protection, the revenue loss of SP2 is higher than the gain in the other two measures. Therefore, regulators may face a trade-off between overall welfare and consumer welfare.

V. Conclusions

We studied the market impacts of a model of spectrum sharing with relaxed incumbent protection. In this model, SPs reduce their traffic when the incumbent is present due to an increase in the congestion costs incurred during this time. Consumers always benefit from relaxation but it could adversely affect the revenue of SPs. The degree of relaxation requires careful selection as a too conservative relaxation could negatively impact the overall market or a too aggressive

[5] In practice, the FCC offers "bidder credits" to some firms participating in auctions.



(a) $B_1 \approx B_2$. (b) $B_1 \rightarrow 0$.

Fig. 7: Social welfare versus C.

relaxation could significantly hurt the revenue of SPs that do not have access to shared spectrum. Moreover, if an additional shared band is allocated through an auction, the regulator may consider subsidizing smaller SPs to give them a better chance of winning the auction which may encourage innovation and stimulate competition.

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