Towards imaging-based quantum optomechanics

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ABSTRACT

Quantum optomechanics has led to advances in quantum sensing, optical manipulation of mechanical systems, and macroscopic quantum physics. However, previous studies have typically focused on dispersive optomechanical coupling, which modifies the phase of the light field. Here, we discuss recent advances in "imaging-based" quantum optomechanics – where information about the mechanical resonator's motion is imprinted onto the spatial mode of the optical field, akin to how information encoded in an image. Additionally, we find radiation pressure backaction, a phenomenon not usually discussed in imaging studies, comes from spatially uncorrelated fluctuations of the optical field. First, we examine a simple thought experiment in which the displacement of a membrane resonator can be measured by extracting the amplitude of specific spatial modes. Torsion modes are naturally measured with this coupling and are interesting for applications such as precision torque sensing, tests of gravity, and measurements of angular displacement at and beyond the standard quantum limit. As an experimental demonstration, we measure the angular displacement of the torsion mode of a Si_3N_4 nanoribbon near the quantum imprecision limit using both an optical lever and a spatial mode demultiplexer. Finally, we discuss the potential for future imaging-based quantum optomechanics experiments, including observing pondermotive squeezing of different spatial modes and quantum back-action evasion in angular displacement measurements.

Keywords: cavity optomechanics, quantum optomechanics, optical lever, quantum imaging, nanomechanics

1. INTRODUCTION

The field of quantum optomechanics[1] has focused on the coupling between mechanical and optical degrees of freedom, including the effects of radiation pressure on mechanical motion. Notable demonstrations include manipulation of mechanical degrees of freedom through optical spring and damping effects,[2] ground state preparation of mechanical modes via coherent cavity cooling,[3] and measurements at and below the standard quantum limit.[4] Key advances facilitated by cavity optomechanics include gravitational wave detectors[5] and emerging quantum technologies such as optical-to-microwave transducers.[6]

Quantum optomechanics typically focuses on the dispersive interaction between mechanical and optical modes, characterized by the coupling of mechanical modes to the phase of the light field. However, some

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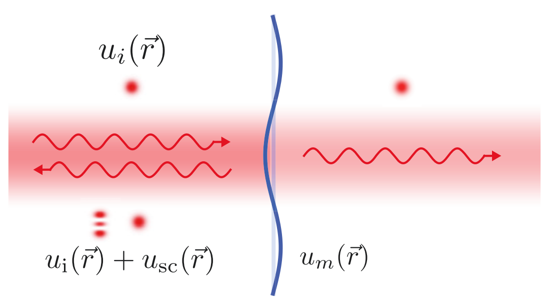

Figure 1. Spatial optomechanical coupling. Light reflected from a surface with modeshape u_m has an orthogonal spatial mode u_{sc} added in the reflected field. The amplitude of the scattered modes can be detected and used to estimate the mechanical mode displacement.

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mechanical modes, including metrologically significant torsion modes,[7] may only experience weak dispersive coupling in some situations due to their net-zero center of mass motion. Inspired by the discovery of high-Q torsion modes in silicon nitride (Si_3N_4) nanoribbons,[8] we explore coupling nanomechanical torsion modes to the spatial modes of an optical field. Drawing on inspiration from the field of quantum imaging,[9,10] we examine how to optimally extract the mechanical displacement from the spatial modes.[11] Unlike most studies within quantum imaging, we study the effects of quantum backaction, which is due to the spatially uncorrelated intensity noise of the optical field. Here, we present preliminary steps towards imaging-based quantum optomechanics experiments, including quantum imprecision noise limited readout of the motion of a torsion mode with an optical lever and spatial mode demultiplexer. In the future, by utilizing low mass, high-Q nanomechanical oscillators, we hope to move into the radiation pressure-dominated regime and observe effects such as quantum backaction and ponderomotive squeezing.

2. SPATIAL OPTOMECHANICAL COUPLING AND RADIATION PRESSURE FORCE

First, we examine how light reflected from a surface introduces additional spacial modes to the reflected field, illustrated in Fig. 1. The reflection of light from a surface with modeshape $u_m(x, y)$ produces a position-dependent phase shift $e^{ikA_m(t)u_m(x,y)}$, where $A_m(t)$ is the amplitude of the mode and k is the wavenumber. In the limit of small displacements ($A_m \ll \lambda$), the reflected field $u_r(x, y)$ is a sum of the initial mode $u_i(x, y)$ plus the scattered mode $u_{\rm sc}(x, y)$, which depends on the mechanical modeshape:

$$u_r(x, y) = A_i u_i(x, y)e^{iku_m(x,y)} \approx u_i(x, y)(1 + iku_m(x, y)) = A_i u_i(x, y) + A_{\rm sc}u_{\rm sc}(x, y). \tag{1}$$

In general, the scattered mode is orthogonal to the input mode. Therefore, by measuring $A_{\rm sc}$, one can make an estimate of the mechanical displacement. We characterize the strength of the coupling via

$$\beta = \int u_i(x, y)u^*_{\rm sc}(x, y)\phi(x, y)dxdy, \tag{2}$$

which can be then used to express the shot noise limited imprecision noise

$$S_z^{\rm imp} = \frac{\hbar c\lambda}{16\pi\beta^2 P_i\eta} \tag{3}$$

which is found by noting the fundamental limiting noise source is the vacuum fluctuations of the scattered mode.

The radiation pressure force on the mechanical mode can be found via[12]

$$F_{\rm RP}(t) = 2\hbar k\langle|E(x, y, t)|^2 u_m(x, y, t)\rangle, \tag{4}$$

where $\langle...\rangle$ denotes an average over space. Evaluating Eq. 4 for a coherent input state, the fluctuating component includes a contribution from the amplitude quadrature noise $X^{\rm vac}_{\rm sc}$:

$$F_{\rm QBA}(t) = 2\hbar k\beta E_i X^{\rm vac}_{\rm sc}(t), \tag{5}$$

which is responsible for the quantum backaction. In terms of a power spectral density, the backaction force is then

$$S_F^{\rm QBA} = \frac{16\pi\hbar\beta^2 P_i}{c\lambda}. \tag{6}$$

Notably, Eqs. 3 and 6 satisfy the imprecision-backaction product[1]

$$S_z^{\rm imp} S_F^{\rm QBA} \geq \hbar^2, \tag{7}$$

signaling that an imaging measurement can add a minimal amount of noise in a measurement of the mechanical displacement.

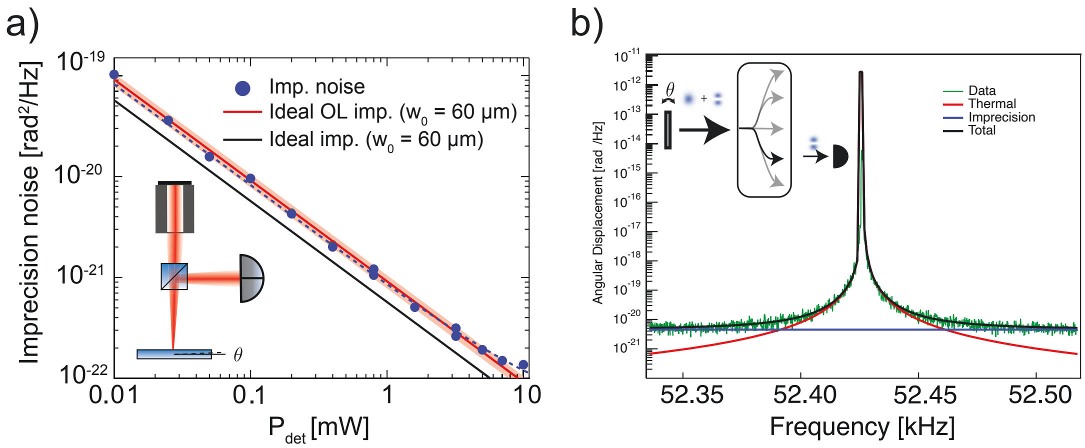

Figure 2. Imaging-based measurements of a torsion mode. a) Optical lever with split photodetector measurements near the quantum imprecision noise limit (red line). The blue points are the measured imprecision noise while sweeping the optical power on the photodetector. The black line is the absolute quantum imprecision noise limit for an input coherent state. Notably, the optical lever is a factor of $\pi/2$ away from this limit. b) Spatial mode demultiplexer measurement of the torsion mode. The power in the HG_{10} mode is detected to determine the mechanical mode amplitude.

As an example, we look at measuring the angular displacement of a torsion oscillator with two different methods, both of which rely on sorting the spatial modes of the reflected field. Specifically, we measure the torsion mode of a Si_3N_4 nanoribbon. The modeshape of the fundamental torsion mode is given by[8]

$$\theta(y) = \sin(\pi y/L) \tag{8}$$

where θ is the angle of rotation around the y-axis and L is the length of the ribbon. We perform our measurements in the center of the ribbon, where $\theta(y)$ is maximized, so that an optical field reflected off the ribbon with an amplitude reflectivity of r_f experiences a phase shift $e^{ik\theta x}$. For an incident field in the fundamental Hermite-Gauss (HG) mode, the reflected field is

$$E(x,y,z) = A_{00} r_\mathrm{f} \frac{\sqrt{2/\pi}}{w(z)} e^{\frac{-(x^2+y^2)}{w^2(z)}} e^{ik\theta x} \approx (1 + ik\theta x) A_{00} r_\mathrm{f} \frac{\sqrt{2/\pi}}{w(z)} e^{\frac{-(x^2+y^2)}{w^2(z)}}, \tag{9}$$

where A_{00} is the incident electric field amplitude, with power equal to $|A_{00}|^2 = P$, $w(z)$ is the spot size and $k = 2\pi/\lambda$ is the wavenumber, and we have assumed that the displacement relative to the wavelength λ is small. In particular, a field centered on the nanoribbon experiences a net zero phase shift, implying a negligible dispersive optomechanical coupling $G = \partial\omega/\partial z \approx 0$, where ω is the optical field frequency. Instead, we can estimate θ by examining the spatial mode of the optical field. Writing the reflected field as an expansion of HG modes shows that the reflection produces a superposition of two orthogonal modes

$$\begin{aligned} E(x,y,z) &= A_{00} r_f \frac{\sqrt{2/\pi}}{w(z)} e^{\frac{-(x^2+y^2)}{w^2(z)}} + A_{10}^{\theta} \frac{2x}{w(z)} \frac{\sqrt{2/\pi}}{\pi w(z)} e^{\frac{-(x^2+y^2)}{w^2(z)}}, \\ &= A_{00} r_f E_{00}(x,y,z) + A_{10}^{\theta} E_{10}(x,y,z), \end{aligned} \tag{10}$$

where

$$A_{10}^{\theta} = ikw_0 \theta A_{00} r_\mathrm{f}, \tag{11}$$

is the amplitude of the HG_{10} mode and $E_{00,10}(x,y,z)$ describe the modeshapes of the $HG_{00,10}$ modes.

3. EXPERIMENT

In Fig. 2, we show measurements of the torsion mode displacement via two different methods: first via an optical lever with a split photodetector along with utilizing a spatial mode demultiplexer that sorts modes in the Hermite-Gauss basis. The inset of Fig. 2a shows a simplified diagram of the optical lever setup, in which laser light is focused onto the nanoribbon and collected on a split photodetector. The torsion mode applies an angular displacement to the reflected field, and the split photodetector measures the lateral displacement after the beam propagates the lever arm distance. The split photodetector can be seen as a mode sorter that measures the amplitude of the HG_{10} mode, albeit with a quantum efficiency that is a factor of $2/\pi$ below the quantum limit.[13] In turn we are able to measure the angular displacement with a shot noise limited sensitivity of[8]

$$S_\theta^{\text{imp}} = \frac{1}{w_0^2}\frac{\hbar c\lambda}{8P}. \tag{12}$$

In Fig. 2a, we show measurements of the torsion mode near the limit of Eq. 12 when accounting for the loss between the nanoribbon and the detector.

We then use a spatial mode demultiplexer (Cailabs Proteus-C) to measure the torsion mode displacement. We use a photodetector to make a direct detection measurement of the HG_{10} intensity, which is proportional to the angular displacement (inset of Fig. 2b). We are able to observe the thermal motion of the torsion mode near the frequency predicted by a finite element analysis model. In Fig. 2b, we fit the observed signal with a model of the thermal noise, which follows a Lorenzian lineshape,[14] plus corresponding a constant white noise term corresponding to the imprecision noise.

4. CONCLUSION

We discuss readout of mechanical motion via spatial coupling to the optical field, in which the information about the displacement and backaction are in orthogonal spatial modes of the field. Additionally, we experimentally demonstrate readout of a nanomechanical resonator with this type of coupling via two different methods. In the first, the angular displacement of a torsion mode is measured using a split photodetector, and in the second, we move to a more general scheme and measure the angular displacement by utilizing a spatial mode demultiplexer. In the future, we aim to extend this work into the regime where radiation pressure backaction plays an important role, which is an important consideration for sensing applications and can lead to new physics such as ponderomotive squeezing of higher-order spatial modes.

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REFERENCES

[1] Aspelmeyer, M., Kippenberg, T. J., and Marquardt, F., "Cavity optomechanics," *Rev. Mod. Phys.* **86**(4), 1391–1452.

[2] Corbitt, T., Chen, Y., Innerhofer, E., Müller-Ebhardt, H., Ottaway, D., Rehbein, H., Sigg, D., Whitcomb, S., Wipf, C., and Mavalvala, N., "An All-Optical Trap for a Gram-Scale Mirror," *Phys. Rev. Lett.* **98**(15), 150802.

[3] Chan, J., Alegre, T. P. M., Safavi-Naeini, A. H., Hill, J. T., Krause, A., Gröblacher, S., Aspelmeyer, M., and Painter, O., "Laser cooling of a nanomechanical oscillator into its quantum ground state," **478**(7367), 89–92.

[4] Mason, D., Chen, J., Rossi, M., Tsaturyan, Y., and Schliesser, A., "Continuous force and displacement measurement below the standard quantum limit," *Nat. Phys.* **15**(8), 745–749.

[5] Collaboration, L. S. and Collaboration, V., "Observation of Gravitational Waves from a Binary Black Hole Merger," *Phys. Rev. Lett.* **116**(6), 061102.

[6] Barzanjeh, S., Xuereb, A., Gröblacher, S., Paternostro, M., Regal, C. A., and Weig, E. M., "Optomechanics for quantum technologies," *Nat. Phys.* **18**(1), 15–24.

[7] Adelberger, E., Gundlach, J., Heckel, B., Hoedl, S., and Schlamminger, S., "Torsion balance experiments: A low-energy frontier of particle physics," *Progress in Particle and Nuclear Physics* **62**(1), 102–134.

[8] Pratt, J. R., Agrawal, A. R., Condos, C. A., Pluchar, C. M., Schlamminger, S., and Wilson, D. J., "Nanoscale Torsional Dissipation Dilution for Quantum Experiments and Precision Measurement," **13**(1).

[9] Lugiato, L. A., Gatti, A., and Brambilla, E., "Quantum imaging," **4**(3), S176.

[10] Magaña-Loaiza, O. S. and Boyd, R. W., "Quantum imaging and information," **82**(12), 124401.

[11] Gessner, M., Treps, N., and Fabre, C., "Estimation of a parameter encoded in the modal structure of a light beam: A quantum theory," **10**(8), 996.

[12] Pinard, M., Hadjar, Y., and Heidmann, A., "Effective mass in quantum effects of radiation pressure,"

[13] Barnett, S., Fabre, C., and Maıtre, A., "Ultimate quantum limits for resolution of beam displacements," **22**(3), 513–519.

[14] Saulson, P. R., "Thermal noise in mechanical experiments," **42**(8), 2437–2445.