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An Augmented Reality Application and Experiment for Understanding and Learning Spatial Transformation Matrices

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Abstract

Understanding spatial transformations and their mathematical representations is essential in computer-aided design, computer graphics, robotics, etc. This research has developed and tested an Augmented Reality (AR) application (BRICKxAR/T) to enhance students' learning of spatial transformation matrices. BRICKxAR/T leverages AR features, including information augmentation, physical-virtual object interplay, and embodied learning, to create a novel and effective visualization experience for learning. In this paper, we evaluated the BRICKxAR/T as a learning intervention using LEGO models for physical and virtual manipulatives in an experiment. The experiment compared AR (N=29) vs. non-AR (N=30) learning workshops with pre- and post-tests on Purdue Visualization of Rotations Test and math questions to assess students' learning gains. All participants math scores significantly improved with the AR workshop tending to show greater improvements. The post-workshop survey showed students were inclined to think BRICKxAR/T an interesting and useful application, and they spent more time learning in AR than non-AR.

Keywords

Augmented Reality, Spatial Transformations, Transformation Matrices, Visualization, Embodied Learning

Statements and Declarations

Conflicts of interest/Competing interests: no conflicts of interest/competing interests

Ethical Statement

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Understanding spatial transformations and their mathematical representations is essential in computer-aided design, computer graphics, robotics, etc. This research has developed and tested an Augmented Reality (AR) application (BRICKxAR/T) to enhance students' learning of spatial transformation matrices. BRICKxAR/T leverages AR features, including information augmentation, physical-virtual object interplay, and embodied learning, to create a novel and effective visualization experience for learning. In this paper we evaluated the BRICKxAR/T as a learning intervention using LEGO models for physical and virtual manipulatives in an experiment. The experiment compared AR (N=29) vs. non-AR (N=30) learning workshops with pre- and post-tests on Purdue Visualization of Rotations Test and math questions to assess students' learning gains. All participants math scores significantly improved with the AR workshop tending to show greater improvements. The post-workshop survey showed students were inclined to think BRICKxAR/T an interesting and useful application, and they spent more time learning in AR than non-AR.

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1 INTRODUCTION

Spatial and mathematical thinking are closely allied [1]–[3]. Also, physical model interactions have shown a significant impact on spatial visualization and reduction of extraneous cognitive load in learning spatial and geometry-based problems [4]–[6]. Understanding tightly coupled spatial transformations and mathematical concepts significantly contributes to STEM (Science, Technology, Engineering, and Mathematics) learning in fields of geometric modeling, computer graphics, computer-aided design (CAD), computer vision, robotics, video games, quantum mechanics, and more; and helps students consider mathematics as an interconnected discipline, assisting in higher-level reasoning activities [7]. Literature has clearly acknowledged the difficulty of learning and teaching spatial transformations and the associated math representations such as matrices [7], [8]. Despite the development of Computer Assisted Learning (CAL) in geometry, spatial transformations, and related mathematics, many students still face challenges in solving geometric problems and rely on a trial-and-error process [9]. Furthermore, it is very challenging to maintain students' attention in CAD workshops, and students often drop the course before it ends [10].

Augmented Reality (AR) as a mediator tool with the ability to superimpose digital content and information over a physical environment supports a context to integrate embodied learning and virtual augmentation of abstract information. AR immersive environment may allow students from different fields of STEM to learn the mathematical logics and abstractions pertained to 3D modeling functions through spatiotemporal experiments while removing the extraneous cognitive load of using keyboards and mice.

The intellectual merit of this research lies in the developed innovative learning environment that enables physical and virtual interplays to engage students in embodied learning through interacting with physical models. In this study, the inherent capability of AR is adopted to realize novel spatial strategies for understanding mathematical concepts through visualizing the concepts and graphical information that are registered (aligned) and synchronized with physical motions in the physical environment. Computer graphics (e.g., arrows, tags, highlighting, etc.) matched with the user's view could effectively draw students' attention [11] and improve their mental imagery by visualizing difficult invisible concepts. The developed AR app - BRICKxAR/T - realizes the synchronized visualization of mathematical concepts with the physical actions. This assists students in perceiving transformation scenarios (motions and the corresponding mathematical functions) within a single comprehensive application.

As a broader impact, this research has the potential to support many students who struggle with spatial and math reasoning, especially those from underrepresented groups in STEM. Gaining a fundamental understanding of spatial transformations can uniquely contribute to students' learning and development of spatial reasoning and allied mathematical skills, leading to improved STEM coursework, STEM retention, and degree attainment, and thus supports students' future development of expertise and career success across STEM disciplines.

This research intends to address the mentioned challenges by answering these **research questions**:

1. Can an AR-powered learning system improve students' spatial visualization of rotations and their mathematical representations, specifically, matrices?
2. Does the AR system help students focus on the targeted learning subject and decrease the task load compared to a non-AR system?

3. Can the AR system stimulate students' motivation and engagement in learning mathematical concepts of geometric transformations?

2 LITERATURE

Studies report that the geometry courses through the US school grades trend more toward recognizing and naming new geometric objects rather than a deep level of analysis [12]. Hence, many educators in STEM professions strive to improve academic competency in these fields. Researchers believe geometric transformations help students think of essential mathematical concepts through new approaches with higher-level reasoning [7]. About 20% of students experience difficulty in problems dealing with spatial skills, such as manipulating figures in space and analyzing complex shapes [9], [13]. Literature indicates that when students engage in physical interaction, it boosts their creativity in generating design ideas [4][14], reduces unnecessary cognitive burden during the creative design process [15], enhances their spatial skills in comprehending size relationships among geometries [16], and fosters improved collaboration and communication during group work [17]–[19].

2.1 AR for Learning Geometry and Mathematics

Several AR applications have been developed for learning descriptive geometry and mathematics [10], [20]–[22], which demonstrate the positive impact of AR intervention in geometry perception. Most of the apps have used AR applications as visualization tools, displaying 3D geometries and different representations (for example, images of unfolded geometry) in a spatial environment to help students' spatial visualization skills [10], [22]–[26]. The mathematical representations in Construct 3D app are limited to certain graphics only [27] and the mathematical representations in the AR app developed by Cahyono et al., 2018 are static text with no real-time interactions [28].GeoGebra AR is one of the recent AR applications in learning geometry and mathematics [21]. However, GeoGebra AR does not provide any major physical interaction or interplay of physical and digital environments in the learning process. Also, the mathematical section in GeoGebra AR requires advanced pre-knowledge of mathematical equations to generate forms in AR; otherwise, it will be a trial-and-error process. Hence, it maybe a challenging application for a self-learning process.

Although the potential of AR in learning mathematics and improving spatial skills has been studied in STEM learning, the technology has not yet been explored deeply in learning geometric transformations and their mathematics.

2.2 AR vs. CAD and Virtual Reality (VR)

While CAD and VR may also assist in learning similar spatial and math concepts, compared to AR, they have major limitations. For example, the 2D images of 3D objects in CAD models do not match the real-world perspective view and require a mental model alignment as well as appropriate use of a keyboard and mouse to navigate through the scene. VR can specifically be helpful for simulating experiences that do not exist in the real world (e.g., a fictional environment) or are not easily accessible (e.g., walking through a space) [29], [30]. However, the VR experiments detach a user from the physical environment by replacing it with a complete virtual surrounding [31]. Modeling a whole new environment to simulate a real experience for VR application could be time-consuming and computationally expensive [32]. The interactions in a VR environment, realized through external hardware controllers, are neither immediate nor natural, which may impose extraneous cognitive loads. Also, some VR experiments report health issues such as motion sickness and injuries due to wearing VR headsets such as Oculus [29][33]. In addition, there are health concerns with sharing a VR headset, especially for children during a pandemic such as COVID-19. These cognitive loads, usability, and health and safety concerns associated with VR may be reduced with AR. For example, the AR-enabled tablets can generally be more affordable than the VR headsets for personal use without the need of sharing. Furthermore, AR supports automatic perspective view alignment with the user's relative position, embodied learning, and physical interactions, which facilitate tangible manipulatives in 3D space, and contribute to improving mental 3D visualization [5], revising mental model misconceptions [14], enhancing spatial cognition and design creativity [4][15], improving idea generation [14][4], and encouraging epistemic action and memory retrieval [34].

3 APPLICATION PROTOTYPE

The BRICKxAR/T app [35], [36] is an AR prototype for learning spatial transformations and matrix algebra. An equivalent non-AR version of the app is also developed with the same visualization functions without AR features. The app has been developed based on the progressive learning method introduced in literature for learning spatial transformations [37]. In this technique, students will learn spatial transformations in three levels of *motion*, *mapping* and *function* [37] as follows:

- Motion: AR supports physical actions and embodied learning.
- Mapping: AR supports visualizing the transformation mapping process through demonstration of the image and re-image of transformations.
- Functions: AR supports augmentation of mathematical functions of transformation matrices.

We have leveraged the AR technology to realize this process for spatiotemporal experiments. Using AR intervention, physical interaction (motion), along with physical and virtual models' interplay (mapping), is supported in the application. Graphical illustrations are displayed in the AR 3D environment, representing the mapping operations. Mathematical functions are synchronized with motion and mapping. In the current experiment, half of the participants interacted with the AR version of the app and the other half interacted with a non-AR version.

3.1 AR App

When the user starts the workshop, the app registers two virtual models, one visible and one hidden, through the image marker and aligned with the physical model. When the physical model is moved by the user, the visible virtual model stays in the original location, representing the ***pre-image*** of the transformation, and the hidden virtual model with a visible coordinate system follows the physical model, together representing the ***image*** of the transformation (Figure 1). The physical model's transformations (translation and rotation) are thus visualized by the registered and synchronized distance line and rotation angle graphics as well as the transformation matrix functions (top).

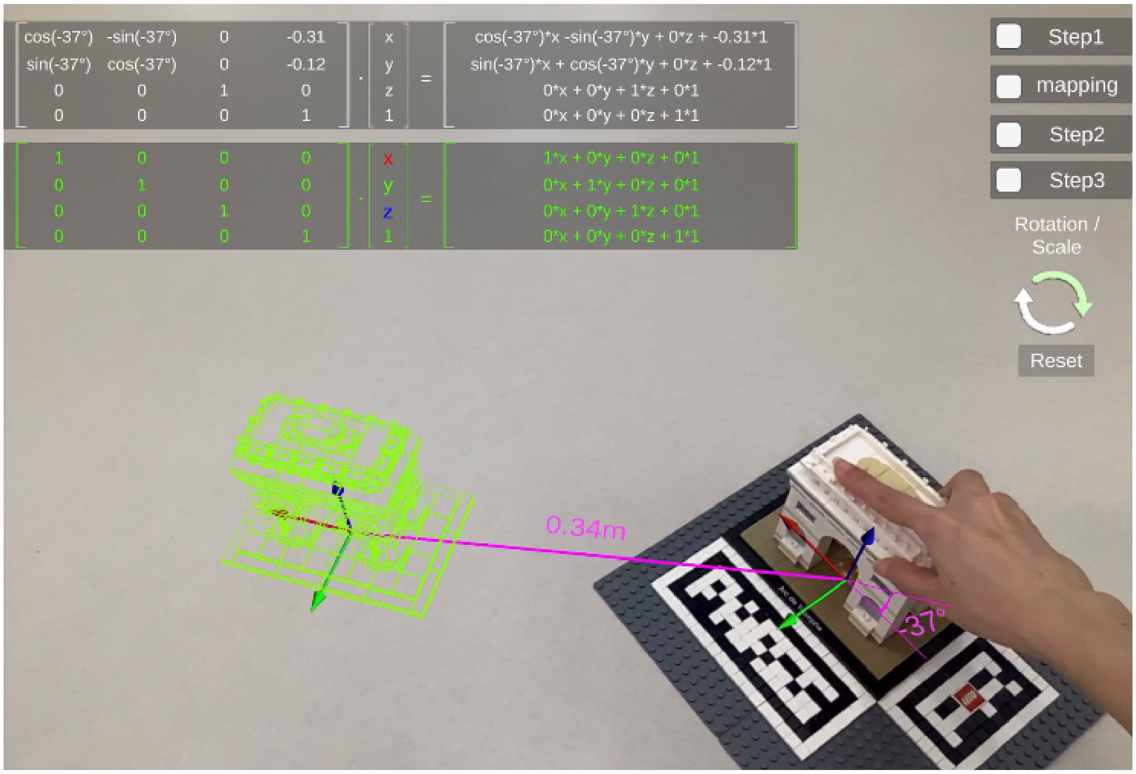


Figure 1: Seen through an AR-enabled iPad, virtual model (pre-image of transformations) and physical model (image of transformations) in the AR scene, with the transformations (translation and rotation) illustrated by synchronized distance line and rotation angle graphics as well as transformation matrix functions (top)

The BRICKxAR/T app has been developed on the AR-enabled iOS mobile device (iPads). In BRICKxAR/T, the AR registration (alignment between virtual and physical models) is highly accurate, and the occlusions between the physical and virtual models are created to realistically reveal the spatial relationships among the models [38]. Hand occlusion is also activated to the AR environment to augment the virtual renderings with correct depth perception with respect to the physical objects (Figure 2).

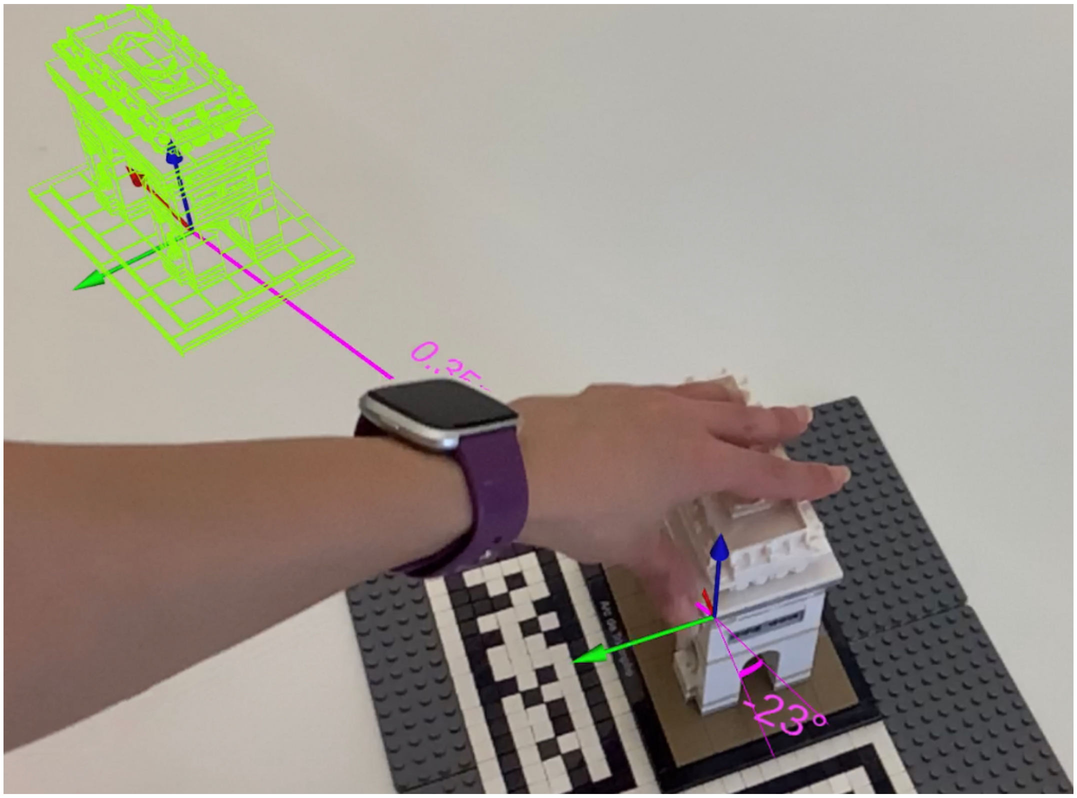


Figure 2: A zoom-in view of the distance line and rotation arc and angle between the physical and virtual models. Occlusion is applied between the real hand and virtual objects

Figure 3 illustrates that the visible virtual model can also be moved through the user interface menus. The 3D Cartesian coordinate system (#1) is the World Coordinate System (WCS) in the AR scene. The WCS gets instantiated in the beginning but never updated during the play. The 3D Cartesian coordinate system (#2) represents the local coordinate system of the physical LEGO model (and an aligned hidden virtual model). This coordinate system gets updated seamlessly by the AR camera tracking the attached image marker and follows the movement of the physical model applied by the user. The wireframe virtual model (#3) can be transformed (translated, rotated, and scaled) through its parameter controls on the AR screen by the user.

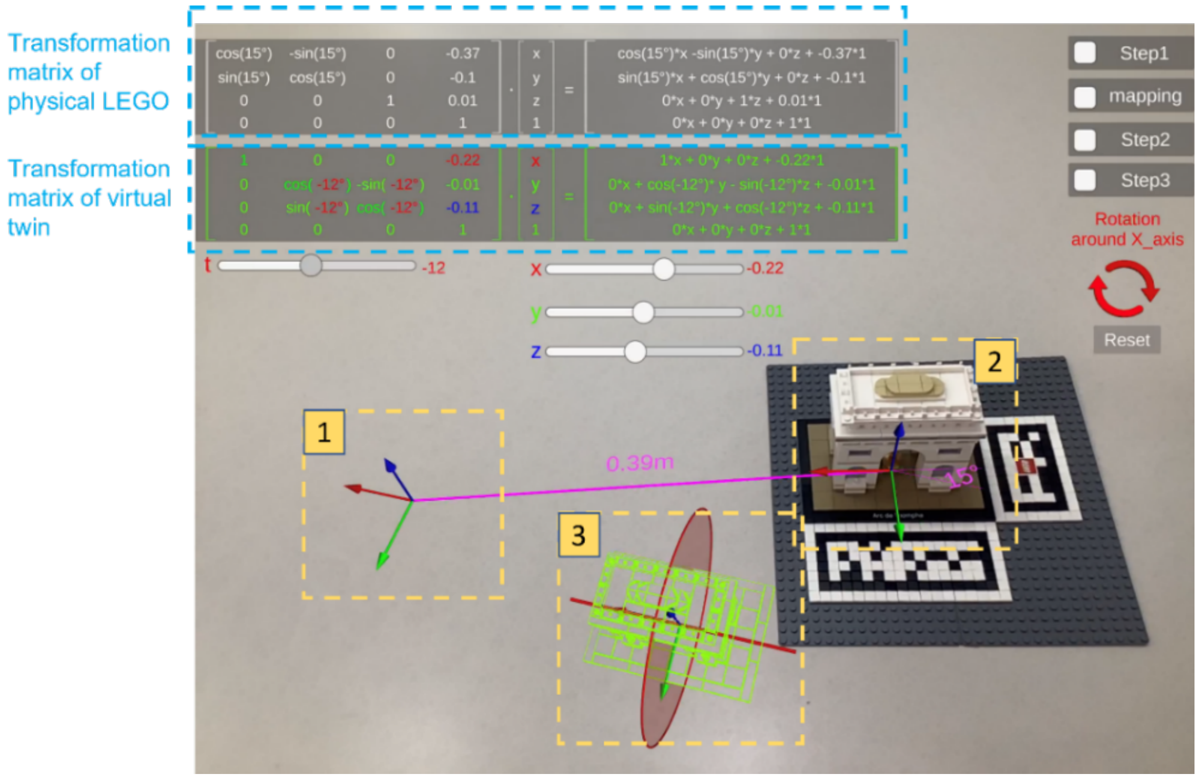


Figure 3: AR scene: physical and virtual models and their corresponding transformation matrices for both models (graphics and notations in yellow and cyan colors are added on top of the AR scene manually for better explanation)

Figure 4 depicts the relations among the AR camera and the three components: physical model, virtual model, and the WCS.

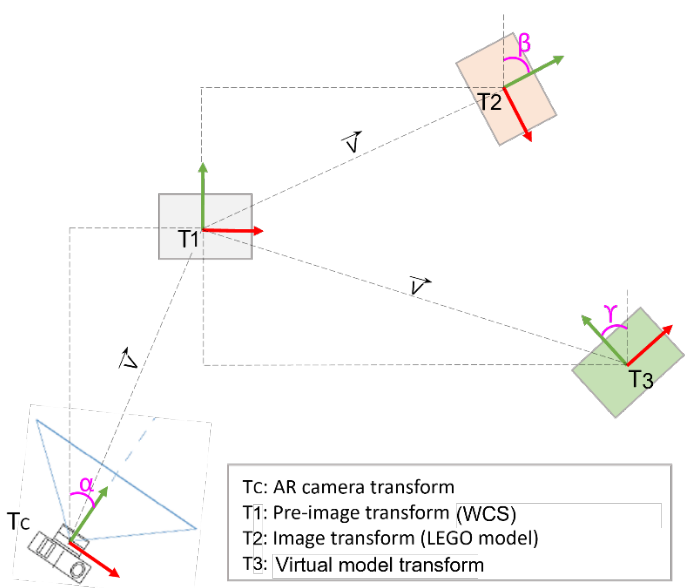


Figure 4: Relations among the AR camera and the three components: physical model, virtual model, and the WCS in the AR scene

The prototype is an RTS (Rotation, Translation, and Scale) puzzle game that helps students understand mathematical notions of spatial transformations along with the mathematical components of transformations, such as variables, parameters, and functions [35]. The mathematical functions matched with the actual motions in real-time assist students to perceive the geometric reasonings behind transformation matrices in an integrated spatial scenario. Students can play with the physical LEGO model, translate and rotate it and observe the corresponding translation and rotation matrices (first row matrices in Figure 3). The distance line and the rotation arc and angle show the graphical representation of the mapping function where the numbers (distance and angle) are matched with the elements in transformation matrices (first row).

Students can also play with the virtual model to numerically compose the transformation matrices (second row) by directly interacting with the corresponding function parameters and associated menu sliders. Translating the virtual model in all three axes can be done at the same time. The associated sliders display when touching x, y, and z parameters of the point vector (representing a point on the geometry) shown in red, green, and blue, respectively (Figure 3).

To rotate or scale the virtual model, students can tap the Rotation/Scale control. Then, the associated transformation matrix and the corresponding slider to change the selected parameter (i.e., rotation angle or scale factor) appears on the screen. The parametric changes applied on the sliders will be reflected in the 4×4 transformation matrix of the virtual model as well as the algebraic equations (results of the matrix multiplication), as shown in Figure 5. To keep the equations of the matrix multiplication simple for the students to understand, rotation around other axes gets zeroed out each time the student chooses a new axis.

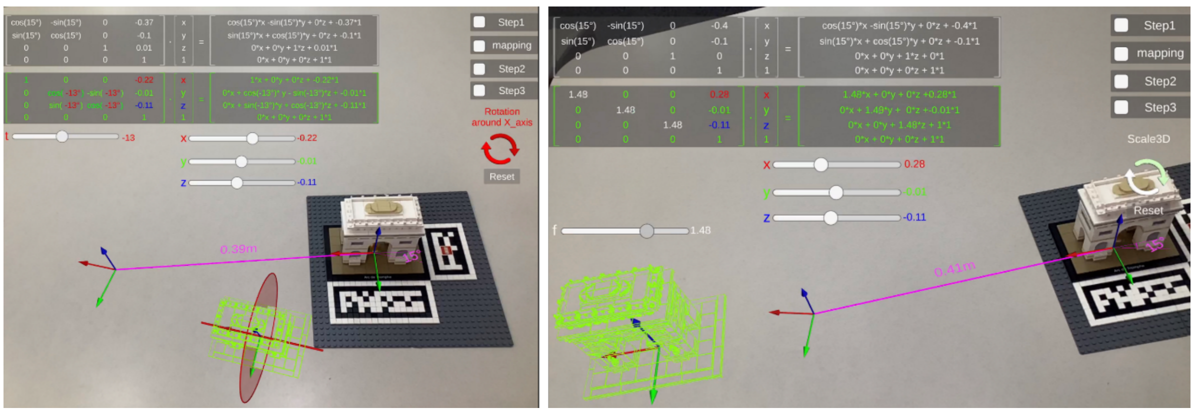


Figure 5: Rotating the virtual model around y-axis (left) and scale in x, y, and z-axes (right)

Finally, students can learn to compose the transformation matrix of the virtual model so that it matches the transformations of the physical model at its current location and orientation (Figure 6). This step intends to intuitively describe the concept behind AR registration (physical and virtual model alignment) as one of the applications of transformation matrices, which is normally done through the AR technology automatically using camera and motion sensors on the AR device.

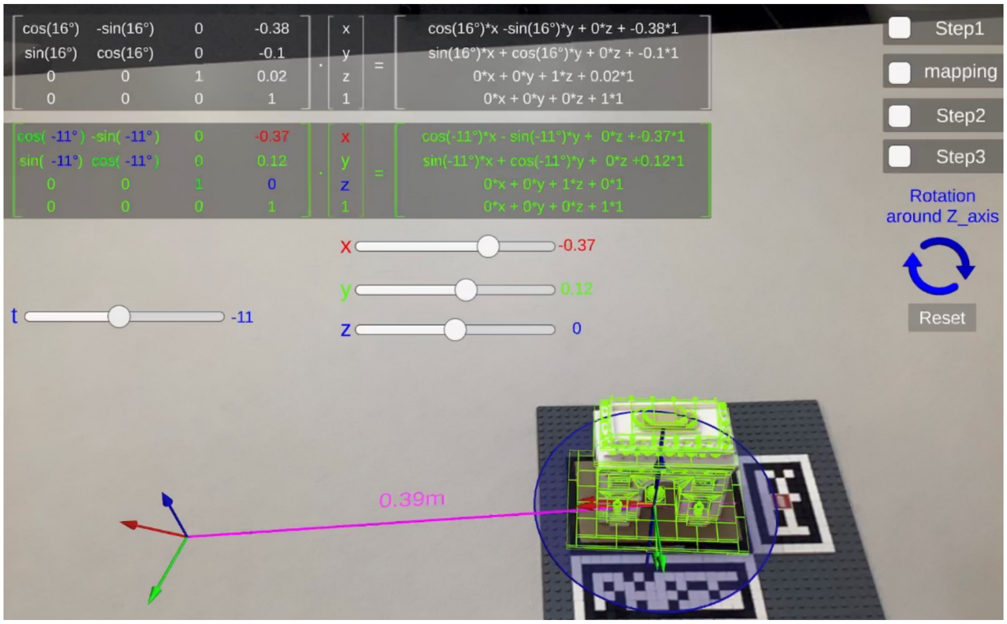


Figure 6: Practicing AR registration through playing with function parameters

3.2 Non-AR App

The features and functions of the equivalent non-AR version of the app are similar to BRICKxAR/T except that physical model interaction is disabled in this version. Due to the lack of a physical model interaction in a non-AR environment, only two virtual models get rendered in the application, including a shaded LEGO model and a wireframe model (i.e., a digital twin). The shaded model remains fixed with a Cartesian coordinate system representing the WCS, and the wireframe model can be transformed using function parameters. In the AR environment of BRICKxAR/T, students can freely move around the table and interact with the physical model; however, the non-AR version is similar to a typical desktop app which only allows common screen navigation as follows:

- Panning: sliding on the screen with two fingers
- Zooming: pinching two fingers together or apart to zoom-out or zoom-in, respectively
- Orbiting: sliding on the screen with one finger towards left and right to orbit the screen clockwise or counterclockwise.

Because the physical-virtual model interaction is not possible in the non-AR environment, this version is restricted to the relations between the shaded model as the pre-image and the wireframe model as the image. Also, the non-AR version does not support an automatic perspective alignment. Therefore, students need to manually navigate through the scene (pan, zoom, orbit) for different perspective views and better visualization of the rendered graphics, such as distance lines, rotation arcs, and corresponding notations. Figure 7 shows a scene from the non-AR versions of workshop1 with manual navigations applied.

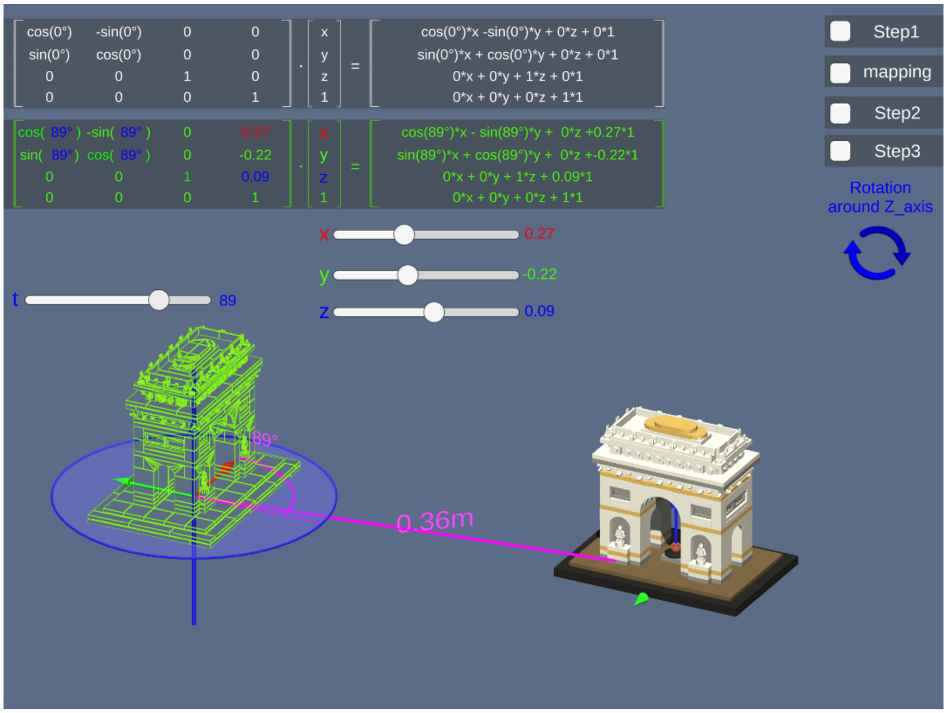


Figure 7: Playing with function parameters in the non-AR version of Prototype 1.

4 RESEARCH OBJECTIVES AND HYPOTHESES

The primary objective of this research is to address the challenges associated with teaching and learning geometric transformations. The aim is to leverage AR as a tool to enhance students' spatial and math skills in understanding the concepts of transformation matrices. The ultimate goal is to provide students with a comprehensive understanding of these concepts, helping them bridge the gap between mathematical abstractions and real-world applications.

Through the development and evaluation of the BRICKxAR/T application, this research aims to contribute to more effective pedagogical approaches for teaching mathematical concepts, ultimately benefiting students in various STEM disciplines.

Based on the identified research problem and questions, the study's hypotheses are as follows:

1. BRICKxAR/T may help students in solving mathematical problems, specifically matrices for geometric transformations (better improvement in AR than non-AR).
2. The AR version of BRICKxAR/T may impose fewer task loads compared to the non-AR version of the app.
3. The AR version of BRICKxAR/T is more exciting and engaging to use compared to the non-AR version of the app.

In this paper, we present an experiment in which the BRICKxAR/T has been evaluated as a learning intervention using LEGO models as examples of physical and virtual manipulatives. We conducted an experiment to compare participants' learning gains (pre-test to post-test) in spatial visualization and math skills after completing an AR or non-AR workshop. Additionally, we conducted surveys to compare the apps' performance and measure students' motivation and play-time.

5 METHODS

5.1 Participants

Upon the human subject research (Institutional Review Boards, IRB#2020-1213M) approval, we invited undergraduate students, through bulk email messages across Texas A&M University. Fifty-nine undergraduate students with normal vision participated in the experiment voluntarily. The students had at least high school-level knowledge in algebra and geometry. The participants were randomly split into AR and non-AR sessions (N_{AR} = 29, N_{non}-AR = 30). Data collection occurred between Fall 2021 and Spring 2022.

Twenty-nine (29) students participated in the AR workshop (Figure 8), among which 31.03% (n=9) were either very familiar or moderately familiar, and 68.97% (n=20) were either slightly or not familiar with digital modeling. 34.48% (n=10) of the students were either very familiar or moderately familiar, and 65.52% (n=19) were either slightly or not familiar with the AR technology.

Figure 8: Students playing with the AR version of the app

Thirty (30) students participated in the non-AR workshop (Figure 9), among which 46.67% (n=14) were either very familiar or moderately familiar and 53.33% (n=16) were either slightly or not familiar with digital modeling (familiarity with AR technology was not asked from the non-AR group).

Figure 9: Students playing with the non-AR version of the app

Table 1 shows that female and males students participated in the AR workshop have a ratio of approximately 3 to 1. About 51.73% of the AR participants were from STEM colleges: 24.14% from engineering and 27.59% from science. The remaining (48.27%) were affiliated with other colleges: architecture (10.34%) and other non-STEM fields (37.93%). In the non-AR workshop, female and male participated with a ratio of approximately 1 to 1. Most participants (73.33%) of the non-AR were from STEM colleges: 50% from engineering and 23.33% from science. Only 26.66% of the non-AR participants were from other colleges: architecture (3.33%) and other non-STEM domains (23.33%).

Table 1: Demographic information of the participant of AR and non-AR workshops

Group	Number	Gender (%)	Field of Study (%)

		Female	Male	Non-Binary	Engineering	Science	Architecture	Other
AR	29	72.41 (n=21)	24.14 (n=7)	3.45 (n=1)	24.14 (n=7)	27.59 (n=8)	10.34 (n=3)	37.93 (n=11)
Non-AR	30	46.67 (n=14)	53.33 (n=16)	0 (n=0)	50 (n=15)	23.33 (n=7)	3.33 (n=1)	23.33 (n=7)

5.2 Design

The current experiment used a mixed design. Participants were randomly assigned to interact with either the AR version or non-AR version of the BRICKxAR/T, which is a between-subjects independent variable. All participants completed a pre-test before interacting with the app and a post-test afterward, which is a within-subjects independent variable. Dependent measures were math test, Purdue Visualization of Rotations test, NASA TLX ratings, and the Motivated-Strategies-for-Learning-Questionnaire.

5.3 Materials

In order to have an integrated testing platform, all tests and surveys were implemented through the online Qualtrics survey application [39].

5.3.1 Purdue Visualization of Rotation Test (PVRT)

We used the 20-item version of PVRT [40] to evaluate students' spatial visualization skills before participating in the workshops to guarantee that students of both groups had similar spatial visualization skills prior to the workshops. Since prior research shows that spatial skills may only improve through repetitive sessions over a long period of time [41], [42], we did not focus on students' PVRT post-test scores and their improvement in a short session workshop. This test consists of 20 multiple choice questions in which students are asked to study how an object is rotated in the sample and select the option that has rotated in the same manner (Figure 10). In our analysis, final scores were calculated using an average of correctly answered questions, scaled to 100 like a percentage (range = 0 to 100, ratio scale).

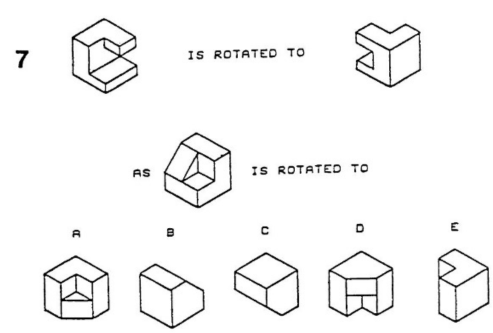


Figure 10: An example question from the 20-item PVRT [40]

5.3.2 Math Test

We designed a math test with 13 questions on transformation matrices, based on learning materials of the Khan Academy [43] (Figure 11). In our analysis, final scores were calculated using an average of correctly answered questions, scaled to 100 like a percentage (range = 0 to 100, ratio scale).

Q9. Which is the best description for the transformation given by the following matrix

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & \cos(t) & -\sin(t) & 2 \\ 0 & \sin(t) & \cos(t) & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}?$$

a) Combined matrix of rotation and move in 3D space

b) Combined matrix of rotation and reflection in 3D space

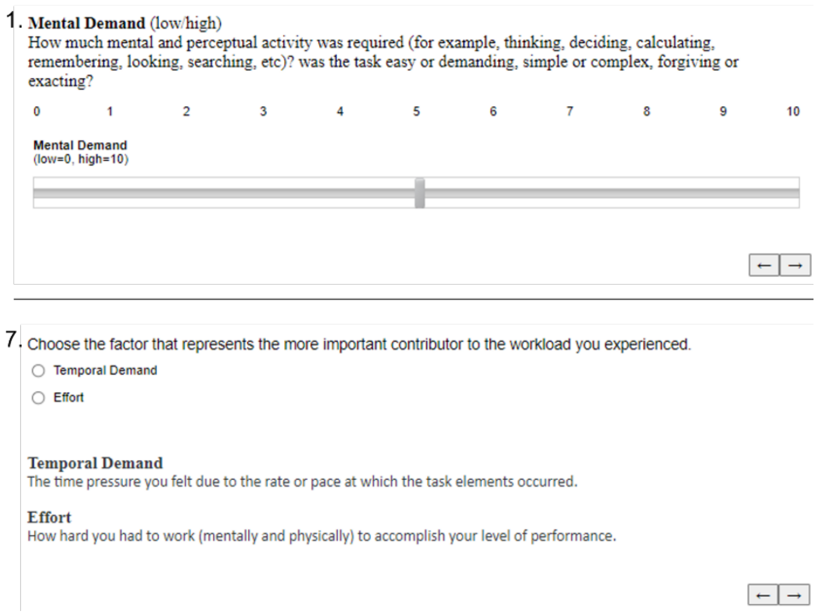
c) Combined matrix of scale and move in 3D space

d) Combined matrix of scale and rotation in 3D space

Figure 11: An example of the math test designed based on [43]

5.3.3 *NASA_TLX Survey*

We measured the application task workload through the NASA-TLX survey [44] on six dimensions: mental, physical, temporal, effort, frustration, and performance. The survey contains 6 ranking questions in which each dimension is graded on an interval scale ranging from low 0 to high 10, and 15 pairwise questions in which subscale pairs are compared (Figure 12).


1. **Mental Demand** (low/high)
How much mental and perceptual activity was required (for example, thinking, deciding, calculating, remembering, looking, searching, etc)? was the task easy or demanding, simple or complex, forgiving or exacting?

0 1 2 3 4 5 6 7 8 9 10

Mental Demand
(low=0, high=10)

← →

7. Choose the factor that represents the more important contributor to the workload you experienced.
- Temporal Demand
- Effort

Temporal Demand
The time pressure you felt due to the rate or pace at which the task elements occurred.

Effort
How hard you had to work (mentally and physically) to accomplish your level of performance.

← →

Figure 12: Top: An example of the rating questions; Bottom: an example of the pairwise questions; adopted from NASA_TLX [44]

Based on [44], the adjusted score of each factor is calculated based on its ranking (0 to 100) and its weight (the number of times that the factor is selected in the pairwise questions:

$$Adjusted\ score\ per\ factor = \frac{rating\ score \times weight}{15} \qquad (1)$$

The maximum rating score is 100, while the maximum time that a factor can ever be selected (weight) is 5; hence, the minimum adjusted score is 0 and the maximum adjusted score that a factor may achieve is 33.3 (ratio scale) based on Equation (1).

5.3.4 *Motivated-Strategies-for-Learning-Questionnaire (MSLQ)*

We assessed students' motivations and interests to play with the app through a questionnaire based on the MSLQ survey [45] in three categories of intrinsic value, task anxiety, and self-regulated-learning. Each category consists of multiple questions to evaluate students' subjective viewpoints regarding the corresponding item. The questions are scaled in 5 steps starting from 1 to 5 representing “strongly agree” to “strongly disagree”. The mean scores of questions within each category are used in the analyses of this study. Figure 13 shows three example questions from the intrinsic value category. In our analysis, items in each category were averaged together (range = 1 to 5, ordinal scale).

	strongly agree	somewhat agree	neither agree or disagree	somewhat disagree	strongly disagree
I like what I learnt and I think the subject is interesting.	○	○	○	○	○
I think that what I learnt in this workshop is useful for me to know.	○	○	○	○	○
Understanding this subject is important to me.	○	○	○	○	○

Figure 13: Questions of the intrinsic value of the motivation questionnaire inspired by MSLQ [9]

5.4 Procedure

Figure 14 illustrates the workflow that the researchers followed to conduct the experiment for evaluating the AR vs. non-AR version of BRICKxAR/T.

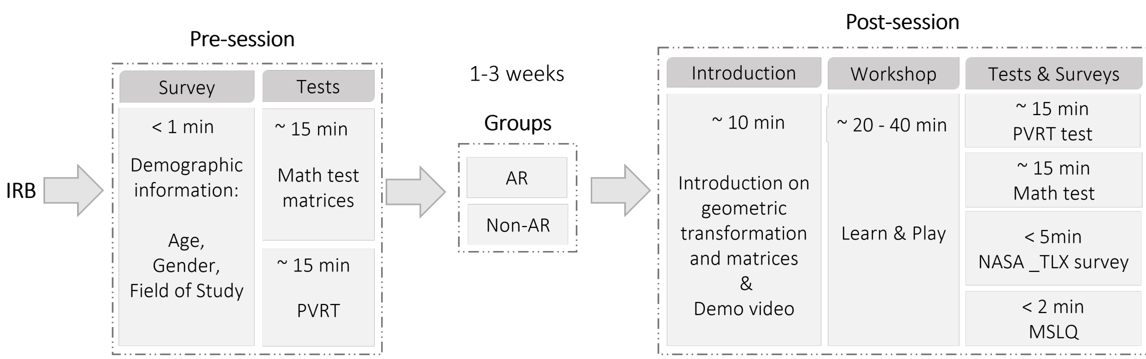


Figure 14: Experiment framework

Once recruited, participants completed the pre-session: an online survey (demographic questionnaire) and pre-tests of Purdue Visualization of Rotation Test (PVRT) and a matrix algebra math test. To ensure unbiased results, the tests' answers were not disclosed to the students during the pre-session, and randomization was applied to the answer choices.

To mitigate the potential Testing and History Threats [35], we conducted the post-sessions one to three weeks after the pre-session. This time interval, as recommended by existing literature, strikes a balance by preventing the participants from recalling specific questions (from pre-tests) while also minimizing the likelihood of any unintentional influence from external events or history on their skills.

Participants were randomly assigned to either the AR or non-AR face-to-face learning workshops. In the post-session, students first watched an introduction video on the concepts of the learning materials and an instruction demo video on using the corresponding apps (AR or non-AR version). The demo videos were used to minimize the “*Instrument and Misidentification Threat”* [35]. Then, students were asked to play with the apps for 20 to 40 minutes and record their screens during the play. To minimize the “*Cross-Group Contamination*” effects [36], AR and non-AR workshops were held separately, and groups were not aware of each other.

It is worth noting that the educational content was equal in the AR and non-AR version of the application. Physical LEGO models were provided for both groups. However, the AR environment allowed students to play and learn from abstract information integrated with the physical environment. This means that they could move around the room and move the LEGO model, while the AR-version of the BRICKxAR/T app responded to those movements and dynamically overlaid abstract information. This is because the LEGO model was attached to image markers for the AR version only. In contrast, the non-AR/desktop version of the app only allowed students to interact with the digital version of the LEGO model. Any body movement or movement of the physical LEGO model was not reflected in the digitally displayed information.

Following their participation in the workshops, the students took the same PVRT and math tests (with randomized answer choices), along with two surveys (NASA_TLX and Motivated-Strategies-for-Learning-Questionnaire or MSLQ).

6 RESULTS

Before conducting any of the statistical tests, the outliers corresponding to each test were excluded using whiskers box plot. Also, the assumptions, specifically, normality and homogeneity of variances, were verified to determine whether to apply parametric or an equivalent non-parametric test. If the tests' pre-assumptions were met, parametric tests were applied, otherwise the equivalent non-parametric tests were conducted. We set 0.05 as a threshold for the p-value to accept or reject the null hypothesis associated to each test. Note that the data illustrated in all tables of this section reflect descriptive statistics before removing the outliers while statistical tests were conducted after removing the outliers.

6.1 Demographics and PVRT Scores

All participants of AR and non-AR group were in the age range of 15 to 30 years old. A chi-squared test comparing gender in the AR (males = 7, females = 21) and non-AR (males = 16, females = 14) groups found a significant difference, $X^2{}_1 = 4.9$, $p = 0.03$. This means that the two groups were statistically different in gender distribution, with the AR group having more females than the non-AR group. In this test the only non-binary participant of AR group was excluded to avoid type I error (caused by a very small sample size in one category).

Table 2 shows the pre-test PVRT mean scores of the AR and non-AR groups. One outlier was excluded from the AR data ($N_{AR} = 28$, $N_{non\text{-}AR} = 30$).

Table 2: PVRT pre-test scores

Group	AR	Non-AR
	Pre-test score (%)	Pre-test score (%)
Min	20	20
Max	100	100
Mean	64.14	65
SD	23.64	23.27

The between-subjects ANOVA conducted on the PVRT pre-test scores did not show a significant difference between the AR and non-AR groups ($F(1, 56) = 0.68$, $p = 0.8$), which indicates that students of the two groups had similar mental visualization skills before participating in the workshops for learning spatial transformation matrices.

6.2 Math Scores

Figure 15 left and right illustrates students' math scores in the pre- and post-sessions of AR and non-AR, respectively.

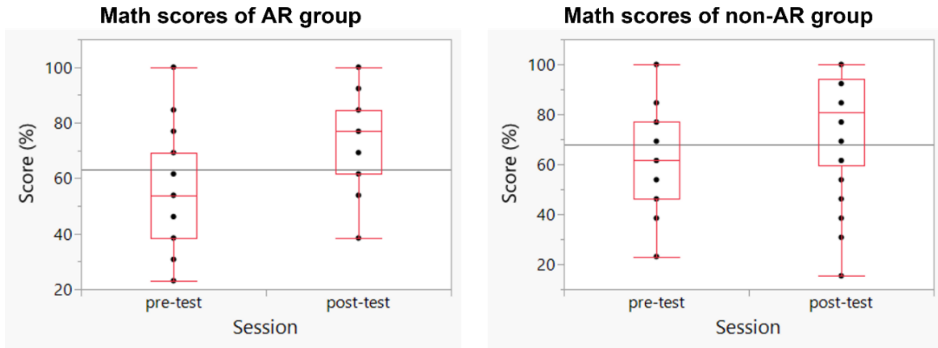


Figure 15. Students' math scores in pre- and post-sessions of workshop 1 AR (left) and non-AR (right).

Table 3 shows that the math mean scores of the AR have increased after the workshops with more score improvement in the AR group (29.33%) compared to the non-AR group (20.83%). The maximum scores (100) have not changed, and they represent a ceiling effect (which will be addressed in future work).

Table 3: Descriptive statistical information of the math scores

Math scores						
Group	**AR**			**Non-AR**		
	Pre-test score (%)	Post-test score (%)	Score improvement (%)	Pre-test score (%)	Post-test score (%)	Score improvement (%)
Min	23.1	38.5	+66.7	23.1	15.4	-33.3
Max	100	100	0	100	100	0
Mean	55.17	71.35	+29.33	61.54	74.36	+20.83
SD	20.15	20.44	N/A	20.15	23.67	N/A

The between-subjects ANOVA t-test showed a near statistically significant difference on students' math pre-test scores between AR vs. non-AR groups ($F(1,56) = 1.6$, $p = 0.06$). The non-AR group ($M = 61.54\%$) scored marginally higher than the AR group ($M = 55.17\%$), which means that the non-AR group had slightly better knowledge about matrix algebra. Because of this nearly statistically significant difference in pre-test scores, we ran two ANCOVAs below that allow us to account for the impact of pre-test scores in investigating learning gains differences between the two conditions.

1. ANCOVA on math post-test, with group as the independent variable

This model examined if the post-test score was a function of the pre-test score and whether the function changed based on the group (AR vs. non-AR). The ANCOVA model had an adjusted $R^2 = 0.40$, which revealed that 40% of the variability of the response variable, i.e., math post-test scores, can be explained by the linear model fitted to the data. The results show that the math pre-test is significantly predictive of the math post-test, $F(1, 56) = 40.06$, $p < 0.001$. This means that students with high pre-test scores tended to have high post-test scores, as expected. However, group (AR vs. non-AR) did not have a significant impact, $F(1, 56) = 0.56$, $p = 0.46$, on the post-test scores when considering the pre-test as the covariate. This means that when we statistically control for the impact of pre-test scores on post-test scores, there is no difference between the two groups. Therefore, with AR showing 29% improvement and non-AR showing 21% improvement, both learning workshops improved matrix algebra learning similarly.

2. ANCOVA on math post-test, with group as the independent variable and field of study and gender as the extraneous variables

This model examined if the post-test score was a function of the pre-test score and whether the function changed based on the group (AR vs. non-AR), considering field of study (STEM vs. non-STEM) and gender as the extraneous explanatory variables. The summary of fit for this ANCOVA model showed adjusted $R^2 = 0.42$, meaning that 42% of the variability of the response variable, i.e., post-test scores, can be explained by this model. The results show that pre-test is highly correlated with the post-test score (again, as expected), $F(1, 55) = 25.56$, $p < 0.001$, while neither group, field of study, nor gender has a statistically significant impact on the post-test scores. This means that when we statistically control for the impact of pre-test scores on post-test scores, there is no difference between the two groups, two genders, or field of study. Both groups, both genders, and both fields of study showed similar improvement in math scores.

Table 4 reports the results of another ANCOVA, which allowed for differences in pre-test scores or other relevant characteristics, like gender or field of study in our case, to be statistically controlled for.

Table 4: Effect test result of the ANCOVA model on post-test scores having pre-test as the covariate, with group, field of study, and gender as independent variables.

Variables	F Ratio	p-value
Math pre-test	25.56	<0.001
Group	1.89	0.18
Field of study	0.99	0.4
Gender	0.09	0.77

6.3 Task Load Analysis

We compared the task load of the AR vs. non-AR versions of the app through the NASA_TLX survey. Figure 16 shows students' adjusted scores calculated for the corresponding six factors of NASA_TLX in AR (left image) vs. non-AR (right image) sessions.

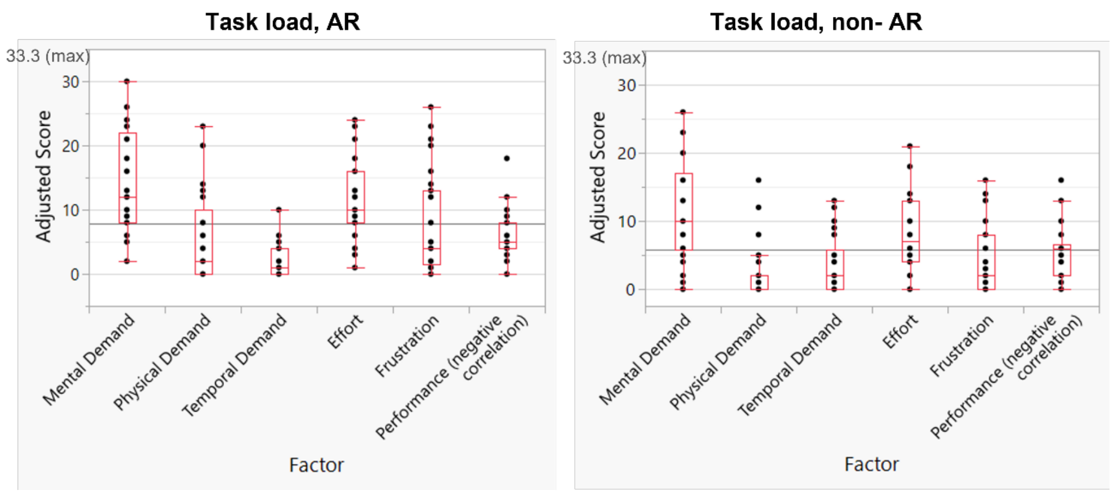


Figure 16. Students' adjusted scores of the six factors of the NASA_TLX survey corresponding to AR (left) vs. non-AR (right) groups.

Error! Reference source not found. shows that students generally rated the prototypes as a low demanding task as the means of the adjusted scores associated with all factors were less than 50% of the maximum possible value of 33.3 (see Equation 1).

Table 5. Minimums, maximums, means, and standard deviations (std) of adjusted scores per factor between AR and non-AR

factors	Group	Min	Max	Mean	SD
Mental Demand	AR	2	30	14.4	14.41
	non-AR	0	26	11.4	7.34
Physical Demand	AR	0	23	5.4	5.38
	non-AR	0	16	2.2	3.93
Temporal Demand	AR	0	10	2.4	2.41
	non-AR	0	13	3.5	4.07
Effort	AR	1	24	11.8	11.83
	non-AR	0	21	8.4	6.1
Frustration	AR	0	26	7.4	7.41
	non-AR	0	16	4.2	5.15

Performance (negative correlation)	AR	0	18	5.6	5.62
	non-AR	0	16	5.3	4.04
Overall task load (%)	AR	25	83	47.1	14.55
	non-AR	0	68	35	17.28

Based on the tests' pre-assumptions, either ANOVA or the non-parametric Wilcoxon Rank Sum test (also known as Mann–Whitney U test) with normal approximation were conducted on the adjusted scores corresponding to each factor. Outliers were removed if existed (4 outliers were removed from the data of physical demand factor and 1 outlier was removed from the data corresponding to the performance factor). The results showed that students adjusted scores were not significantly different in mental demand ($F(1,57) = 2.22$, $p = 0.14$), temporal demand ($Z= -0.76$, $p = 0.45$), frustration ($Z = 1.68$, $p = 0.09$), and performance ($Z= -0.1$, $p = 0.92$) between AR and non-AR groups. However, the data was significantly different in physical demand ($Z= -2.8$, $p = 0.005$), effort ($Z = 2.27$, $p = 0.02$). This outcome reveals that students felt a substantially higher task load, specifically regarding physical demand and effort, in the AR group compared to their peers in the non-AR group, which may be derived from the physical load of holding the device and playing with the physical model at the same time (in the AR workshop) and their unfamiliarity with the AR environment.

6.4 Motivation Analysis

We evaluated students' motivations in three categories through a motivation questionnaire (adopted from MSLQ [45]) (Table 6).

Table 6: Mean values of students' ratings to the questions of the MSLQ survey (range 1 strongly agree to 5 strongly disagree)

Category	Item	Mean value	
		AR	**non-AR**
Intrinsic-value	I like what I learned, and I think the subject is interesting.	1.7	1.9
	I think that what I learned in this workshop is useful for me to know.	2.1	2
	Understanding this subject is important to me.	2.6	2.9
	Mean of the intrinsic value category questions	2.1	2.3
Task-anxiety	I was so nervous during the workshop that I could not remember the material I had learned.	3.6	3.5
	I had an uneasy, upset feeling when I was participating in the workshop.	4.3	4.6
	When I was performing the task in the workshop, I thought about how poorly I was doing.	3.6	3.9
	Mean of the task-anxiety category questions	3.8	4
Self-regulated-learning	When I was taking the tests (math and mental rotation test), I put together what I learned in the workshop and lecture.	1.6	1.8
	When I was taking the math test, I used visual imagery to visualize the geometric transformations and what I experienced in the workshop.	1.6	1.9
	Visualization of matrix representations during the workshop helped me in solving the math test.	1.8	2.2
	Mean of the self-regulated-learning category questions	1.7	2

The result of the non-parametric Wilcoxon Rank Sum tests on students' answers to each category did not show any significant difference between AR vs. non-AR groups with the following test results: intrinsic-value: $Z = -1.02$, $p = 0.31$; Task anxiety: Z = -0.6, $p = 0.55$; self-regulated-learning result of $Z = -1.62$, $p = 0.11$. For both groups, the results of the intrinsic-value category show that students agreed (total M_{AR} = 2.1, SD_{AR} = 0.2, total $M_{non\text{-}AR}$ = 2.3, $SD_{non\text{-}AR}$ = 0.2) that what they learned in the workshop was an interesting, useful, and important subject to learn, with the AR group agreed more than the non-AR group. The task-anxiety category results reveal that students somewhat disagreed (total M_{AR} = 3.8, SD_{AR} = 0.2, and total $M_{non\text{-}AR}$ = 4, $SD_{non\text{-}AR}$= 0.2) about how uneasy and upset they felt during the workshop, with the non-AR group disagreed more than the AR group. The results from the self-regulated-learning category show that students agreed (total M_{AR} = 1.7, SD_{AR} = 0.1, total $M_{non\text{-}AR}$ = 2, $SD_{non\text{-}AR}$= 0.1) that the learning materials of the workshops were helpful in self-learning and answering the questions, with AR group agreed more than the non-AR group.

6.5 Play Time

During the workshops, students were encouraged to play with the application for 20 to 40 minutes; however, they were free to quit when they did not feel like playing more. The results from the iPads' screen records showed that, on average, students in the AR sessions were more willing to play with the application (M = 25.33 minutes) compared to their peers in the non-AR sessions (M = 15.99 minutes).

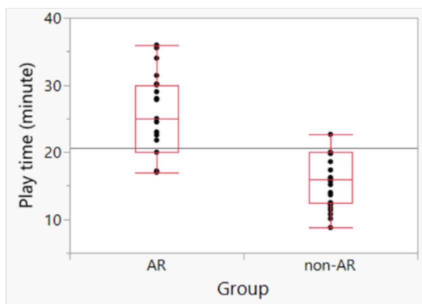


Figure 17. Students' play time with the application in the AR vs. non-AR sessions.

We used the data of screen recordings to compare the length of students' play time in the AR vs. non-AR workshops (Table 7).

Table 7: Descriptive statistical information of students play time with the apps

	Play time (minute)	
Group	**AR**	**non-AR**
Min	17	8.78
Max	35.95	22.65
Mean	25.33	15.99
SD	5.56	3.8

The result of the non-parametric Wilcoxon Rank Sum tests on students' play time with the apps showed that students played more with the AR app during the workshops than the non-AR app, with a significant difference (Z = -5.51, p < 0.001). Based on the play time we may interpret that the AR app was more interesting and engaging for the students to learn through play.

7 DISCUSSIONS

This paper presented BRICKxAR/T and the experiment with test cases to evaluate the apps in the AR vs. non-AR environments. BRICKxAR/T is an AR educational tool for learning the geometric reasoning behind mathematical representations of spatial transformations. In the test cases, we first guaranteed that both groups had similar spatial visualization skills prior to participation in the workshops through PVRT. Then, we assessed students' learning gains in math skills through the math test on transformation matrices before and after participating in the workshops and compared the results between the AR and non-AR groups. The application task load and participants' motivations were also evaluated through NASA_TLX and MSLQ surveys, respectively.

Based on the between-subjects ANOVA result of the PVRT pre-test scores, students had similar spatial visualization skills before participating in the workshops. Based on the between-subjects ANOVA result of the math pre-test scores, students of non-

AR group had better knowledge (near statistically significant difference) than the AR group before the workshops. Students' math scores for matrix algebra improved significantly after the workshops in both of the AR and non-AR groups, with more improvements in the AR group. Hence, the researchers conclude that the features integrated in BRICKxAR//T interventions (in both AR and non-AR, and especially in AR) may improve students' understanding of mathematical representations of spatial transformations.

This research has not conducted a comparative study between the AR group and a conventional "control group" (for example, a group that learns the same topic through conventional methods, such as lectures, handbooks, or videos). The reason is that CAL has been practiced widely and has already shown significant improvement compared to the conventional methods due to literature. AR can be considered as a novel CAL method, and thus we decided to compare BRICKxAR/T with a higher bar, which is our non-AR setting, and the results are promising. Especially, students in the AR group agreed more than the non-AR group on that what they learned in the workshop was an interesting, useful, and important subject to learn, and that the learning materials of the workshops were helpful in self-learning and answering the questions. More importantly, the data from the screen recordings showed that students were willing to spend significantly more time playing with AR than non-AR app. This suggests that students are more interested and engaging in the BRICKxAR/T learning environment. This result is also aligned with researchers' observations: most students of AR group were more excited and curious to play with the AR app and spent more time to play and learn, while many students in the non-AR group got bored early and stopped playing and learning.

8 CONCLUSIONS AND FUTURE WORK

BRICKxAR/T has the potential to support many students who struggle with spatial and math reasoning, especially those from underrepresented groups in STEM. Gaining a fundamental understanding of spatial transformations can uniquely contribute to students' learning and development of spatial reasoning and allied mathematical skills, leading to improve STEM coursework, STEM retention, and degree attainment, and thus supports students' future development of expertise and career success across STEM disciplines.

The contributions of this study include the following computer-human interaction technology and learning innovations:

- BRICKxAR/T is an innovative learning environment that enables physical and virtual interplays to engage students in embodied learning (by transforming physical models by hand).
- BRICKxAR/T integrates spatial transformation matrices and related math information with the physical model movement controlled by students in AR, making difficult invisible concepts visible for supporting an intuitive and formal understanding of spatial reasoning and mathematical formulation.
- .BRICKxAR/T showed the potential to help students conceive, connect, and compare math conceptions of motions, mappings, and functions in AR to help overcome well-documented difficulties students face when learning spatial transformations and allied mathematical representations.
- The functions of BRICKxAR/T help students see relationships between spatial manipulations and mathematical operations, bridging the spatial-mathematical divide.
- BRICKxAR/T's motion tracking of physical object transformations and the student's hand that controls the objects showed the potential to help collect fine-grained behavior data to enhance learning analytics, for example, what the rotation axes and angles are when a hand rotates a physical model.

The results from the NASA_TLX showed that students perceived significantly more physical load and effort while playing in the AR workshop compared to their peers in the non-AR workshop. The researchers believe that the outcome may be driven by the current limitations of BRICKxAR/T, which require students to hold the iPad (an affordable AR device for learning) with one hand and manipulate the physical model or the screen user interface with the other hand. Also, the unfamiliarity of students with the AR environment may impact their subjective assessment of the effort factor, meaning that they felt more effort in learning the new environment. Leveraging 3D model registration (such as Vuforia Model Target [46]) and exceling the implementation through immersive devices (such as the more convenient and immersive future AR glasses) may improve the task load rating by the AR users. Addressing the limitations of the current application in future work is expected to significantly improve the learning of spatial transformations and their mathematical representations utilizing AR technologies.

ACKNOWLEDGMENTS

[Omitted for blind review]

DATA AVAILABILITY

Sample data generated or analysed during this study are included in this published article. (A supplementary video demo and the developed math test for matrix algebra, submitted with the manuscript, will be available publicly upon acceptance of the manuscript for publishing). Other datasets generated and/or analysed during the current study are available from the corresponding author on reasonable request.

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