

Dynamic excitations in periodic fish scale inspired structures

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ABSTRACT

Fish scale-like structures on substrates, arranged periodically, work together to create unique mechanical and optical behaviors. These include nonlinear stiffness, anisotropic deformation, and eventually, locking behavior. Fabrication of scale-like biomimetic examples involves embedding stiffer, plate-like sections into softer substrates. Previously, research has focused on their static qualities. The dynamic response is just as fascinating, showing remarkable interplay between geometry and materials, along with anisotropies. The damping behavior observed here significantly diverges from the conventional damping seen in mechanical frameworks, often modeled as Rayleigh damping. Here we discuss the origin of some of these behaviors that include material-geometry distinction in damping, multiple damping modes and interplay of dissipation possibilities. We have shown a derivation of simple mathematical laws estimating nonlinear spring damper system that govern architecture-dissipation relationships and can help guide design. We conclude by noting the different type of structural damping with other forms of dissipation typically encountered in mechanical behavior.

Keywords: Biomimetic scales, Nonlinear dynamics, Dissipation behavior, Metamaterials, Emergent behavior

1. INTRODUCTION

Fish scale inspired structures have been under intense scrutiny as lightweight and smart alternatives to traditional materials.¹⁻⁴ Their structural advantages are now already well established. However, their behavior in dynamic regime, especially damping has been somewhat poorly understood. An archetype of these materials have been slender substrates with scales adorned on one or both sides. Several new studies have shed light on these biomimetic scale systems.^{5,6} These studies emphasize the dynamic properties and damping mechanisms of thin plates and membranes, attributed to the sliding motion of scales.^{3,5,6}

Given the complexity and the high cost associated with detailed analysis due to the continuous variation of properties across the structure, a more accessible approach can be passively considered through the use of lumped parameter models. Lumped parameter models simplify the representation of a system by condensing its continuous properties—mass, stiffness, and damping—into a finite set of concentrated parameters. Lumped parameter model has widely been used in many applications.⁷⁻⁹ This enables the dynamic behavior of these intricate fish scale-inspired structures to be efficiently captured with ordinary differential equations. Such a methodology not only aids in achieving a more profound comprehension and predictability of the materials' dynamic responses but also significantly reduces the need for extensive resources, effectively narrowing the divide between the complexity of biological inspirations and the feasibility of engineering applications.

Non-linear oscillators mark a significant frontier in the field of mechanical engineering and solid mechanics, offering innovative solutions to challenges in energy harvesting, vibration isolation, and system miniaturization. Studies found that, a piezoelectric generator incorporating into nonlinear spring oscillator can be designed to enhance energy conversion efficiency by enabling operation across multiple resonant frequencies. Employing nonlinear springs with distinct stiffness values, the generator demonstrated superior performance by producing significantly higher output power across three resonant frequencies compared to traditional linear systems. This approach confirmed the efficacy of utilizing nonlinear mechanics for energy harvesting but also opened avenues for optimizing such systems to adapt to varying ambient vibrations through potential frequency tuning mechanisms.^{10,11} In a study, an experimental setup was developed to explore the dynamics of a nonlinear oscillator

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equipped with a nonlinear negative magnetic stiffness (NNMS). Through direct testing on an air track platform, the research demonstrated how NNMS can significantly influence the system's dynamic responses, showcasing behaviors consistent with Duffing's equation. Key findings revealed the potential of NNMS in enhancing vibration isolation capabilities by effectively reducing resonance frequencies and broadening the bandwidth for vibration control, underscoring its utility in designing advanced mechanical systems for complex dynamic scenarios.¹² Besides, in an experimental study an X-shaped anti-vibration structure with tunable contact mechanisms (NXSC) was developed which significantly enhances low-frequency vibration isolation. Their findings demonstrate that the NXSC structure markedly improves loading capacity, extends the quasi-zero stiffness zone without sacrificing performance, and offers superior adjustability and stability compared to traditional systems.¹³ Additionally, the study on micromechanical oscillators with non-linear springs hints at the feasibility of miniaturization in non-linear oscillatory systems, suggesting a promising direction for developing highly sensitive and efficient mechanical sensors and actuators crucial in micro-electromechanical systems (MEMS) and beyond.¹⁴ These investigations advance our understanding of non-linear oscillatory systems and also also underscore the transformative potential of non-linear spring mechanisms in enhancing the efficiency, versatility, and performance of mechanical systems across a wide array of applications.

Despite some studies have explored the dynamic behavior of fish-scale structures, several challenges have limited the use of full scale FE models and experimental analysis leading us to explore lumped parameter models for these systems. In this paper, we present a novel lumped parameter model and its consequences on the nonlinear dynamics of bio-mimetic scale systems. Specifically we discover that the models predict existence of chaotic vibrations under forced regime for various geometric parameter.

2. METHODS AND MODELS

Our lumped parameter nonlinear spring-mass-damper model is based upon close investigation of the behavior of prior fish scale systems, capable of reproducing nonlinear strain stiffening¹ and jamming or locking behavior¹ as well as emergent viscosity.⁵ Fig. 1 (a) shows a schematic view of a scale-covered beam under various conditions: no bending (left), positive curvature (middle), and negative curvature (right). Figure 1b depicts a schematic view of a nonlinear spring. In Figure 1c (left), a force-displacement plot demonstrates how variations in Γ enhance the nonlinearity within the spring-mass-damper system. This effect contrasts with observations from prior study,⁴ which illustrated in Figure 1c (right), where an increase in the frictional coefficient within the moment-curvature analysis indicates a higher structural stiffness. It is well known that the origin of jamming and nonlinear strain stiffening arises due to scales sliding. Such jammed spring can be modeled using a functional approximation of the restoring force of the scaled substrate using a two variable tangent function $F_{sp} = \alpha \tan(\zeta x)$ where x is analogous to curvature of the substrate. α measures the strength of non-linearity which is dependent on the amount of scale embedding, scales density/overlap and elastic modulus of the substrate. The parameter ζ is also a function of scale overlap since, structural models indicate the locking curvature is inversely related to scale overlap, with denser scales leading to earlier locking. Thus one can say that at lock $x \rightarrow x_L$, $\zeta x_L \rightarrow \pi/2$ and hence $\zeta = \frac{\pi}{2x_L}$. Thus one can write the spring restoring force as $F_{sp} = \alpha \tan \frac{\pi x}{2x_L}$. Second interesting point to note is the nature of damping. It is well known that even dry frictional effects give rise to linear drag like forces due to scales sliding effect.⁵ Hence the damper will be modeled as a linear dashpot with some drag coefficient c . Assuming a harmonic force of frequency ω and amplitude f_0 on the system, the overall equation of motion can be written as:

$$m\ddot{x}(t) + c\dot{x}(t) + \alpha \tan\left(\frac{\pi x}{2x_L}\right) = f_0 \cos(\omega t) \quad (1)$$

Following non-dimensionalization can be done for this system $\bar{x} = x/x_L$, which leads to the following spatially-non dimensional equation:

$$\ddot{\bar{x}}(t) + \delta \dot{\bar{x}}(t) + \Gamma^2 \tan\left(\frac{\pi \bar{x}}{2}\right) = \gamma \cos(\omega t) \quad (2)$$

where $\delta = c/m$, $\Gamma = \sqrt{\alpha/mx_L}$, $\gamma = f/mx_L$. The physical Interpretation of these parameters are as follows - δ is specific damping coefficient that measures the amount of damping per unit mass, Γ is a measure of specific nonlinear stiffness and γ is the ratio of applied force to inertia. Note that despite the commonality of these

systems to fish scale substrate, critical distinction exists - that of not having the asymmetry due to scales being present on only one direction of the substrate. Thus the system is fundamentally both more symmetric and more nonlinear with nonlinear stiffening present on both directions of bending. This is however, more apt for scale covered organisms executing motion in a submerged media. We now take advantage of this lumped parameter system to carry out parameter analysis to explore the effect of these parameters on the overall dynamics of vibration.

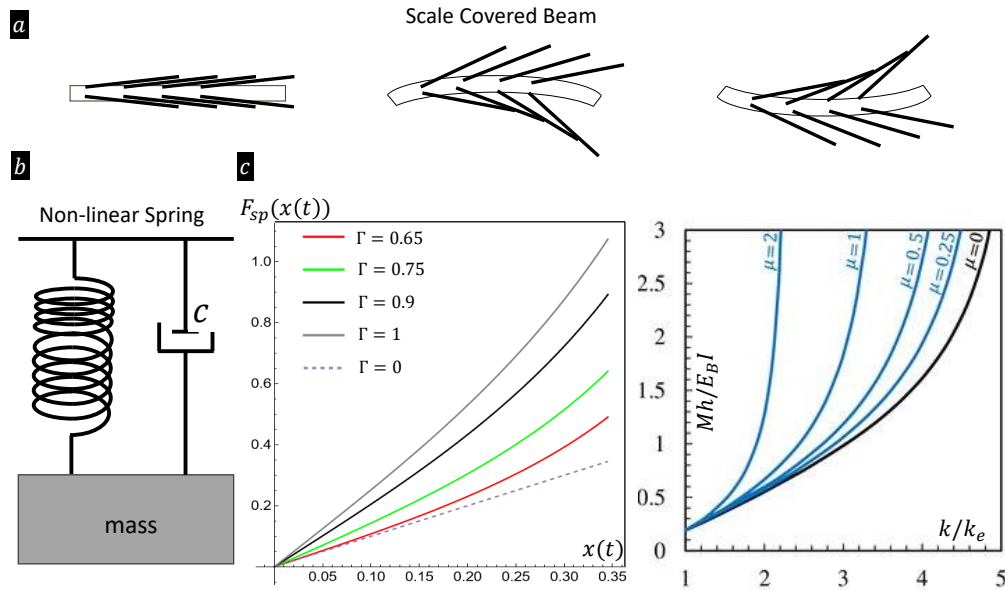


Figure 1. a) The structure depicts a fish scale-covered structure where the scales are initially engagement (left). in addition, it is illustrated the deformation of the intended structure and the sliding of the scales on one side and their separation on the other side (middle and right). b) Non-linear spring that is used for estimation of analytical model. c) The force-displacement diagram for the scaled structure as the parameter Γ increases, which represents the non-linear stiffness of the system, gradually making the system stiffer with its increment. $\Gamma = 0$ is depicted for linear type of the equation assuming $k = 1$ which k is the stiffness constant. On the right side, the normalized moment-curvature diagram of the scaled beam is observed, which has been obtained from.⁴ As a result, the nonlinear behavior of the structure is visible and comparable in both figures.

3. RESULTS AND DISCUSSION

We first investigate the effect of various system parameters on the non-dimensional displacement-time profile of the system. In Fig.2(a) we plot the displacement profile of the oscillator for various values of specific damping parameter δ . We find that an increase in damping leads to a reduction of amplitude of vibration. This is expected and mirrors linear systems. As damping term is decreased, there is an increase in the amplitude of vibration but at the same time, the waveforms undergo change from a regular sinusoidal shape to more irregular shape reflecting an increased non linearity in response. The increase in amplitude is also accompanied by an increase in frequency of vibration. These trends amplify as the damping is further decreased. As the damping decreases, higher frequency terms become more prominent. Next, in Fig.2(b) we plot the non-displacement-time profile of the oscillator with various values of nonlinear stiffness parameter Γ . An unmistakable trend emerges where higher nonlinear stiffness leads to lower amplitude and higher frequency. Similarly, plotting the dimensional displacement with time for various values of angular frequency of the applied force ω clearly shows that higher ω leads to lower amplitude and higher frequency, Fig.2(c). This shows that more frequent load leads to more vibration of the system. Finally in Fig.2(d), we plot the non-dimensional displacement with time for various amplitudes of forcing parameters Γ . The forcing has a profound effect on the displacement profile. Higher forcing amplitudes lead to higher time periods as clear from Fig.2(d). This is a classic route to chaotic vibrations well

known in literature. The higher amplitude forcing also leads to much more complex waveforms with pronounced peak splitting. These factors strongly indicate the potential chaotic vibrations in the system.

Next, to investigate the potential for chaos in the system, we make use of plotting Poincare maps.

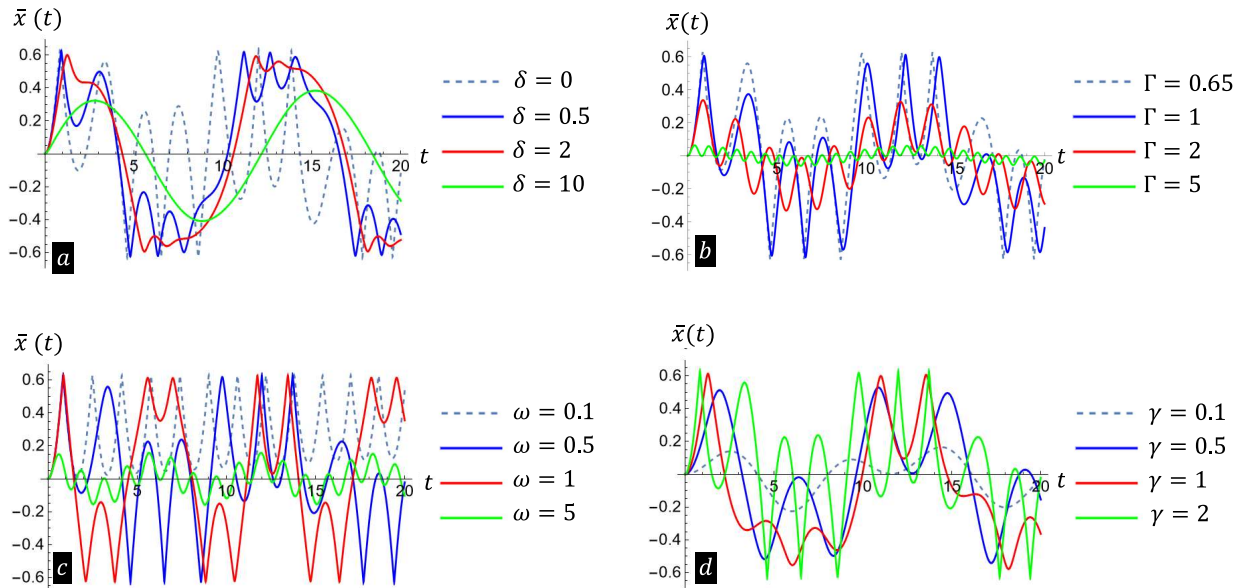


Figure 2.

The dynamic response of the scaled structure under the influence of parameter changes in the equation, including a) damping (δ), b) nonlinear coefficient of the equation or tangent coefficient (Γ), c) the angular frequency of the periodic external force (ω), and d) the amplitude of the periodic driving force (γ). The default coefficients are defined as $\delta = 0.02$, $\Gamma = 0.65$, $\gamma = 2$, $\omega = 0.5$.

To this end, in Fig. 3, both the Poincaré section and the phase plot are depicted. The Poincaré map, a fundamental tool in the study of dynamical systems, represents the intersection points of a periodic orbit in the phase space with a certain lower-dimensional subspace, typically employed to analyze the qualitative behavior of systems and to detect chaos. In this figure, an array of Poincare maps are presented with progressively increasing specific damping coefficient. It is clear from the map that for higher damping, vibrations are more regular as confirmed by the previous displacement-time plots. However, as the damping is reduced, the fixed points shift, leading to complex phase plots. These plots indicate that the period of oscillations have begun to increase. Finally, the oscillation periods increase more and more till the tell tale signature of chaos is clearly emergent. Thus these systems are capable of exhibiting chaos for sufficiently low damping. In the real world system, damping can be modulated through the adjustment of material properties, inter scale lubricant and the geometric configuration of the structure, including factors such as the overlap ratio of scales, the initial angle of scales relative to the substrate, friction between scales, the embedded length of the scales, and others.

Exploring Poincare maps further, in Fig. 4, we plot the evolution of Poincare maps with the evolution of both damping and nonlinear stiffness magnitude. We discover that for even low damping which would otherwise lead to chaotic vibrations, an increased stiffness leads to chaos suppression. Broadly, this trend holds as an increase in both damping and stiffness can lead to decrease in chaotic vibration. Similarly, for even relatively higher

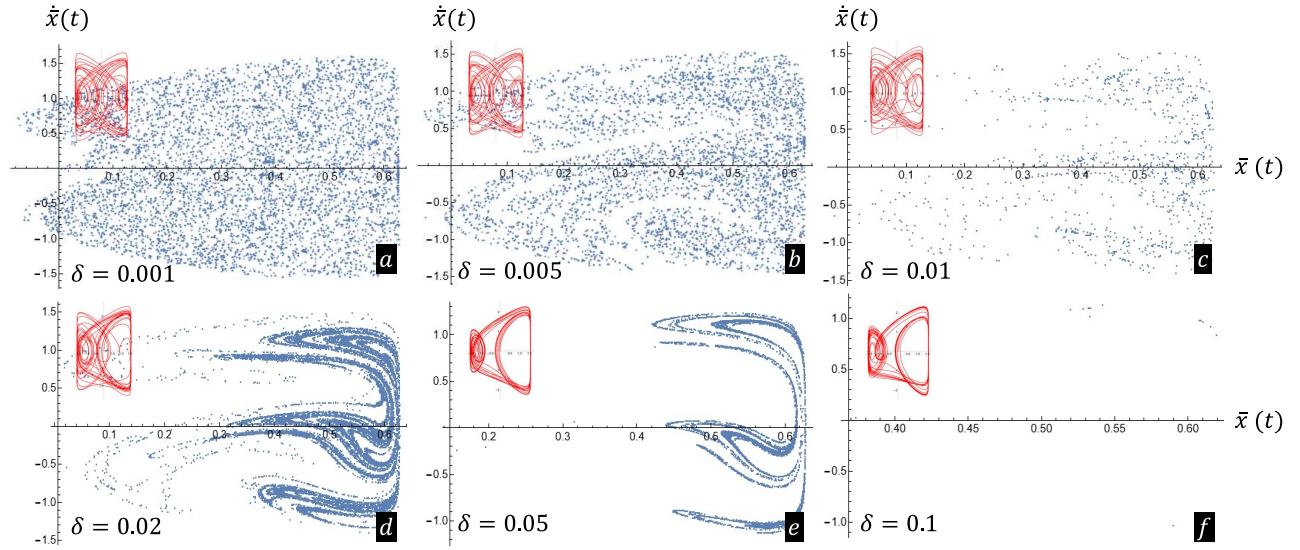


Figure 3.

The perturbation section of the forced vibration equation for a substrate covered with scales, which illustrates chaotic behavior and also depicts the influence of the damping parameter on the changes in this chaos, is presented. The remaining parameters of the equation for this case are equal to $\Gamma = 0.65$, $\gamma = 2$, $\omega = 0.5$, and these plots are drawn for the periodicity of $2\pi/\omega = 4\pi$. Also, in the corner of each plot, the phase plots are also completely depicted. Increasing the damping coefficient of the system leads to the emergence of a specific form of points in Poincaré section which represent chaos in the system. As the damping is increased, these chaotic flow are reduced and eventually eliminated.

damping, a decreased nonlinear stiffening can activate chaotic states as depicted in this figure, Fig. 4 (top left). Note that in a real system, Γ is influenced by both the material and geometric properties of the system. Similarly δ is also influenced by both scale overlap, friction and also presence of lubricating function. Our models suggest that by adjusting geometric parameters, we can effectively control the chaotic characteristics of the system.

4. CONCLUSION

In conclusion, we introduce a new lumped parameter, reduced order model to investigate the nonlinear dynamics of bio mimetic scale substrate that is capable of going beyond the limits of the computational inefficiencies of lumped parameter models. The models mimic the behavior of actual bio mimetic scale systems reported in earlier literature. The results indicate for the first time the potential route to chaotic vibration of these substrate. Such chaos can be controlled and tuned geometrically using scale distribution, material and interface properties, leading to new avenues of smart materials development. Hence, a combined material-geometry co-design with active materials can lead to novel nonlinear dynamic behavior not observed in typical beams. Note that these nonlinear behavior do not rely on large deformation. Model limitations include lack of introduced asymmetries, spatial variations and comparison with experimental systems. However, despite these limitations, this study lays ground for the first time to evaluate these biomimetic scale systems for complex, chaotic vibrations.

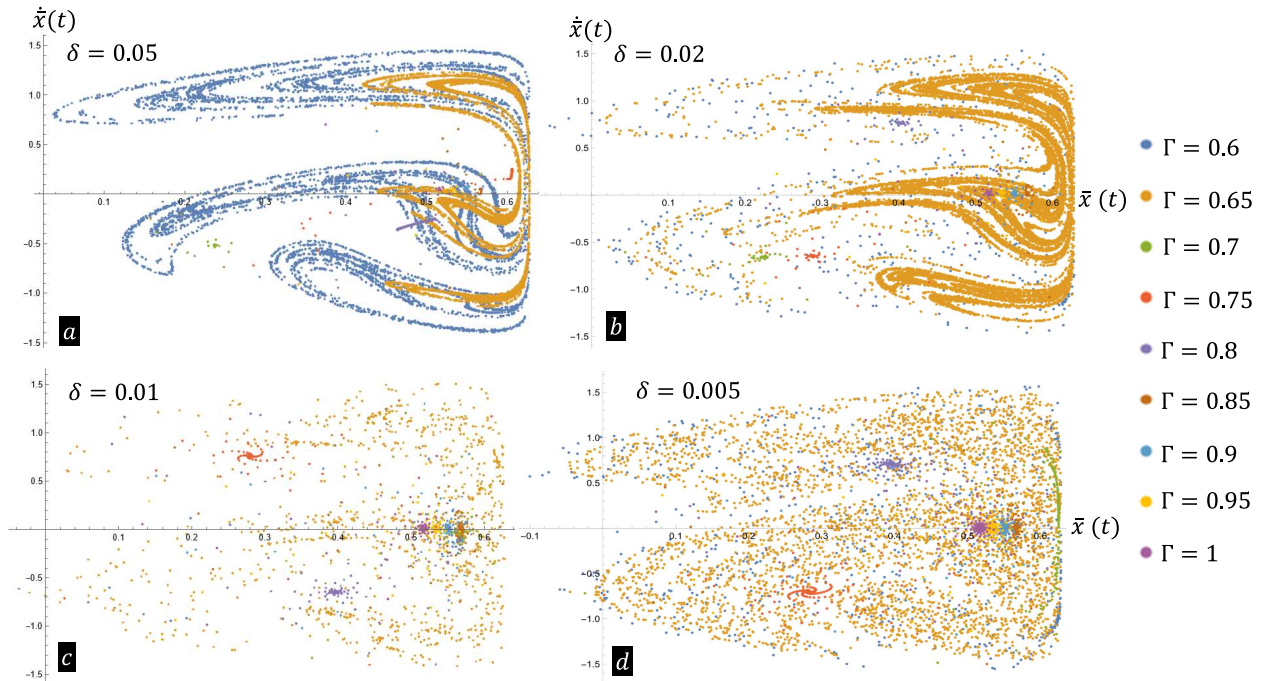


Figure 4. Displaying the Poincaré section for different values of the nonlinear parameter of the equation for four different damping conditions. The colors represent different Γ values, and it is evident that within the range of $\Gamma = 0.6$ and $\Gamma = 0.65$, there are more instances of chaos. Other parameters are as $\gamma = 2$, $\omega = 0.5$. As observed in the figures, for each damping coefficient, chaos disappeared with increasing non-linear stiffness. In fact, the occurrence of chaos with the damping coefficient, non-linear stiffness, and locking of the scales is correlated.

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