A Coupled Game Theory and Lyapunov Optimization Approach to Electric Vehicle Charging at Fast Charging Stations

Mohammad Hossein Abbasi, Ziba Arjmandzadeh, Jiangfeng Zhang, Venkat Krovi, Bin Xu, Dillip Kumar Mishra

Abstract—**The development of electric vehicle (EV) charging stations has been a key consideration for enabling the evolution of EV technology and continues to support the fosterage of this technology. Notably, fast charging enhances the EV user's adaptability by reducing the charging time and supporting long-mile travel. The optimal operation and erratic power demand of a fast charging station (FCS) are still challenging. It is necessary to understand EV charging scheduling and FCS management, which can jointly overcome the problem of EV users on account of optimal operation. However, joint optimization needs detailed future information, which is a formidable task for prediction. This paper aims to address the joint optimization issue using combined game theory and the Lyapunov optimization approach. This hybrid approach could ease the data forecast requirement and minimize the operating costs of FCSs while optimally dispatching EVs to FCSs and satisfying their energy demand. Further, the problem is decomposed into three subproblems. The first subproblem addresses a network of FCSs that try to maximize their revenue through a dynamic pricing game with EV customers who have different behavioral responses to the prices. The pricing game determines the electricity selling prices in a distributed manner as well as the energy demand of users. Subsequently, EVs are assigned to local FCSs, taking into account the distance from and the queue at the stations. Finally, the third subproblem exploits Lyapunov optimization to control the operation cost of each FCS, considering the impact of demand charges. In this paper, the proposed method is validated through a numerical analysis using the real data of FCSs in Boulder, Colorado. Moreover, the presented results revealed that the proposed method is efficient regarding dynamic pricing and optimal allocation of EVs to stations.**

Index Terms—**EV Charging Scheduling, Fast Charging Station Optimization, Game Theory, Lyapunov Optimization.**

I. Introduction

OVER the last few decades, the energy sector has undergone unprecedented transformation concerning several factors, including energy security, energy equity, and environmental sustainability. Indeed, several countries have fostered their development in the transportation sector to achieve Net Zero through electric vehicle (EV) technology [1]. Transportation sectors driven by electricity could significantly help decrease gasoline use, and contrary to gasoline, EVs would be plugged into the electric station, which creates pathways for devising clean and sustainable energy. Moving to an electric transportation system, the difficult part of EV technology lies in the charging speed, whereas gasoline vehicles have a very short refueling time. This difficulty is addressed through fast charging stations (FCSs), which improve charging speed and enable long-mile travel. In this respect, managing EV demand and FCS supply through joint optimization of EV scheduling and FCS management at the individual station level and across a regional system of FCSs is critical [2]. However, joint management of EV charging scheduling and FCSs requires detailed future information, which is challenging to obtain. Therefore, we propose a hybrid approach for optimizing EV charging scheduling and FCS management to tackle the above challenges while relaxing the need for future information.

As FCS is a significant part of EV technology, which has increased the attention from academia over the last few years, there have been a number of investigations involving FCSs. The authors in [3] design an FCS with adaptable charging ports aiming to maximize the station's profit and user satisfaction. However, fixed selling prices are considered in [3]. The research in [4] develops a distributed EV charging mechanism where EVs decide whether to charge their batteries based on information, such as the charging probability of other EVs, electricity prices, etc. However, the work in [4] relies on predicted data and does not optimize the FCS management problem. Ref. [5] presents a novel method to operate an FCS with a power cap policy while maintaining a high quality of service experienced by clients. A hybrid charging management strategy to address the under-voltage problem caused by FCSs at a radial distribution network is explored in [6]. EVs choose to participate in the voltage regulation plan or not; they receive a subsidy upon participation. The methodologies in [5, 6] assume fixed electricity prices, as opposed to dynamic prices, and require considerable forecasted factors such as arrival time, charging demand, and queue at stations. In [7], an optimal charging planning strategy for an electric bus system is proposed considering rapid on-route and terminal charging infrastructure. It is demonstrated that increasing buses' battery capacity reduces the total system cost to some extent, while after a certain threshold, the battery capacity does not affect costs. The study is limited to electric buses and cannot be

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extended to other EV types. The work in [8] proposes an approach to reduce the operation costs of an FCS and the waiting time of EVs. However, Ref. [8] does not consider energy storage system (ESS) or renewable generation at the FCS.

Dynamic pricing is found to be effective in increasing FCS profit [2]. In addition, joint optimization of FCS management and EV charging scheduling deserves more attention, and its potentials to improve the results require better exploration [9]. The researchers in [2] present a real-time management strategy for a system of FCSs using dynamic pricing, where both EV routing and FCS optimization problems are solved. However, the study lacks consideration of the operation costs of FCSs. The authors in [10] propose profit-optimal management of an FCS considering energy storage unit. Despite using dynamic pricing, EV allocation to stations is missing in [10] due to the consideration of a single FCS. The proposed strategy in [11] finds the best FCS for EVs to visit while trying to maximize the profit of FCSs. The FCSs compete with each other using dynamic prices determined in an auction-based fashion where FCSs set a base selling price, and then EVs bid higher prices to gain charging priority based on their arrival time. The problem is designed so that no queue is built up at the stations, but the catch is that EVs may finally purchase electricity at higher prices than the offered prices by FCSs. The study in [12] explores an approach based on game theory to manage a charging station optimally. The first game dynamically determines the selling prices considering user response to prices. On the other hand, the second game distributes power among charging EVs. Solely analyzing the approach in the case of slow parking lot charging is the limitation of this study. An optimal navigation planning algorithm is presented in [13], where EVs are assigned to FCSs and charging ports at each FCS through Harris Hawks optimization [14] and fuzzy inference system, respectively. Recent work in [15] investigates multi-agent DDPG reinforcement learning to solve the problem of maximizing the revenue of FCSs, fulfilling the charging demand of EVs, and optimally dispatching EVs to stations. The study supposes the selling prices are fixed, and the transportation model requires predicting EV flow.

Previous studies rely on forecasted information, such as arrival time and arrival state of charge (SOC), which suffer from the limited accuracy of the existing forecast-based methods. Further, the accuracy of the forecasted data impacts the results of conventional optimization algorithms. Thus, the Lyapunov optimization approach [16] is employed that relaxes the need for future information and decomposes the problem into subproblems such that the problem for each time step is separately solved. Being independent of future forecasts makes the solution robust to prediction inaccuracies, while the decomposition lowers the compute time. Lyapunov optimization approach has been employed in various studies for optimization and control purposes. Ref. [17] applies Lyapunov optimization to control the usage of the battery energy storage of a commercial building. The authors in [18] propose a Lyapunov optimization-based model to maximize a charging station's long-term profit through a dynamic pricing mechanism. However, the impact of demand charges, renewable generation, as well as ESS on the FCS profit is ignored. The assignment of EVs to charging stations is proposed based on the Lyapunov optimization technique in [19]. The goal is to minimize the average time the user spends from requesting the charging service to accessing it. In [20], the Lyapunov optimization is used to optimally divide the available power among charging EVs at an FCS.

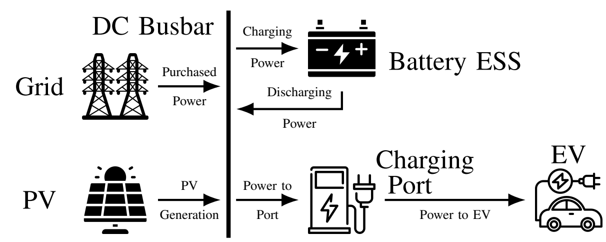

Fig. 1. The components of each FCS in the system under study.

Despite the development presented by the foregoing attempts, the existing study mainly emphasized joint optimization of EV charging planning, EV charging navigation, and FCS cost minimization. As noted, separate studies have been reported on EV charging scheduling and EV assignment to stations on account of cost optimization. This lack of joint optimization can create an imbalance between FCSs supply and EVs demand [2]. On the other hand, FCS cost minimization through EV charge planning and its navigation relies on future forecasting, which needs large key information that is challenging to predict. Considering the above challenges, this paper proposes a combined approach to the optimization problem as game theory and Lyapunov for EV charging scheduling, navigation, and FCS cost (of purchasing power) minimization. The key contribution of this paper is as follows:

- A hybrid game theory-Lyapunov optimization algorithm is proposed that does not require future forecasting, and the convergence of the Lyapunov optimization approach is proved.
- The developed Lyapunov approach decomposes the original FCS management problem into many subproblems, where each subproblem has far fewer variables and can be calculated very fast without relying on future forecast data.
- Both energy and maximum demand charges are factored in when modeling the FCS operation cost problem. Hence, the problem is numerically analyzed for one month to reflect the impact of demand charges on costs.

The remaining sections of the paper are structured as follows. The problem formulation is introduced in Section II. In Section III, extensive numerical results are explored and discussed. Finally, conclusions are drawn in Section IV.

II. Problem Formulation

In this paper, the dynamic pricing problem of a network of FCSs is solved with a focus on minimizing the operation cost of stations. Each FCS is equipped with a PV system and an ESS, as shown in Fig. 1. The proposed problem of this manuscript comprises three subproblems as visualized in

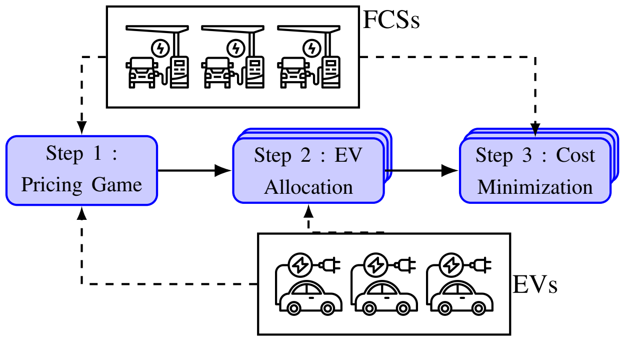

Fig. 2. Step 1 is solved once, accounting for all EVs and FCSs. Step 2 and Step 3 are separately solved for each EV and FCS, respectively. The solid lines show the order of running subproblems, while the dashed lines point to the subproblems where EVs/FCSs are targeted.

Fig. 2. The first subproblem addresses the dynamic pricing part of the model. Using a non-cooperative Stackelberg game, the stations increase the selling price, and the requesting EVs react to the higher price by reducing their charging demand. Eventually, a selling price is found beyond which the overall revenue of the FCSs would drop. In the second subproblem, the EVs are allocated to the FCSs based on distance and queue at each station. Lastly, the third subproblem deals with minimizing the cost of purchasing power at each FCS. The solution is obtained by utilizing Lyapunov optimization, eliminating the need for future information about uncertain model parameters.

A. Pricing Game Model

This subproblem determines the stations' selling price and the EV's final energy demand. The objectives of FCSs and EVs conflict in that each FCS aims to maximize its revenue, while EVs intend to minimize the cost of purchasing electricity. The interaction between EVs and FCSs is modeled by a non-cooperative Stackelberg game where FCSs are leaders, and EVs are followers. The procedure of the pricing game is as follows. Firstly, we assume all FCSs belong to a company who offers the same selling price. Therefore, the selling prices of all FCSs are identical. Every EV that intends to visit a station in the next time step submits its energy requirement, which is a rough estimation of its energy need. At this point, the EV has not chosen a station yet, and it may visit any station. On the contrary, EVs planning to visit an FCS during the current time step should adjust their demand according to the already settled selling price. We focus on the customers visiting a station in the following time step. The original approximate energy requirement of such customers, $E^o_{n,t}$, is expressed as,

$$E^o_{n,t} = (z_n^{\text{end}} - z_{n,t})C_n, \tag{1}$$

where z_n^{end} is SOC of nth EV at the end of charging session, $z_{n,t}$ shows the initial SOC of nth EV whose charging event occurs at time slot t, and C_n is nth EV battery capacity.

The goal of each FCS is to maximize its profit, which is stated as follows,

$$\Gamma_{k,t} = \sum_{n=1}^{N_{k,t}} \left(\lambda_t^s E_{n,t} - \lambda_t^p E_{n,t}\right), \tag{2}$$

where $\Gamma_{k,t}$ represents the profit of kth FCS obtained during t, $N_{k,t}$ denotes the number of EVs that visit station k at time slot t, λ_t^s and λ_t^p are the price of electricity sold to the EVs by the FCS and the purchasing price from the grid at time t, respectively, and $E_{n,t}$ is the delivered energy to nth EV at t.

In addition, each EV tries to minimize its associated charging cost. In order to model EV price-sensitive behavior, the following utility function is employed [12],

$$\Omega_{n,t} = -\frac{1}{2} S_{n,t} E_{n,t}^2 + (\lambda_n^{\max} - \lambda_t^s) E_{n,t}, \tag{3}$$

where $S_{n,t}$ is the client's sensitivity to selling price, and $\lambda_n^{\max}$ displays the maximum acceptable price beyond which the driver opts out of utilizing the charging station. By taking the derivative of (3) with respect to $E_{n,t}$, the point where $\Omega_{n,t}$ is maximum is found,

$$E^*_{n,t} = \frac{\lambda_n^{\max} - \lambda_t^s}{S_{n,t}}. \tag{4a}$$

Hereafter, it is assumed that user behavior is optimal, i.e., $E_{n,t} = E^*_{n,t}$. Note that $E_{n,t} \le E^o_{n,t}$. This charging energy is computed based on EV sensitivity to prices and dynamic pricing. The user's price sensitivity is presented as,

$$S_{n,t} = \frac{S_n^b}{(z_n^{\text{end}} - z_{n,t}) B_{n,t}}, \tag{4b}$$

$$S_n^b = \frac{\lambda_n^{\max} - \lambda_n^b}{C_n}, \tag{4c}$$

$$B_{n,t} = \begin{cases} \dfrac{e^{\alpha_{n,t}} - 1}{e - 1}, & \text{HS} \\ \alpha_{n,t}, & \text{MS} \\ \ln[\alpha_{n,t}(e-1) + 1], & \text{LS} \end{cases}, \tag{4d}$$

$$\alpha_{n,t} = \max\left(\min\left[1 - \frac{\lambda_t^s - \lambda_n^b}{\lambda_n^{\max} - \lambda_n^b}, 1\right], 0\right), \tag{4e}$$

where S_n^b is the base sensitivity of the EV driver, λ_n^b indicates the minimum price when the EV driver becomes sensitive to the selling price by reducing its energy demand, and $B_{n,t}$ denotes the behavioral response of the EV to the prices. In (4d), three behavioral responses are considered, namely, high sensitive (HS), medium sensitive (MS), and low sensitive (LS). Lastly, note that $\alpha_{n,t}, B_{n,t} \in [0, 1]$.

By substituting (4b) and (4c) into (4a), the following expression is obtained for the settled energy demand of nth EV based on λ_n^b and the electricity selling price as follows,

$$E_{n,t} = \begin{cases} \dfrac{\lambda_n^{\max} - \lambda_t^s}{\lambda_n^{\max} - \lambda_n^b} (z_n^{\text{end}} - z_{n,t}) C_n B_{n,t}, & \lambda_t^s > \lambda_n^b \\ (z_n^{\text{end}} - z_{n,t}) C_n, & \lambda_t^s \le \lambda_n^b \end{cases}, \tag{5}$$

which is equivalent to

$$E_{n,t} = \begin{cases} \dfrac{\lambda_n^{\max} - \lambda_t^s}{S_{n,t}}, & 0 \le B_{n,t} < 1 \\ (z_n^{\text{end}} - z_{n,t}) C_n, & B_{n,t} = 1 \end{cases}. \tag{6}$$

The concavity of $\Omega_{n,t}$ and the linearity of $\Gamma_{k,t}$ ensures the existence and uniqueness of the Nash equilibrium [21]. The dynamic pricing game can be iteratively solved as represented in Algorithm 1.

Algorithm 1 Pricing Game

Initialization:

1: price = initial price
2: set $\Delta\lambda$
3: $\mathit{\Gamma}_t = 0$

Loop:

1: **while** True **do**
2: price $\leftarrow$ price + $\Delta\lambda$
3: **for** all EVs **do**
4: collect $E_{n,t}$ from user
5: **end for**
6: **if** $\mathit{\Gamma}_t$ is reduced **then**
7: return previous $E_{n,t}$ and price $\leq$ price $-$ $\Delta\lambda$
8: terminate
9: **end if**
10: **end while**

The pseudocode of the iterative process to obtain the Nash equilibrium is outlined in Algorithm 1. The algorithm is initialized by setting the selling price and considering a step size to change it. The initial prices are not less than the purchasing prices because that would mean negative profit, i.e., loss, for the FCS. In line 3 of initialization, $\mathit{\Gamma}_t = \sum_k \mathit{\Gamma}_{k,t}$. We use the sum of the profit of all FCSs because, at this point, EVs are not assigned to stations. In other words, each EV may visit any station. Afterward, the selling price is raised by $\Delta\lambda$, the energy demand of EVs is updated, and $\mathit{\Gamma}_t$ is recomputed. The iterative procedure continues until the maximum $\mathit{\Gamma}_t$ is reached. Note that $\Delta\lambda$ is the increment in the sales price λ^s_t, and it is consistent with the time step. That is, λ^s_t cannot be less than the electricity price at that time step, while $\Delta\lambda$ is a small increment of ¢0.1/kWh used to update the sales price, i.e., $\lambda^s_t \leftarrow \lambda^s_t + \Delta\lambda$ in each iteration. It is worth mentioning that Algorithm 1 does not require the possibly private information of EVs, such as SOC.

Algorithm 1 can be executed automatically through an app. The consumer only needs to set their price sensitivity level, i.e., select one of the options in Eq. (4d), and choose the maximum price at which they are willing to charge, i.e., $\lambda^{\max}_n$. After announcing that the said customer wants to visit a station in the next hour and determining the initial demand $E^o_{n,t}$ in (1), the program within the app updates the energy demand until maximum $\mathit{\Gamma}_t$ is reached.

B. EV Assignment to FCSs

This subsection allocates EVs to FCSs according to the distance between the EV and FCS and the station queue. Similar to the previous subproblem, the problem is solved iteratively. It is assumed that there are no queues at the stations at the beginning of the day. Then, customers start to submit their requests to visit the stations. The assignment of EVs to stations is performed with the help of the multinomial logit model that finds the best station to which an EV is assigned based on the value of $\pi_{n,k}$ [2]. The selected station has the highest $\pi_{n,k}$,

$$\pi_{n,k} = \frac{e^{\beta_1 d_{n,k} + \beta_2 q_k}}{\sum_{j=1}^{N_s} e^{\beta_1 d_{n,j} + \beta_2 q_j}}, \tag{7}$$

where β_1 and β_2 are weighting coefficients, $d_{n,k}$ shows the distance between nth EV and kth FCS, q_k indicates the queue at station k, and N_s denotes the total number of stations that nth EV can reach based on its remaining battery charge. Note that (7) is independent from the electricity price as all FCSs offer identical selling prices.

C. Operation Cost Minimization

In this subsection, we employ the Lyapunov optimization approach to minimize the costs incurred by the FCSs for purchasing power. Additionally, this subsection focuses only on managing a single station, as the previous section has allocated EVs to each station. Thus, the presented algorithm in this subsection is performed separately for each FCS. Lyapunov optimization relaxes the need for future information and further decomposes the problem into subproblems that are independently solved [20]. The former property of this method is beneficial in cases where the future cannot be accurately predicted, while the latter characteristic enhances the speed of solving the problem. More importantly, Lyapunov optimization mathematically guarantees its suboptimal solution is within a controllable boundary around the global optimal solution.

1) Formulation: Since FCS costs depend on demand charge and energy charge, the objective function is constructed considering total station energy demand and maximum power demand. The problem is formulated in a general format as follows. For station k we have,

$$\min \quad \frac{1}{T}\mathbb{E}\left\{\Delta t \sum_{t=1}^{T} \lambda^p_t P_t + \lambda^d \max\left[P_t | t = 1, \cdots, T\right]\right\}, \tag{8a}$$

subject to

$$P_t = \sum_{i=1}^{N_p} P_{i,t} - P^r_t + P^c_t - P^d_t, \quad \forall t \tag{8b}$$

$$\sum_{n=1}^{N^i_e} E^i_{n,t}\sigma_{n,i} = \eta P_{i,t}\Delta t, \quad \forall t, i \tag{8c}$$

$$\sum_{i=1}^{N_p}\sum_{n=1}^{N^i_e} E^i_{n,t}\sigma_{n,i} = \sum_{n=1}^{N_{k,t}} E_{n,t}, \quad \forall t \tag{8d}$$

$$\sum_{i=1}^{N_p} \sigma_{n,i} = 1, \quad \forall n \tag{8e}$$

$$\sigma_{n,i} \in \{0, 1\}, \quad \forall n, i \tag{8f}$$

$$0 \leq P^r_t \leq P^R_t, \quad \forall t \tag{8g}$$

$$0 \leq P_t \leq P^s_t, \quad \forall t \tag{8h}$$

$$0 \leq P_{i,t} \leq P^b_{i,t}, \quad \forall t, i \tag{8i}$$

$$0 \leq P^c_t, P^d_t \leq P^{\max}, \quad \forall t \tag{8j}$$

$$E^{\min} \leq E^{\text{ESS}}_t \leq E^{\max}, \quad \forall t \tag{8k}$$

$$E^{\text{ESS}}_{t+1} = E^{\text{ESS}}_t + \left(P^c_t\eta - \frac{P^d_t}{\eta}\right)\Delta t, \quad \forall t \tag{8l}$$

where P_t^c, P_t^d, P_t^r, $P_{i,t}$, $E_{n,t}^i$ and $\sigma_{n,i}$ are the problem's decision variables. $\mathbb{E}\{\cdot\}$ means expectation, t shows time, P_t is the FCS power demand, λ^d indicates demand charge rate in \$/kW, N_p is the number of charging ports at the FCS, $P_{i,t}$ represents the charging power of an EV drawn from the ith charging port, P_t^r expresses the renewable power generation during t at the FCS, P_t^c and P_t^d are charging and discharging power of ESS, respectively, N_e^i is the number of EVs that connect to port i to be charged, $E_{n,t}^i$ displays the energy demand of n-th EV that uses port i, P_t^R exhibits the maximum renewable generation, P_t^s displays the available power of the station at t, $P_{i,t}^b$ indicates the available power of charging port i at time instance t, $P^{\max}$ denotes (dis)charging limit of ESS, E_t^{ESS} presents the energy stored in ESS which is limited to $E^{\min}$ and $E^{\max}$, and the charging and discharging efficiencies of ESS are assumed to be equal and both are η.

The objective function in (8) is the expectation of monthly station costs. The loss cost of ESS is not considered in (8) because it is much smaller than electricity purchasing price (see Fig. 6 of [22]). Constraints (8b) state the station power is equal to the sum of the powers of all charging ports and the charging power of ESS minus the delivered renewable power and discharging power of the ESS. Moreover, (8c) states that the energy demand of all EVs who use port i is equal to the energy absorbed from that port, while (8d) expresses the total energy drawn from all ports equals the energy demand of EVs who visit station k. In addition, (8e) and (8f) guarantee each EV is charged through one charging port. Constraints (8g) to (8j) limit the renewable generation power to P_t^R, station power to P_t^s, charging port power to $P_{i,t}^b$, and ESS (dis)charging power to $P^{\max}$, respectively. Finally, constraints (8l) formulate the remaining energy of ESS.

In order to cast the problem into the framework of the Lyapunov optimization, the ESS depth of discharge (DOD) is defined as queue backlog as follows,

$$Q_t \triangleq E^{\max} - E_t^{\text{ESS}}, \tag{9}$$

which, using (8l), can be expressed as

$$Q_{t+1} = Q_t + \left(\frac{P_t^d}{\eta} - P_t^c\eta\right)\Delta t = Q_t + \varrho_t, \tag{10}$$

where $\varrho_t = P_t^d\Delta t/\eta - P_t^c\eta\Delta t$.

Furthermore, the objective function in (8a) should be modified to be used in the Lyapunov optimization. Hence, the immediate cost is calculated for each time slot as the following,

$$u_t \triangleq \lambda_t^p P_t \Delta t + \lambda^d \max\left[P_t - P_t^{\text{pre}}, 0\right], \tag{11}$$

where the first part calculates the energy charge of the current time slot, and the second part finds the demand charge increase due to the station's power demand at time slot t. In the second term, P_t^{pre} shows the maximum power so far drawn by the FCS during the current billing cycle, which can be expressed as $P_t^{\text{pre}} = \max\left[P_{t-1}^{\text{pre}}, P_{t-1}\right]$. By summing over all time slots, (11) becomes

$$\sum_{t=1}^{T} u_t = \Delta t \sum_{t=1}^{T} \lambda_t^p P_t + \lambda^d \max\left[P_t | t = 1, \cdots, T\right]. \tag{12}$$

Therefore, $\frac{1}{T}\mathbb{E}\left\{\sum_{t=1}^{T} u_t\right\}$ equals the objective function in (8a). In the following, we will define a Lyapunov function, form the Lyapunov drift-plus-penalty term, find the upper bound of the drift-plus-penalty term, and define problem (17) that minimizes the upper bound of the drift-plus-penalty term through Algorithm 2. Then, we prove that solving problem (17) through Algorithm 2, for each time slot separately, converges to the true optimal solution of (8) and satisfies constraints (8l).

2) Lyapunov Optimization: Next, the Lyapunov function is defined as the following quadratic function,

$$L[Q_t] \triangleq \frac{1}{2}Q_t^2, \tag{13}$$

which is a positive-definite function and is used to define the Lyapunov drift [18] as,

$$\Delta[Q_t] \triangleq \mathbb{E}\{L[Q_{t+1}] - L[Q_t] | Q_t\}, \tag{14}$$

which presents the temporally expected change of the Lyapunov function. Finally, the drift-plus-penalty term is defined as [18],

$$\Delta[Q_t] + V\mathbb{E}\{u_t | Q_t\}, \tag{15}$$

where V is a non-negative constant that adjusts the weight between minimizing the queue or the power demand of the station. According to the Lyapunov optimization theory, if the Lyapunov drift function in (14) has an upper bound for all t, then minimizing the upper bound at each time slot t solves problem (8).

The following theorem derives the upper bound for the Lyapunov drift.

Theorem 1. *At any time slot t, the Lyapunov drift-plus-penalty term has the following upper bound,*

$$\Delta[Q_t] + V\mathbb{E}\{u_t|Q_t\} \leq B + Q_t\mathbb{E}\{\varrho_t|Q_t\} + V\mathbb{E}\{u_t|Q_t\}, \tag{16}$$

where B is a positive constant as follows,

$$B = \frac{1}{2}\left[\frac{P^{max}}{\eta}\Delta t\right]^2.$$

Proof. Using (13), we can write,

$$L[Q_{t+1}] - L[Q_t] = \frac{1}{2}\left(Q_{t+1}^2 - Q_t^2\right).$$

With the help of (9), we obtain,

$$\begin{aligned} L[Q_{t+1}] - L[Q_t] &= \frac{1}{2}\left[\left(E^{\max} - E_{t+1}^{\text{ESS}}\right)^2 - \left(E^{\max} - E_t^{\text{ESS}}\right)^2\right] \\ &= \frac{1}{2}\left[(E_{t+1}^{\text{ESS}})^2 - (E_t^{\text{ESS}})^2 - 2E^{\max}\left(E_{t+1}^{\text{ESS}} - E_t^{\text{ESS}}\right)\right] \\ &= \frac{1}{2}\left(E_{t+1}^{\text{ESS}} - E_t^{\text{ESS}}\right)\left(E_{t+1}^{\text{ESS}} + E_t^{\text{ESS}} - 2E^{\max}\right). \end{aligned}$$

Combining (8l) and (10), we have $E_{t+1}^{\text{ESS}} = E_t^{\text{ESS}} - \varrho_t$, thus,

$$L[Q_{t+1}] - L[Q_t] = \frac{1}{2}\varrho_t\left(\varrho_t - 2E_t^{\text{ESS}} + 2E^{\max}\right).$$

Leveraging (9), we have $2E^{\max} - 2E_t^{\text{ESS}} = 2Q_t$, thus,

$$L[Q_{t+1}] - L[Q_t] = \frac{1}{2}\varrho_t\left(\varrho_t + 2Q_t\right) = \frac{1}{2}\varrho_t^2 + Q_t\varrho_t.$$

Algorithm 2 Minimizing FCS Operation Cost

Inputs: V, P_1^s, Δt, η, $P^{\max}$, $E^{\max}$, $E^{\min}$, and E_1^{ESS}
Outputs: P_t^c, P_t^d, P_t^r, $P_{i,t}$ and $E_{n,t}^i$
Initialization: $Q_1 \leftarrow E^{\max} - E_1^{\text{ESS}}$

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for t ∈ [1, T] do
    find P_t^c, P_t^d, P_t^r, P_{i,t} and E_{n,t}^i by solving (17)
    ϱ_t = P_t^d Δt/η − P_t^c η Δt
    update P_t^s
    Q_t ← Q_t + ϱ_t
end for
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Utilizing (8j) and the definition of ϱ_t, we have,

$$-P^{\max}\eta\Delta t \le \varrho_t \le \frac{P^{\max}}{\eta}\Delta t.$$

Since $0 < \eta \le 1$, then $\varrho_t^2 \le [P^{\max}\Delta t/\eta]^2$. Therefore,

$$L[Q_{t+1}] - L[Q_t] \le B + Q_t\varrho_t.$$

By taking conditional expectations from both hand-sides with respect to Q_t, we arrive at,

$$\mathbb{E}\{L[Q_{t+1}] - L[Q_t]|Q_t\} \le B + Q_t\mathbb{E}\{\varrho_t|Q_t\}.$$

Using (14) and adding $V\mathbb{E}\{u_t|Q_t\}$ to both hand-sides of the inequality, yields,

$$\Delta[Q_t] + V\mathbb{E}\{u_t|Q_t\} \le B + Q_t\mathbb{E}\{\varrho_t|Q_t\} + V\mathbb{E}\{u_t|Q_t\}.$$

■

We can minimize the upper bound of the drift-plus-penalty in (16) through the following problem,

$$\begin{aligned} \min \quad & Q_t\varrho_t + Vu_t, \\ \text{subject to } & \text{(8b)-(8k)}. \end{aligned} \tag{17}$$

Since (17) is independently solved for each time slot t, the solving procedure can be implemented through Algorithm 2, where for each time slot t, Q_t is a fixed value. In other words, problem (17) is solved for each t by fixing Q_t. Once the values of $P_{i,t}$ are obtained, Q_t is updated for the next time slot, i.e., Q_{t+1} is calculated.

In the following, we demonstrate that constraint (8l) is met by the solution of problem (17), which is solved through Algorithm 2.

$$Q_{t+1} = Q_t + \varrho_t = Q_t + \frac{P_t^d\Delta t}{\eta} - P_t^c\eta\Delta t.$$

By replacing (9) and rearranging, we obtain (8l)

$$E^{\max} - E_{t+1}^{\text{ESS}} = E^{\max} - E_t^{\text{ESS}} + \frac{P_t^d\Delta t}{\eta} - P_t^c\eta\Delta t,$$

$$E_{t+1}^{\text{ESS}} = E_t^{\text{ESS}} + P_t^c\eta\Delta t - \frac{P_t^d\Delta t}{\eta}.$$

Subsequently, the next theorem proves the solution of Algorithm 2 converges to the true optimal solution of (8), denoted as p^*.

Theorem 2. *For all $T > 1$, the optimal solution of* (17)*, denoted by $\{\hat{P}_t^c, \hat{P}_t^d\}$, and the optimal solution of* (8)*, denoted by $\{P_t^{c*}, P_t^{d*}\}$, satisfy the following inequality,*

$$\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\hat{u}_t\} \le \frac{B_1}{V} + p^*, \tag{18}$$

where p^* is the optimal objective function value of (8) *and $\hat{u}_t$ is calculated by* (11) *using $\{\hat{P}_t^c, \hat{P}_t^d\}$ and*

$$B_1 = B + \frac{1}{T}\mathbb{E}\{L[\hat{Q}_1]\} + \frac{1}{\eta}E^{max}P^{max}\Delta t. \tag{19}$$

Proof. For $\{P_t^d, P_t^c\}$ satisfying the constraints of (8), define $A_t = P_t^d\Delta t/\eta$ and $\xi_t = P_t^c\eta\Delta t$. Consequently, based on (10) and (8l) we derive $Q_{t+1} = Q_t + A_t - \xi_t$ and $E_{t+1}^{\text{ESS}} = E_t^{\text{ESS}} - A_t + \xi_t$, respectively. By taking expectation on (8l) and summing over $\{1, 2, \cdots, T\}$, we have

$$\mathbb{E}\{E_{T+1}^{\text{ESS}}\} - \mathbb{E}\{E_1^{\text{ESS}}\} = \sum_{t=1}^{T}\left(-\mathbb{E}\{A_t\} + \mathbb{E}\{\xi_t\}\right), \tag{20}$$

where the alteration of the left-hand side is due to the law of telescoping sums. This implies that all feasible solutions $\{P_t^d, P_t^c\}$ satisfying the constraints of (8) will satisfy the constraint (20). Now, by replacing (8l) with (20), we define the following new optimization problem

$$\begin{aligned} \min \quad & \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{u_t\} \qquad \text{(21a)} \\ & \text{subject to (8b)-(8k), (20)} \end{aligned}$$

It is worth mentioning that since expectation of sum is sum of expectation, from (12) we have,

$$\sum_{t=1}^{T}\mathbb{E}\{u_t\} = \mathbb{E}\left\{\Delta t\sum_{t=1}^{T}\lambda_t^p P_t + \lambda^d\max[P_t|t = 1, \cdots, T]\right\},$$

which means the objective of (21) is identical to (8a). Moreover, because all $\{P_t^d, P_t^c\}$ satisfying the constraints of (8) will also satisfy (20) and thus all constraints of (21), problem (21) can be called a relaxed version of problem (8) in the sense that the optimal solution of (8) is a feasible solution of (21), and the optimal objective of (21) is less than or equal to the optimal objective value of (8). Assume $\tilde{p}$ denotes the optimal objective function value of (21) under the optimal solution $\{\tilde{P}_t^c, \tilde{P}_t^d\}$, thus, $\tilde{p} \le p^*$. Hence,

$$\tilde{p} = \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\tilde{u}_t\} \le p^* = \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{u_t^*\}, \tag{22}$$

where $\tilde{u}_t$ and u_t^* are calculated by the optimal solutions of (21) and (8) using (11), respectively. Additionally, because (17) minimizes the upper bound of the drift-plus-penalty term, the solutions of (17) yield the minimum of $Q_t \cdot \varrho_t + V \cdot u_t$, which equals to the following,

$$\hat{Q}_t \cdot \hat{\varrho}_t + V \cdot \hat{u}_t = \hat{Q}_t\left(\hat{A}_t - \hat{\xi}_t\right) + V \cdot \hat{u}_t, \tag{23}$$

where $\hat{A}_t$ and $\hat{\xi}_t$ correspond to the values of A_t and ξ_t evaluated at $\{\hat{P}_t^c, \hat{P}_t^d\}$, respectively. Evaluating $Q_t \cdot \varrho_t + V \cdot u_t$ with any other values satisfying the constraints of (17) (i.e., (8b)-(8k)) results in a value that is greater than or equal to (23). Note that the solution $\{\tilde{P}_t^c, \tilde{P}_t^d\}$ of the relaxed problem (21) also satisfies (8b)-(8k), thus

$$\hat{Q}_t\left(\hat{A}_t - \hat{\xi}_t\right) + V \cdot \hat{u}_t \leq \tilde{Q}_t\left(\tilde{A}_t - \tilde{\xi}_t\right) + V \cdot \tilde{u}_t, \quad (24)$$

where $\tilde{A}_t$ and $\tilde{\xi}_t$ correspond to the values of A_t and ξ_t evaluated at $\{\tilde{P}_t^c, \tilde{P}_t^d\}$, respectively. By taking expectations with respect to $\hat{Q}_t$ of (23), we have

$$\hat{Q}_t\mathbb{E}\{\hat{A}_t - \hat{\xi}_t|\hat{Q}_t\} + V\mathbb{E}\{\hat{u}_t|\hat{Q}_t\}.$$

Plugging the result into (16) yields

$$\Delta[\hat{Q}_t] + V\mathbb{E}\{\hat{u}_t|\hat{Q}_t\} \leq B + \hat{Q}_t\mathbb{E}\{\hat{A}_t - \hat{\xi}_t|\hat{Q}_t\} + V\mathbb{E}\{\hat{u}_t|\hat{Q}_t\}.$$

By taking expectations from both hand sides, we obtain

$$\mathbb{E}\{\Delta[\hat{Q}_t]\} + V\mathbb{E}\{\hat{u}_t\} \leq B + \mathbb{E}\{\hat{Q}_t\}\mathbb{E}\{\hat{A}_t - \hat{\xi}_t\} + V\mathbb{E}\{\hat{u}_t\}, \quad (25)$$

where the condition with respect to $\hat{Q}_t$ is removed through the law of iterated expectations. Using (24) and (25), we obtain the following

$$\begin{aligned}\mathbb{E}\{\Delta[\hat{Q}_t]\} + V\mathbb{E}\{\hat{u}_t\} &\leq B + \mathbb{E}\{\hat{Q}_t\}\mathbb{E}\{\hat{A}_t - \hat{\xi}_t\} + V\mathbb{E}\{\hat{u}_t\} \\ &\leq B + \mathbb{E}\{\tilde{Q}_t\}\mathbb{E}\{\tilde{A}_t - \tilde{\xi}_t\} + V\mathbb{E}\{\tilde{u}_t\}.\end{aligned}$$

Now, using (14) and by summing both hand sides over all time slots, we arrive at the following,

$$\mathbb{E}\{L[\hat{Q}_{T+1}]\} - \mathbb{E}\{L[\hat{Q}_1]\} + V\sum_{t=1}^{T}\mathbb{E}\{\hat{u}_t\} \leq$$
$$TB + \sum_{t=1}^{T}\mathbb{E}\{\tilde{Q}_t\}\mathbb{E}\{\tilde{A}_t - \tilde{\xi}_t\} + V\sum_{t=1}^{T}\mathbb{E}\{\tilde{u}_t\}.$$

Since $L[Q_t]$ is a positive definite function, we can drop $\mathbb{E}\{L[\hat{Q}_{T+1}]\}$ from the left-hand side of the inequality, thus,

$$-\mathbb{E}\{L[\hat{Q}_1]\} + V\sum_{t=1}^{T}\mathbb{E}\{\hat{u}_t\} \leq$$
$$TB + \sum_{t=1}^{T}\mathbb{E}\{\tilde{Q}_t\}\mathbb{E}\{\tilde{A}_t - \tilde{\xi}_t\} + V\sum_{t=1}^{T}\mathbb{E}\{\tilde{u}_t\}.$$

Noting that $Q_t \leq E^{\max}$, $A_t - \xi_t \leq \max\{A_t\}$, and $\max\{A_t\} = P^{\max}\Delta t/\eta$, we have,

$$\sum_{t=1}^{T}\mathbb{E}\{\tilde{Q}_t\}\mathbb{E}\{\tilde{A}_t - \tilde{\xi}_t\} \leq TE^{\max}P^{\max}\Delta t/\eta.$$

Therefore, we obtain,

$$-\mathbb{E}\{L[\hat{Q}_1]\} + V\sum_{t=1}^{T}\mathbb{E}\{\hat{u}_t\} \leq$$
$$TB + \frac{T}{\eta}E^{\max}P^{\max}\Delta t + V\sum_{t=1}^{T}\mathbb{E}\{\tilde{u}_t\}.$$

which by rearranging and dividing by TV,

$$\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\hat{u}_t\} \leq \frac{B}{V} + \frac{1}{TV}\mathbb{E}\{L[\hat{Q}_1]\}$$

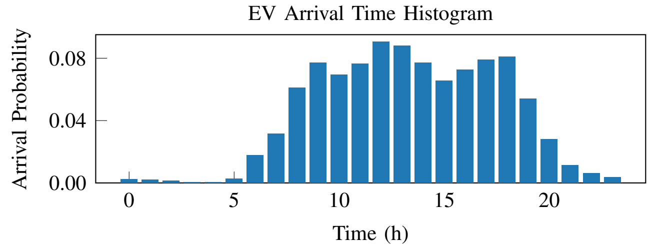

Fig. 3. Arrival histogram of EVs based on FCSs in Boulder city in Colorado. The data is gathered from 2018 to 2022 [23].

$$+ \frac{1}{V\eta}E^{\max}P^{\max}\Delta t + \frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\tilde{u}_t\}.$$

Based on (22) and (19), we have

$$\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}\{\hat{u}_t\} \leq \frac{B_1}{V} + p^*,$$

which is identical to (18). It is worth noting that as V increases, the left-hand side of (18) converges to p^*. ■

The summary of theorems 1 and 2 means that instead of solving problem (8), we can minimize the upper bound of the drift-plus-penalty term for each time slot. The result will be a suboptimal solution with at most $\mathcal{O}(1/V)$ deviation from the true optimal solution.

Ultimately, the proposed approach in this paper boils down to Algorithm 1, Algorithm 2, and Eq. (7). Firstly, Algorithm 1 finds the selling electricity price and energy demand of EVs for each time slot. Then, Eq. (7) assigns each EV to the best FCS. Lastly, Algorithm 2 minimizes the cost of purchasing power for each station by determining the charging power of EVs at each charging port, the charging and discharging power of ESS, and the generated renewable power. Note that Algorithm 1 integrates all the FCSs, while Algorithm 2 is independently executed for each station.

III. Simulation Results and Analysis

This section presents simulation results to evaluate the proposed approach. In simulations, 20 DC fast charging stations are considered, where each FCS is equipped with 20 charging ports with a power rating of 350 kW. Moreover, the data of charging stations in Boulder city in Colorado is used, which spans from 2018 to 2022 [23]. The data are further used to find the EV arrival time distribution, as shown in Fig. 3, which is, in turn, exploited to simulate EV charging request scenarios. Since the data belongs to Boulder city, for the sake of coherence, the location of 20 charging stations in Boulder is chosen to calculate the distance each EV has to travel to reach an FCS. These locations are marked in Fig. 4, which is obtained from Google Maps.

Since the problem is decomposed and solved for each time step separately, we assume EV arrival, energy demand, and solar PV availability are known. EV energy demand is known through the pricing game, and we assume EVs follow the value obtained in the pricing game step. In addition, we assume EVs can report their arrival in the Lyapunov optimization step, or

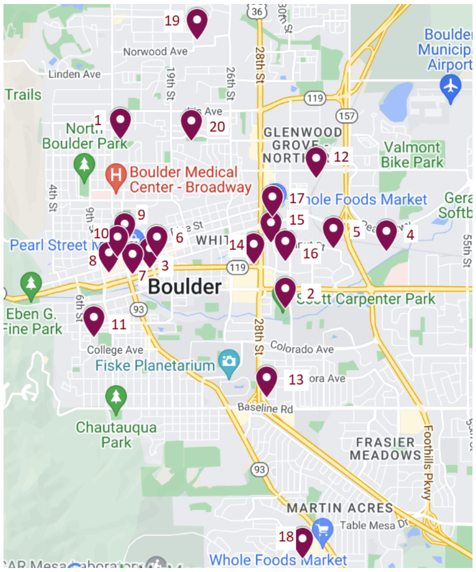

Fig. 4. The locations of 20 selected stations on the map.

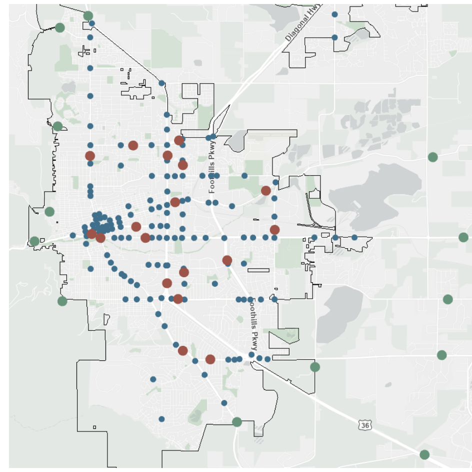

Fig. 5. City of boulder traffic count data.

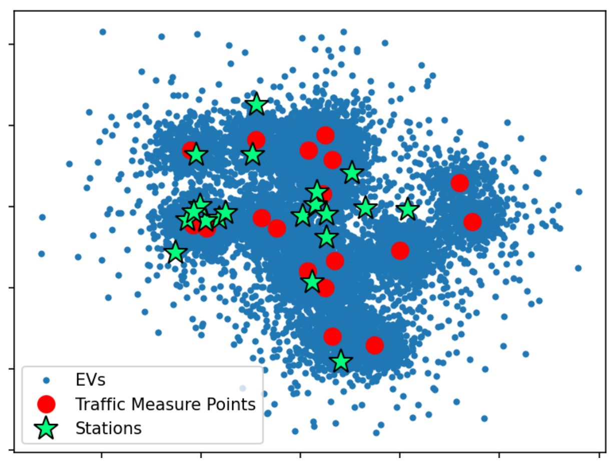

Fig. 6. EV population distribution around Boulder.

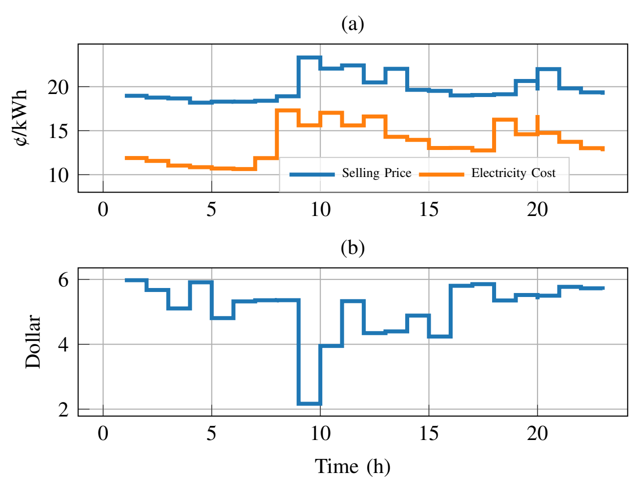

Fig. 7. Pricing game results. The results are monthly averages. (a) The hourly cost of purchasing electricity from the grid and the selling price to EVs. (b) The average hourly cost per charging session.

EV arrival can be forecasted with acceptable accuracy. Either way, EV arrivals for the current interval are a known input. A similar argument applies to solar PV availability, assuming that solar PV generation is accurately forecasted for the current time step.

The hourly electricity prices at which FCSs purchase energy from the grid are obtained from PJM's hourly real-time prices [24]. We consider a case with high EV penetration where 10,000 electric cars visit the 20 charging stations during the day. The 10,000 EVs are selected from 10 top-selling EVs in 2022. Every EV is assigned a different probability of selection according to its U.S. sales percentage in 2022 [25].

EV population is distributed around the city based on traffic count data in Boulder city [26]. As Fig. 5 displays, three types of traffic count are used, namely, Arterial Counts Program (ART), Boulder Valley Counts Program (BVP), and Turning Movement Counts (TMC). The red, blue, and green spots represent ART, BVP, and TMC. In this paper, ART is used to simulate EVs' locations around the city. Assume that EV traffic flow follows the overall traffic flow. Each EV's location is randomly generated with the help of ART. First, one ART point is randomly selected based on its selection probability, where the traffic count in each red point is utilized to find the selection probability. Then Gaussian noise is added to the coordinates of the selected red point to simulate EV's location. The result is presented in Fig. 6, where the blue dots show EVs' simulated locations, the red circles represent ART measuring points, and the green stars are 20 selected FCSs.

The first part of the problem, which is based on game theory, is discretized in such a way that each time slot is one hour. In addition, although the prices are dynamic, all the FCSs have similar selling prices. Hence, the allocation of EVs to stations is performed based on distance and queue at FCSs.

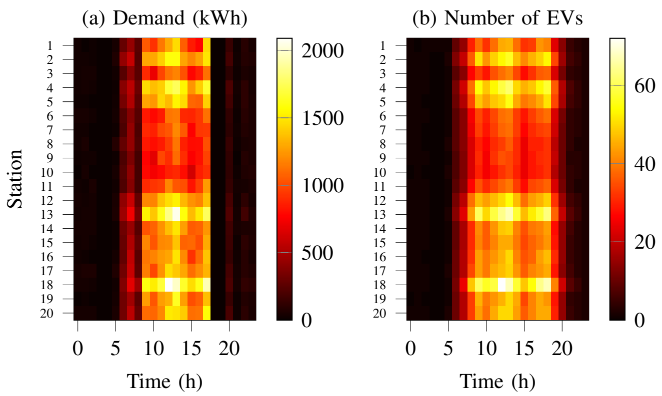

Fig. 8. Allocation of EVs to stations for a test day. (a) Illustrates the energy demand from each station throughout the day. (b) Shows the number of EVs that visit each FCS every hour of the day.

A. Pricing Game

Fig. 7 depicts the result of determining the prices of every hour of the test day through Algorithm 1. The results in Fig. 7 are monthly averages for each hour. Fig. 7 (a) displays the hourly cleared selling price as opposed to the purchasing price from the grid. While the electricity cost varies throughout the day from around 10 ¢/kWh to around 18 ¢/kWh, the selling prices fluctuate around 20 ¢/kWh. The results are obtained by setting the maximum acceptable prices by each customer to a randomly selected value between 25 and 30 ¢/kWh. On the other hand, the minimum price beyond which an EV user becomes sensitive to the selling prices, λ_n^b, is set between 15 and 20 ¢/kWh. These values adjust the user behavior so that the final cleared prices fall well below 40 and 70 ¢/kWh, which is the average cost of charging an EV in public charging stations [27]. Note that, the cleared price is the electricity price at which the station purchases the electricity from the grid or the market. The selling price is the price at which the energy is sold to EVs by the FCS. Furthermore, Fig. 7 (a) shows the peak and trough of the cleared selling prices that follow those of the electricity cost. Thus, the selling prices drop with the decrease of the electricity cost even though the electricity cost is not integrated into the customer sensitivity model. Fig. 7 (b) represents the average hourly cost per charging session, calculated by averaging the sum of money each customer pays to charge their EV in each time slot. It is noticeable that a reduction in the average amount of money the user pays for a charging session follows the increase in selling prices. In other words, since customers lower their charging demand as selling prices rise, the average payment by users drops. However, due to the high number of customers in peak hours, the FCSs still gain maximum profit.

B. EV Assignment to FCSs

A test day results of allocating EVs to stations are plotted in Fig. 8, where the hourly energy demand from each station during the day and the total number of EVs allocated to each station per hour are depicted. As seen in Fig. 8, the FCSs are busy from 09:00 am to 05:00 pm because EV charging demand is high in this span. Moreover, according to the heatmaps, stations 18, 13, and 4 are the busiest FCSs in terms of energy demand and the number of visiting EVs. Comparing these results with Fig. 4 and Fig. 6, it is clear that stations 18, 13, and 4 are located in regions where no other FCS is nearby, and two or three red spots surround each of them. Hence, the customers prefer to wait in the queue rather than travel miles to another station. Regarding crowdedness, from the heatmaps, FCSs 18, 13, and 4 are followed by stations 2, 5, and 12. It can be verified from Fig. 4 that there are few stations in the vicinity of the said FCSs. Moreover, it is noticeable from the heatmaps that stations 1, 19, and 20 do not host as many customers as stations 2, 5, and 12, while they have comparable requested demand. This is because fewer EVs with higher demand visited stations 1, 19, and 20. Since the high-demand users occupied the chargers, the remaining customers were assigned to other FCSs.

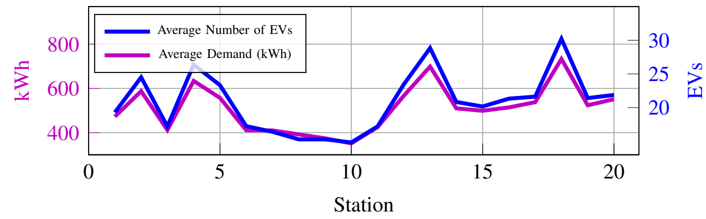

Fig. 9. The daily average energy demand from each station and the average number of EVs visiting every FCS. The results are monthly averages.

The maximum energy request is equal to 2.09 MWh. It occurs between 12:00 pm and 01:00 pm at station 18, corresponding to 68 EVs intending to visit station 18, where 68 is the maximum number of cars visiting an FCS during the day. Note that the order of energy demand does not necessarily follow the number of EVs that visit a station. For example, between 10:00 am and 11:00 am, the energy demand at stations 19 and 20 are 1034 kWh and 1082 kWh, respectively. However, the number of EVs who visit stations 19 and 20 in the same hour is 37 and 36, respectively. Fig. 9 represents the average daily demand from each FCS and the average daily number of vehicles that visited each station. As seen, the EVs are equally allocated to the FCSs close to each other, while farther away stations experience a higher number of visits.

C. Cost Minimization

In this step, the operation cost of each FCS is minimized through Algorithm 2. This algorithm's results are controlled by coefficient V, which is a user input. It is reported that the demand charges account for 73.7% of the average monthly bill of charging stations in the United States [28]. Therefore, demand charges are also integrated into the problem formulation. The proposed Lyapunov optimization is a trade-off between minimizing the operation cost and the DOD queue at the stations. The trade-off is controlled via arbitrary coefficient $V \geq 0$.

In this regard, Fig. 10 presents the impact of coefficient V on the net profit of the FCSs and the maximum power

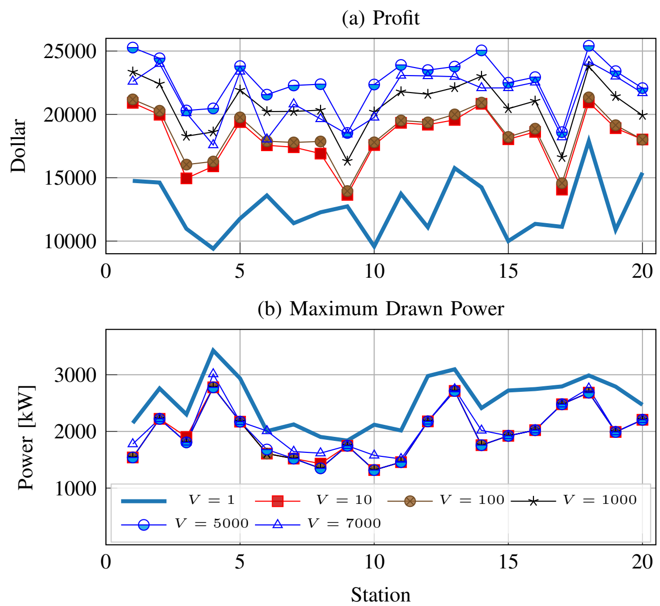

Fig. 10. The impact of coefficient V on results. (a) The variation of the net profit of each FCS as V changes. (b) The alterations of the maximum power that each FCS draws in the billing cycle, i.e., one month, as V changes.

that each FCS draws from the grid during the one-month billing cycle. As expected, the increase in V enhances the profit while reducing the maximum drawn power. The reason lies in the fact that the Lyapunov optimization attempts to lower the costs, including demand charges; thus, the maximum drawn power is reduced. Fig. 10 (a) illustrates the increase in profit as V rises. However, the profit improvement stops at roughly $V \approx 5000$, where the results begin to deteriorate due to the violation of the ESS capacity limit by queue backlog size. Besides, Fig. 10 (b) displays the evolution of maximum drawn power with V where the highest power is drawn if $V = 1$. Note that this power level only occurs in one time slot, and this plot does not illustrate the average power demand. Moreover, increasing V from 1 to 10 leads to a drop in maximum drawn power. Furthermore, higher values of V, up to the $V \approx 5000$ threshold, have an imperceptible effect on the maximum drawn power. Afterward, the maximum drawn power increases with V. Comparing results in Fig. 10 (b), we observe that in most cases, by increasing V, a power demand reduction of 500 kW is seen (bringing the maximum power demand to less than 2.5 MW). A drop of 500 kW may lead to $5000 decreased monthly electricity bill, seeing that the average demand charges across the U.S. are $10/kW [28]. However, it is achieved at the cost of higher operation of ESS, which results in its degradation. Lastly, it is noticeable that, regardless of the value of V, the stations' profits are not much different, which indicates that Section III-B properly assigned EVs to FCSs. Interestingly, station profit was not a factor in allocating EVs to FCSs, yet they are assigned in a way that the overall profit is divided among stations fairly homogeneously.

The profit improvement stems from the optimal exploitation of the ESS and renewable generation. Table I shows the monthly average of ESS DOD, average PV generation, and expected profit at stations. The ESS capacity is considered 1000 kWh at each station, and the daily average available PV generation is 213 kWh. As discussed earlier, the best performance is attained with $V = 5000$. In addition, according to the Lyapunov optimization theory, increasing V renders the problem (17) to converge to the optimal solution of (8) at the cost of a larger queue backlog. Consequently, as seen in Table I, values of V greater than zero yield higher utilization of ESS and PV generation. However, after the threshold of $V = 5000$, over-utilization of ESS occurs, and the profit deteriorates.

TABLE I
THE IMPACT OF COEFFICIENT V ON ESS UTILIZATION, PV GENERATION, AND PROFIT

V	Avg. ESS DOD [kWh]	Avg. PV Gen. [kWh]	Avg. Profit
1	10.31	197.04	$12,631
10	10.22	197.05	$18,088
100	11.47	197.69	$18,442
1000	113.29	205.83	$20,682
2000	243.29	209.53	$21,625
3000	369.74	211.53	$22,252
4000	497.38	212.75	$22,592
5000	624.70	213.65	$22,629
6000	750.34	214.16	$22,153
7000	850.69	214.36	$21,362
10000	992.08	214.67	$18,917

TABLE II
COMPARISON OF THE PROPOSED APPROACH WITH THE BENCHMARK

	Proposed Approach	Benchmark
Average price/hour	$5.14	$7.28
Lost customers	590 (daily avg.)	0
Avg. Maximum Drawn Power	1977 [kW]	2550 [kW]
Avg. Demand Charges	$19,770	$25,500
Average system profit	87.34 [$/MWh]	72.21 [$/MWh]
Average FCS profit	4.37 [$/MWh]	3.61 [$/MWh]
Avg. profit per charger	0.22 [$/MWh]	0.18 [$/MWh]
Avg. profit per 100 customers	0.87 [$/MWh]	0.73 [$/MWh]

D. Comparison with Benchmark

In this section, the proposed approach is compared with a benchmark model as follows. In the benchmark model, the pricing game is removed, and the electricity is sold to the customers at a fixed price. The selling price is 0.05 [$/kWh] higher than the hourly electricity cost. For example, if FCSs buy electricity for 0.1 [$/kWh], they sell energy to users at 0.15 [$/kWh]. The 0.05 [$/kWh] margin is selected since the difference between electricity cost and selling price in the proposed approach is around 0.05 [$/kWh] in Fig. 7. Similar to the proposed approach, EVs in the benchmark model, are assigned to FCSs using the multinomial logit model in (7), and the operation cost is minimized through Lyapunov optimization.

Table II represents the comparison between the proposed algorithm and the benchmark model. As seen in Table II, the average hourly price for a charging session is increased from \$5.14 in the proposed approach to \$7.28 in the benchmark model. Moreover, there are no lost customers in the benchmark approach because customers are assumed to purchase electricity regardless of the price. Further, the average maximum drawn power across all stations is increased by 13.8% from 1977 to 2550 [kW], leading to a similar percentage increase in demand charges. In addition, the average system profit is dropped when using the benchmark model. Note that since the price sensitivity of the users is removed in the benchmark approach, the final amount of energy purchased by each user may be different than the final amount of energy purchased by the same user when their price sensitivity is accounted for in the proposed approach. Therefore, in order to compare the profits obtained by the proposed approach and the benchmark algorithm, the profits are normalized by the total energy demand from the users. For instance, in the case of the proposed approach, the total average daily demand for EVs is 259.1 [MWh]. Thus, the normalized \$22,629 profit becomes $22629/259.1 = 87.34$. Finally, by dividing the normalized profit by the number of FCSs, total number of chargers in all FCSs, and number of customers, the average FCS profit, average profit per charger, and average profit per customer are obtained.

IV. Conclusion

This paper studies the charging scheduling and charging navigation of EVs as well as the operation cost minimization of FCSs. The problem is decomposed into three subproblems to help enhance the computation speed and remove dependency on future information. The first subproblem determines the hourly selling prices and EV energy demands through a pricing game for a network of FCSs by integrating user behavioral responses into the model. The second subproblem assigns EVs to nearby FCSs with the help of a multinomial logit model, while the third subproblem leverages the Lyapunov optimization to minimize the operation cost of FCSs. The proposed algorithm is fast due to its decomposed nature and requires around 10 minutes to run the simulation for a month, and the charging scheduling of 10,000 EVs is planned daily. The dynamic pricing subproblem yields low selling prices; subsequently, the Lyapunov optimization guarantees high station revenue. Lastly, the proposed model is benchmarked against a recent study to showcase its performance.

The work of this manuscript can be extended in the following ways for future work. First, the competition among different FCS companies can be considered by integrating rival companies' prices. Further, the proposed approach can be studied under limited available power at FCSs, where the available power should be optimally distributed among charging EVs. In addition, users can be prioritized such that those with top priorities, such as law enforcement vehicles or ambulances, are permitted to jump in the queue.

Acknowledgments

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